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Assignment – Probability.

1) Two dice are rolled at once. Find out the probability for sum of numbers being even and one of the die shows 6.

→ favourable event – sum of numbers – even and one of them – 6.

i.e (2,6) (6,2) (4,6) (6,4) (6,6) – $\frac{5}{36}$

2) Two dice are rolled at once. Find out probability for sum of numbers being less than 7.

→ (1,1) (1,2) (1,3) (1,4) (1,5)
(2,1) (2,2) (2,3) (2,4)
(3,1) (3,2) (3,3)
(4,1) (4,2)
(5,1)

$\frac{\cancel{15}^{5}}{\cancel{36}_{12}} = \frac{5}{12}$

3) You toss a fair coin 3 times. Given that you have observed atleast one heads, what is probability of observing atleast 2 heads.

→ H H T
T H H
H T H
H H H

→ $\frac{1}{2}\left(\frac{1}{2}\times\frac{1}{2}\right) = \frac{1}{8}\times 4$

probability of atleast 2 heads. = $\frac{4}{8}$

$\frac{5}{12}$

probability of atleast 1 heads = $1-(TTT) = \frac{7}{8}$.

$P(2 \text{ heads} / 1 \text{ heads}) = \frac{4}{8}\times\frac{8}{7} = \boxed{\frac{4}{7}}$

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4) A and B are a married couple with 2 kids. One of them is a girl. What is probability other kid is also a girl.

→ 2 kids – G B/G → $\frac{1}{2}$ = 50% chance.

5) In my town, it's rainy for one third of the days. Given that it is rainy, there will be heavy traffic with 1/2 chance and given that it is not rainy, there will be traffic with probability 1/4.
If its rainy & there is traffic, I arrive late for work with probability 1/2.
On other hand, the probability of being late is 1/8 if there is no rain & traffic.
In other situation the probability of being late is 1/4
You pick random day.

a) What is probability that it's not raining and there is heavy traffic & I am not late.

→ Probability of not raining – $\left(1-\frac{1}{3}\right) = \frac{2}{3}$

Probability of traffic when not raining = $\frac{1}{4}$

Probability of not being late when not raining and heavy traffic = $\left(1-\frac{1}{4}\right) = \frac{3}{4}$.

∴ Overall probability = $\frac{\cancel{2}^{1}}{\cancel{3}} \times \frac{1}{4} \times \frac{\cancel{3}}{\cancel{4}_{2}} = \frac{1}{8}$.

b) Probability that I am late.

→ Probability of Rain | Heavy traffic | late − $1/3 \times 1/2 \times 1/2$

No rain | Heavy traffic | Late − $2/3 \times 1/4 \times 1/4$

Rain | No traffic | Late − $1/3 \times 1/2 \times 1/4$

No Rain | No traffic | Late − $2/3 \times 3/4 \times 1/8$

i.e $\frac{1\times2}{12\times2} + \frac{1}{24} + \frac{1}{24} + \frac{1}{16} = \frac{4}{24} + \frac{1}{16} = \frac{1}{6} + \frac{1}{16}$

$= \frac{1\times8}{6\times8} + \frac{1\times3}{16\times3}$

$= \boxed{\frac{11}{48}}$

c) Given that I arrived late, what is probability it rained.

Probability of Rain | Traffic | Late − $1/3 \times 1/2 \times 1/2$

Rain | No Traffic | Late − $1/3 \times 1/2 \times 1/4$

i.e $\frac{1\times2}{12\times2} + \frac{1}{24} = \frac{3}{24} = \boxed{\frac{1}{8}}$

Note

1) Joint Probability − two events occuring simultaneously

2) Marginal Probability − event's probability irrespective of the outcome of another variable

3) Conditional Probability − one event occuring in the presence of second event.

$\frac{5}{12}$

6) A box contains 3 coins - 2 regular and 1 fake
You pick a coin at random and toss it.

a) What is probability it lands head.

Reg Reg Fake $\rightarrow P(H) = \frac{1}{\cancel{2}} \times \frac{\cancel{2}}{3} + 1 \times \frac{1}{3}$

$= \frac{\cancel{2}}{4}\ \frac{1}{3} + \frac{1}{3}$

$= \frac{2}{3}.$

It will $P(H) \times P(\text{Coin})$

b) You pick a coin at random, toss it and get heads
What is probability that it is 2 headed coin (fake)

$\rightarrow$ Using Bayes Rule $\rightarrow P(\text{Fake} | \text{head}) = \frac{P(\text{Fake} / \text{head})}{P(\text{head})}$

$= \frac{\frac{1}{3} \times 1}{\frac{2}{3}}$

$= \frac{1}{2}$

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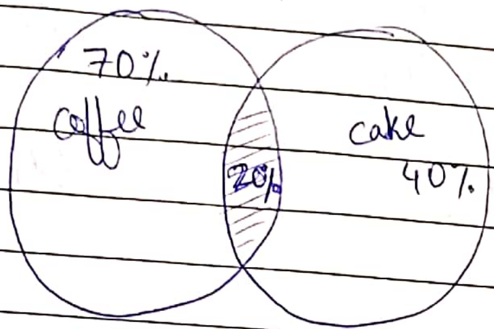
7) Suppose that, of all the customers at a coffee shop
a) 70% purchase a cup of coffee
b) 40% purchase a piece of cake
c) 20% purchase both a cup of coffee and piece of cake.
Given that randomly chosen customer purchased a piece of cake, what is probability that he/she also purchased a cup of coffee.



$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{20/100}{40/100}$$

$$= \frac{1}{2}.$$

8) A probability of A to tell the truth in 5 cases out of 6 and he states that white ball was drawn from bag containing 8 blacks and 1 white ball. Find the probability that white ball was drawn.

→ $P(\text{White ball}) = \frac{1}{9}$ $P(\text{black ball}) = \frac{8}{9}$

$P(\text{Truth}) = \frac{5}{6}$ $P(\text{False}) = \frac{1}{6}$.

$\frac{5}{12}$

$P(\text{White ball} \mid \text{Truth}) = \frac{1}{9} \times \frac{5}{6} = \frac{5}{54}$

$P(\text{White ball} \mid \text{False}) = \frac{8}{9} \times \frac{1}{6} = \frac{8}{54}$

$\frac{4}{84} = \frac{1}{9}$.

As per Bayes Thm,

$$P(\text{white}) = \frac{\frac{5}{6} \times \frac{1}{9}}{\frac{5}{6} \times \frac{1}{9} + \frac{1}{6} \times \frac{8}{9}}$$

$$= \frac{\frac{5}{54}}{\frac{5}{54} + \frac{8}{54}} = \frac{\frac{5}{\cancel{54}}}{\frac{13}{\cancel{54}}} = \frac{5}{13}$$

Q) A speaks truth 4 out of 5 times. A dice is rolled. A reports 6. What are chances it is 6.

→ $P(6) = \frac{1}{6}$, $P(\text{rest}) = \frac{5}{6}$.

$P(\text{truth}) = \frac{4}{5}$, $P(\text{false}) = \frac{1}{5}$

$$P(6) = \frac{\frac{4}{5} \times \frac{1}{6}}{\left(\frac{4}{5} \times \frac{1}{6}\right) + \left(\frac{1}{5} \times \frac{5}{6}\right)} \Bigg\} \frac{P1}{(P1 + P2)}$$

$$= \frac{\frac{4}{30}}{\frac{4}{30} + \frac{5}{30}} = \frac{\frac{4}{\cancel{30}}}{\frac{9}{\cancel{30}}} = \frac{4}{9}.$$

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10) In a class 40% of student study math and science 60% of students study math. What is probability of student studying science given she is already studying math?

→ P(Math) = 0.6   P(Math & Sci) = 0.4

P(Science | Math) = $\frac{P(Sci) \cap P(Math)}{P(Math)}$

$= \frac{0.4}{0.6} = \frac{2}{3}$

11) Below is table of graduates & PGs.

	Graduate	Post Graduate	Total
Male	19	41	60
Female	12	28	40
Total	31	69	100

a) Probability that a randomly selected individual is a male and a graduate? What kind of probability it is?

→ Joint probability → P(M & G) = $\frac{19}{100} = 0.19$

b) Probability that randomly selected individual is male.

→ $\frac{60}{100} = 0.6$ $\frac{5}{12}$

c) Probability of randomly selected individual being a graduate

→ $\frac{31}{100} = 0.31$, it is marginal probability

d) Probability that a randomly selected person is female given that selected person is PG. What kind of probability is this?

→ $P(\text{female} \mid \text{post graduate}) = \dfrac{P(\text{female} \cap PG)}{P(PG)}$

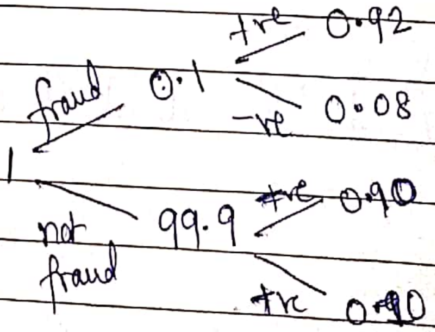
$= \dfrac{28}{69}$

This is conditional probability.

Bayes Theorem.

12) You need to figure out whether a company is fraud based on the legal charges they filed. We have the knowledge that, the chances a company submitting fraudulen filing is 0.1. There exists an algorithm that can predict fraud. This returns a +ve result 92% of the cases in which fraud is present & correct negative in 90% of the cases. Suppose we observe a company for whom the algorithm returns a fraud result. Calculate posterior probability company truly did fraud.

→



$$P(\text{fraud} \mid \text{tve}) = \frac{P(\text{tve} \mid \text{fraud})\, P(\text{fraud})}{P(\text{tve})}$$

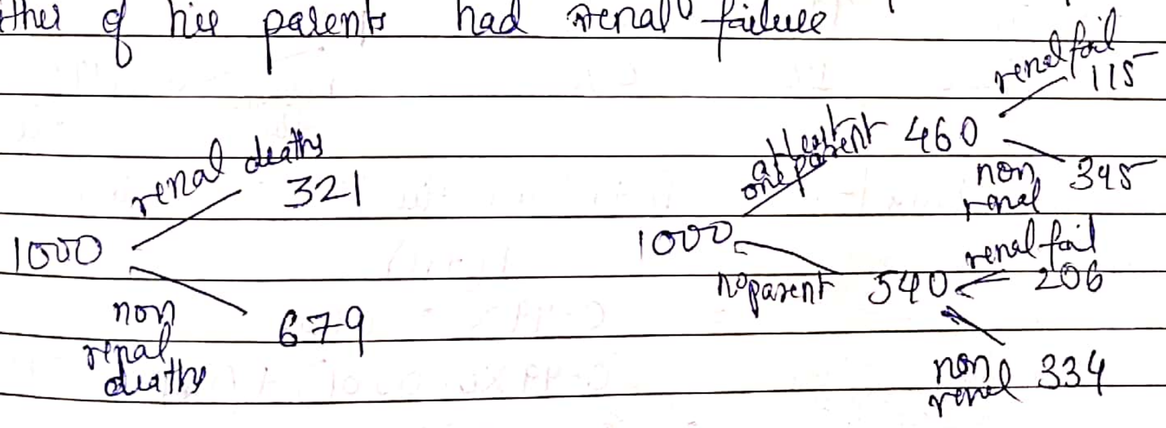
$$= \frac{0.92 \times 0.1}{(0.92 \times 0.1) + (0.10 \times 99.9)}$$

$$= \frac{0.092}{10.082}$$

$$P(\text{fraud} \mid \text{tve}) = 0.0091$$

13) In a particular region during 1 year period, there were 1000 deaths. It was observed that 321 people died of renal failure & 460 ~~ment~~ had one parent with renal failure. Of these 460 men, 115 died of renal failure. Calculate that a man dies of renal failure, if neither of his parents had renal failure.

→



P(man dies of renal failure | neither parents had renal failure) $= \dfrac{P(\text{neither parents renal failure} \mid \text{man dies of renal failure}) \times P(\text{man dies of renal failure})}{P(\text{neither parents have renal failure})}$ 5/12

$$= \frac{\frac{206}{540} \times \frac{540}{1000}}{\left(\frac{206}{540} \times \frac{540}{100}\right) + \left(\frac{115}{460} \times \frac{460}{1000}\right)}$$

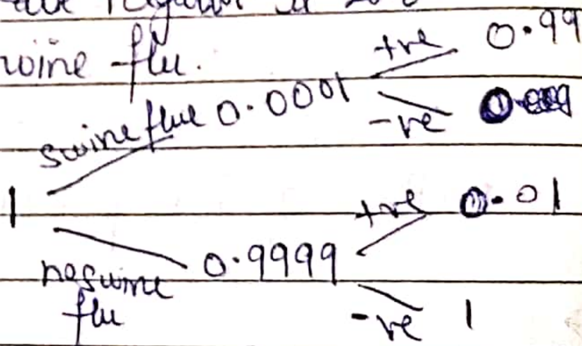
$$= \frac{\frac{206}{1000}}{\frac{206+115}{1000}}$$

$$= \frac{206}{321}$$

$= \underline{0.64}$ – probability man dies of renal failure neither of his parents have renal failure

14) You go to see the doctor about ingrowing toe nail. The doctor selects you at random to have blood test for swine flu for purpose of this exercise is 1 in 10000. The test is 99% accurate, false positive is 1%. False negative is zero. What is probability you have swine flu.

	Positive	Negative
→ True	99%	100%
False	1%	0%



$$P(\text{swine} \mid +ve) = \frac{P(+ve \mid \text{swine flu}) \times P(\text{swine flu})}{P(+ve)}$$

$$= \frac{0.99 \times 0.0001}{(0.99 \times 0.0001) + (0.01 \times 0.9999)}$$

$$= 0.0099$$