

Convolution

Convolution is an operation that applies a filter (kernel) to an input (like an image) to extract features such as edges, textures, or patterns.

Modes of Convolution:

- **Valid Convolution:**

- No padding is added, so the output is smaller than the input.
- The filter moves horizontally and vertically without going outside the input's borders.

Example: Input size 5x5 convolved with a 3x3 filter results in a 3x3 output.

- **Same Convolution:**

- Padding is added to keep the output size the same as the input.
- The filter slides horizontally and vertically, including padded borders.

Example: Input size 5x5 with a 3x3 filter and padding of 1 results in a 5x5 output.

- **Full Convolution:**

- Padding is added so that every element of the input is visited by the filter, resulting in a larger output.
- The filter starts outside the input boundary, moving horizontally and vertically over fully padded edges.

Example: Input size 5x5 with a 3x3 filter produces a 7x7 output.

Dropout layers:

A regularization technique that randomly drops a fraction of neurons during training to prevent overfitting.

Purpose:

- Prevents over-reliance on specific neurons.
- Improves model generalization.

Batch Normalization Layer:

Normalizes the inputs of a layer to maintain a stable distribution of activations, speeding up training.

Purpose:

- Accelerates convergence and improves stability.
- Reduces sensitivity to initialization and allows higher learning rates.

Pooling Layer:

Pooling layers reduce the spatial dimensions (height and width) of feature maps, retaining important features while minimizing computational complexity.

Types of Pooling Layer:

- **Max Pooling:** Selects the maximum value within a pool, highlighting the most prominent features.
- **Average Pooling:** Computes the average value within a pool, preserving background information.

Purpose:

- Reduces overfitting by downsampling.
- Increases computational efficiency and translation invariance.

The general formula to calculate the result of a single neuron in a layer is:

$$z = (w_1 \cdot x_1 + w_2 \cdot x_2 + \dots + w_n \cdot x_n) + b$$

Where:

z: The weighted sum of the inputs plus the bias. This is the value that will be passed to the activation function.

w₁, w₂, ..., w_n: The weights connecting the neuron to the inputs from the previous layer.

x₁, x₂, ..., x_n: The inputs from the previous layer.

b: The bias term.

In matrix form:

$$Z = W \cdot X + B$$

Where:

Z = Weighted sum (before activation)

W = Weight matrix

X = Input vector

B = Bias vector



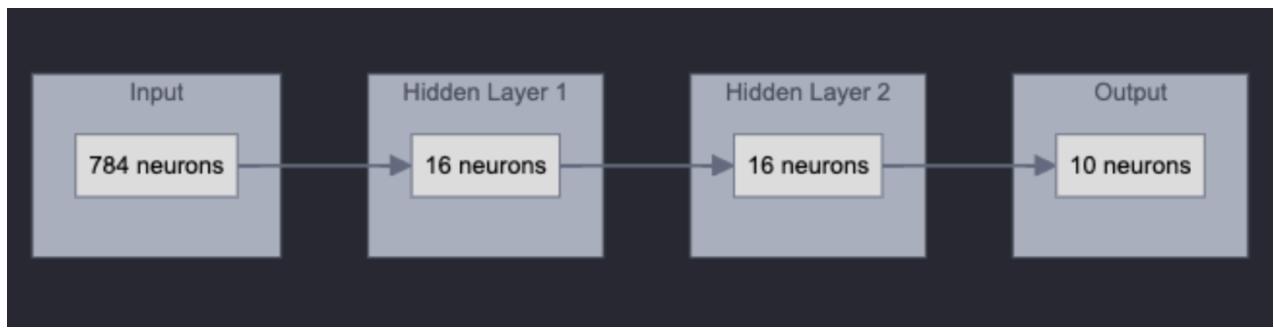
Calculation of Number of Parameters:

Consider a layer with:

1 input layer with 784 neurons (from an image of 28x28 pixels)

2 hidden layers with 16 neurons each

1 output layer with 10 neurons



Weights: (neurons of input layer x neurons of hidden layer 1 + neurons of hidden layer 1 x neurons of hidden layer 2 + neurons of hidden layer 2 x neurons of output layer)

$$= (784 \times 16 + 16 \times 16 + 16 \times 10) = 12544 + 256 + 160 = 12960$$

Bias: (16 + 16 + 10) = 42

Total number of parameters : $12960 + 42 = 13002$

weights and biases in a convolutional neural network (CNN) :

In CNN, we have **filters (which have weights) and biases.**

If you have a 3×3 filter:

- Weights: 9 parameters (3×3)
- Bias: 1 parameter

Total parameters per filter = $9 + 1 = 10$ parameters

So if you have 32 filters in a convolutional layer:

- Total weights = $32 \times (3 \times 3) = 288$
- Total biases = $32 \times 1 = 32$
- Total parameters = $288 + 32 = 320$

Each filter shares a single bias term across all spatial locations.