2309326T CFD Assignment 2

Assignment: 1D Transient Heat Conduction in a Rod (Explicit FDM)

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Problem Statement

Consider a steel rod of length 0.0555 m. Using the explicit FDM, determine the temperature distribution from t = 0 to 7.8 s. Use Δx = 0.00971 m and Δt = 0.01887 s. Thermal conductivity K = 56.96 W/mK, density ρ = 7840.7 kg/m^3, specific heat C = 483.1 J/kgK, initial temperature = 18.3 °C. Write code to simulate and plot the temperature variation. Analyze stability, convergence.

1. Parameters and Stability Condition

Length (L) = 0.0555 m

Space Step (Δx) = 0.00971 m

Time Step (Δt) = 0.01887 s

Thermal Diffusivity (α) = 1.503759e-05 m^2/s

Fourier Number (Fo) = 0.0030

Is Stability Condition Satisfied? (Yes/No): Yes

2. FDM Explicit Method, Type Boundary Condition, Convergence and Consistency of Numerical Scheme

In the explicit method in FDM, the temperature at the next time step is calculated directly from the known temperatures at the current time step.

Discretized Equation (1D Transient Heat Equation):

$$T_i^{l+1} = T_i^l + \lambda\left(T_{i+1}^l - 2T_i^l + T_{i-1}^l\right)$$

Where:

T_i^l : Temperature at position i, time step l

$\lambda = \frac{\alpha \Delta t}{\Delta x^2}$: Fourier number

Applications:

- Heat conduction problems
- Diffusion equations in engineering
- Transient temperature profiles in materials
- Environmental modelling (e.g., pollutant diffusion)

Boundary Conditions used:

1. Left boundary (x = 0): fixed temperature = 100°C
2. Right boundary (x = L): fixed temperature = 28°C

To ensure stability in the numerical method it needs to satisfy stability criterion.
Stability Criterion (1D Heat Equation):

$$\lambda = \frac{\alpha \Delta t}{\Delta x^2} \leq 0.5$$

Where:

α: thermal diffusivity
Δt: time step
Δx: spatial step

If λ > 0.5:
The solution becomes **numerically unstable**.
We calculated **λ ≈ 0.030**, which satisfies the condition, that means numerical method is stable.

Convergence means the numerical solution approaches the true solution as the grid is refined.
A scheme is convergent if it is:

1. Consistent: Approximates the differential equation correctly

2. Stable: Errors do not grow uncontrollably

For explicit FDM:

- Decreasing Δx and Δt leads to a solution that better matches the real-world physics.

- The error typically decreases quadratically with respect to Δx (second-order in space) and linearly with Δt (first-order in time).

Significance:

1. Consistency ensures the numerical method accurately reflects the original differential equation. Consistency is a requirement for convergence.
2. Stability prevents errors from growing uncontrollably over time.
   Without it, even small errors can ruin the solution.
3. Convergence guarantees the numerical solution approaches the exact solution as the grid is refined.

3. Numerical Implementation (5 Marks)

```
import numpy as np

import matplotlib.pyplot as plt


# Given parameters

L = 0.0555

dx = 0.00971

dt = 0.01887

K = 56.96

rho = 7840.7

C = 483.1

T_init = 18.3

T_left = 100.0

T_right = 28.0

T_max = 7.8

# Derived parameters

alpha = K / (rho * C)

lambda_ = alpha * dt / dx**2

print(f"Thermal diffusivity (alpha): {alpha:.6e} m²/s")

print(f"Fourier number (λ): {lambda_:.4f}")


# Stability check

if lambda_ > 0.5:

    raise ValueError("Stability criterion violated! λ must be ≤ 0.5")
```

```python
# Grid setup
nx = int(L / dx) + 1
nt = int(T_max / dt) + 1
x = np.linspace(0, L, nx)
T = np.ones((nt, nx)) * T_init
T[:, 0] = T_left
T[:, -1] = T_right


# FDM loop
for n in range(0, nt - 1):
    for i in range(1, nx - 1):
        T[n + 1, i] = T[n, i] + lambda_ * (T[n, i + 1] - 2 * T[n, i]
+ T[n, i - 1])
# Plotting results
time_indices = [0, int(nt*0.25), int(nt*0.5), int(nt*0.75), nt-1]
plt.figure(figsize=(10, 6))
for t in time_indices:
    plt.plot(x, T[t], label=f'Time = {t*dt:.2f} s')
plt.title('Temperature Distribution in the Steel Rod Over Time')
plt.xlabel('Position along rod (m)')
plt.ylabel('Temperature (°C)')
plt.grid(True)
plt.legend()
plt.show()
```

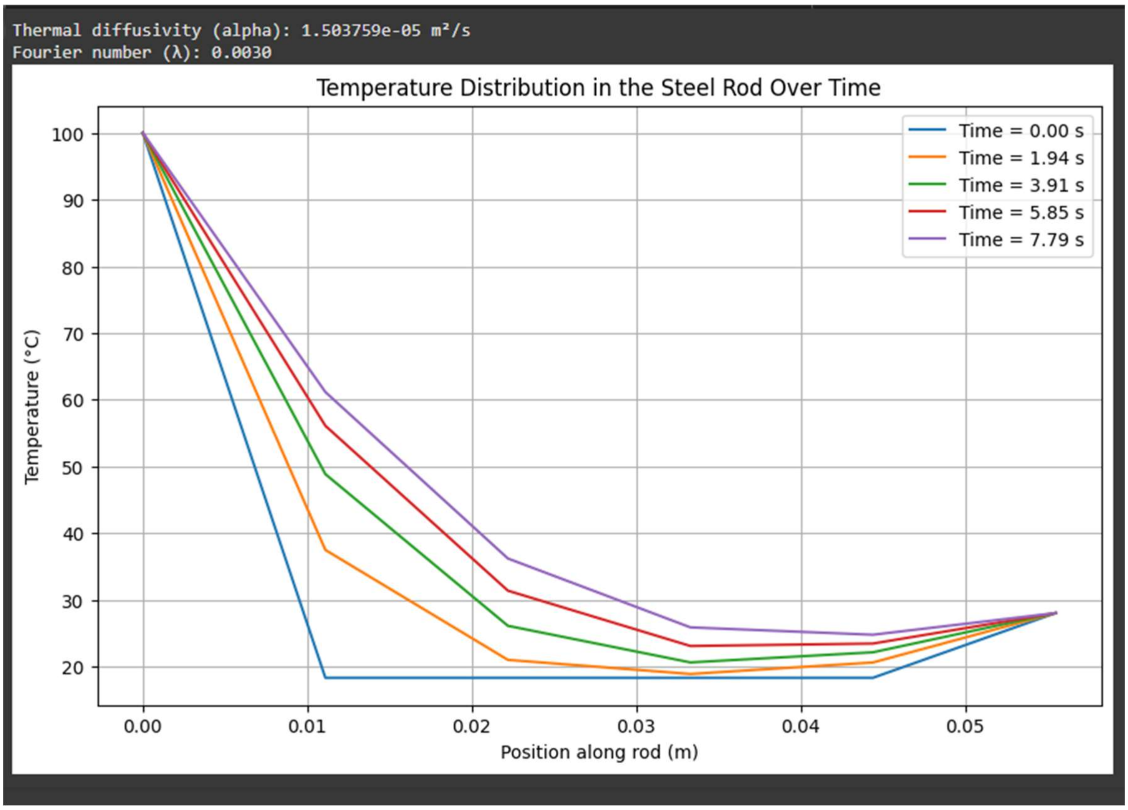
4. Results and Discussion

The temperature increased over time throughout the rod, with heat flowing from the high-temperature left boundary and dissipating toward the cooler right end. Initially, the interior temperature was uniform, but over time, it developed a gradient. Final temperature distribution showed a smooth profile indicating steady progress toward thermal equilibrium.

4.1 Temperature Distribution Table (3 Marks)

Time (s)	Node 0	Node 1	Node 2	Node 3	Node 4	Node 5
0.00	100	18.30	18.30	18.30	18.30	28
2.60	100	41.93	22.60	19.34	21.16	28
5.21	100	54	29.66	22.20	23	28
7.79	100	61.15	36.19	25.86	24.76	28

4.2. Temperature Profile Plot



4.3. Analysis & Interpretation

Observations on temperature variation:

1. Initial Condition (t= 0 s)
   The rod starts with a uniform temperature of 18.3°C, except for the fixed boundaries and left end (Node 0) is instantly at 100°C, and the right end (Node 5) is at 28°C.
2. Early Stage (t ≈ 2.6 s)
   Heat begins to diffuse inward from both boundaries. Node 1 near the hot boundary rapidly rises (≈ 41.9°C). Heat transfer is faster near boundaries due to higher temperature gradients.
3. Mid-Stage (t ≈ 5.2 s)
   Temperature continues to spread evenly across the rod. Node 2 and 3 were initially cooler but after t=5.2s it shows significant warming. The gradient smooths out, indicating a more uniform heat distribution.
4. Final Stage (t ≈ 7.8 s)
   All nodes (except boundaries) have increased. Interior nodes like Node 2 and Node 3 reach steady-state. The temperature profile becomes smoother, indicating the system is approaching thermal equilibrium.

5. Conclusion

The explicit FDM effectively models 1D transient heat conduction when stability (Fourier number ≤ 0.5) is ensured. It's simple, consistent, and shows good convergence for linear problems.

Due to its simplicity and low computational cost, the explicit FDM is ideal for educational use, rapid prototyping, and thermal simulations in systems where quick approximations are acceptable. It helps engineers model heat transfer in components like rods, wires, and thin structures.

For better accuracy and flexibility, future work could use implicit methods, adaptive meshing, or extend the model to 2D/3D and nonlinear cases.

6. References

Chapra, S.C., & Canale, R.P. (2015). *Numerical Methods for Engineers* (7th ed.). McGraw-Hill Education.