Assignment 1 - Probability

Omkar Mukund Raut

Contents

1 Uniform Random Numbers

Let U be a uniform random variable between 0 and 1.

1.1 Generate 10^6 samples of U using a C program and save into a file called uni.dat .
Solution: Download the following files and execute the C program.

```
https://github.com/Omkar1399/
    Assignment-1/tree/main/codes
```

1.2 Load the uni.dat file into python and plot the empirical CDF of U using the samples in uni.dat. The CDF is defined as

$$F_U(x) = \Pr(U \leq x) \tag{1.2.1}$$

Solution: The following code plots Fig. 1.2

```
https://github.com/Omkar1399/
    Assignment-1/blob/main/codes
    /cdf_plot.py
```

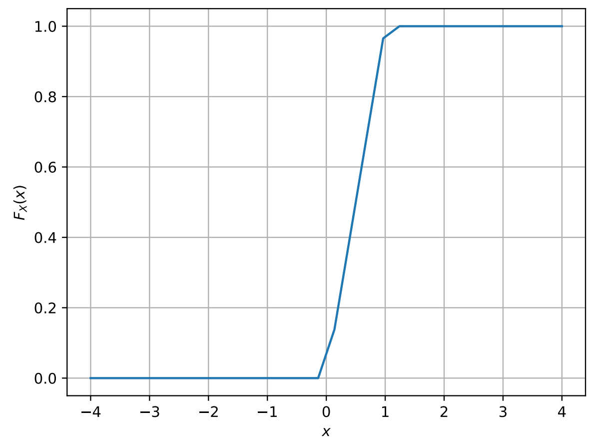

Fig. 1.2. The CDF of U

1.3 Find a theoretical expression for $F_U(x)$.
Solution: Probability density function:

$$\begin{cases} \frac{1}{1-0} & \text{for } x \in [0,1] \\ 0 & \text{otherwise} \end{cases}$$

Cumulative density function:

$$\begin{aligned} F_U(x) = f_U(x) &= \int_{-\infty}^{\infty} f_U(x)(dx) \\ &= \int_0^x f_U(x)(dx) \\ &= \int_0^x \frac{1}{1-0}(dx) \\ &= \frac{1}{1-0}[x]_0^x \\ &= \frac{x-0}{1} \end{aligned}$$

$$\begin{cases} 0 & \text{for } x < 0 \\ \frac{x-0}{1} & \text{for } x \in [0,1) \\ 1 & \text{for } x \geq 1 \end{cases}$$

1.4 The mean of U is defined as

$$E[U] = \frac{1}{N}\sum_{i=1}^{N} U_i \tag{1.4.1}$$

Solution: mean of uniform= 0.500007
and its variance as

$$\text{var}[U] = E[U - E[U]]^2 \tag{1.4.2}$$

Solution: variance of uniform= 0.083301
Write a C program to find the mean and variance of U.
Solution:

```
codes/mean_variance.c
codes/mean_variance.h
```

1.5 Verify your result theoretically given that

$$E\left[U^k\right] = \int_{-\infty}^{\infty} x^k dF_U(x) \tag{1.5.1}$$

Solution: $E[U - E[U]]^2$

$= E[U^2 - 2UE[U] + E[U]^2]$
$= E[U^2] - E[U]^2$

$$
\begin{aligned}
E[u] &= \int_0^1 xF(x)dx \\
&= \frac{1}{1-0}\int_0^1 xdx \\
&= \frac{[x^2]_0^1}{2} \\
&= 0.5 \\
E[u^2] &= \int_0^1 x^2F(x)dx \\
&= \frac{1}{1-0}\int_0^1 x^2dx \\
&= \frac{[x^3]_0^1}{3} \\
&= \frac{1}{3}
\end{aligned}
$$

Now substituting values of $E[U]$ and $E[U^2]$

$$
\begin{aligned}
&= \frac{1}{3} - \left(\frac{1}{2}\right)^2 \\
&= \frac{1}{3} - \frac{1}{4} \\
&= \frac{1}{12} \\
&= 0.083
\end{aligned}
$$