Q4) a) Empirically, we observe that, in certain time periods, when the value of stocks goes up, the value of bonds goes down. What can explain this negative correlation between changes in stock and bond prices? Could such effects have destabilizing effects in the stock market?

This inverse relationship between the equity and debt markets has been studied comprehensively, and it is the result of investors hedging against volatility. In cycles of economic downturn, when the stock market tends to move downwards, both retail and institutional investors tend to invest in bonds and commodities such as gold. This is known as the flight to safety phenomena where investors are especially cautious about risk during these economic downturns, leading them to pull their money our of the stock market, which leads to the equity market falling, and investing this money in debt instruments, which results in the debt market rallying. This is especially true for 'rare events' such the .com bust, the 2008 financial crisis, and the Covid pandemic. This suggests that the prices of bonds are impacted by the willingness of investors to hold stocks.

Yet, to argue to that this relationship has a destabilising effect on the market implying that the act of money leaving the stock market leads to the money entering the stock market is flawed. Investors pulling out of the stock market would result in the market falling, irrespective of whether they later invest in bonds. Although, the existence of bonds as a safe hedge against volatile stock markets could be said to influence investors decision to exist the stock market when risk increases.

Q1a i) No Arbitrage condition if :-

$$\frac{GBP}{LTC} \times \frac{LTC}{EUR} \times \frac{EUR}{USD} \times \frac{USD}{GBP} = 1$$

In this case, $132.76 \times \frac{1}{153.94} \times 0.8281 \times \frac{1}{0.7162}$

$= 0.99715 < 1$, hence Clockwise Strategy will be used to arbitrage.

Sell 1 GBP for LTC = $GBP\ 1 \times \frac{LTC}{GBP}\ \frac{1}{132.76} =$

$= 0.007532$

Sell 0.007532 LTC for EUR = LTC 0.007532

$\times \frac{EUR}{LTC}\ 153.94 = 1.15954$

Sell 1.15954 EUR for USD = EUR 1.15954 $\times$

$\frac{USD}{EUR}\ \frac{1}{0.8281} = 1.40024$

Sell 1.40024 USD for GBP = USD 1.40024 $\times \frac{GBP}{USD}$

$0.7162 = 1.002850$

Net Profit = $1.002850 - 1 \Rightarrow 0.002850$ GBP (Per unit of GBP)

Ask Spread ⇒

(Q1a) ii) $132.86 \times \frac{1}{154.04} \times 0.8283 \times \frac{1}{0.7166} = 1.0086$

Bid Spread ⇒

$132.16 \times \frac{1}{153.84} \times 0.8279 \times \frac{1}{0.7158} = 0.9943$

Clockwise Strategy

⇒ Sell 1 GBP per LTC

$\frac{1}{132.86} = 0.0075264$ LTC (Buy LTC at Ask)

⇒ Sell 0.0075264 LTC per EUR

$0.0075264 \times 154.04 = 1.15942$ (Buy EUR at Ask)

⇒ Sell 1.15942 EUR per USD

$1.15942 \times \frac{1}{0.8283} = 1.39975$ (Buy USD at Ask)

⇒ Sell 1.39975 USD for GBP

$1.39975 \times 0.7158 = 1.00194$ (Sell USD at bid)

Net Profit = $1.00194 - 1 = 0.00194$ (per unit GBP)

Question 4

B I 1000 ; 6% CR , 3y , Y = 6%

$$P = \frac{60}{1+.06} + \frac{60}{1+0.06^2} + \frac{60}{1+0.06^3} + \frac{1000}{1+1.06^3}$$

$$\cancel{P = \frac{60}{0.06} \times \left(1 - \frac{1}{1+0.06^3}\right) \times \frac{1000}{1+1.06^3}}$$

$= 1000$

$$\text{Duration} \Rightarrow \frac{\left(\frac{60\times1}{1.06} + \frac{60\times2}{1.06^2} + \frac{1060\times3}{1.06^3}\right)}{1000}$$

$= 2.84$ years

$$\text{Mod Duration} \Rightarrow D^* = \frac{D}{(1+y)} = \frac{2.84}{1.06}$$

$= 2.68$ years

II Convexity $\Rightarrow \frac{1}{P \times (1+y)^2} \Sigma_{t=1}^{T} \left[\frac{CF_t \cdot (t^2+t)}{1+y^2}\right]$

$$\Rightarrow \left[\frac{60 \times (1^2+1)}{1.06} + \frac{60 \times (2^2+2)}{1.06^2} + \frac{1060 \times (3^2+3)}{1.06^3}\right] \div 1000 \times (1.06^2)$$

$\Rightarrow 9.84$ years

iii Assuming a 1% increase in Y% from 6% to 7%, the bond decreases in price P by -2.68×0.01 from 1000 to $(1 - 2.68 \times 0.01) \times 1000$

$\Rightarrow 973.2$
(using Modified D)

The convexity of the bond implies a price correction of 1% would result in:-

$9.84 \times 0.5 \times 1\%^2 = 0.000492$

$973.2 \times (1.000492) = £973.68$

C The value of liabilities is given as :-

$6M/(1+0.06)^2 + 4M/(1+0.06)^4 = 5.34 + 3.1684$

$= 8.5084\,M$

The duration of liabilities is given as :-

$2\left(\frac{5.34}{8.5084}\right) + 4\left(\frac{3.1684}{8.5084}\right)$

$= 2.7448$ years

Mod Duration :-

$2.7448/1.06 = 2.59$ years

Let X_2 be the value of the investment in 2 year bonds and X_4 in 4 years bonds =>

$$X_2 + X_4 = 8.5084 \text{ M}$$

Duration of Portfolio P => $W_2 D_2 + (1 - W_2) D_4$

$$W_2 = \frac{A_2}{8.5084 \text{ M}}$$

Modified Duration of 2 years bonds =>

=> $\frac{2}{1.06}$

" " " " 4 years bonds =>

=> $\frac{4}{1.06}$

Duration of the portfolio is the duration of liabilities

$$\Rightarrow 2.59 = W_2 D_2 + (1 - W_2) D_4$$
$$W2(2/1.06) + (1 - W_2) \times (4/1.06)$$
$$2.59 = 1.89 W_2 + (1 - W_2) 3.77$$
$$2.59 = 1.89 W_2 + 3.77 - 3.77 W_2$$
$$1.88 W_2 = 1.18 \Rightarrow W_2 = 0.6277$$

$$A_2 = 0.6277 \times 8.5084$$
$$= \underline{5.3407 \text{ M}}$$

$$A_4 = 8.5084 - 5.3407$$
$$= \underline{3.1677 \text{ M}}$$

Q2

A In the CAPM, the variance of return or risk is determined by idiosyncratic risk and systematic risk: $\sigma_i^2 = B_i^2 \sigma_M^2 + \sigma_\varepsilon$

Yet, the CAPM assumes that risk is only determined by systematic risk ($B_i^2 \sigma_M^2$). Hence it does not take into account other variables that result in systematic variance which the CAPM would consider idiosyncratic risk.

Smaller Cap stocks are more sensitive to economic conditions as they do not have stability. Although these are considered systematic risk, the CAPM does not measure it as such.

A portfolio of low B/M stocks (Book value/Market value) would generate $\alpha > 0$ when tested against CAPM but $\alpha = 0$ when tested against FF3 as α no longer captures systematic risk.

8. a) $E(r_i) = \alpha_i + r_f + \beta_i[E(r_m) - r_f]$

$\beta_A = 0.75$, $\beta_B = 1.2$

$\alpha_A = 4\%$ (Undervalued) $\alpha_B = -3\%$ (overvalued)

$E[r_m] = 5\%$, $r_f = 1\%$

In order for an arbitrage to occur, there must be no risk exposure and no capital outlay.

We need to short sell B and buy A in order to arbitrage. In order to do this, we must ensure that the size of the long position is equal to the size of our short.

Assuming short size is 1.84 (given) :-

$-1.84 \times 1.2 + 1.60 \times 0.75 - 0.60 \times 0 \Rightarrow 0$

Hence, Arbitrage Condition Satisfied

The Beta of r_f is 0

b. $E[r_A] = 4\% + 1\% + 0.75[5\% - 1\%]$

$= 8\%$

$E[r_B] = -3\% + 1\% + 1.2[5\% - 1\%]$

$= 2.8$

Input into formula $\Rightarrow 1.60 \times [1\% + 4\% + 0.75 \times R_m - 1\%] - 1.84 \times [1\% - 3\% + 1.2 \times R_m] - 0.60 \times 1\%$

In the previous equation, there are idiosyncratic risks present that are not hedged against. This makes it risky since CF's can vary based on the utility of A and B.

(ii) $\mathcal{E}_i = 0$, $SD = 0.03$, $Cov = 0$

Var of $\mathcal{E}_i = 0.03^2 = 0.0009$

Q5

A) According to Put-Call theory, $S + P = PV(x) + C$

Hence $7.71 + 80 = 80 / (1.05)^{0.5} + 10$

$\Rightarrow 87.71 = 88.04$

Hence Put Call is mispriced according to PCT.

B Brook could sell a call and put which has the same strike price of 60.

$$S + P = PV(x) + C$$

$$5 + 60 = \frac{60}{(1.04)^{0.5}} + C \Rightarrow C = £6.17$$

Hence cost of straddle will be $5 + 6.17 = £11.17$

This is Brooke's profit. But if the stock price increases then the call option will be exercised due to which her profit will reduce by $[-(S_T - x), 0]$

Hence if at the time of expiration, the stock price is equal to $60 + 11.17 = 71.17$ or greater, then her profit will be zero or make a loss.

~~Vice Versa with price decrease and put options.~~

~~If the stock trade~~

If the stock price decreases then the put option will be exercised due to which the payoff will reduce by $[-(x - S_T), 0]$.

Hence if at expiration the stock price is $60 - 11.17 = 48.83$ or less, she will BE or experience loss.

If price is in 71.17 – 48.83 range, she will profit.

c) If the stock price takes the 'up' value (230) in 1 year, then the call option will be in the money ⇒

$$P = \max(230-180, 0) \Rightarrow 50$$

But if stock price takes down value (140) in 1 year, the call option will be out of money with payoff = 0.

Hence $uS_0 = 230$, $P_u = 50$
$dS_0 = 140$, $P_d = 0$

$$\text{Hedge ratio} = \frac{P_u - P_d}{uS_0 - dS_0} = \frac{50}{60} = \frac{5}{6}$$

$$\text{Strike price of REM is } \frac{10}{H} = \frac{10}{5/6} = \underline{12}$$

Fdsfssf