**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

ANS (**B) 0.2676**

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.
3. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

**ANS**  **A. False. We know that the distribution of ages is normal with mean μ = 38 and standard deviation σ = 6. We can use this information to calculate the z-scores for the ages of 44 and 38, as follows:**

**z\_44 = (44 - 38) / 6 = 1**

**z\_38 = (38 - 38) / 6 = 0**

**Using a standard normal table or calculator, we can find that the proportion of employees with ages greater than 44 is approximately 0.1587, and the proportion of employees with ages between 38 and 44 is approximately 0.3413. Therefore, there are more employees between 38 and 44 than older than 44.**

**B. We can use the information given to find the proportion of employees who are under 30. Let X be the age of an employee, then we want to find P(X < 30). We can find the z-score for 30 as follows:**

**z = (30 - 38) / 6 = -1.33**

**Using a standard normal table or calculator, we can find that the proportion of employees with ages less than 30 is approximately 0.0918. Therefore, we would expect about 0.0918 \* 400 = 36.72 employees under the age of 30. So, the statement is true. However, we should keep in mind that this is an expected value, and the actual number of employees under the age of 30 could be different.**

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

**ANS The Normal Distribution has its link with the Central Limit Theorem, which states that ‘Any large sum of independent identically distribution random variables are approximately Normal then (X1 + X2) and (2X1) tends to have Normal distribution only If X1 and X2 are i.i.d and n is Large.**

1. Let X ~ N (100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

**ANS** **two values symmetric about the mean, such that the probability of X taking a value between them is 0.99, the answer is (E) 90.1 and 109.9.**

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

**(ANS) (327.2, 752.8) million Rupees**

1. Specify the 5th percentile of profit (in Rupees) for the company

**(ANS) 375.9 million Rupees**

1. Which of the two divisions has a larger probability of making a loss in a given year?

**(ANS) 0.19**