

Assignment No - 10

Title :- Travelling Salesman Problem
(Dynamic Programming)

Problem Statement :-

Write a program to solve the Travelling Salesman problem and to print the path & the cost using dynamic programming

Objective :-

To understand and implement dynamic programming algorithm for solving travelling sales person problem & study dynamic programming.

Theory :-

Dynamic programming strategy without loss of generality, regard a tour to be a simple path that start & ends at Vertex 1.

Every tour consist of an edge $\langle 1, k \rangle$ for some $k \in V - \{1\}$ & a path from vertex k to vertex 1. The path from vertex k to vertex 1 must be a shortest k to 1 path going through all vertices in $V - \{1, k\}$. Hence, the principle of optimality holds at $g(i, s)$ be the length of a shortest path starting at vertex i , going through

all vertices in S & terminating at vertex 1
 $g(1, V - \{1\})$ is the length of an optimal
 Sales person tour from the principle of
 optimality

Algorithm for TSP using DP.

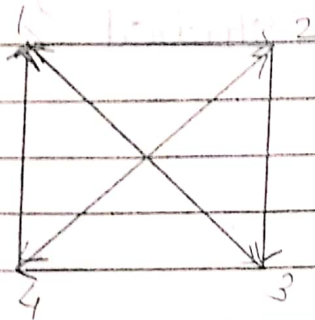
- Step 1 $g(1, \{1\}) = 0$
- Step 2 for $S = 2$ to n do
- Step 3 for all subsets S of size s & containing 1 \rightarrow
- Step 4 for $g(1, S) = \infty$
- Step 5 for all $j \in S$; $j \neq 1$ do
- Step 6 $g(1, S) = \min_{j \in S} \{c_{1j} + g(j, S - \{j\})\}$
- Step 7 return $\min(g(1, S))$

By dynamic programming

There are at most $2^n \times n$ subproblems
& each one takes linear time to solve

The total running time is therefore
 $O(n^2 2^n)$

example



0	10	15	20
5	0	9	10
6	13	0	12
8	8	9	0

$$g(2, \emptyset) = (2| = 5; \quad g(3, \emptyset) = (3| = 6) \quad \& \quad g(4, \emptyset) = (4| = 8$$

$$g(2, \{3\}) = (23 + g(3, \emptyset) = 15; \quad g(2, \{4\}) = 18$$

$$g(3, \{2\}) = 18; \quad g(3, 4) = 20$$

$$g(4, \{2\}) = 13; \quad g(4, \{3\}) = 15$$

$$g(2,$$

$$g(2, \{3, 4\}) = \min\{(23 + g(3, \{4\})), (24 + g(4, \{3\}))\} = 25$$

$$g(3, \{2, 4\}) = \min\{(32 + g(2, \{4\})), (34 + g(4, \{2\}))\} = 28$$

$$g(4, \{2, 3\}) = 25$$

finally

$$g(1, \{2, 3, 4\}) = \min\{(12 + g(2, \{3, 4\})), (13 + g(3, \{2, 4\})), (14 + g(4, \{2, 3\}))\}$$

$$= \min\{35, 40, 43\}$$

$$= 35$$

$$T(1, \{2, 3, 4\}) = 2$$

$$T(2, \{3, 4\}) = 4$$

$$T(4, \{3\}) = 3$$

Final optimal tour : 1-2-4-3-1 &
Cost of tour : 35

Conclusion :-

The Dynamic programming Strategy
for TSP is studied & implemented.