

MLA Theory Assignment 2

Q.1. Consider following data where

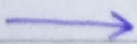
X_i = Rating for movie "Bahubali-part 1" by the person.

Y_i = Rating for movie "Bahubali-part 2" by the person.

where rating is to be done on the scale of 1 to 5 & 1 is lowest rating & 5 is highest rating.

- Find values of B_0 & B_1 w.r.t. linear regression model which best fits given data.
- Interpret & explain equation of regression line.
- If new person votes "Bahubali part-1" as 3 then predict the rating of same person for "Bahubali part-2".

Person	X_i = Rating for movie "Bahubali-part 1" by i th the person	Y_i = Rating for movie "Bahubali-part 2" by i th the person
1	4	3
2	2	4
3	3	2
4	5	5
5	1	3
6	3	1



i.

$$X_i = Y_i$$

$$\bar{X} = \frac{4+2+3+5+1+3}{6} = \frac{18}{6} = 3$$

$$\bar{Y} = \frac{3+4+2+5+3+1}{6} = \frac{18}{6} = 3$$

X_i	Y_i	$X_i - \bar{X}$	$(X_i - \bar{X})^2$	$(Y_i - \bar{Y})$	$(X_i - \bar{X})(Y_i - \bar{Y})$
4	3	1	1	0	0
2	4	-1	1	1	-1
3	2	0	0	-1	0
5	5	2	4	2	4
1	3	-2	4	0	0
3	1	0	0	-2	0
			$\Sigma = 10$		$\Sigma = 3$

$$B_1 = \frac{\Sigma (X_i - \bar{X})(Y_i - \bar{Y})}{\Sigma (X_i - \bar{X})^2} = \frac{3}{10} = 0.3$$

$$B_0 = \bar{Y} - B_1 \bar{X}$$

$$= 3 - 0.3 \times 3$$

$$= 3 - 0.9$$

$$B_0 = 2.1$$

ii. Equation of regression line is $Y = 2.1 + 0.3X$

Interpretations:-

① For increase in value of X by 1 unit there is increase in value of Y in 0.3 units.

② Even if $X=0$ value of independent variable, it is expected that value of Y is 2.1.

iii. We know, eqⁿ of regression line is $Y = 2.1 + 0.3X$.
Using this we can predict rating for "Bahubali-part 2" if we know rating for "Bahubali-part 1".

Here, $X = 3$

$$Y = 2.1 + 0.3 \times 3$$

$$= 2.1 + 0.9$$

$$Y = 3$$

Thus, the rating prediction of same person for "Bahubali-part 2" is 3.

Q.2. Define Regularized regression. Explain Lasso & Ridge regression.

→ • The least-squares regression can turn out to be unstable as it is highly reliant on the training data. Instability is a pointer of a tendency to over fit. Regularization is a broad-spectrum method to avoid such over fitting

by applying supplementary constraints to the weight vector.

- A general approach is to construct such that the weights are, on average, small in magnitude; this is referred to as shrinkage. This regularized problem still has a closed-form solution:

$$\hat{W} = (X^T X + \lambda I)^{-1} X^T y$$

where I stands for the identity matrix with 1's on the diagonal & 0's everywhere else.

Lasso :-

- It is a substitute form of regularized regression which means 'least absolute shrinkage & selection operator'. It replaces the ridge regularization term $\sum w_i^2$ with the sum of absolute weights $\sum |w_i|$.
- It uses L_1 regularization technique.
- It is normally used with more number of features, for the reason that it automatically does feature selection.
- It does not work well when features are highly correlated.

Ridge Regression:-

- The regularization amounts to adding λ to the diagonal of $X^T X$, a renowned trick to get better the numerical stability of matrix inversion. This form of least-squares regression is known as ridge regression.
- It shrinks the parameters, so it is normally used to prevent multicollinearity.
- It reduces the model complexity by coefficient shrinkage.
- It uses L2 regularization technique.
- It is majorly used to prevent overfitting.
- It is not useful in case of high no. of features.
- It works well when features are highly correlated.

Q.3. Consider the following dataset consisting of the scores of two variables on each of seven individuals:

Subject	A	B
1	1.0	1.0
2	1.5	2.0
3	3.0	4.0
4	5.0	7.0
5	3.5	5.0
6	4.5	5.0
7	3.5	4.5

Consider subject 1 & subject 4 as initial centroids.
Apply k-means algorithm & show step-by-step generation of clusters. Write the centroids final clusters.

→ $M_1 = (1.0, 1.0)$ $M_2 = (5.0, 7.0)$

Step 1:	Subject	Mean Vector
Group 1	1	(1.0, 1.0)
Group 2	4	(5.0, 7.0)

$$\text{Euclidian distance} = \sqrt{|M_{1x} - x_i|^2 + |M_{2y} - y_i|^2}$$

$$d(M_1, 1) = \sqrt{|1.0 - 1.0|^2 + |1.0 - 1.0|^2} = 0$$

$$d(M_2, 1) = \sqrt{|5.0 - 1.0|^2 + |7.0 - 1.0|^2} = 7.21$$

$$d(M_1, 2) = \sqrt{|1.0 - 1.5|^2 + |1.0 - 2.0|^2} = 1.12$$

$$d(H_{2,2}) = \sqrt{|5.0 - 1.5|^2 + |7.0 - 2.0|^2} = 6.10$$

$$d(H_{1,3}) = \sqrt{|1.0 - 3.0|^2 + |1.0 - 4.0|^2} = 3.61$$

$$d(H_{2,3}) = \sqrt{|5.0 - 3.0|^2 + |7.0 - 4.0|^2} = 3.61$$

$$d(H_{1,4}) = \sqrt{|1.0 - 5.0|^2 + |1.0 - 7.0|^2} = 7.21$$

$$d(H_{2,4}) = \sqrt{|5.0 - 5.0|^2 + |7.0 - 7.0|^2} = 0$$

$$d(H_{1,5}) = \sqrt{|1.0 - 3.5|^2 + |1.0 - 5.0|^2} = 4.72$$

$$d(H_{2,5}) = \sqrt{|5.0 - 3.5|^2 + |7.0 - 5.0|^2} = 2.50$$

$$d(H_{1,6}) = \sqrt{|1.0 - 4.5|^2 + |1.0 - 5.0|^2} = 5.32$$

$$d(H_{2,6}) = \sqrt{|5.0 - 4.5|^2 + |7.0 - 5.0|^2} = 2.06$$

$$d(H_{1,7}) = \sqrt{|1.0 - 3.5|^2 + |1.0 - 4.5|^2} = 4.30$$

$$d(H_{2,7}) = \sqrt{|5.0 - 3.5|^2 + |7.0 - 4.5|^2} = 2.92$$

Step 2: Thus, we obtain 2 clusters containing $\{1, 2, 3\}$ & $\{4, 5, 6, 7\}$ as $d(H_{1,i}) \leq d(H_{2,i})$ for subjects $\{1, 2, 3\}$.

Their new centroids are:

$$H_1 = \left(\frac{1}{3} (1.0 + 1.5 + 3.0), \frac{1}{3} (1.0 + 2.0 + 4.0) \right) = (1.83, 2.33)$$

$$H_2 = \left(\frac{1}{4} (5.0 + 3.5 + 4.5 + 3.5), \frac{1}{4} (7.0 + 5.0 + 5.0 + 4.5) \right)$$

$$H_2 = (4.12, 5.38)$$

Subject	A	B	$d(H_1, S_i)$	$d(H_2, S_i)$
1	1.0	1.0	1.57	5.38
2	1.5	2.0	0.47	4.28
3	3.0	4.0	2.04	1.78
4	5.0	7.0	5.64	1.84
5	3.5	5.0	3.15	0.73
6	4.5	5.0	3.78	0.54
7	3.5	4.5	2.74	1.08

Step 3: We obtain 2 new clusters $\{1, 2\}$ & $\{3, 4, 5, 6, 7\}$ as $d(H_1, i) < d(H_2, i)$ for subjects 1 & 2.

New centroids are:

$$H_1 = \left(\frac{1}{2} (1.0 + 1.5), \frac{1}{2} (1.0 + 2.0) \right) = (1.25, 1.5)$$

$$H_2 = \left(\frac{1}{5} (3.0 + 5.0 + 3.5 + 4.5 + 3.5), \frac{1}{5} (4.0 + 7.0 + 5.0 + 5.0 + 4.5) \right)$$

$$H_2 = (3.9, 5.1)$$

Subject	A	B	$d(H_1, S_i)$	$d(H_2, S_i)$
1	1.0	1.0	0.56	5.02
2	1.5	2.0	0.47	3.92
3	3.0	4.0	2.04	1.42
4	5.0	7.0	5.64	2.20
5	3.5	5.0	3.15	0.41
6	4.5	5.0	3.78	0.61
7	3.5	4.5	2.74	0.72

Step 4: Since, there is no change in the cluster, the k-means algorithm comes to a halt here & final result consists of 2 clusters $\{1, 2\}$ & $\{3, 4, 5, 6, 7\}$.

Q.4. For the following transactions data set generate association rules using Apriori algorithm. Consider minimum support as 50% & confidence as 75%.

Transaction ID	Items
1	Bread, Cheese, Eggs, Juice
2	Bread, Cheese, Juice
3	Bread, Milk, Yogurt
4	Bread, Juice, Milk
5	Cheese, Juice, Milk

→
Step 1:

$$\text{Support}(\text{item}) = \text{Frequency of item} / \text{Number of transactions}$$

Item	Frequency	Support (in %)
Bread	4	$4/5 = 80\%$
Cheese	3	$3/5 = 60\%$
Eggs	1	$1/5 = 20\%$
Juice	4	$4/5 = 80\%$
Milk	3	$3/5 = 60\%$
Yogurt	1	$1/5 = 20\%$

Step 2: Remove all the items whose support is below given minimum support i.e. 50%.

Item	Frequency	Support (in %)
Bread	4	80%
Cheese	3	60%
Juice	4	80%
Milk	3	60%

Step 3: Now, form the 2 items candidate set & their frequencies.

Items Pair	Frequency	Support (in %)
Bread, Cheese	2	$2/5 = 40\%$
Bread, Juice	3	$3/5 = 60\%$
Bread, Milk	2	$2/5 = 40\%$
Cheese, Juice	3	$3/5 = 60\%$
Cheese, Milk	1	$1/5 = 20\%$
Juice, Milk	2	$2/5 = 40\%$

Step 4: Remove all the items whose support is below given minimum support i.e. 50%.

Items Pair	Frequency	Support (in %)
Bread, Juice	3	60%
Cheese, Juice	3	60%

Step 5: Now, form 3 items candidate set & frequency.

Items Pair	Frequency	Support (in %)
Bread, Juice, Cheese	2	$2/5 = 40\%$

The 2 item subsets are {Bread, Juice}, {Juice, Cheese} & {Bread, Cheese}. But {Bread, Cheese} is not a member of table in step 4 & hence it is not frequent & it is violating Apriori

property. Thus {Bread, Juice, Cheese} is not considered.

Step 6: Generate Rules.

For rules, we consider item pairs:

① {Bread, Juice} Bread \rightarrow Juice & Juice \rightarrow Bread

② {Cheese, Juice} Cheese \rightarrow Juice & Juice \rightarrow Cheese

$$\text{Confidence}(A \rightarrow B) = \text{Support}(A \cup B) / \text{Support}(A)$$

Thus,

$$\begin{aligned} \text{① Confidence}(\text{Bread} \rightarrow \text{Juice}) &= \text{Support}(\text{Bread} \cup \text{Juice}) / \text{Support}(\text{Bread}) \\ &= 60/80 \\ &= 75\% \end{aligned}$$

$$\begin{aligned} \text{② Confidence}(\text{Juice} \rightarrow \text{Bread}) &= \text{Support}(\text{Juice} \cup \text{Bread}) / \text{Support}(\text{Juice}) \\ &= 60/80 \\ &= 75\% \end{aligned}$$

$$\begin{aligned} \text{③ Confidence}(\text{Cheese} \rightarrow \text{Juice}) &= \text{Support}(\text{cheese} \cup \text{Juice}) / \text{Support}(\text{cheese}) \\ &= 60/60 \\ &= 100\% \end{aligned}$$

$$\begin{aligned}\textcircled{4} \text{ Confidence (Juice} \rightarrow \text{Cheese)} &= \frac{\text{Support (Juice} \cup \text{Cheese)}}{\text{Support (Juice)}} \\ &= 60/80 \\ &= 75\%\end{aligned}$$

All the above rules are valid because the confidence of each rule is greater than or equal to the confidence given i.e. 75%.