Assignment 2: MS2300/MS5140

- Submit your assignment solutions (to all questions) as a *single* jupyter notebook.
- Name your jupyter notebook as **GroupNo_assign2.ipynb**.
- For **group number** refer to *the shared spreadsheet* on the google classroom.
- Only *one* submission per group on the google classroom.
- Use matplotlib for plots and markdown option to explain your answers.
- Due date **20 March 2024**.

1. (15 marks)

   (a) Plot the function: $f(x) = (3x - 1.4)sin(18x)$ and write down a code to determine the global minima as well as all the local minima in the interval $x \in [0, 1.22]$.

   (b) Is it possible to use the method such as **Brent, Golden or Bounded** within `scipy.optimize.minimize_scalar` to find the global minima of $f(x)$ in $x \in [0, 1.22]$? If yes, write down your code to demonstrate.

   (c) If not, which approach will you choose in `scipy.optimize` to find the global minima? Write down your code to demonstrate.

2. (15 marks) Given a function of two-variables $f(x, y) = 100(y - x^2)^2 + (0.5 - x)^2$.

   (a) Plot the 2D contour plot of $f(x, y)$ on x and y axes.

   (b) Write down the Jacobian and Hessian Matrices for $f(x, y)$. You can use `sympy`.

   (c) Use the following methods: **Nelder-Mead, Powell, CG, BFCG, Newton-CG, SLSQP** in `scipy.optimize.minimize` to find (x, y) that minimize $f(x, y)$. Choose a tolerance value of 10^{-6} and choose a starting point, randomly, in the search domain $-2 \leq x, y \leq 2$. Plot a table for your results, keeping (x_0, y_0) same for all methods:

method	starting (x_0, y_0)	solution	no. of (#) iterations	# function evaluations	# Jacobian evaluations	# Hessian evaluations

3. (15 marks)

   (a) Plot the 2D contour plot and use `scipy.optimize.basinhopping` to find the global minima of the function: $f(x, y) = (x^2 + y - 10)^2 + (x + y^2 - 1)^2$, in, $-6 \leq x, y \leq 6$.

   (b) Does $f(x, y)$ has any local minima(s) in the above search domain?

   (c) use `scipy.optimize.minimize` to find the minima of $f(x, y)$ under the constraint $0 < y < x^2$.

4. (20 marks) Given data of size $N = 4$:

x	y
1	1.0
2	1.2
3	1.5
4	1.8

(a) Construct and plot a least squares quadratic polynomial ($\hat{y}$) for the given data.

(b) Find the least-squares approximation of the form $\hat{y} = ce^{bx}$ for the given data.

(c) Compare the error measure, $E = \sum_i^N (y_i - \hat{y}_i)^2$ for (a) and (b).

(d) Assume a polynomial of the form: $F(x) = a_0\phi_0(x) + a_1\phi_1(x)$

where, $\phi_0(x) = 1$, and $\phi_1(x) = x - 2.5$, and parameter, $a_k = \frac{\sum_{i=1}^N \phi_k(x_i)y_i}{\sum_{i=1}^N (\phi_k(x_i))^2}$.

i. Find $F(x)$

ii. Show that $\phi_0(x)$ and $\phi_1(x)$ are orthogonal such that: $\sum_{i=1}^N \phi_j(x_i)\phi_k(x_i) = 0$, whenever $j \neq k$

5. (15 marks) Given a set of points as shown in the figure. Can you write a Python code that maps the locus of points that are closer to the origin $(0, 0)$ than any other point in the figure?

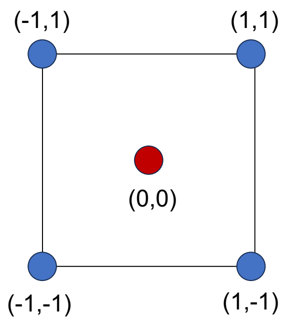

Figure 1:

6. (20 marks) The nonlinear system:

$$\begin{aligned} 1 + x_1^2 - (x_2 + 1)^3 &= 0, \\ 4x_1 - x_2 - 5 &= 0 \end{aligned} \tag{0.1}$$

(a) Can be transformed into the fixed-point form:

$$\begin{aligned} x_1 &= g_1(x_1, x_2) = \frac{1}{4}(x_2 + 5), \\ x_2 &= g_2(x_1, x_2) = (1 + x_1^2)^{1/3} - 1. \end{aligned} \tag{0.2}$$

Using (0.2) to find the solution of (0.1), starting with $\boldsymbol{x}^{(0)} = (1.5, 0.5)$. Does the Seidel approach accelerate convergence? Iterate until $\|\boldsymbol{x}^{(k)} - \boldsymbol{x}^{(k-1)}\|_\infty < 10^{-2}$.

(b) For $x_1 \in [1, 2]$ and $x_2 \in [0.25, 0.75]$, calculate the maximum value of $\|J_G(\boldsymbol{x})\|_\infty$, where, $J_G(\boldsymbol{x})$ is the Jacobian matrix.

(c) Solve the set of non-linear equations in (0.1) by Newton's method. Iterate until $\|\boldsymbol{x}^{(k)} - \boldsymbol{x}^{(k-1)}\|_\infty < 10^{-2}$, starting with $\boldsymbol{x}^{(0)} = (1.5, 0.5)$.