Topics: Normal distribution, Functions of Random Variables

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1. **The time required for servicing transmissions is normally distributed with μ = 45 minutes and σ = 8 minutes. The service manager plans to have work begin on the transmission of a customer's car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?**

   A. **0.3875**
   B. **0.2676**
   C. **0.5**
   D. **0.6987**

Answer :

X = 60 , Mean = 45+10 = 55 , Std. Deviation = 8

```
from scipy import stats
round(1-stats.norm.cdf(60,loc=55,scale=8),5)
```

Output : 0.26599

Option B is the correct answer.

2. **The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean μ = 38 and Standard deviation σ=6. For each statement below, please specify True/False. If false, briefly explain why.**

   A. **More employees at the processing center are older than 44 than between 38 and 44.**

Answer :

False ,

mean = 38 & SD = 6

means that, most of the ages are lying between 32 and 44

```
#Z-score for 44
from scipy import stats
round(1-stats.norm.cdf(44,loc=38,scale=6),4)
```

Output : 0.1587

i.e. 63 employees out of 400

```
#Z-score between 38 and 44
from scipy import stats
round(stats.norm.cdf(44,loc=38,scale=6) - stats.norm.cdf(38,loc=38,scale=6),4)
```

Output : 0.3413
i.e. 137 employees out of 400

therefore , 137 > 63 hence given condition is false.

B. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Answer :

True ,

```
from scipy import stats
round(stats.norm.cdf(30,loc=38,scale=6),4)
```

Output : 0.0912
i.e. 36 employees out of 400
hence given condition is True.

3. If $X_1 \sim N(\mu, \sigma^2)$ and $X_2 \sim N(\mu, \sigma^2)$ are *iid* normal random variables, then what is the difference between 2 X_1 and $X_1 + X_2$? Discuss both their distributions and parameters.

Answer :

We know that ,

if $X_1 \sim N(\mu, \sigma^2)$ and $X_2 \sim N(\mu, \sigma^2)$ are two independent random variables then ,

$X_1 + X_2 \sim N(\mu + \mu , \sigma^2 + \sigma^2)$

Similarly if $Z = aX_1 + bX_2$, where X and Y are as defined above, i.e Z is linear combination of X_1 and X_2 , then $Z \sim N(a\mu + b\mu, a^2 \sigma^2 + b^2 \sigma^2)$.

Therefore from the question ,

$2X1 \sim N(2 \mu , 4 \sigma^2)$ &

$X1+X2 \sim N(2 \mu, 2\sigma^2)$

$2X1 - (X1+X2) = N(0,2\sigma^2)$

4. **Let X ~ N(100, 20^2). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.**

   A. **90.5, 105.9**
   B. **80.2, 119.8**
   C. **22, 78**
   D. **48.5, 151.5**
   E. **90.1, 109.9**

Answer :

Since a and b are symmetric about mean,
It is two tailed test ,
Hence , for 99% , we need to add 0.5% on either side = 0.99+0.005 = 0.995

Z value of 0.005 is ,

```
from scipy import stats
stats.norm.ppf(0.005)
```

Z value of 0.005 = -2.57

Now, Z value of 0.995 is ,

```
from scipy import stats
stats.norm.ppf(0.995)
```

Z value of 0.995 = 2.57
Hence ,

$$Z = \frac{x - Mean}{SD}$$

Hence ,

$$x = SD * Z + Mean$$

$$x = 20*Z + 100$$

therefore ,

$$a = (20 * (-2.57)) + 100$$

$$a = 48.5$$

$$b = (20* 2.57) + 100$$

$$b = 151.5$$

Option D. (48.5, 151.5) is correct answer.

5. **Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions $Profit_1 \sim N(5, 3^2)$ and $Profit_2 \sim N(7, 4^2)$ respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45**

   A. **Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.**

Answer :

```
import numpy as np
from scipy import stats
mean = 5+7
print('Mean Profit is Rs', mean*45,'Million')
sd = np.sqrt((9)+(16))
print('Standard Deviation is Rs', sd*45, 'Million')
print('Range is Rs',(stats.norm.interval(0.95,540,225)),'in Millions')
```

Output :
```
Mean Profit is Rs 540 Million
Standard Deviation is Rs 225.0 Million
Range is Rs (99.00810347848784, 980.9918965215122) in Millions
```

   B. **Specify the 5^{th} percentile of profit (in Rupees) for the company**

Answer :

we know that , Z value for 5^{th} percentile is = -1.645

X = SD*Z + Mean is ,

```
X= 540+(-1.645)*(225)
print('5th percentile of profit is',round(X),'(in Million Rupees)')
```

Output : 5th percentile of profit is 170 (in Million Rupees)

C. Which of the two divisions has a larger probability of making a loss in a given year?

Answer :

Making loss , i.e X<0

Division 1 :

stats.norm.cdf(0,5,3)

Output : 0.04779035

Division 2 :

stats.norm.cdf(0,7,4)

Output : 0.04005915

Hence ,

Division 2 will face more loss.