Assignment 1

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Download all python codes from

https://github.com/Omkaradithya/EE3900-Linear-Systems-and-Signal-Processing/blob/main/Assignment%201/ramsey%201.1.5.py

and latex codes from

https://github.com/Omkaradithya/EE3900-Linear-Systems-and-Signal-Processing/blob/main/Assignment%201/assgn1.tex

Problem

(Ramsey - 1.1.5) Plot the points $\begin{pmatrix}0\\2\end{pmatrix}$, $\begin{pmatrix}1\\1\end{pmatrix}$, $\begin{pmatrix}4\\4\end{pmatrix}$ and $\begin{pmatrix}3\\5\end{pmatrix}$ and prove that they are vertices of a rectangle.

Solution

Let,

$$\mathbf{A} = \begin{pmatrix}0\\2\end{pmatrix}, \tag{0.0.1}$$

$$\mathbf{B} = \begin{pmatrix}1\\1\end{pmatrix}, \tag{0.0.2}$$

$$\mathbf{C} = \begin{pmatrix}4\\4\end{pmatrix} \tag{0.0.3}$$

$$\mathbf{D} = \begin{pmatrix}3\\5\end{pmatrix} \tag{0.0.4}$$

The direction vector are calculated as follows:

$$\mathbf{B}-\mathbf{A} = \begin{pmatrix}1-0\\1-2\end{pmatrix} = \begin{pmatrix}1\\-1\end{pmatrix} \tag{0.0.5}$$

$$\mathbf{C}-\mathbf{B} = \begin{pmatrix}4-1\\4-1\end{pmatrix} = \begin{pmatrix}3\\3\end{pmatrix} \tag{0.0.6}$$

$$\mathbf{D}-\mathbf{C} = \begin{pmatrix}3-4\\5-4\end{pmatrix} = \begin{pmatrix}-1\\1\end{pmatrix} \tag{0.0.7}$$

$$\mathbf{A}-\mathbf{D} = \begin{pmatrix}0-3\\2-5\end{pmatrix} = \begin{pmatrix}-3\\-3\end{pmatrix} \tag{0.0.8}$$

Since the directional vectors AB (B-A) and CD (D-C) are in the same ratio, AB and CD are parallel and opposite to each other. Similarly, directional vectors BC (C-B) and DA (A-D) are also parallel and opposite to each other. Since the opposites sides are parallel, the given points are vertices of a parallelogram. Also,

$$(\mathbf{B}-\mathbf{A})^T(\mathbf{C}-\mathbf{D}) = \begin{pmatrix}1&-1\end{pmatrix}\begin{pmatrix}1\\-1\end{pmatrix} \tag{0.0.9}$$

$$= 0 \tag{0.0.10}$$

Therefore

$$\angle ABC = 90^o$$

and hence, the points A,B,C and D are vertices of a rectangle.

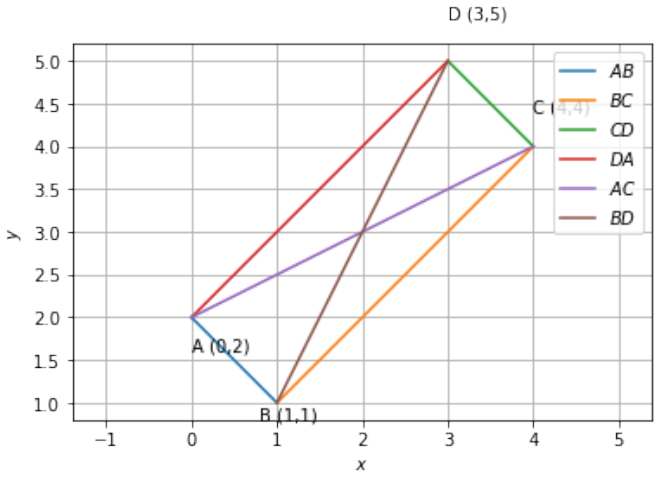

Fig. 0: Plot of the points