Assignment 1

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Problem

(Ramsey - 1.1.5) Plot the points $\begin{pmatrix}0\\2\end{pmatrix}$, $\begin{pmatrix}1\\1\end{pmatrix}$, $\begin{pmatrix}4\\4\end{pmatrix}$ and $\begin{pmatrix}3\\5\end{pmatrix}$ and prove that they are vertices of a rectangle.

Solution

Let,

$$\mathbf{A} = \begin{pmatrix}0\\2\end{pmatrix}, \tag{0.0.1}$$

$$\mathbf{B} = \begin{pmatrix}1\\1\end{pmatrix}, \tag{0.0.2}$$

$$\mathbf{C} = \begin{pmatrix}4\\4\end{pmatrix} \tag{0.0.3}$$

$$\mathbf{D} = \begin{pmatrix}3\\5\end{pmatrix} \tag{0.0.4}$$

The direction vector are calculated as follows:

$$\mathbf{AB} = \begin{pmatrix}1-0\\1-2\end{pmatrix} = \begin{pmatrix}1\\-1\end{pmatrix} \tag{0.0.5}$$

$$\mathbf{BC} = \begin{pmatrix}4-1\\4-1\end{pmatrix} = \begin{pmatrix}3\\3\end{pmatrix} \tag{0.0.6}$$

$$\mathbf{CD} = \begin{pmatrix}3-4\\5-4\end{pmatrix} = \begin{pmatrix}-1\\1\end{pmatrix} \tag{0.0.7}$$

$$\mathbf{DA} = \begin{pmatrix}0-3\\2-5\end{pmatrix} = \begin{pmatrix}-3\\-3\end{pmatrix} \tag{0.0.8}$$

Since the directional vectors AB and CD are in the same ratio, AB and CD are parallel and opposite to each other. Similarly, directional vectors BC and DA are also parallel and opposite to each other. Since the opposites sides are parallel, the given points are vertices of a parallelogram. Also,

$$(\mathbf{B}-\mathbf{A})^T(\mathbf{C}-\mathbf{D}) = \begin{pmatrix}1&-1\end{pmatrix}\begin{pmatrix}1\\-1\end{pmatrix} \tag{0.0.9}$$

$$= \begin{pmatrix}0\end{pmatrix} \tag{0.0.10}$$

Therefore

$$\angle ABC = 90^o$$

and hence, the points A,B,C and D are vertices of a rectangle.

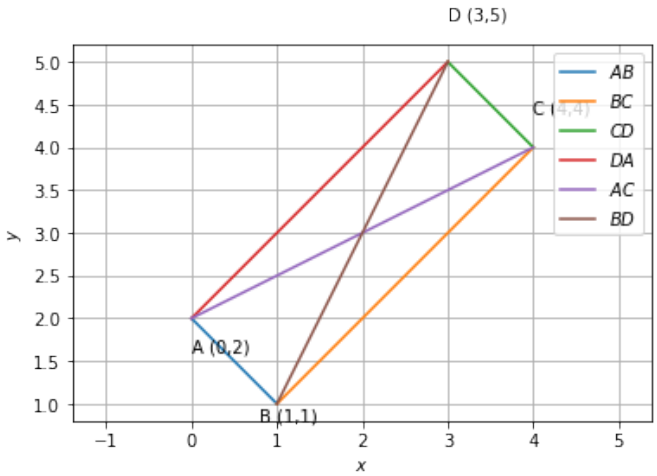

Fig. 0: Plot of the points