Assignment 1

Omkaradithya R Pujari - AI20BTECH11017

Download all python codes from

and latex codes from

PROBLEM

(Ramsey - 1.1.5) Plot the points $\begin{pmatrix} 0 \\ 2 \end{pmatrix}$, $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$, $\begin{pmatrix} 4 \\ 4 \end{pmatrix}$ and $\begin{pmatrix} 3 \\ 5 \end{pmatrix}$ and prove that they are vertices of a rectangle.

SOLUTION

Let,

$$\mathbf{A} = \begin{pmatrix} 0 \\ 2 \end{pmatrix}, \tag{0.0.1}$$

$$\mathbf{B} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \tag{0.0.2}$$

$$\mathbf{C} = \begin{pmatrix} 4 \\ 4 \end{pmatrix} \tag{0.0.3}$$

$$\mathbf{D} = \begin{pmatrix} 3 \\ 5 \end{pmatrix} \tag{0.0.4}$$

The direction vector are calculated as follows:

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} 1 - 0 \\ 1 - 2 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \tag{0.0.5}$$

$$\mathbf{C} - \mathbf{B} = \begin{pmatrix} 4 - 1 \\ 4 - 1 \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \end{pmatrix} \tag{0.0.6}$$

$$\mathbf{D} - \mathbf{C} = \begin{pmatrix} 3 - 4 \\ 5 - 4 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \end{pmatrix} \tag{0.0.7}$$

$$\mathbf{A} - \mathbf{D} = \begin{pmatrix} 0 - 3 \\ 2 - 5 \end{pmatrix} = \begin{pmatrix} -3 \\ -3 \end{pmatrix} \tag{0.0.8}$$

Since the directional vectors AB (B-A) and CD (D-C) are in the same ratio, AB and CD are parallel and opposite to each other. Similarly, directional vectors BC (C-B) and DA (A-D) are also parallel and opposite to each other. Since the opposites sides are parallel, the given points are vertices of a parallelogram. Also,

$$(\mathbf{B} - \mathbf{A})^T (\mathbf{C} - \mathbf{D}) = \begin{pmatrix} 1 & -1 \end{pmatrix} \begin{pmatrix} 1 \\ -1 \end{pmatrix} \tag{0.0.9}$$

$$= \begin{pmatrix} 0 \end{pmatrix} \tag{0.0.10}$$
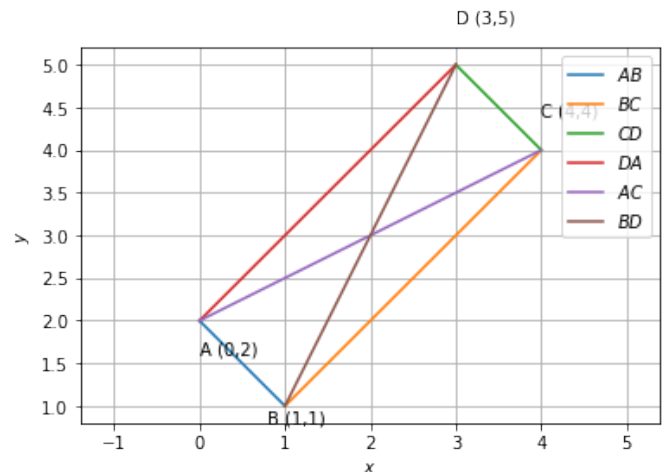
Therefore

$$\angle ABC = 90^o$$

and hence, the points A,B,C and D are vertices of a rectangle.

Fig. 0: Plot of the points