Seat No.	

F. E. (All Branches) (Part-I) (Semester-I) (Revised) Examination, December - 2019
ENGINEERING MATHEMATICS - I
Sub. Code : 59177

Day and Date : Tuesday, 10 - 12 - 2019 **Total Marks : 100**
Time : 2.30 p.m to 5.30 p.m.

Instructions :
1) **Solve any three questions from each Section.**
2) **Use of Non-programmable calculator is allowed.**
3) **Figures to the right indicate full marks.**

SECTION-I

Q1) Attempt any Three **[15]**

a) Find Rank of Matrix

$$A = \begin{bmatrix} -1 & 2 & 3 & -2 \\ 2 & -5 & 1 & 2 \\ 3 & -8 & 5 & 2 \\ 5 & -12 & -1 & 6 \end{bmatrix}$$

b) Test the consistency of the following equations and solve them if possible
$3x + 3y + 2z = 1, x + 2y = 4, 2x - 3y - z = 5$

c) Use Matrix Method to Solve the homogenous system of equations.
$4x + 3y - z = 0,$ $3x + 4y + z = 0,$ $5x + y - 4z = 0$

d) Determine Constant 'b' such that the system of homogenous equations
$2x + y + 2z = 0,$ $x + y + 3z = 0,$ $4x + 3y + bz = 0$
Has a Non trivial solution. Find Non trivial solution using matrix method.

Q2) Attempt any Three. **[15]**

a) Examine for linear dependence or independence of vectors. If dependent find the relation between them.
$x_1 = [1, -1, 1], x_2 = [2, 1, 1]$ and $x_3 = [3, 0, 2]$

b) Find eigen values and eigen vactor of smallest eigen value of matrix A

$$\text{where } A = \begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}.$$

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c) Verify Cayley Hamilton theorem for the matrix A and hence find

A^{-1} Where $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$.

d) Find Eigen values of the following matrix A and hence find Eigen values of A^{-1}

$$A = \begin{bmatrix} 3 & 10 & 5 \\ -2 & -3 & -4 \\ 3 & 5 & 7 \end{bmatrix}$$

Q3) Attempt any four. **[20]**

a) Simplify $\dfrac{(\cos 2\theta - i\sin 2\theta)^7 (\cos 3\theta + i\sin 3\theta)^{-5}}{(\cos 4\theta + i\sin 4\theta)^{12} (\cos 5\theta - i\sin 5\theta)^{-6}}$

b) Prove that $(1 + i)^{100} + (1-i)^{100} = -2^{51}$

c) Express $\cos 7\theta$ in terms of powers of $\sin\theta$ and $\cos\theta$

d) Solve the equation $x^3 - 1 = 0$

e) If $\sin(\alpha + i\beta) = x + iy$ Then prove that

i) $\dfrac{x^2}{\cosh^2\beta} + \dfrac{y^2}{\sinh^2\beta} = 1$ ii) $\dfrac{x^2}{\sin^2\alpha} - \dfrac{y^2}{\cos^2\alpha} = 1$

SECTION-II

Q4) Attempt any three of the following: **[15]**

a) Prove that $e^x \sin x = x + x^2 + \dfrac{1}{3}x^3 + \ldots$

b) By using Taylor's Theorem arrange $7+(x + 2)+3(x+2)^3+(x+2)^4-(x + 2)^5$ in powers of x

c) Evaluate $\lim\limits_{x\to 0} \dfrac{e^x - 1 - x}{\log(1+x) - x}$

d) Evaluate $\lim\limits_{x\to 1} \left(1 - x^2\right)^{\frac{1}{\log(1-x)}}$

Q5) Attempt any four of the following: **[20]**

a) If $u = \log(x^2 + y^2)$, Prove that $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$

b) If $u = x^2 \tan^{-1}\left(y/x\right) + y^2 \sin^{-1}\left(x/y\right)$,

Prove that $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = 2u$

c) If $x = v^2 + w^2, y = w^2 + u^2, z = u^2 + v^2$, Find $\frac{\partial(x,y,z)}{\partial(u,v,w)}$

d) Find percentage error in the area of an ellipse due to 1% error in major nad minor axes.

e) Find the maximum and minimum values of $x^3 + 3xy^2 - 3x^2 - 3y^2 + 4$

Q6) Attempt any three of the following: **[15]**

a) Solve by Gauss elimination method
$4x + y + z = 4; x + 4y - 2z = 4; 3x + 2y - 4z = 6$

b) Apple Gauss-Jordon Method to solve
$x + 2y + z = 8; 2x + 3y + 4z = 20; 4x + 3y + 2z = 16$

c) Use Gauss Seidel Iterative method to Solve following equations up to three iterations.
$10x + y + z = 12; 2x + 10y + z = 13; 2x + 2y + 10z = 14$

d) Determine the largest eigen value and the corresponding eigen vector of the following matrix A using power method,

Where $A=\begin{bmatrix} 1 & 3 & -1 \\ 3 & 2 & 4 \\ -1 & 4 & 10 \end{bmatrix}$, take initial eigen vector $X=\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

(up to four iterations)

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