Portfolio Risk Simulation: Efficient Frontier Analysis

1. Introduction

The objective of this project was to simulate portfolio performance using a Monte Carlo approach in C++ and to visualise the resulting efficient frontier in Python. The efficient frontier represents the set of optimal portfolios offering the highest expected return for a given level of risk, as described in Modern Portfolio Theory.

2. Data and Assumptions

- **Tickers:** SPY (S&P 500 ETF), QQQ (Nasdaq-100 ETF), GLD (Gold ETF)
- **Period:** 2 years of daily adjusted close prices
- **Source:** Yahoo Finance via the `yfinance` Python package
- **Risk-free rate:** 2% annualised
- **Trading days per year:** 252 (used for annualisation)

Daily returns were calculated as percentage changes in adjusted close prices. These returns were stored in a CSV file (`returns.csv`) and used as the input to the C++ simulation.

3. Methodology

3.1 Monte Carlo Simulation in C++

- Read historical daily returns from `returns.csv`
- Calculate mean returns vector and covariance matrix
- Generate $N = 10{,}000$ random portfolios with weights summing to 1
- For each portfolio, compute:
  - Expected return
  - Volatility (standard deviation)
  - Sharpe ratio: $(\mu - r_f)/\sigma$
- Save results to `portfolios.csv`

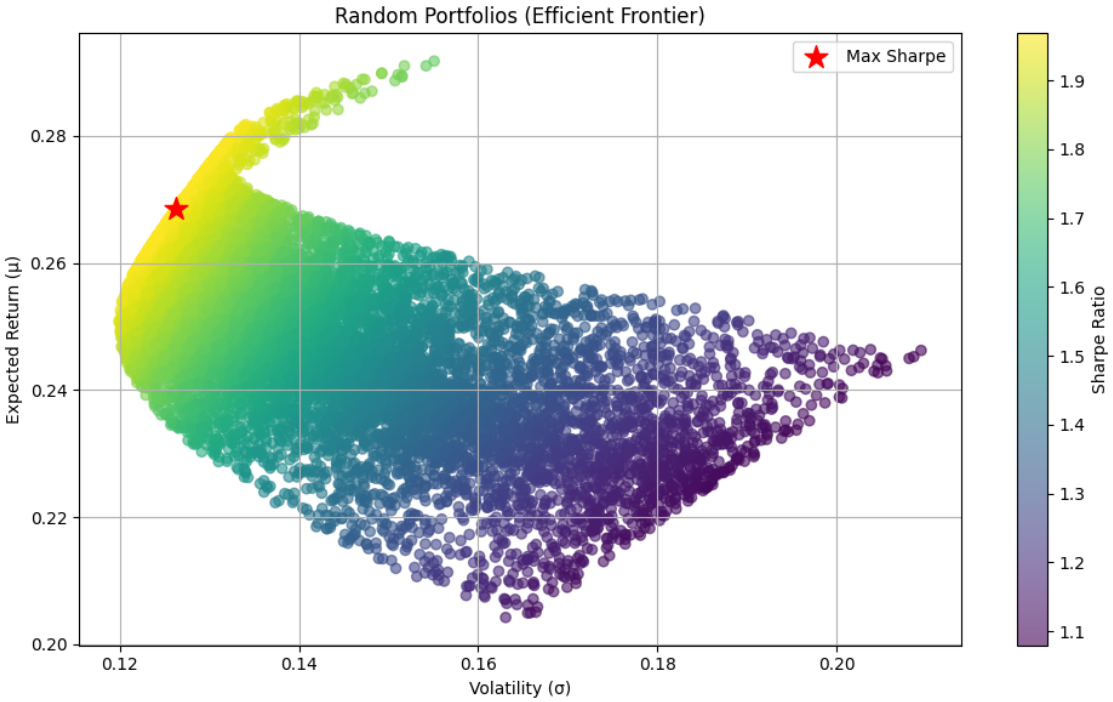
3.2 Visualisation in Python

The `portfolios.csv` file was loaded into Python, and `matplotlib` was used to:

- Scatter plot portfolios coloured by Sharpe ratio
- Highlight the portfolio with the maximum Sharpe ratio
- Label axes as Volatility (σ) vs. Expected Return (μ)

4. Results

The figure below shows the simulated portfolios forming the efficient frontier. The yellow region contains high-Sharpe portfolios, while the red star marks the portfolio with the maximum Sharpe ratio given the inputs.



5. Conclusion

This simulation illustrates the trade-off between risk and return. Portfolios on the upper-left edge of the frontier provide the best risk-adjusted returns. The maximum Sharpe portfolio in this simulation offered a strong balance of return and volatility, but results are entirely dependent on historical data and chosen assumptions. Extensions could include:

- Using a larger universe of assets
- Incorporating transaction costs
- Running sensitivity analysis on the risk-free rate