

Homework3

Instructions

All solutions must be written in your own words, and you must program the algorithms yourself. **You can use sklearn to implement PCA and Pytorch to implement autoencoder.**

Your programs are suggested to be written in Python. **Code is NOT required to submit. But you may get extra bonus if you turn in your code, and it is easy to read.** If you solve any problems by hand just digitize that page and submit it (make sure the problem is labeled).

In this homework, you will be implementing two dimension reduction methods: PCA and auto-encoder for airplane images. Please download the dataset from <https://www.cs.toronto.edu/~kriz/cifar.html>. It has 10 classes in total and one of those is airplane. There are 5K airplane images for training and 1K airplane images for testing. The images are 32 by 32 color images. You may need to convert it to a long vector with $32 \times 32 \times 3$ elements as your input.

- 1) Describe the process of PCA in your own words. Use the 5K training data to find the principal components. What's the number of principal components (N) if the cumulative variance ratio is at least 70%? With this N, what's the average reconstruction errors on training and test data. Show three pairs of images (the original images and its reconstructed images) in your test set with largest reconstructed errors and three pairs of images in the test set with smallest reconstructed errors.

Hint: the cumulative ratio can be calculated as,

$$\frac{\lambda_i}{\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6}$$

- 2) Implement autoencoder to reduce feature dimensions and reconstruct images. The network has three layers with linear activation functions as shown in Fig.1. The number of hidden units is the same as the number of principal components you get from problem 1). What's the average reconstruction errors on training and test data? Compare with the PCA based method, which method has lower reconstruction error? Show three pairs of images in your test set with largest reconstructed errors and three pairs of images in the test set with smallest reconstruction errors. Are the encoder weights pairwise correlated or not? Could you please randomly pick three pairs of encoder weight to verify it (for example w^1 and w^2 in Fig.1)?

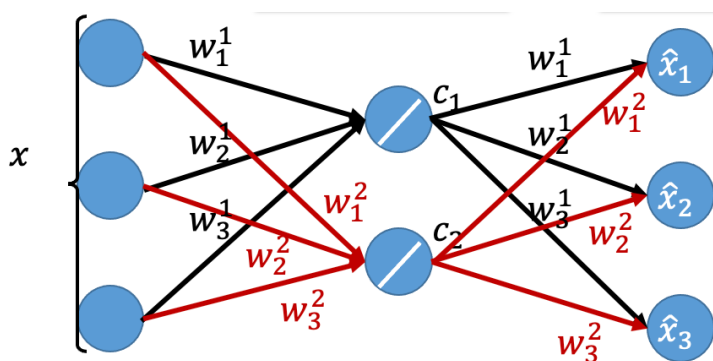


Figure1: Autoencoder with one hidden layer and 2 hidden units. $w^1 = [w_1^1, w_2^1, w_3^1]^T$, $w^2 = [w_1^2, w_2^2, w_3^2]^T$

- 3) (Bonus question): Can you further decrease the autoencoder reconstruction error by revising the network architecture? The only thing you cannot change here is the number of hidden units c_i (still needs to be N) for a fair comparison (All the methods in this homework reduce the number of dimensions from $32*32*3$ to N).