Mechatronics and robotics

INTRO TO BIOTRONICS ASSIGNMENT

Forward Kinematics Sheet

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FORWARD KINEMATICS ASSIGNMENT

2. Consider the three-link planar manipulator shown in Figure 3.12. Derive

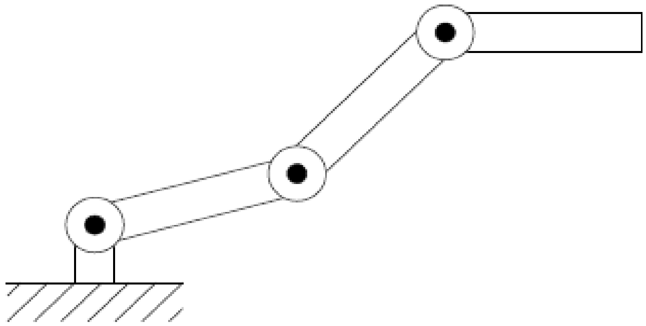
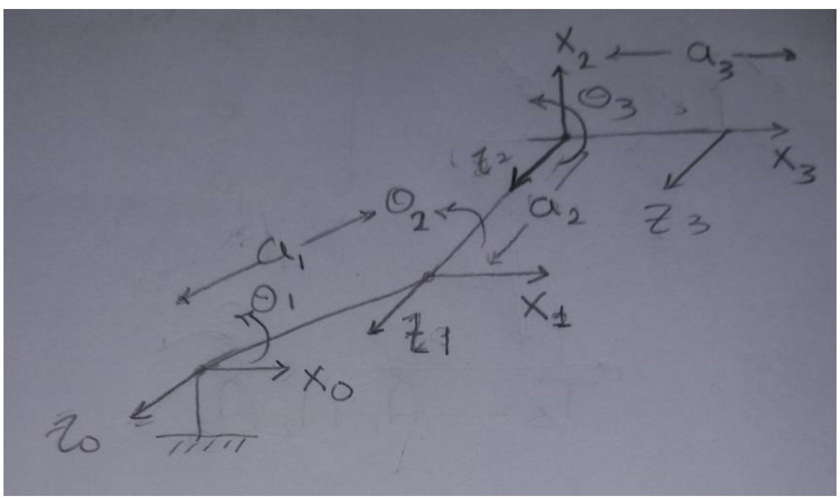


Figure 3.12: Three-link planar arm of Problem 3-2.

the forward kinematic equations using the DH-convention.



D-H Table:

fram	α	a	d	θ	
1	0	0	$-a_1$	θ_1	A_1
2	0	0	$-a_2$	θ_2+90	A_2
3	0	0	$-a_3$	θ_3-90	A_3

- ## Forward Kinematics Equation:

$$^{0}T_1 = A_1 = \begin{bmatrix} C(\theta_1) & -S(\theta_1) & 0 & 0 \\ S(\theta_1) & C(\theta_1) & 0 & 0 \\ 0 & 0 & 1 & -a_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^{1}T_2 = A_2 = \begin{bmatrix} C(\theta_2+90) & -S(\theta_2+90) & 0 & 0 \\ S(\theta_2+90) & C(\theta_2+90) & 0 & 0 \\ 0 & 0 & 1 & -a_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^{2}T_3 = A_3 = \begin{bmatrix} C(\theta_3-90) & -S(\theta_3-90) & 0 & 0 \\ S(\theta_3-90) & -S(\theta_3-90) & 0 & 0 \\ 0 & 0 & 1 & -a_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^{0}T_3 = A_1 A_2 A_3$$

- ## MATLAB Code:

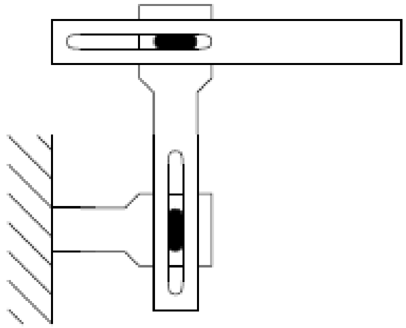
```
clear,clc

% Ensure you have the Symbolic Math Toolbox installed and available
syms theta1 theta2 theta3 real;
syms d1 a1 real;
syms d2 a2 real;
syms d3 a3 real;
syms alpha1 alpha2 alpha3 real;



% Assume some typical values for d, a, alpha or leave them as symbols
% Link 1
alpha1 =0;
a1_val = 0;
d1 = -a1;
theta1_val = theta1;


% Link 2
alpha2 = 0;
a2_val = 0;
d2 = -a2;
theta2_val = theta2+(pi/2);


% Link 3
alpha3 = 0;
a3_val = 0;
d3 = -a3;
theta3_val = theta3-(pi/2);


% Compute transformation matrices for each link
A1 = DhTable(a1_val, alpha1, d1,theta1_val);
A2 = DhTable(a2_val, alpha2, d2,theta2_val);
A3 = DhTable(a3_val, alpha3, d3,theta3_val);

% Compute the overall transformation matrix from base to end-effector
T = A1 * A2 * A3;


T = simplify(T);

% Display results
disp('0T1 = A1 = ');
disp(A1);
disp('1T2 = A2 = ');
disp(A2);
```

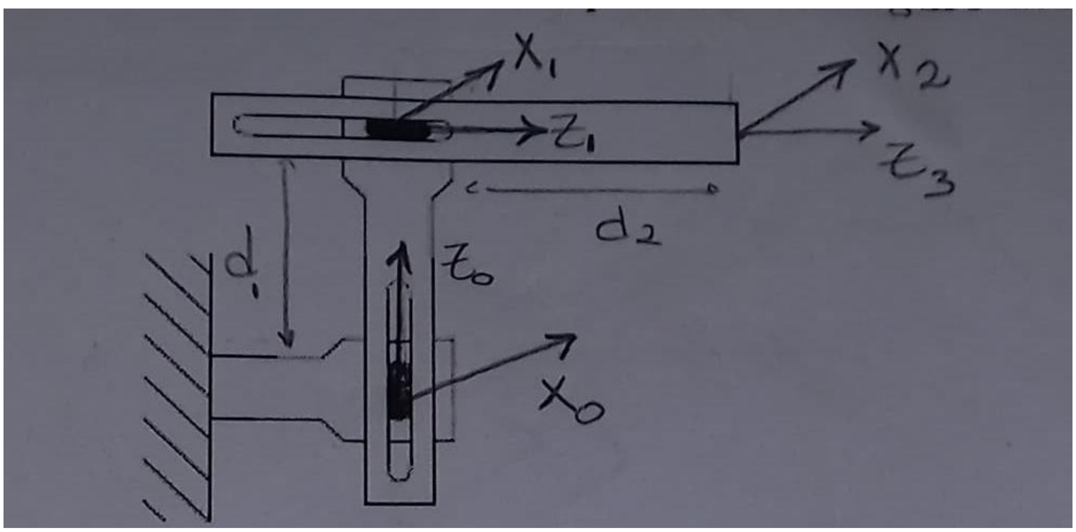
```
disp('2T3 = A3 = ');
disp(A3);
disp('0T3 = ');
disp(T);



function T = DhTable(a,alpha,d,theta)
    % Define the DH transformation matrix as an anonymous function
    DHMatrix = [cos(theta) -cos(alpha)*sin(theta) sin(alpha)*sin(theta)
a*cos(theta);
                sin(theta) cos(alpha)*cos(theta) -sin(alpha)*cos(theta)
a*sin(theta);
                0 sin(alpha) cos(alpha) d;
                0 0 0 1];

     T = DHMatrix;
end
```

➢ Output:

```
0T1 = A1 =
[cos(theta1), -sin(theta1), 0,   0]
[sin(theta1),  cos(theta1), 0,   0]
[          0,            0, 1, -a1]
[          0,            0, 0,   1]

1T2 = A2 =
[cos(theta2 + pi/2), -sin(theta2 + pi/2), 0,   0]
[sin(theta2 + pi/2),  cos(theta2 + pi/2), 0,   0]
[                 0,                   0, 1, -a2]
[                 0,                   0, 0,   1]

2T3 = A3 =
[cos(theta3 - pi/2), -sin(theta3 - pi/2), 0,   0]
[sin(theta3 - pi/2),  cos(theta3 - pi/2), 0,   0]
[                 0,                   0, 1, -a3]
[                 0,                   0, 0,   1]

0T3 =
[cos(theta1 + theta2 + theta3), -sin(theta1 + theta2 + theta3), 0,              0]
[sin(theta1 + theta2 + theta3),  cos(theta1 + theta2 + theta3), 0,              0]
[                            0,                              0, 1, - a1 - a2 - a3]
[                            0,                              0, 0,              1]
```

3. Consider the two-link cartesian manipulator of Figure 3.13. Derive





DH-Table

fram	α	a	d	θ
1	90°	0	d_1	0
2	0	0	d_2	0

- ## Forward Kinematics Equation:

$$^{0}T_1 = A_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^{1}T_2 = A_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^{0}T_2 = A_1 A_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & -d_2 \\ 0 & 1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

• MATLAB Code:

```
clear,clc

% Ensure you have the Symbolic Math Toolbox installed and available
syms theta1 theta2 theta3 real;
syms d1 a1 real;
syms d2 a2 real;
syms d3 a3 real;
syms alpha1 alpha2 alpha3 real;



% Assume some typical values for d, a, alpha or leave them as symbols
% Link 1
alpha1 =pi/2;
a1_val = 0;
d1_val = d1;
theta1_val = 0;


% Link 2
alpha2 = 0;
a2_val = 0;
d2_val = d2;
theta2_val = 0;



% Compute transformation matrices for each link
A1 = DhTable(a1_val, alpha1, d1_val,theta1_val);
A2 = DhTable(a2_val, alpha2, d2_val,theta2_val);

% Compute the overall transformation matrix from base to end-effector
T = A1 * A2 ;


T = simplify(T);

% Display results
disp('0T1 = A1 = ');
disp(A1);

disp('1T2 = A2 = ');
disp(A2);

disp('0T2 = ');
disp(T);
```

```
function T = DhTable(a,alpha,d,theta)
    % Define the DH transformation matrix as an anonymous function
    DHMatrix = [cos(theta) -cos(alpha)*sin(theta) sin(alpha)*sin(theta)
a*cos(theta);
                sin(theta) cos(alpha)*cos(theta) -sin(alpha)*cos(theta)
a*sin(theta);
                0 sin(alpha) cos(alpha) d;
                0 0 0 1];

     T = DHMatrix;
end
```

➢ Output:

0T1 = A1

[1, 0, 0, 0]
[0, 0, -1, 0]
[0, 1, 0, d1]
[0, 0, 0, 1]

1T2 = A2 =
[1, 0, 0, 0]
[0, 1, 0, 0]
[0, 0, 1, d2]
[0, 0, 0, 1]

0T2 =

[1, 0, 0, 0]
[0, 0, -1, -d2]
[0, 1, 0, d1]
[0, 0, 0, 1]

4. Consider the two-link manipulator of Figure 3.14 which has joint 1 revolute and joint 2 prismatic. Derive the forward kinematic equations using the DH-convention.

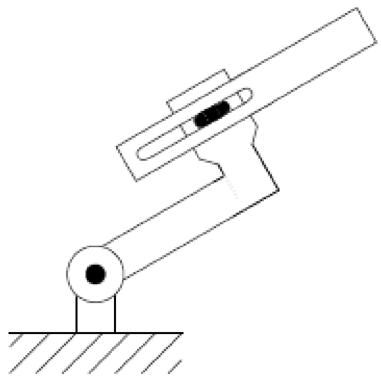
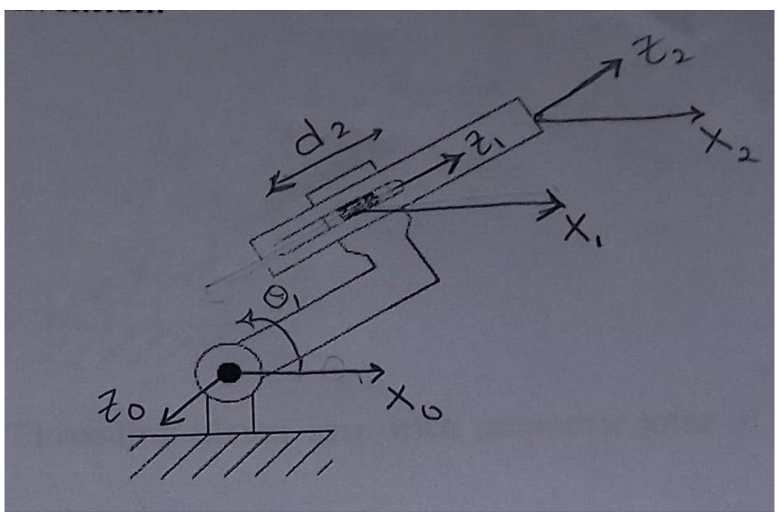


Figure 3.14: Two-link planar arm of Problem 3-4.



DH-Table:-

fram	α_i	a_i	d_i	θ_i
1	180°	0	0	θ_1
2	0	0	d_2	0

- ## Forward Kinematics Equation:

$$
{}^{0}T_1 = A_1 = \begin{bmatrix} C(\theta_1) & S(\theta_1) & 0 & 0 \\ S(\theta_1) & -C(\theta_1) & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{1}T_2 = A_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
T_2 = A_1 A_2 = \begin{bmatrix} C(\theta_1) & S(\theta_1) & 0 & 0 \\ S(\theta_1) & -C(\theta_1) & 0 & 0 \\ 0 & 0 & -1 & -d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

➢ Output:

```
A1 =

[cos(theta1),  sin(theta1),  0, 0]
[sin(theta1), -cos(theta1),  0, 0]
[          0,            0, -1, 0]
[          0,            0,  0, 1]


A2 =

[1, 0, 0,  0]
[0, 1, 0,  0]
[0, 0, 1, d2]
[0, 0, 0,  1]


T =

[cos(theta1),  sin(theta1),  0,   0]
[sin(theta1), -cos(theta1),  0,   0]
[          0,            0, -1, -d2]
[          0,            0,  0,   1]
```

5. Consider the three-link planar manipulator of Figure 3.15 Derive the forward kinematic equations using the DH-convention.

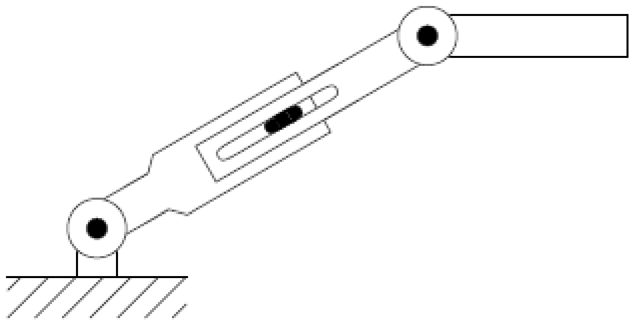
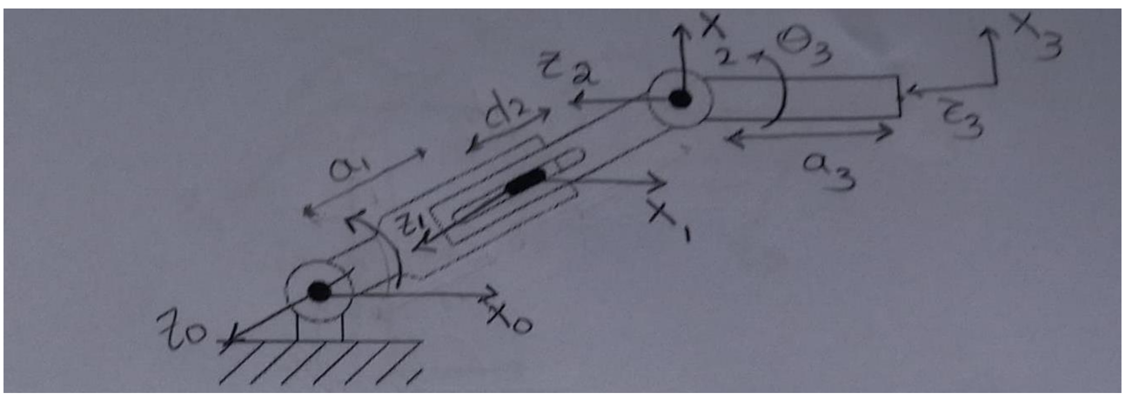


Figure 3.15: Three-link planar arm with prismatic joint of Problem 3-5.



DH-Table:

frame	α_i	a_i	d_i	θ_i
1	0	0	0	θ_1
2	-90	0	a_1+d_1	0
3	0	0	$-a_3$	θ_3

- ## Forward Kinematics Equation:

$$
{}^{0}T_{1} = A_{1} = \begin{bmatrix} C(\theta_1) & -S(\theta_1) & 0 & 0 \\ S(\theta_1) & C(\theta_1) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{1}T_{2} = A_{2} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & a_1+d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{2}T_{3} = A_{3} = \begin{bmatrix} C(\theta_3) & -S(\theta_3) & 0 & 0 \\ S(\theta_3) & C(\theta_3) & 0 & 0 \\ 0 & 0 & 1 & -a_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

➢ Output:

```
A1 =

[cos(theta1), -sin(theta1), 0, 0]
[sin(theta1),  cos(theta1), 0, 0]
[          0,            0, 1, 0]
[          0,            0, 0, 1]


A2 =

[1,  0, 0,       0]
[0,  0, 1,       0]
[0, -1, 0, a1 + d1]
[0,  0, 0,       1]


A3 =

[cos(theta3), -sin(theta3), 0,   0]
[sin(theta3),  cos(theta3), 0,   0]
[          0,            0, 1, -a3]
[          0,            0, 0,   1]


T =

[cos(theta1)*cos(theta3), -cos(theta1)*sin(theta3), -sin(theta1),  a3*sin(theta1)]
[cos(theta3)*sin(theta1), -sin(theta1)*sin(theta3),  cos(theta1), -a3*cos(theta1)]
[           -sin(theta3),             -cos(theta3),            0,         a1 + d1]
[                      0,                        0,            0,               1]

fx >> 
```

6. Consider the three-link articulated robot of Figure 3.16. Derive the forward kinematic equations using the DH-convention.

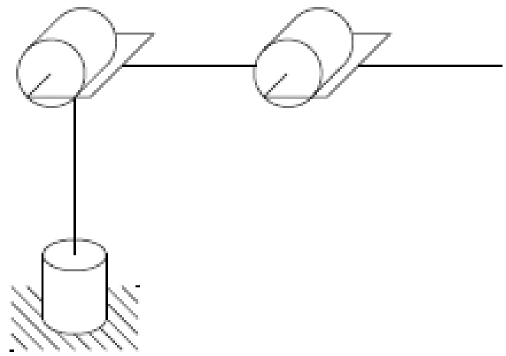
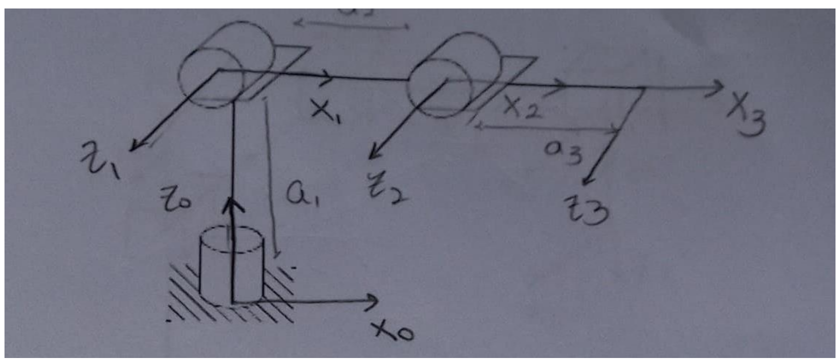


Figure 3.16: Three-link articulated robot.



DH-Table:

Frame	α_i	a_i	d_i	θ_i
1	90	0	a_1	θ_1
2	0	a_2	0	θ_2
3	0	a_3	0	θ_3

- ## Forward Kinematics Equation:

$$^{0}T_{1} = A_{1} = \begin{bmatrix} C(\theta_1) & 0 & S(\theta_1) & 0 \\ S(\theta_1) & 0 & -C(\theta_1) & 0 \\ 0 & 1 & 0 & a_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^{1}T_{2} = A_{2} = \begin{bmatrix} C(\theta_2) & -S(\theta_2) & 0 & a_2 C(\theta_2) \\ S(\theta_2) & C(\theta_2) & 0 & a_2 S\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^{2}T_{3} = A_{3} = \begin{bmatrix} C(\theta_3) & -S(\theta_3) & 0 & a_3 C(\theta_3) \\ S(\theta_3) & C(\theta_3) & 0 & a_3 S\theta_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^{0}T_{3} = A_1 A_2 A_3$$

➢ Output:

```
A1 =

[cos(theta1), 0,  sin(theta1),  0]
[sin(theta1), 0, -cos(theta1),  0]
[           0, 1,            0, a1]
[           0, 0,            0,  1]


A2 =

[cos(theta2), -sin(theta2), 0, a2*cos(theta2)]
[sin(theta2),  cos(theta2), 0, a2*sin(theta2)]
[          0,            0, 1,              0]
[          0,            0, 0,              1]


A3 =

[cos(theta3), -sin(theta3), 0, a3*cos(theta3)]
[sin(theta3),  cos(theta3), 0, a3*sin(theta3)]
[          0,            0, 1,              0]
[          0,            0, 0,              1]
```

T =

[cos(theta1)*cos(theta2)*cos(theta3) - cos(theta1)*sin(theta2)*sin(theta3),

- cos(theta1)*cos(theta2)*sin(theta3) - cos(theta1)*cos(theta3)*sin(theta2), sin(theta1), a2*cos(theta1)*cos(theta2) + a3*cos(theta1)*cos(theta2)*cos(theta3) - a3*cos(theta1)*sin(theta2)*sin(theta3)]

[cos(theta2)*cos(theta3)*sin(theta1) - sin(theta1)*sin(theta2)*sin(theta3),

- cos(theta2)*sin(theta1)*sin(theta3) - cos(theta3)*sin(theta1)*sin(theta2), -cos(theta1), a2*cos(theta2)*sin(theta1) + a3*cos(theta2)*cos(theta3)*sin(theta1) - a3*sin(theta1)*sin(theta2)*sin(theta3)]

[cos(theta2)*sin(theta3) + cos(theta3)*sin(theta2),

cos(theta2)*cos(theta3) -sin(theta2)*sin(theta3), 0, a1 + a2*sin(theta2) + a3*cos(theta2)*sin(theta3) + a3*cos(theta3)*sin(theta2)]

[0, 0, 0, 1]

7. Consider the three-link cartesian manipulator of Figure 3.17. Derive the forward kinematic equations using the DH-convention.

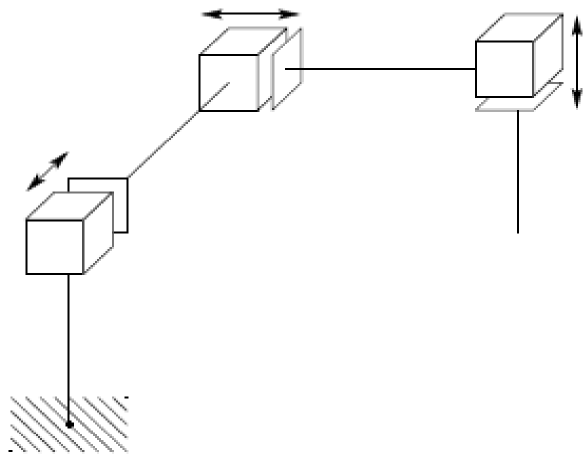
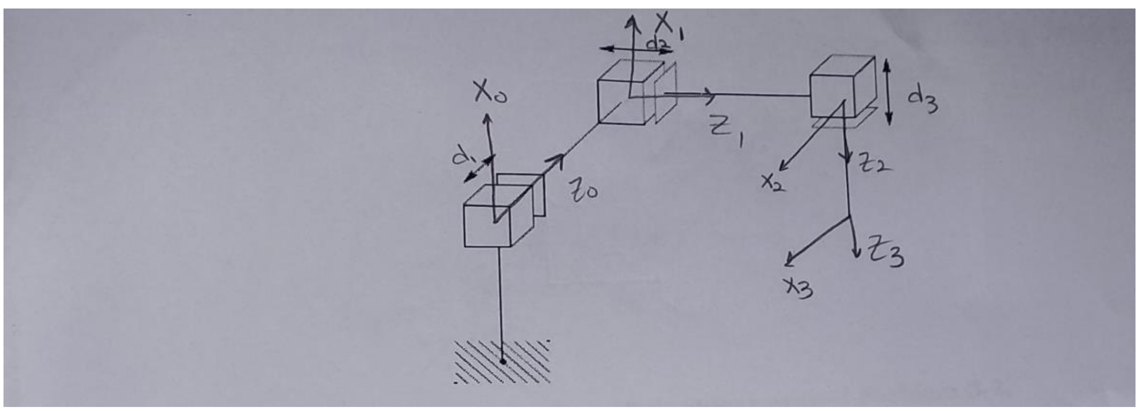


Figure 3.17: Three-link cartesian robot.



DH-Table:

frame	α_i	a_i	d_i	θ_i
1	90	0	d_1	0
2	-90	0	d_2	0
3	0	0	d_3	0

- Forward Kinematics Equation:

$$ {}^{0}T_{1} = A_{1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{1}T_{2} = A_{2} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{2}T_{3} = A_{3} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

➢ Output:

```
A1 =

[1,  0, 0,  0]
[0,  0, 1,  0]
[0, -1, 0, d1]
[0,  0, 0,  1]


A2 =

[1, 0,  0,  0]
[0, 0, -1,  0]
[0, 1,  0, d2]
[0, 0,  0,  1]


A3 =

[1, 0, 0,  0]
[0, 1, 0,  0]
[0, 0, 1, d3]
[0, 0, 0,  1]


T =

[1, 0, 0,       0]
[0, 1, 0,      d2]
[0, 0, 1, d1 + d3]
[0, 0, 0,       1]
```