CS 221
Logic Design

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REMEMBER OUR RULES

Boolean functions: Canonical and standard forms

Lecture 3

BOOLEAN FUNCTIONS

- **A Boolean function**
  - Expresses the logical relationship between binary variables and
  - is evaluated by determining the binary value of the expression for all possible values of the variables

DIFFERENT REPRESENTATIONS OF A BOOLEAN FUNCTION

i. Algebraic expression

(a) $F_2 = x'y'z + x'yz + xy'$

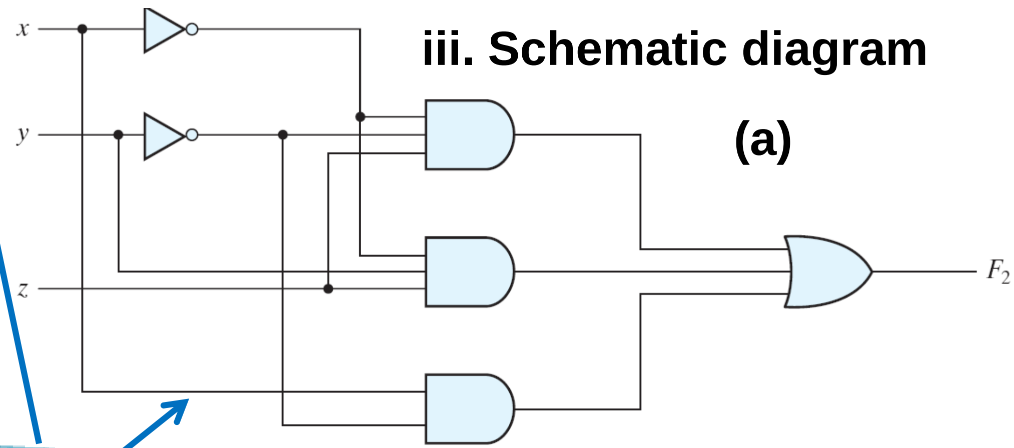
(b) $F_2 = x'z\,(y' + y) + x\,y'$

$= x'z \,.\, 1 + xy'$

$= x'z + xy'$

iii. Schematic diagram

(a)

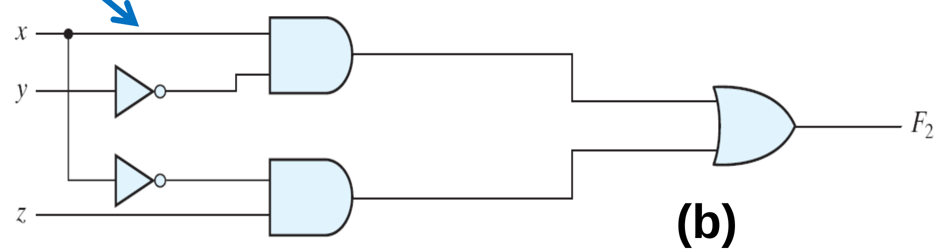


(a) $F_2 = x'y'z + x'yz + xy'$

function F_2

ii. Truth table

x	y	z	F_2
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	0



(b)

(b) $F_2 = xy' + x'z$

CANONICAL AND STANDARD FORMS

It is useful to specify Boolean functions in one of these forms

- **Canonical forms:**
  – Sum of Minterms (SOM)
  – Product of Maxterms (POM)
- **Standard forms:**
  – Sum of products (SOP)
  – Product of sums (POS)

Canonical Forms: Minterms & Maxterms

- $F_1 = x + y'z$

terms? **x and y'z**

when y'z = 1? **if y=0 and z=1**

when F_1 = 1? **if x=1 or y'z=1**

Minterm = Anding term is 1

Maxterm = Oring term is 0

Canonical Forms: Minterms & Maxterms

n **variables form:**
2^n minterms
2^n maxterms

Table 2.3
Minterms and Maxterms for Three Binary Variables

			Minterms		Maxterms	
x	**y**	**z**	**Term**	**Designation**	**Term**	**Designation**
0	0	0	$x'y'z'$	m_0	$x + y + z$	M_0
0	0	1	$x'y'z$	m_1	$x + y + z'$	M_1
0	1	0	$x'yz'$	m_2	$x + y' + z$	M_2
0	1	1	$x'yz$	m_3	$x + y' + z'$	M_3
1	0	0	$xy'z'$	m_4	$x' + y + z$	M_4
1	0	1	$xy'z$	m_5	$x' + y + z'$	M_5
1	1	0	xyz'	m_6	$x' + y' + z$	M_6
1	1	1	xyz	m_7	$x' + y' + z'$	M_7

literals are **AND**ed in a **term**
0→ primed, 1→ unprimed

literals are **OR**ed in a **term**
1→ primed, 0→ unprimed

SUM OF MINTERMS (SOM)

- From a given truth table, Boolean function can expressed into SOM:
1) Forms minterms that produce a **1** in the function
2) **OR**ing all those terms

x	y	z	Function f_1
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

Section 2.6

- Ex: Consider this truth table: $f_1 = ?$

$f_1 = x'y'z + xy'z' + xyz$

$f_1 = m_1 + m_4 + m_7$

$f_1\,(x, y, z) = \Sigma\,(1, 4, 7)$

SOM forms

PRODUCT OF MAXTERMS (POM)

- From a given truth table, Boolean function can expressed into POM:

1) Forms maxterms that produce a **0** in the function
2) **AND**ing all those terms

x	*y*	*z*	Function f_1
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

Section 2.6

- Ex: Consider this truth table: $f_1 = ?$

$f_1 = (x+y+z)(x+y'+z)\mathbf{(x+y'+z')}(x'+y+z')(x'+y'+z)$

$f_1 = M_0 \,.\, M_2 \,.\, M_3 \,.\, M_5 \,.\, M_6$

$f_1\,(x, y, z) = \Pi\,(0, 2, 3, 5, 6)$

POM forms

"Mistake in Book" p.52

PRODUCT OF MAXTERMS (POM) CONT.

- **Another way** to obtain POM from a given truth table:

1) Get the minterms that produces a 0 in the function
2) **OR**ing all those terms
3) Take the complement of function

x	y	z	Function f_1
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

Section 2.6

- Ex: Consider this truth table: $f_1 = ?$

1) $f'_1 = x'y'z' + x'yz' + x'yz + xy'z + xyz'$
2) $f_1 = (x+y+z)(x+y'+z)(x+y'+z')(x'+y+z')(x'+y'+z)$
3) $f_1 = M_0 \,.\, M_2 \,.\, M_3 \,.\, M_5 \,.\, M_6$

Important property

- Any Boolean function can be expressed as:
  - A sum of minterms (SOM)
    - Where "sum" means ORing of terms
  - A product of maxterms (POM)
    - where "product" means ANDing of terms
- Boolean functions expressed as SOM and POM are said to be in ***canonical form***
- ***Example:***

Express the Boolean function $F = A + B'C$ in a sum of minterms.

- To express a Boolean function as a sum of minterms:

<u>Method 1:</u>

(1) It must be expanded into a sum of AND terms

(2) Any missing variable *x* in each AND term is ANDed with (***x + x'***)

<u>Method 2:</u>

Using truth table

Method 1

Example 2.4 (p.69)

Express the Boolean function $F = A + B'C$ in a sum of minterms.
The first term A is missing two variables; therefore:

$$A = A(B + B') = AB + AB' \quad \text{still missing one variable:}$$

$$A = AB(C + C') + AB'(C + C')$$

$$= ABC + ABC' + AB'C + AB'C'$$

The second term $B'C = B'C(A + A') = AB'C + A'B'C$

Combining all terms, we have

$$F = ABC + ABC' + \underline{AB'C} + AB'C' + \underline{AB'C} + A'B'C$$

But $AB'C$ appears twice, and according to theorem 1 $(x + x = x)$
Rearranging the minterms in ascending order, we finally obtain

$$F = A'B'C + \underline{AB'C'} + AB'C + ABC' + ABC$$

$$= m_1 + m_4 + m_5 + m_6 + m_7$$

"Mistake in Book" p.53

EXAMPLE 2.4 (P.69) CONT.

<u>Method 2</u>

- **<u>Another procedure</u>** for deriving the minterms of a Boolean function:

1) Obtain the truth table of the function from the algebraic expression
2) Read minterms from the truth table

Table 2.5

1) *Truth Table for $F = A + B'C$*

A	*B*	*C*	*F*
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

2) From the truth table:

$F = A'B'C + AB'C' + AB'C + ABC' + ABC$

$= m_1 + m_4 + m_5 + m_6 + m_7$

Method 3

Example 2.4 (p.69) Cont.

- **Another procedure** for deriving the minterms of a Boolean function:

1) Obtain different combination of missing literals from algebraic expression in their places

- To express a Boolean function as a product of maxterms:

Method 1:

(1) It must be brought into of OR terms

(2) Any missing variable x in each OR term is ORed with xx'

Method 2:

Using truth table and canonical conversion procedure

EXAMPLE 2.5 (P.70)

Method 1

Express the Boolean function $F = xy + x'z$ in a product of maxterm form.

First, convert the function into OR terms using the distributive law:

$$F = xy + x'z = (xy + x')(xy + z)$$
$$= (x + x')(y + x')(x + z)(y + z)$$
$$= (x' + y)(x + z)(y + z)$$

Each OR term is missing one variable;

therefore:

$$x' + y = x' + y + zz' = (x' + y + z)(x' + y + z')$$
$$x + z = x + z + yy' = (x + y + z)(x + y' + z)$$
$$y + z = y + z + xx' = (x + y + z)(x' + y + z)$$

Combining all the terms and removing those that appear more than once, we finally obtain:

$$F = (x + y + z)(x + y' + z)(x' + y + z)(x' + y + z')$$
$$= M_0M_2M_4M_5$$

EXAMPLE 2.5 (P.70) CONT.

<u>Method 2</u>

- **<u>Another method</u>** for deriving the maxterms of a Boolean function:

1) Derive the truth table of the function from the algebraic expression
2) Read minterms from the truth table
3) Use canonical conversion procedure to get product of maxterms

Table 2.6

1) *Truth Table for $F = xy + x'z$*

x	*y*	*z*	*F*
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

2) $F(x,y,z) = \Sigma(1,3,6,7)$

3) $F(x,y,z) = \Pi(0,2,4,5)$

CANONICAL CONVERSION PROCEDURE

- To convert from canonical form to another:

  (1) Interchange the symbols Σ and Π

  (2) List the missing numbers from the original form.

Note:

The total number of minterms and maxterms is 2^n, where ***n*** is the number of binary variables in the function.

CANONICAL CONVERSION PROCEDURE: EXAMPLE

- Example1:

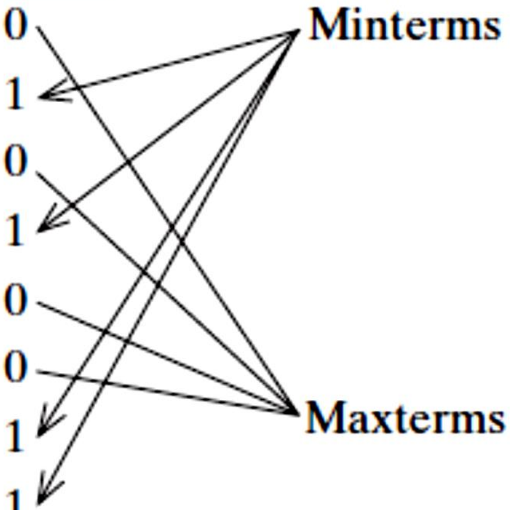
F (x,y,z)=Σ(1,3,6,7)

Other canonical form:

F(x,y,z)= π(0,2,4,5)

Table 2.6
Truth Table for $F = xy + x'z$

x	y	z	F
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1



- Example2:

F(x,y,z,m) = Π (0,2,4,5,8,9,11,13,14)

Other canonical form: F(x,y,z,m) = Σ (1,3,6,7, 10,12,15)

COMPLEMENT OF FUNCTION

- Example1:

$F\ (x,y,z)=\Sigma(1,3,6,7)$

Its complement: $F\ (x,y,z)=\pi(1,3,6,7)$

- Example2:

$F_1 = x'yz' + x'y'z$

Its complement: $F1' = (x+y'+z)(x+y+z')$

- Example3:

$F1= \pi\ (0,3,4,5,6,7)$

Its complement: $F1'= \Sigma\ (0,3,4,5,6,7)$

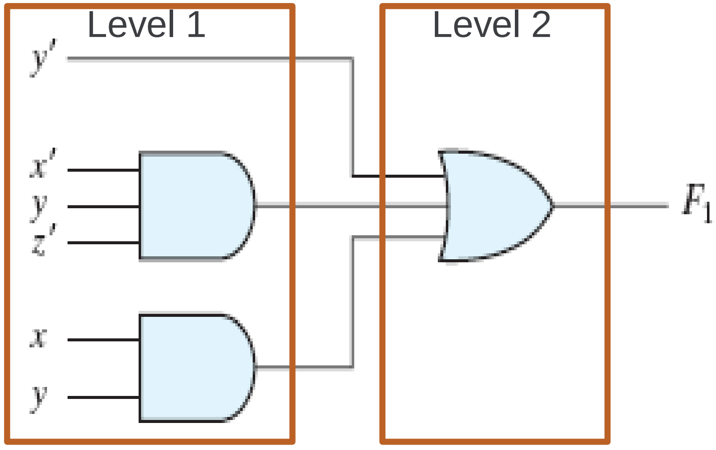
STANDARD FORMS

There are two types of standard forms:

- Sum Of Product (SOP)
- Product Of Sum (POS)

STANDARD FORMS: SUM OF PRODUCTS (SOP)

- Is a Boolean expression containing AND terms, called *product terms*, with one or more literals each.
- Sum Of Minterms (SOM) is a special case from SOP.
- Ex: $F_1 = y' + xy + x'yz'$
- The logic diagram of a SOP consists of :
  - A group of AND gates
  - Followed by a single OR gate

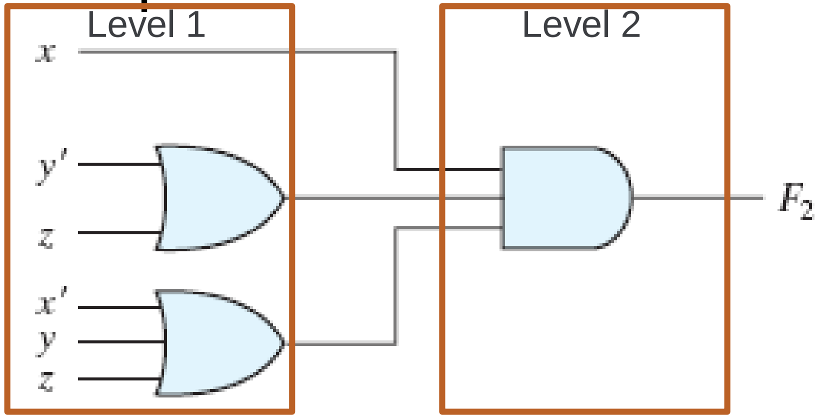


(a) Sum of Products

Two-level implementation

STANDARD FORMS: PRODUCT OF SUMS (POS)

- Is a Boolean expression containing OR terms, called *sum terms*, with one or more literals each.
- Product Of Maxterms (POM) is a special case from POS.
- Ex: $F_2 = x(y' + z)(x' + y + z')$
- The logic diagram of a POS consists of :
  - A group of OR gates
  - Followed by a single AND gate



(b) Product of Sums

Two-level implementation

A NON-STANDARD FORM

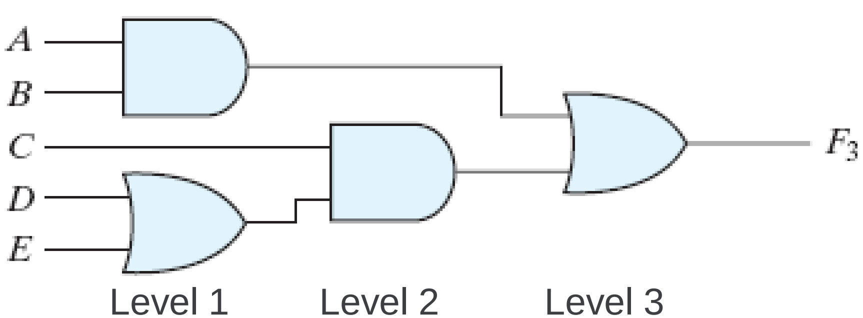
- For example, the function

$$F3 = AB + C(D + E)$$

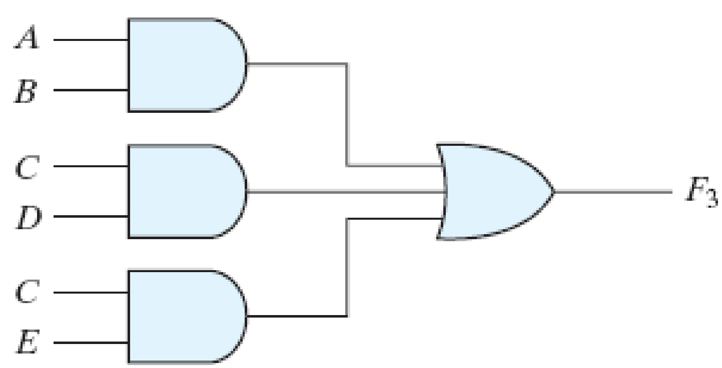
  - Is neither sum of products nor product of sums
  - There are *three level* of gating in this circuit
- It can be changed to a standard form:

$$F3 = AB + CD + CE$$

  - The sum of products expression is implemented in a *two-level* implementation (*least amount of delay*)



(a) $AB + C(D + E)$



(b) $AB + CD + CE$

THANKS

Next week : K-map