CS 221
Logic Design

Fall 2021

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Synchronous sequential logic Analysis & Design

Lecture 8

ANALYSIS OF CLOCKED SEQUENTIAL CIRCUITS

- Analysis of clocked sequential circuits

➔ describes what a given circuit will do under certain operating conditions

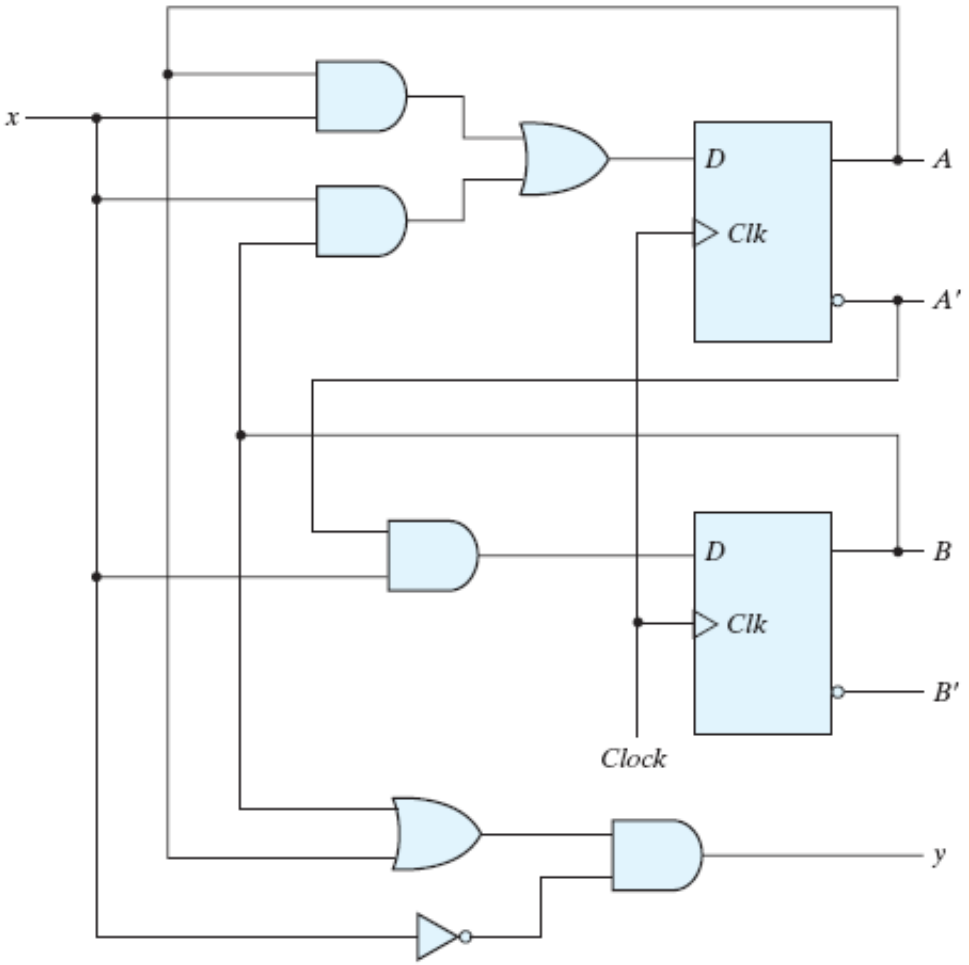
- Behavior of clocked sequential circuit

➔ is determined from the inputs, the outputs, and the state of its flip-flops (present state)

- The analysis of a sequential circuits consists of obtaining:
  - ***State table*** (*transition* table) or
  - ***State diagram***
- It is also possible to write Boolean expressions that describe the behavior of the sequential circuit ➔ ***state equations***

State equations

- The sequential circuit consists of:
  - Two D flip-flops (A and B)
  - An input x
  - An output y
- Determine next state of D flip-flops:
  - A(t+1)= A(t)x(t) +B(t) x(t)
  - B(t+1) = A'(t) x(t)
- The present state of the output y:
  - y(t)= [A(t)+B(t)]x'(t)



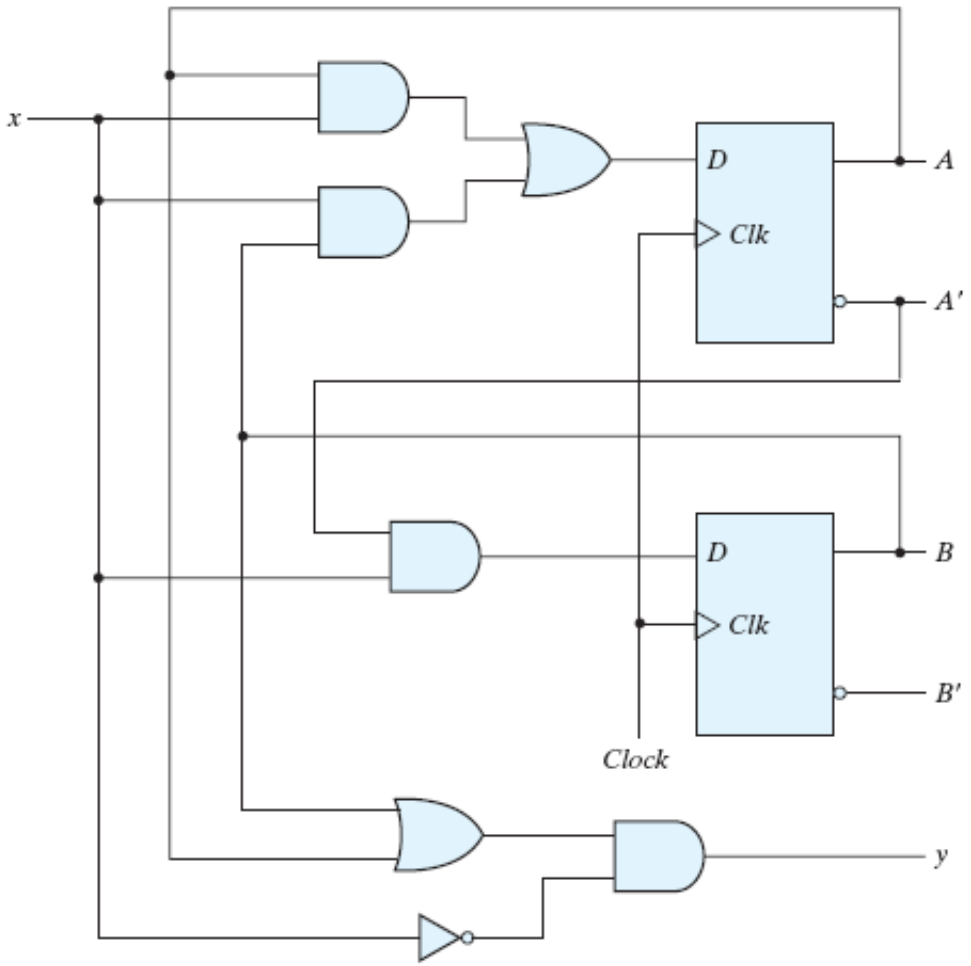
State equation
specifies the next state as a function of the present state and inputs

State Table

- The state equations:
  - A(t+1)= Ax +B x
  - B(t+1) = A' x
  - y= (A+B)x'

Table 5.2
State Table for the Circuit of Fig. 5.15

Present State		Input	Next State		Output
A	B	x	A	B	y
0	0	0	0	0	0
0	0	1	0	1	0
0	1	0	0	0	1
0	1	1	1	1	0
1	0	0	0	0	1
1	0	1	1	0	0
1	1	0	0	0	1
1	1	1	1	0	0



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  - B(t+1) = A' x
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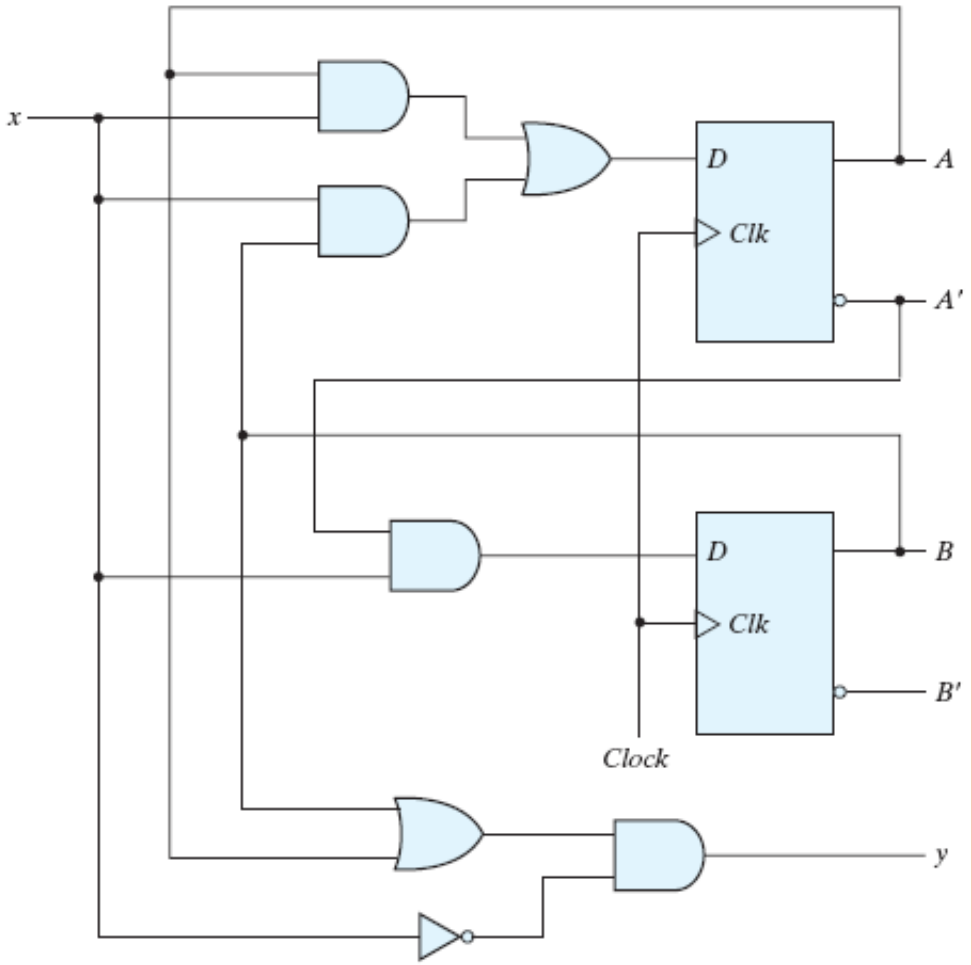
0	0	1	0	1	0
0	1	0	0	0	1
0	1	1	1	1	0
1	0	0	0	0	1
1	0	1	1	0	0
1	1	0	0	0	1
1	1	1	1	0	0



STATE TABLE

- The state equations:
  - A(t+1)= Ax +B x
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  - y= (A+B)x’

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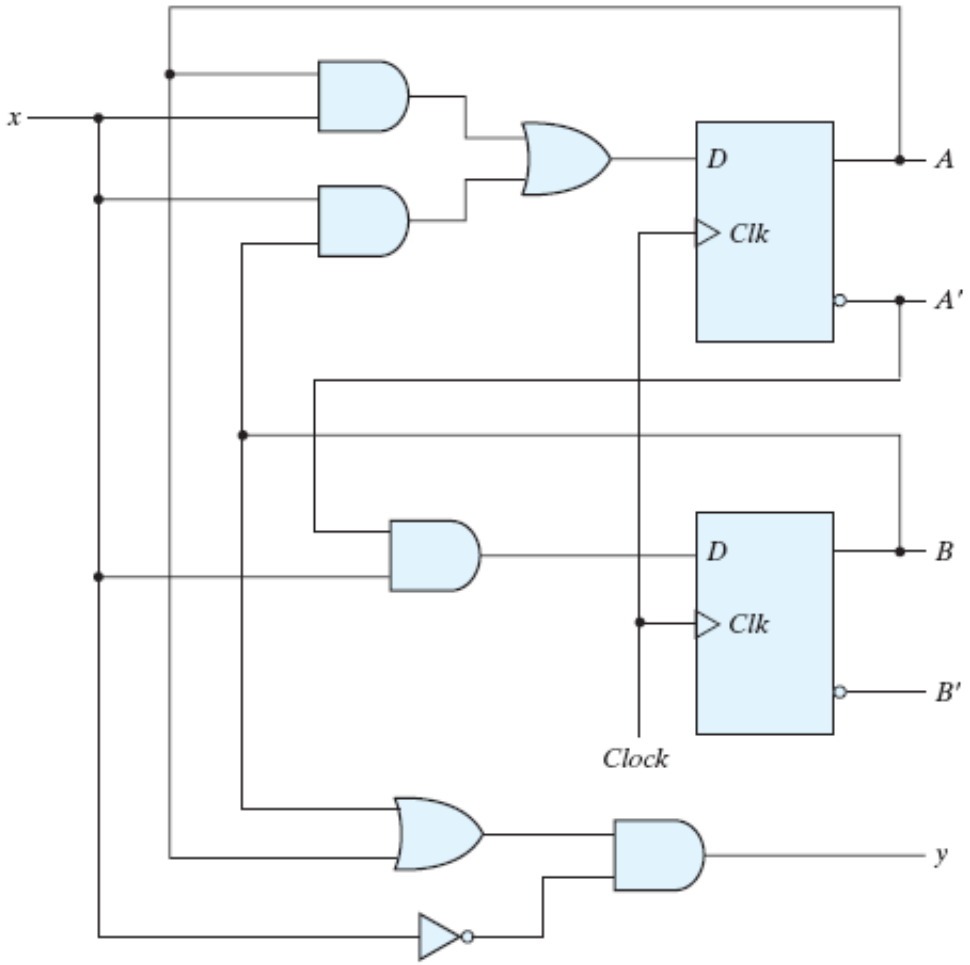
A	B	x	A	B	y
0	0	0	0	0	0
0	0	1	0	1	0
0	1	0	0	0	1
0	1	1	1	1	0
1	0	0	0	0	1
1	0	1	1	0	0
1	1	0	0	0	1
1	1	1	1	0	0



Two forms of State table

Table 5.2
State Table for the Circuit of Fig. 5.15

Present State		Input	Next State		Output
A	B	x	A	B	y
0	0	0	0	0	0
0	0	1	0	1	0
0	1	0	0	0	1
0	1	1	1	1	0
1	0	0	0	0	1
1	0	1	1	0	0
1	1	0	0	0	1
1	1	1	1	0	0

Table 5.3
Second Form of the State Table

Present State		Next State				Output	
		x = 0		x = 1		x = 0	x = 1
A	B	A	B	A	B	y	y
0	0	0	0	0	1	0	0
0	1	0	0	1	1	1	0
1	0	0	0	1	0	1	0
1	1	0	0	1	0	1	0

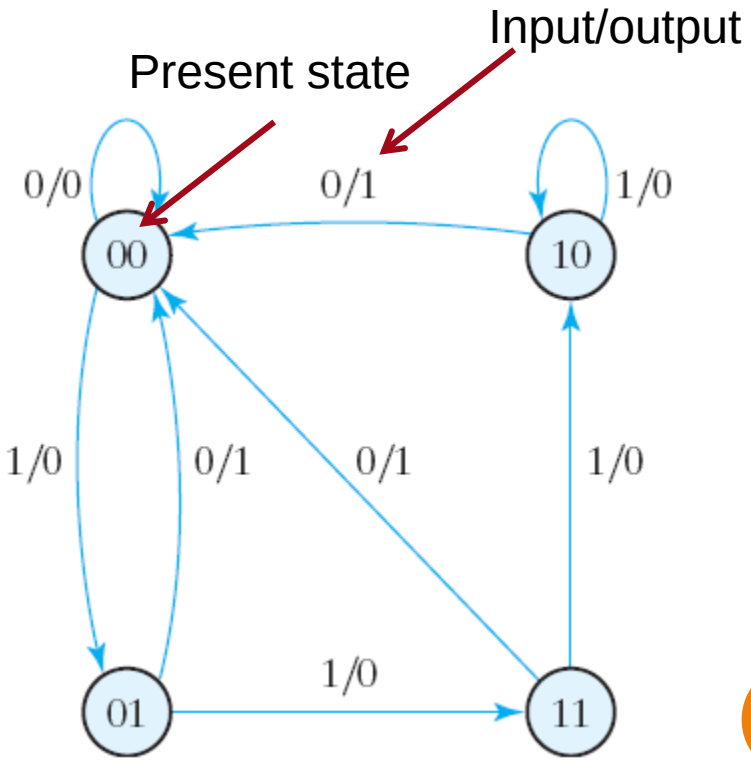
- The state equations:
  - A(t+1)= Ax +B x
  - B(t+1) = A' x
  - y= (A+B)x'

State Diagram

- The information available in a state table can be represented graphically in the form of a state diagram

Table 5.3
Second Form of the State Table

Present State		Next State				Output	
		$x = 0$		$x = 1$		$x = 0$	$x = 1$
A	B	A	B	A	B	y	y
0	0	0	0	0	1	0	0
0	1	0	0	1	1	1	0
1	0	0	0	1	0	1	0
1	1	0	0	1	0	1	0

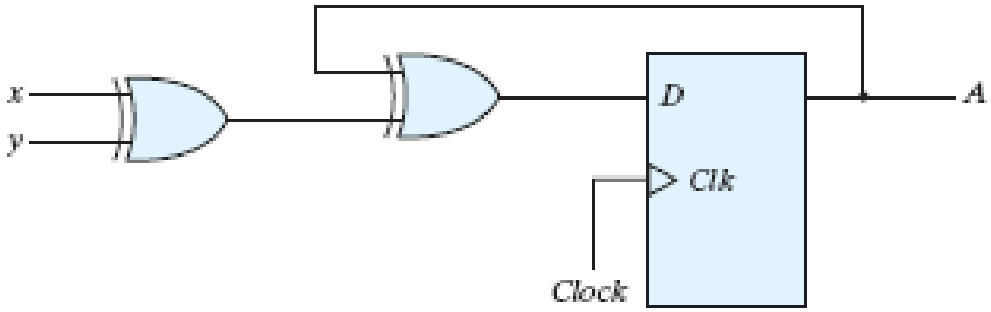


ANALYSIS WITH D FLIP-FLOPS

- Input equation:

$$D_A = A \oplus x \oplus y$$

XOR with 3 inputs = **odd function**



(a) Circuit diagram

Present state	Inputs		Next state
A	*x*	*y*	*A*
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

(b) State table

(c) State diagram

FLIP-FLOP EQUATIONS

- ***Output equations:***
  - A set of Boolean functions describes the part of the combinational circuit that generates external outputs
- ***Flip-flop Input equations*** (***excitation*** equations):
  - A set of Boolean functions describes the part of the circuit that generates the inputs to the flip-flops
- Ex: $D_Q = x+y$
  - The following input equation specifies an OR gate with inputs x and y connected to D input of a flip-flop whose output is labeled with the symbol Q
- Previous example (fig. 5.15)
  - $D_A = Ax + Bx$
  - $D_B = A' x$
  - $y = (A+B)x'$

For D flip-flop, state equation = input equation

Analysis with JK or T flip-flops

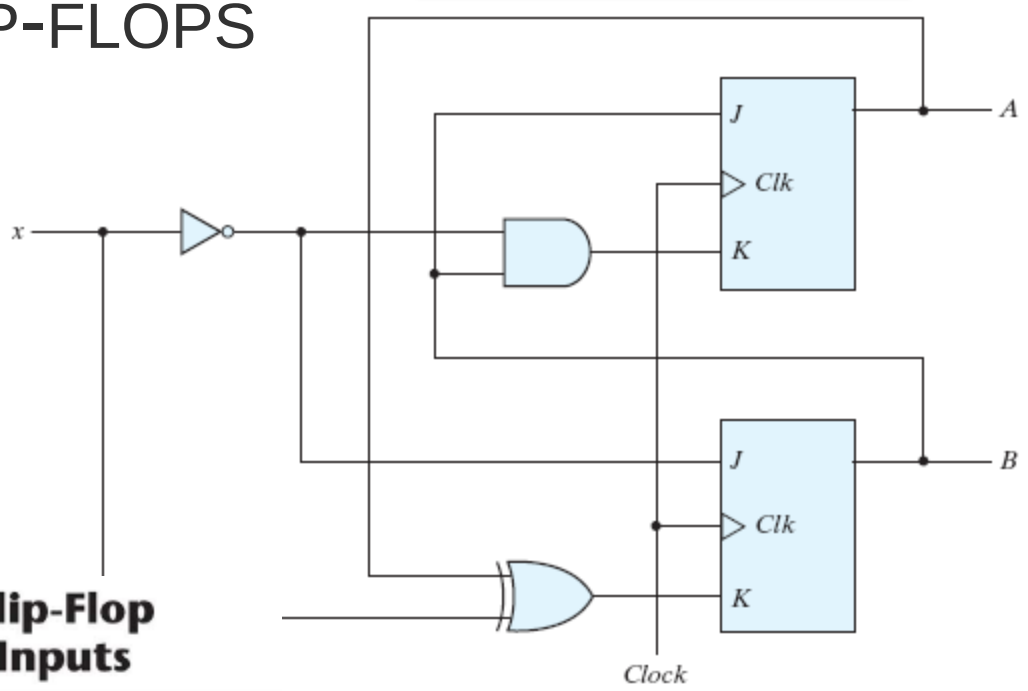
- When JK or T flip-flops are used, it is necessary to refer to the corresponding:
  - characteristic table or
  - characteristic equation

  to obtain the next-state values.

Method 1: Using Characteristic table

ANALYSIS WITH JK FLIP-FLOPS

1. Determine input equations
   $J_A = B$, $K_A = Bx'$
   $J_B = x'$, $K_B = A'x + Ax' = A \oplus x$
2. List binary values of each input equation
3. Use characteristic table to determine the next state



Present State		Input	Next State		Flip-Flop Inputs			
A	B	x	A	B	J_A	K_A	J_B	K_B
0	0	0	0	1	0	0	1	0
0	0	1	0	0	0	0	0	1
0	1	0	1	1	1	1	1	0
0	1	1	1	0	1	0	0	1
1	0	0	1	1	0	0	1	1
1	0	1	1	0	0	0	0	0
1	1	0	0	0	1	1	1	1
1	1	1	1	1	1	0	0	0

Flip-Flop Characteristic Tables

JK Flip-Flop

J	K	Q(t + 1)	
0	0	Q(t)	No change
0	1	0	Reset
1	0	1	Set
1	1	Q′(t)	Complement

Method 2: Using Characteristic eq.

ANALYSIS WITH JK FLIP-FLOPS

JK characteristic eq.

$Q(t+1)= JQ' + K'Q$

➔ $A(t+1)= JA' + K'A$

➔ $B(t+1)= JB' + K'B$

1. Determine input equations
   $J_A = B$, $K_A = Bx'$
   $J_B = x'$, $K_B = A'x + A\,x' = A \oplus x$
2. Substitute input equations into characteristic eq. to obtain state equations

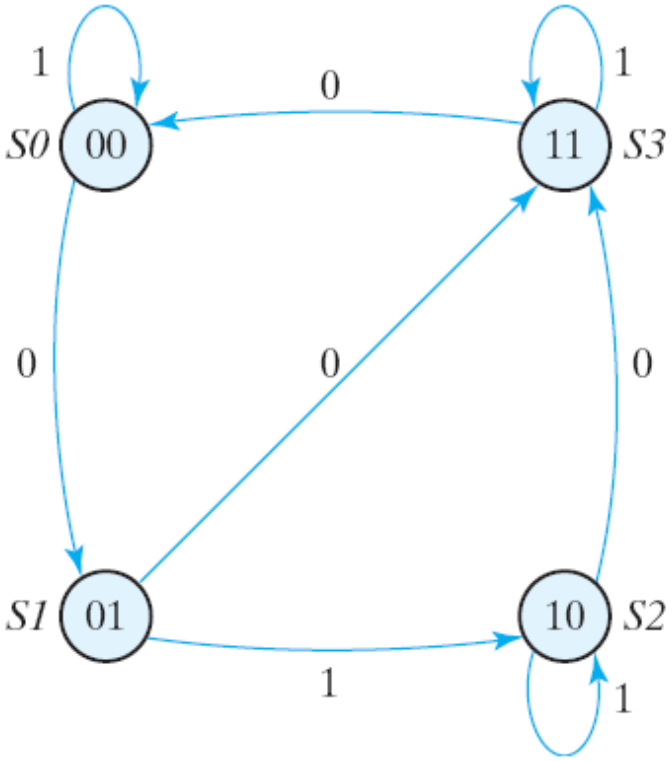
   $A(t+1) = JA' + K'A$
   $= BA' + (Bx')'\,A$
   $= A'B + AB' + Ax$

   $B(t+1) = JB' + K'B$
   $= x'B' + (A \oplus x)'B$
   $= B'x' + AB\,x + A'B\,x'$
3. Use characteristic state eq. to determine the next state values in state table

Present State		Input	Next State	
A	B	x	A	B
0	0	0	0	1
0	0	1	0	0
0	1	0	1	1
0	1	1	1	0
1	0	0	1	1
1	0	1	1	0
1	1	0	0	0
1	1	1	1	1

Analysis with JK flip-flops: State diagram

Present State		Input	Next State	
A	B	x	A	B
0	0	0	0	1
0	0	1	0	0
0	1	0	1	1
0	1	1	1	0
1	0	0	1	1
1	0	1	1	0
1	1	0	0	0
1	1	1	1	1



T characteristic eq.

$Q(t+1) = T \oplus Q = TQ' + T'Q$

ANALYSIS WITH T FLIP-FLOPS

1. Determine input equations

   $T_A = Bx$, $T_B = x$, $y = AB$

2. Substitute input equations into characteristic eq. to obtain state equations

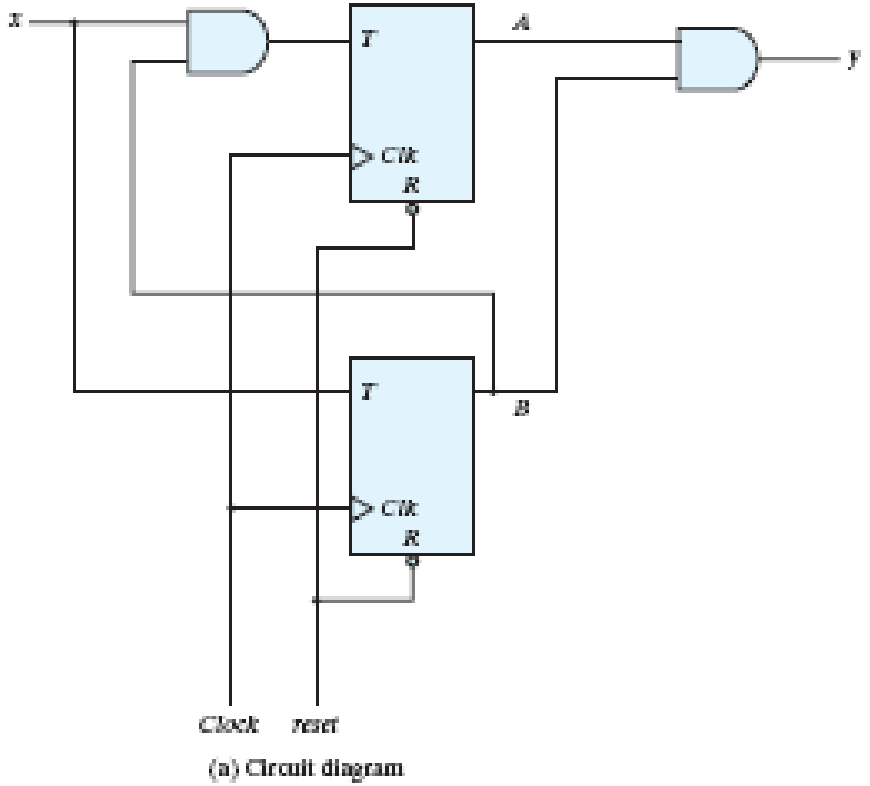
   $A(t+1) = T'A + TA'$

   $= (Bx)'A + (Bx)A'$

   $= AB' + Ax' + A'Bx$

   $B(t+1) = x \oplus B$

3. Use characteristic state eq. to determine the next state values in state table



(a) Circuit diagram

ANALYSIS WITH T FLIP-FLOPS

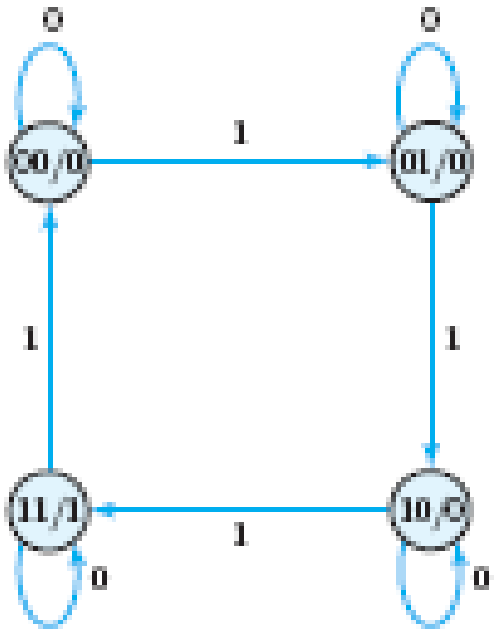
Next state equations:

A(t+1) = AB' +Ax' + A'Bx

B(t+1) = x ⊕ B

State Table for Sequential Circuit with T Flip-Flops

Present State		Input	Next State		Output
A	B	x	A	B	y
0	0	0	0	0	0
0	0	1	0	1	0
0	1	0	0	1	0
0	1	1	1	0	0
1	0	0	1	0	0
1	0	1	1	1	0
1	1	0	1	1	1
1	1	1	0	0	1



Notes:

* As long as x=1, the circuit behaves as a *binary counter*
* Here, the output depends on the present state only (**Moore model**) and independent of the input

➔inside each circle (present state/output)

The design procedure

- Design procedure or methodologies specify hardware that will implement a desired behavior.
- Here, we will illustrate manual efforts for small circuits using D, JK and T flip-flops.

THE DESIGN PROCEDURE (CONT.)

- Procedure for designing synchronous sequential circuits:

1. From word description and specifications, derive a state diagram for the circuit
2. Reduce the number of states if necessary
3. Assign binary values to the states
4. Obtain the binary-coded state table
5. Choose the type of flip-flops to be used
6. Derive the simplified flip-flop input and output equations
7. Draw the logic diagram

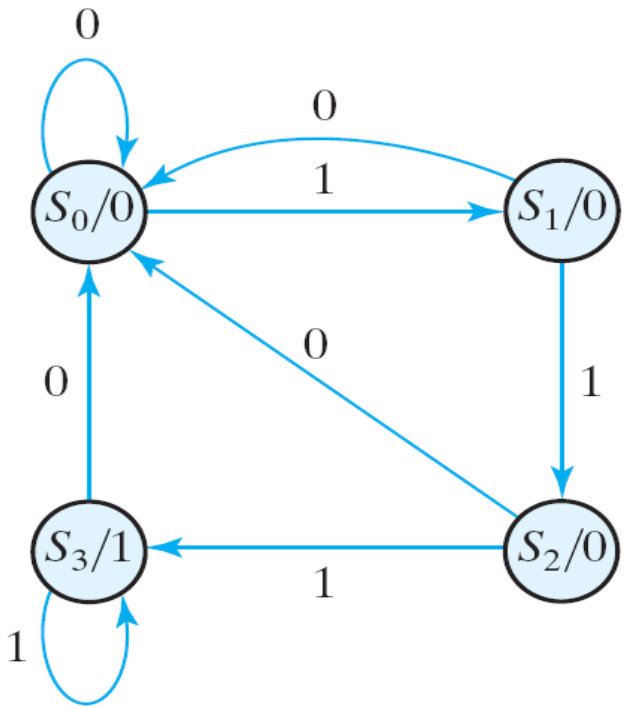
THE DESIGN PROCEDURE (CONT.)

- The part of the design that follows a well-defined procedure is referred to as *synthesis*
- Designers using logic synthesis tools (software) can follow a simplified process that develops an HDL description directly from state diagram
- The first step is a critical part of the process, because succeeding steps depend on it.

Sequence Detector Example

- Circuit's word description :

  We wish to design a circuit that detects a sequence of 3 or more consecutive 1's in a string of bits coming through an input line



Input:

serial bit stream

Output:

Y= 1,

when 3 or more consecutive 1's are coming

Y= 0,

otherwise

➔No. of states=?

➔Moore Model

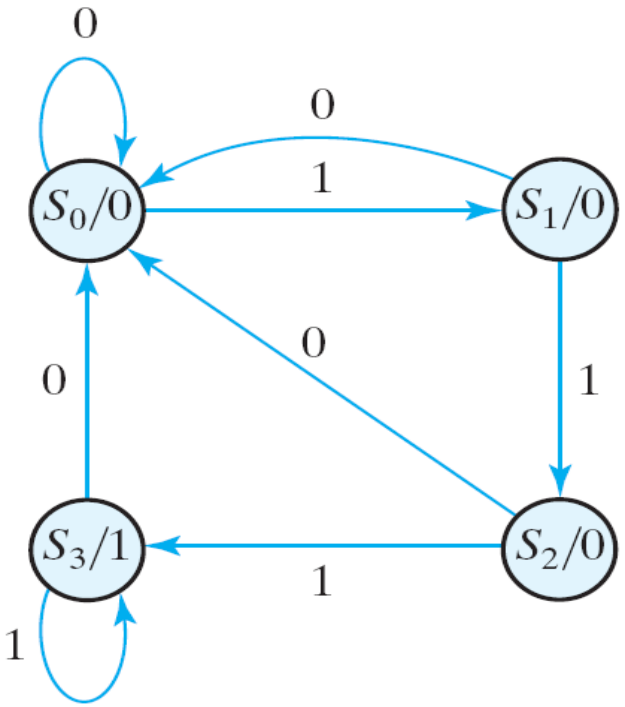
Sequence Detector Example (Cont.)

- Synthesis using D flip-flop

3. Assign binary codes to the states

S_0=00, S_1=01 , S_2=10, S_3=11

4. Obtain the binary coded state table



From state diagram

Present State		Input	Next State		Output
A	B	x	A	B	y
0	0	0	0	0	0
0	0	1	0	1	0
0	1	0	0	0	0
0	1	1	1	0	0
1	0	0	0	0	0
1	0	1	1	1	0
1	1	0	0	0	1
1	1	1	1	1	1

5. Choose the type of flip-flops to be used
   - ➔ 2 D flip-flop
   - ➔ $Q(t+1) = D_Q$
   - ➔ Next state values specify the D input equations

Sequence Detector Example (Cont.)

- Synthesis using D flip-flop

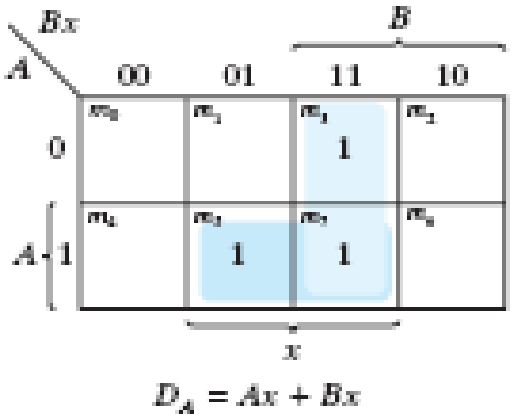
6. Simplify the input and output eq.

$A(t+1) = D_A(A, B, x) = \Sigma(3,5,7)$

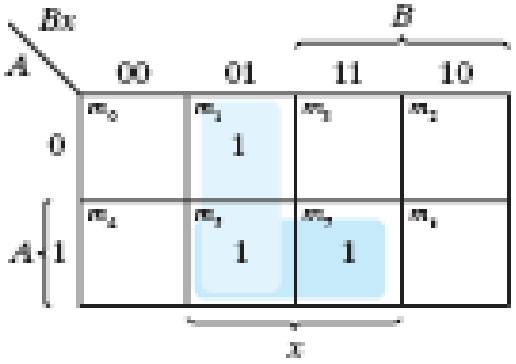
$B(t+1) = D_B(A, B, x) = \Sigma(1,5,7)$

$y(A, B, x) = \Sigma(6,7)$

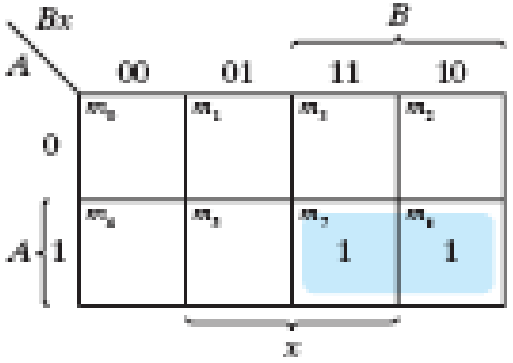
Next State		Output
A	**B**	**y**
0	0	0
0	1	0
0	0	0
1	0	0
0	0	0
1	1	0
0	0	1
1	1	1



$D_A = Ax + Bx$



$D_B = Ax + B'x$



$y = AB$

Sequence Detector Example (Cont.)

Input and output eq.:

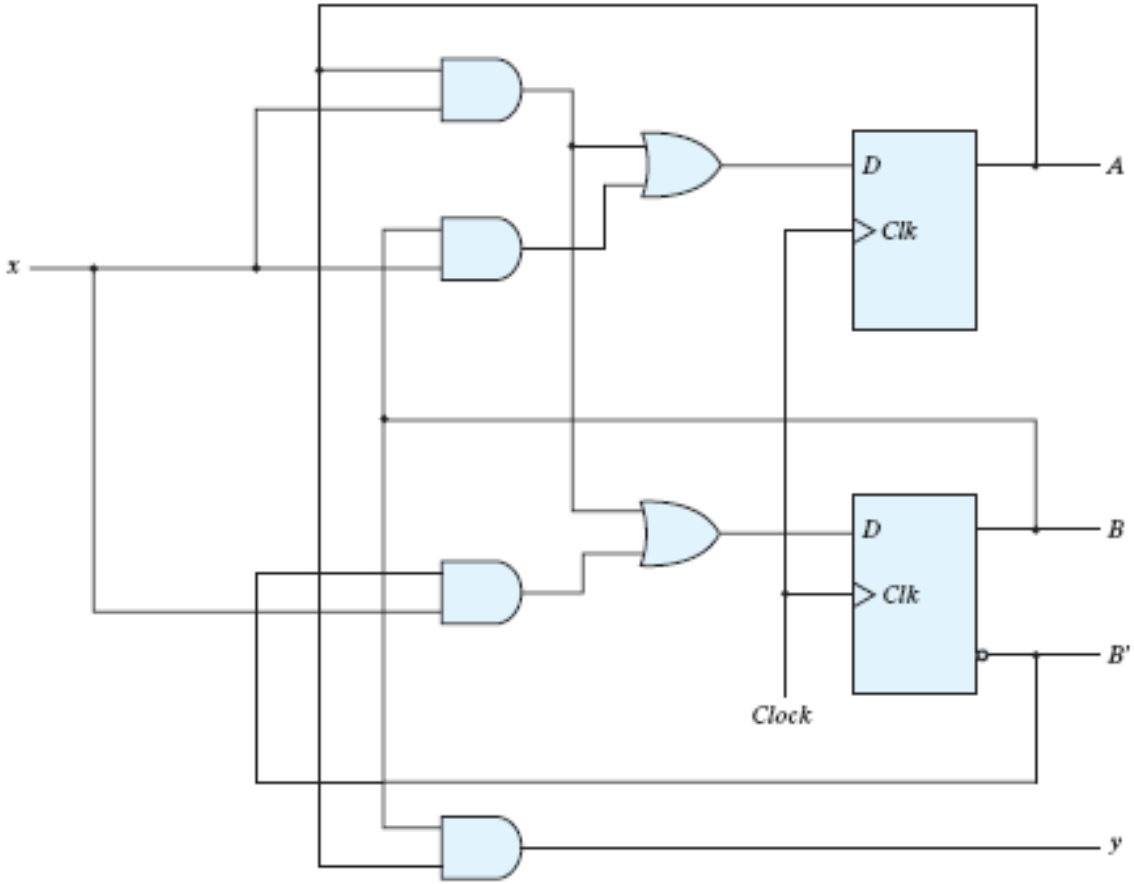
$D_A(A, B, x) = Ax + Bx$

$D_B(A, B, x) = Ax + B'x$

$y(A, B, x) = AB$

- Synthesis using D flip-flop

7. Draw the logic diagram



الذين لم يمتحنوا امتحان الميدترم

- على الطلاب الاتى اسماءهم الحضور يوم الخميس 9 ديسمبر لمكتبى (فى الدور الثالث) الساعة 12 لاعادة امتحان الميدترم:

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Thanks