

# Project 3 Numerical Methods

Dustin O'Brien

September 17, 2024

## Problem 13

Consider the function  $D$  where

$$D = f(x+h) - 2f(x) + f(x-h)$$

Using Taylors Theorem we know that

$$f(x+h) = f(x) + hf'(x) + h^2\frac{1}{2}f''(x) + h^3\frac{1}{6}f'''(x) + O(h^4)$$

$$f(x-h) = f(x) - hf'(x) + h^2\frac{1}{2}f''(x) - h^3\frac{1}{6}f'''(x) + O(h^4)$$

Thus,

$$D = f(x) + hf'(x) + h^2\frac{1}{2}f''(x) + h^3\frac{1}{6}f'''(x)$$

$$+ f(x) - hf'(x) + h^2\frac{1}{2}f''(x) - h^3\frac{1}{6}f'''(x)$$

$$- 2f(x)$$

$$= h^2f''(x) + O(h^4) = f(x+h) - 2f(x) + f(x-h)$$

$$f''(x) = \frac{f(x+h) - 2f(x) + f(x-h) - O(h^4)}{h^2}$$

$$f''(x) = \frac{f(x+h) - 2f(x) + f(x-h)}{h^2} + O(h^2)$$

## Problem 2a