Linear Algebra Answers (Chapter3)

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1

$$3\vec{\alpha} + 4\vec{\beta} = 3\begin{bmatrix}1\\2\\3\\4\end{bmatrix} + 4\begin{bmatrix}0\\2\\4\\6\end{bmatrix} = \begin{bmatrix}3\\6\\9\\12\end{bmatrix} + \begin{bmatrix}0\\8\\16\\24\end{bmatrix} = \begin{bmatrix}3\\14\\25\\36\end{bmatrix}$$

2

$2\vec{\alpha}_1 + 2\vec{\beta} + 3\vec{\alpha}_2$ - $3\vec{\beta} = 2\vec{\alpha}_3 + 2\vec{\beta}$

$$\vec{\beta} = \frac{2}{3}\vec{\alpha}_1 + \vec{\alpha}_2 \text{ - } \frac{2}{3}\vec{\alpha}_3 = \begin{bmatrix}\frac{5}{3}\\\frac{7}{3}\\\frac{5}{3}\\-\frac{4}{3}\end{bmatrix}$$

3

取

$\vec{\alpha} = \Big[x_1, x_2, \cdots, x_n\Big]^T \in \mathbf{V}$

$\vec{\beta} = \Big[y_1, y_2, \cdots, y_n\Big]^T \in \mathbf{V}$

则有

$\vec{\alpha} + \vec{\beta} = \Big[x_1 + y_1, x_2 + y_2, \cdots, x_n + y_n\Big]^T \in \mathbf{V}$

设

$x_i - a_i = c_x, y_i - a_i = c_y,\ x_i + y_i - a = c_{xy}$

则有

$a_i = c_{xy} - c_x - c_y,$

即a_i的值与i无关

这说明

$a_1 = a_2 = \cdots = a_n$

$k\vec{\alpha} \in \mathbf{V}$请自行验证

4

(1)

取

$\vec{\alpha}= \begin{bmatrix}1, 1\end{bmatrix}^T \in \mathbf{V_1}$

则

$(-1)\vec{\alpha}= \begin{bmatrix}-1, -1\end{bmatrix}^T \notin \mathbf{V_1}$

(2)

取

$\vec{\alpha_1}= \begin{bmatrix}1, 0\end{bmatrix}^T \in \mathbf{V_2}$

$\vec{\alpha_2}= \begin{bmatrix}0, -1\end{bmatrix}^T \in \mathbf{V_2}$

则

$\vec{\alpha_1} + \vec{\alpha_2}= \begin{bmatrix}1, -1\end{bmatrix}^T \notin \mathbf{V_2}$

(3)

取

$\vec{\alpha_1}= \begin{bmatrix}1, 0\end{bmatrix}^T \in \mathbf{V_3}$

$\vec{\alpha_2}= \begin{bmatrix}0, 1\end{bmatrix}^T \in \mathbf{V_3}$

则

$\vec{\alpha_1} + \vec{\alpha_2}= \begin{bmatrix}1, 1\end{bmatrix}^T \notin \mathbf{V_3}$

5

取

$\vec{\alpha}= \begin{bmatrix}0, 1, \ldots, 1, 1\end{bmatrix}^T \in \mathbf{W}$

$\vec{\beta}= \begin{bmatrix}1, 1, \ldots, 1, 0\end{bmatrix}^T \in \mathbf{W}$

则

$\vec{\alpha} + \vec{\beta} = \begin{bmatrix}1, 2, \ldots, 2, 1\end{bmatrix}^T \notin \mathbf{W}$

这说明$\mathbf{W}$不构成$\mathbf{R}$上的向量空间

6

(1)
取
$\vec{\alpha_1} = \left[3x_1 + 2x_2, x_1, x_2\right]^T \in \mathbf{W}$
$\vec{\alpha_2} = \left[3y_1 + 2y_2, y_1, y_2\right]^T \in \mathbf{W}$
则
$\vec{\alpha_1} + \vec{\alpha_2} = \left[3(x_1 + y_1) + 2(x_2 + y_2), x_1 + y_1, x_2 + y_2\right]^T \in \mathbf{W}$
$k\vec{\alpha_1} = \left[3(k(x_1)) + 2(k(x_2)), k(x_1), k(x_2)\right]^T \in \mathbf{W}$
说明$\mathbf{W}$为$\mathbf{R}_3$的一个子空间
(2)
观察知，取
$\vec{\beta} = \left[3, 1, 0\right]^T$
$\vec{\gamma} = \left[2, 0, 1\right]^T$
则
$\mathbf{W} = \mathrm{Span}\{\vec{\beta}, \vec{\gamma}\}$