A Report on Barrier Options

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ABTRACT

A barrier option is a derivative contract that is activated or extinguished when the price of the underlying asset crosses a certain level. Most models assume continuous monitoring of the barrier. However, in practice most, if not all, barrier options traded in markets are discretely monitored. Investors may use different types of options to hedge their risks and speculate the movement of the prices of the underlying asset. Options are financial derivatives whose value depends on the value of the underlying asset Options give the holder the right but not the obligation to either buy or sell an underlying asset at a specific price and a specific date That are determined on the day the contract is written giving rise to the call and put option. The project looked at the meaning of barrier options, the different types, benefit of barrier Option and some practical examples. The data used consisted of the strike prices, volatility percentage at those strike prices, and the stock closing price on Friday 25th November, 2022 which was $114.41. No dividend was given so zero dividend was assumed and a barrier of $120 and an interest rate of 12% was also assumed. Finally, a Black Scholes model was built in R to calculate the prices of the different types of barrier options.

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INTRODUCTION

A barrier option is an option where the payoff is conditional upon the underlying asset's price breaching a barrier level during the option's lifetime. Barrier options are exotic options because they are more complex than the regular American or European options. A barrier option is a derivative where the payoff depends on whether the underlying asset has reached or exceeded a predetermined price. Barrier option can either be a knock-out, or knock- in.
A barrier event occurs when the underlying crosses the barrier level. They are also considered a type of path-dependent option because their value fluctuates as the underlying asset value changes during the option's contract term. In other words, a barrier option's payoff is based on the underlying asset's price path. The option becomes worthless or may be activated upon the crossing of a price point barrier. Barrier options are typically classified as either knock-in or knock-out. Apart of the knock-in and knock-out barrier option, there also other types of barrier options such as the rebate barrier option, Turbo warrant barrier option, Parisian options etc.

Problem statement

The goal of this project is to explain barrier options including its different classes using a simple example and to apply real world data which will be gotten from Yahoo finance to find the price of a barrier option in R.

Objectives

1. Understanding barrier option
2. Examining the different types of barrier options and reasons for trading barrier options.
3. Using a simple example to calculate the value of different barrier options using the Black Scholes option valuation method.

UNDERSTANDING BARRIER OPTION

A barrier option is a type of derivative option where the payoff depends on whether the underlying asset has reached or exceeded a predetermined price. A barrier option put and calls either comes into existence or go out of existence the first time the asset price reaches the barrier. If they are in existence at expiration, they are equivalent to ordinary put and calls. Barrier option can either be a knock-in or knock-out. Barrier options are very popular among investors because the barrier features provide investors with protection and leverage. Barrier options are similar in some ways to ordinary options.

For example, an up-and-out call option gives the option holder the payoff of a call option if the price of the underlying asset does not reach a higher barrier level before the expiration date.
On the other hand, an up-and- in call is a regular call that comes into existence only if the barrier is reached. When the barrier H is less than or equal to the strike price K, the value of the up-and-out call is zero and the value of the up-and-in call is equal to the value of the regular call. Similarly, an up-and-out put is a put option that ceases to exist when the barrier H that is greater than the current underlying asset price is reached. On the other hand, an up-and-in put is a put that comes into existence only if the barrier is reached.

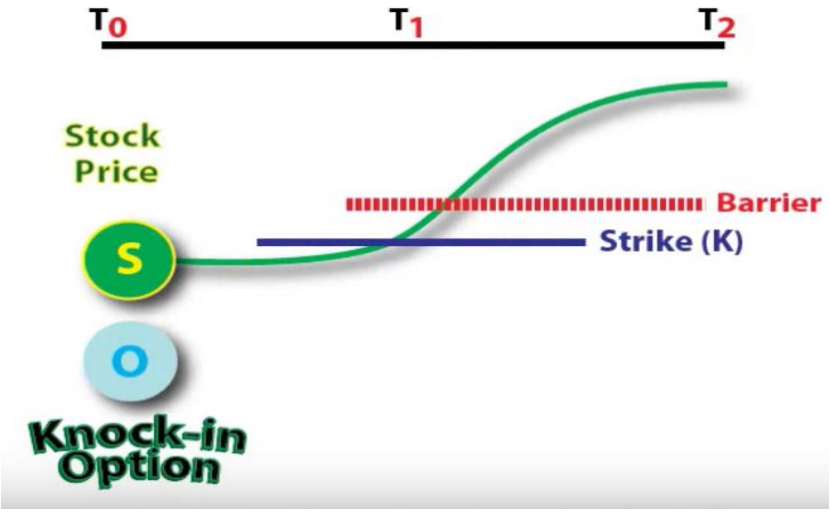
TYPES OF BARRIER OPTIONS

- **Knock in barrier options**

A knock-in option is a type of barrier option where the rights associated with that option only come into existence when the price of the underlying security reaches a specified barrier during the option's life. Meaning that the option does not exist initially until it reaches the barrier. Once a barrier is knocked in, or comes into existence, the option remains in existence until it expires. Knock-In options may be further classified two:

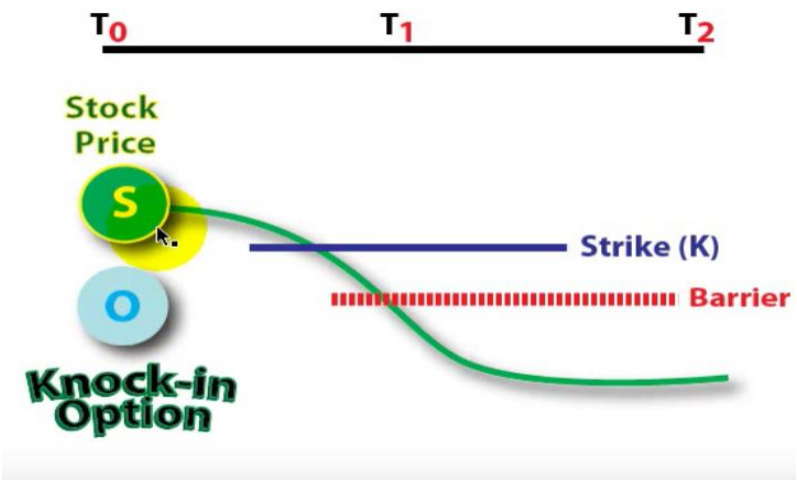
Knock-In Barrier (up-and-in)

For the **knock up-and-in** barrier option, the barrier is set above the price of the underlying asset and the option only comes into existence if the price of the underlying asset rises above the set barrier. If the price of the underlying asset never reaches the barrier until maturity, then the contract expires with no option and worthless as the option only gets to be created when the price of the underlying asset reaches the barrier thereby knocking the option into existence and once the option is knocked into existence it remains in existence until maturity as can be seen in the above figure.



Knock-In Barrier(down-and-in)

For **knock down-and-in** barrier options, the barrier is set below the price of the underlying asset and the option only comes into existence when the price of the underlying asset drops below the barrier. If the price of the underlying asset fluctuates above the barrier until expiration, then the contract expires worthless as can be seen in the below figure.

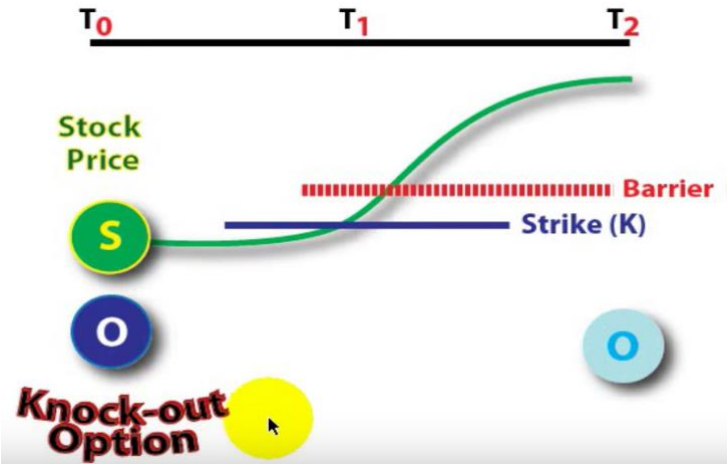


- **Knock-Out Barrier Options**

These go out of expiration if the underlying asset price reaches the barrier. That is the option cease to exist if the underlying asset price reaches the set barrier during the life of the option. Knock-out barrier options can further be classified as up-and-out or down-and-out. An up-and-out option ceases to exist when the underlying security moves above a barrier that is set above the underlying's initial price. A down-and-out option ceases to exist when the underlying asset moves below a barrier that is set below the underlying asset's initial price. If an underlying asset reaches the barrier at any time during the option's life, the option is knocked out, or terminated.

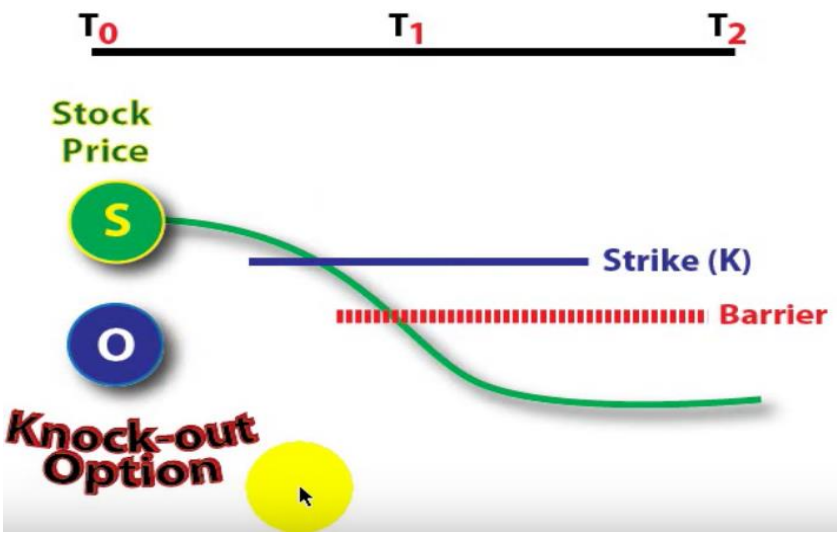
Knock-Out Barrier(up-and-out)

For a knock up-and-out option, the barrier is set above the stock price and if the stock price rises above the barrier, it is knocked out of existence and the contract expires without an option but if the stock price fluctuates below the barrier until maturity, then the option expires with an option and a value.



Knock-Out Barrier (down-and-out)

For a knock down-and-out barrier option, the barrier is set below the price of the underlying asset and if the price of the underlying asset falls below the barrier before expiration, then the option is knocked out of existence and the contract expires without an option.



- **Rebate Barrier Options**: These make a fixed payment if the asset price reaches the barrier. The payment can occur either at the time the barrier is reached, or at the time the option expires, in which case it is a deferred rebate. Rebate options can be either “up rebates” or “down rebates,” depending on whether the barrier is above or below the current price.

- **Turbo Warrant Barrier Options*:*** Mainly traded in Europe and Hong Kong, Turbo warrants are a type of down-and-out option that is highly leveraged and is characterized by low volatility. They are popular in Germany and are used for speculation purposes.

- **Parisian Option*:*** In a Parisian option, reaching the barrier price does not trigger the contract. Instead, the underlying asset's price must spend a pre-defined amount of time beyond the trigger barrier price for the contract to kick in. The amount of time that the underlying asset's price spends outside and inside the barrier price range is measured in this type of option.

How a Barrier Option works

Barrier options are very similar to traditional calls and puts. Two investors agree on the terms of the contract, including the price of the contract, the strike price, barrier price, and the expiration date. The option holder then has the option, but not the obligation, to exercise the contract if they want to.

The major difference is that barrier options place additional restrictions on the contract. With a traditional option, the option holder can exercise the option at any time up until the expiration date. Barrier options add additional rules as to when the contract can be exercised based on changes in the underlying stock's price.

BENEFITS OF BARRIER OPTIONS

The benefits of barrier options is attributable to the fact that they give investors more flexibility to set the terms of their contracts. An investor may be willing to sell a call option but worry about losing a large amount of money, for example, if a stock skyrockets in price. Barrier options that render the option ineffective at a certain price can help the seller limit their losses in a scenario where their potential losses would otherwise be significant.

METHODOLOGY

To calculate the value of the barrier options, we obtained our data for META company from yahoo finance. The data we used consisted of the strike prices, volatility percentage at these strike prices (k), and the price of the stock (s) at the time which was $114.41. No dividend was given so we assumed zero dividend and assumed a barrier of $120 and an interest rate of 12%. We then used the Black Scholes to calculate the prices of the different types of barrier options using R and a powerful derivate library called ‘derivmkts’.

DATA ANALYSIS AND RESULTS

After obtaining the necessary data needed, the ‘derivmkts’ library was used to build a Black Scholes Model which we used to calculate the ordinary option price for Calls (C) and Puts (P), and also the barrier option prices for Calls and Puts knock-out and knock-in options which are:

- Call up and in (Cui)
- Call down and in (Cdi)
- Call up and out (Cuo)
- Call down and out (Cdo)
- Put up and in (Pui)
- Put down and in (Pdi)
- Put up and out (Puo)
- Put down and out (Pdo)

After plugging in our values into the model that was built, the result we got for a META option with a stock price of $111.41, strike price of $120, time to maturity of 0.55 years, volatility of 0.6156, dividend of 0, a barrier price of $100 and a risk free rate of return of 12% was:

Call value	Put value	Up and in call	Up and out call	Up and in put	Up and out put	Down and in call	Down and out call	Down and in put	Down and out put
19.24	20.17	19.74	1.33e-14	20.67	0	2.03	10.30	20.57	0.09

The above results show that as the barrier price is below the strike price, the price of the regular call option is approximately equal to the price of the up and in call. It also shows that as the barrier is below the strike price, the price of the regular put is almost approximately equal to the price of the up and in put, while the price of the down and out put equals zero since the sum of the prices of the up and out put and the up and in put is equal to the price of the regular put.

We went further to test our model to obtain option values and barrier option values for META stock with different strike prices and volatility.

Below are the results obtained from calculating different barrier options value for the same expiry date.

s	K	C	p	Cui	Cdi	Cuo	Cdo	Pui	Pdi	Puo	Pdo
111.41	90	34.32	7.16	34.16	34.37	0.21	-7.11E-15	4.28	7.21	2.94	3.55E-15
111.41	95	31.27	8.79	30.86	30.99	0.13	0.00E+00	5.09	8.51	3.42	-3.55E-15
111.41	100	28.23	10.43	27.70	27.77	0.07	0.00E+00	6.02	9.97	3.95	1.42E-14
111.41	105	25.28	12.16	24.78	24.82	0.03	-1.42E-14	7.16	11.70	4.54	0.00E+00
111.41	110	22.27	13.83	21.98	21.99	0.01	7.11E-15	8.40	13.55	5.16	1.42E-14
111.41	115	19.32	15.56	19.41	19.41	1.24E-03	-7.11E-15	9.84	15.66	5.81	-1.42E-14
111.41	120	16.62	17.55	17.04	17.04	1.33E-14	7.11E-15	11.47	17.97	6.50	4.26E-14

From the result, we can see a trend.

From the ordinary calls and puts, we see that as the strike price (k) gets close to the stock price (s) the price of the call decreases and the price of the put increases. The reverse is the case for when the strike price deviates from the price of the stock. In this case, the price of the call increases and the price of the puts decrease.

We can also see that the price of the ordinary call is very similar, even approximately the same to that of the Call up and in option. Likewise, the ordinary Puts and the Put down and in option.

Another observation we see is that the value of the regular call approximately equals the value of a down and in call plus the value of a down and out call.

CONCLUSION

Barrier options are similar to European and American options except that it involves a barrier and offer cheaper premiums compared to standard options. An investor who wants to maximize returns on investment may take advantage of the low premium it offers. Additionally, the investor may hedge a position with a barrier option in situations where the price of the underlying asset reaches a specific level, opt to use knock-in options

FURTHER RESEARCH

This project is very broad, and unfortunately due to time restrictions, we were unable to do a more in-depth research and data analysis into the topic.

Some of the things we would like to consider in the future is using Monte Carlo Simulations or Black Scholes Merton model and compare the results from each of these models. We would also consider researching about a financial market that has live barrier options for sale and compare the results we have to what they have like we did with the ordinary options in Robinhood which yielded almost exactly similar results in comparison.

Finally, our model can be used to build a scalable mobile or web application that can accept input parameters like the current stock price, time to maturity etc. and output the barrier option value which can be assessable by the public.

References

https://www.investopedia.com/terms/b/barrieroption.asp

Chen, J. (2022, November 6). *What is a barrier option? definition, and knock-in vs. knock-out.* Investopedia. Retrieved November 27, 2022

Carr, P., Ellis, K., & Gupta, V. (1998). Static hedging of exotic options. *The Journal of Finance*, *53*(3), 1165–1190. https://doi.org/10.1111/0022-1082.00048

https://en.wikipedia.org/wiki/Barrier_option

Carr, P., Ellis, K., & Gupta, V. (1998). Static hedging of exotic options. *The Journal of Finance*, *53*(3), 1165–1190. https://doi.org/10.1111/0022-1082.00048

Options, Futures and other Derivatives, Hull, J. C., 2018: 10th edition, Prentice Hall.

Barrier options. (2012). *An Introduction to Exotic Option Pricing*, 161–208. https://doi.org/10.1201/b11589-14

Rich, D. R. (1994). "The mathematical foundations of barrier option pricing theory". Advances in Futures and Options Research (7), pp. 267– 311.

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