Artificial Intelligence

Unit-4 (Propositional Logic)

Artificial Intelligence 01CE0702

Marwadi University

Department of Computer Engineering

Shilpa Singhal

Logic

- One of the prime activities of the human intelligence is reasoning.
- The activity of reasoning involves construction, organization and manipulation of statements to arrive at new conclusions.
- Thus logic can be defined as a scientific study of the process of reasoning.
- Logic is a formal language.
- Logic is basically classified in two main categories
  - Propositional logic
  - Predicate logic

Proposition

- Propositions are elementary atomic sentences.
- Propositions may be either true or false but may take on no other value.
- There are two kinds of proposition
  - Simple
  - compound
- Some examples of simple propositions are
  - It is raining.
  - My car is painted silver.
  - John and sue have five children.
  - Snow is white.
  - People live on the moon.

Propositional Logic

- A proposition (or a statement) is a declarative sentence which is either true or false but not both.

- Imperative, exclamatory, interrogative or open sentences are not statements in logic.

- The **Truth Value** of a proposition is True(denoted as T) if it is a true statement, and False(denoted as F) if it is a false statement.

Some examples of Propositions are given below −

- "Man is Mortal", it returns truth value “TRUE”
- "12 + 9 = 3 – 2", it returns truth value “FALSE”

The following is not a Proposition −

- "A is less than 2".

It is because unless we give a specific value of A, we cannot say whether the statement is true or false.

Statements That Are Not Propositions-

Following kinds of statements are not propositions-

1. Command
2. Question
3. Exclamation
4. Inconsistent
5. Predicate or Proposition Function

Following statements are not propositions-

- Close the door. (Command)
- Do you speak French? (Question)
- What a beautiful picture! (Exclamation)
- I always tell lie. (Inconsistent)
- $P(x) : x + 3 = 5$ (Predicate)

Example 1 : For Example consider, the following sentences.

(i) VSSUT is at Burla.

(ii) $2 + 3 = 5$

(iii) The Sun rises in the east.

(iv) Do your home work.

(v) What are you doing?

(vi) $2 + 4 = 8$

(vii) $5 < 4$

(viii) The square of 5 is 15.

(ix) $x + 3 = 2$

(x) May God Bless you!

- All of them are propositions except (iv), (v),(ix) & (x) sentences (i), (ii) are true, whereas (iii),(iv), (vii) & (viii) are false.
- Sentence (iv) is command, hence not a proposition.
- (v) is a question so not a statement.
- (ix) is a declarative sentence but not a statement, since it is true or false depending on the value of x.
- (x) is a exclamatory sentence and so it is not a statement.

Mathematical identities are considered to be statements. Statements which are imperative, exclamatory, interrogative or open are not statements in logic

Compound Statements

Many propositions are composites that are, composed of sub propositions and various connectives discussed subsequently. Such composite propositions are called compound propositions.

A proposition is said to be primitive if it cannot be broken down into simpler propositions, that is, if it is not composite.

Example : Consider, for example following sentences.

a. "The sun is shining today and it is colder than yesterday"
b. b. "Sita is intelligent and she studies every night."

The propositions in Example 1 are primitive propositions.

Two Normal (Canonical) Forms

All wffs can be expressed in the following to normal forms

1. CNF (Conjunctive Normal Form)

e.g.: $$(A \lor \neg B) \land (B \lor \neg D \lor \neg C)$$

Clause 1 *clause 2*

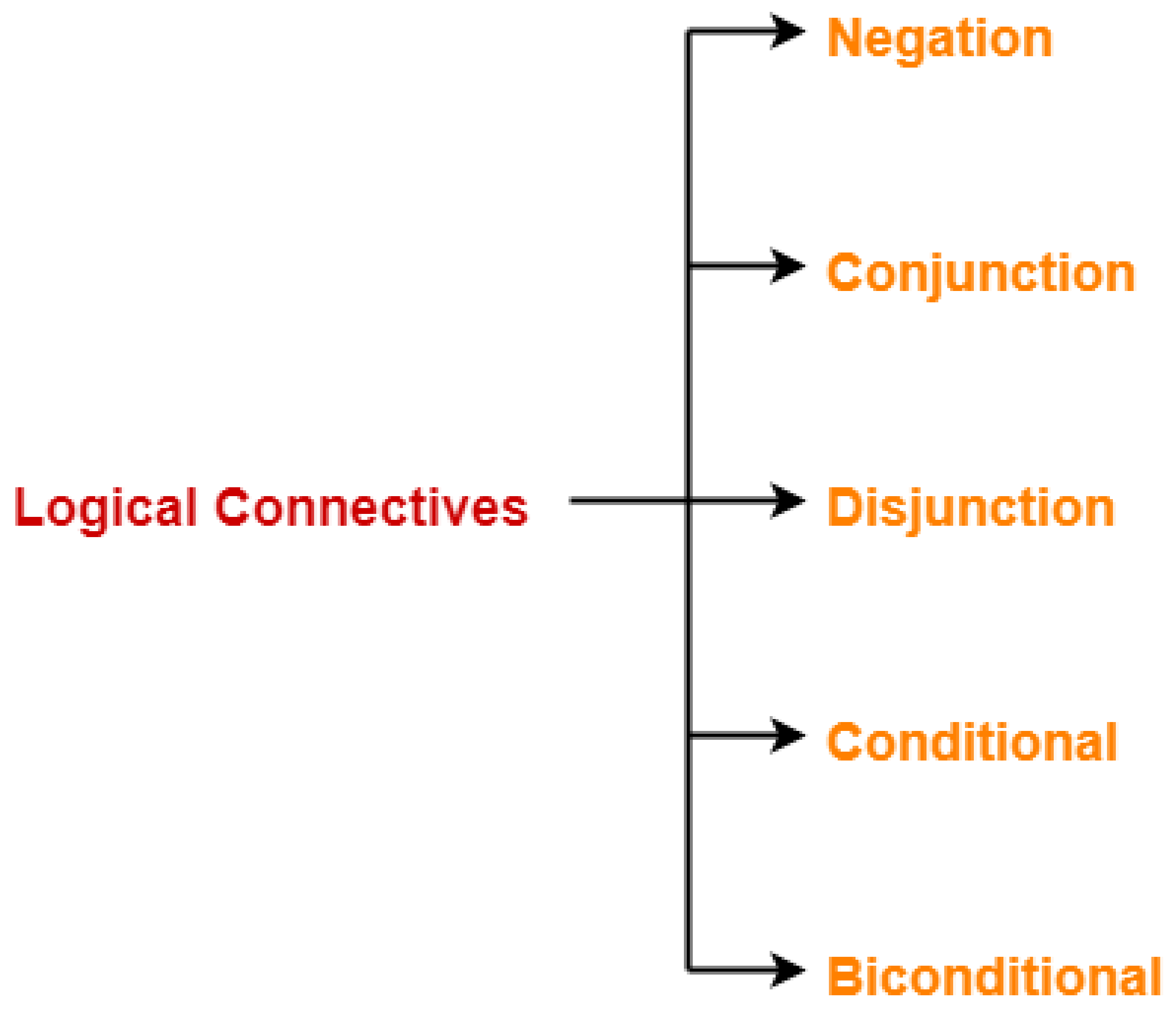
2. DNF (Disjunctive Normal Form)

e.g.: $$(A \land \neg B) \lor (B \land \neg D \land \neg C)$$

models *models*

Some Equivalence Laws

Idempotency	$P \vee P = P$ $P \& P = P$
Associativity	$(P \vee Q) \vee R = P \vee (Q \vee R)$ $(P \& Q) \& R = P \& (Q \& R)$
Commutativity	$P \vee Q = Q \vee P$ $P \& Q = Q \& P$ $P \leftrightarrow Q = Q \leftrightarrow P$
Distributivity	$P \& (Q \vee R) = (P \& Q) \vee (P \& R)$ $P \vee (Q \& R) = (P \vee Q) \& (P \vee R)$
De Morgan's laws	$\sim(P \vee Q) = \sim P \& \sim Q$ $\sim(P \& Q) = \sim P \vee \sim Q$
Conditional elimination	$P \rightarrow Q = \sim P \vee Q$
Bi-conditional elimination	$P \leftrightarrow Q = (P \rightarrow Q) \& (Q \rightarrow P)$


Logical Connectives
Negation
Conjunction
Disjunction
Conditional
Biconditional

LOGICAL OPERATIONS OR LOGICAL CONNECTIVES :

The phrases or words which combine simple statements are called logical connectives. There are five types of connectives. Namely, ‘not’, ‘and’, ‘or’, ‘if…then’, iff etc.

The first one is a unitary operator whereas the other four are binary operators.

In the following table we list some possible connectives, their symbols & the nature of the compound statement formed by them.

Sr. No.	Connective	Symbol	Compound statement
1	AND	$\wedge$	Conjunction
2	OR	$\vee$	Disjunction
3	NOT	$\neg$	Negation
4	If....then	$\rightarrow$	Conditional or implication
5	If and only if (iff)	$\leftrightarrow$	Biconditional

1.Conjunction (AND):

- If two statements are combined by the word “and” to form a compound proposition (statement) then the resulting proposition is called the conjunction of two propositions. Symbolically, if P & Q are two simple statements, then ‘P ∧ Q’ denotes the conjunction of P and Q and is read as ‘P and Q.
- Since, P ∧ Q is a proposition it has a truth value and this truth value depends only on the truth values of P and Q. Specifically, if P & Q are true then P ∧ Q is true; otherwise P ∧ Q is false

Truth Table

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

Example : Let

P: In this year monsoon is very good.

Q: The rivers are flooded.

Then, $P \wedge Q$: In this year monsoon is very good and the rivers are flooded.

2.Disjunction (OR) :

- Any two statements can be connected by the word ‘or’ to form a compound statement called disjunction. Symbolically, if P and Q are two simple statements, then $P \vee Q$ denotes the disjunction of P and Q and read as 'P or Q’ .
- The truth value of $P \vee Q$ depends only on the truth values of P and Q. Specifically if P and Q are false then $P \vee Q$ is false, otherwise $P \vee Q$ is true.

Truth Table

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

Example 4:

P: Paris is in France

Q :2 +3 =6 then

$P \vee Q$: Paris is in France or 2 + 3 = 6.

Here, $P \vee Q$ is true since P is true & Q is False. Thus, the disjunction $P \vee Q$ is false only when P and Q are both false.

3.Negation (NOT)

- Given any proposition P, another proposition, called negation of P, can be formed by modifying it by “not”. Also by using the phrase “It is not the case that or” “It is false that” before P we will able to find the negation.
- Symbolically, $\neg P$ Read as “not P” denotes the negation of P.
- The truth value of $\neg P$ depends on the truth value of P If P is true then $\neg P$ is false and if P is false then $\neg P$ is true.

Truth Table

P	$\neg P$
T	F
F	T

Example : Let P: 3 is a factor of 12.
Then Q = ¬ P: 3 is not a factor of 12.

Here P is true & ¬P is false.

4.Conditional or Implication: (If…then)

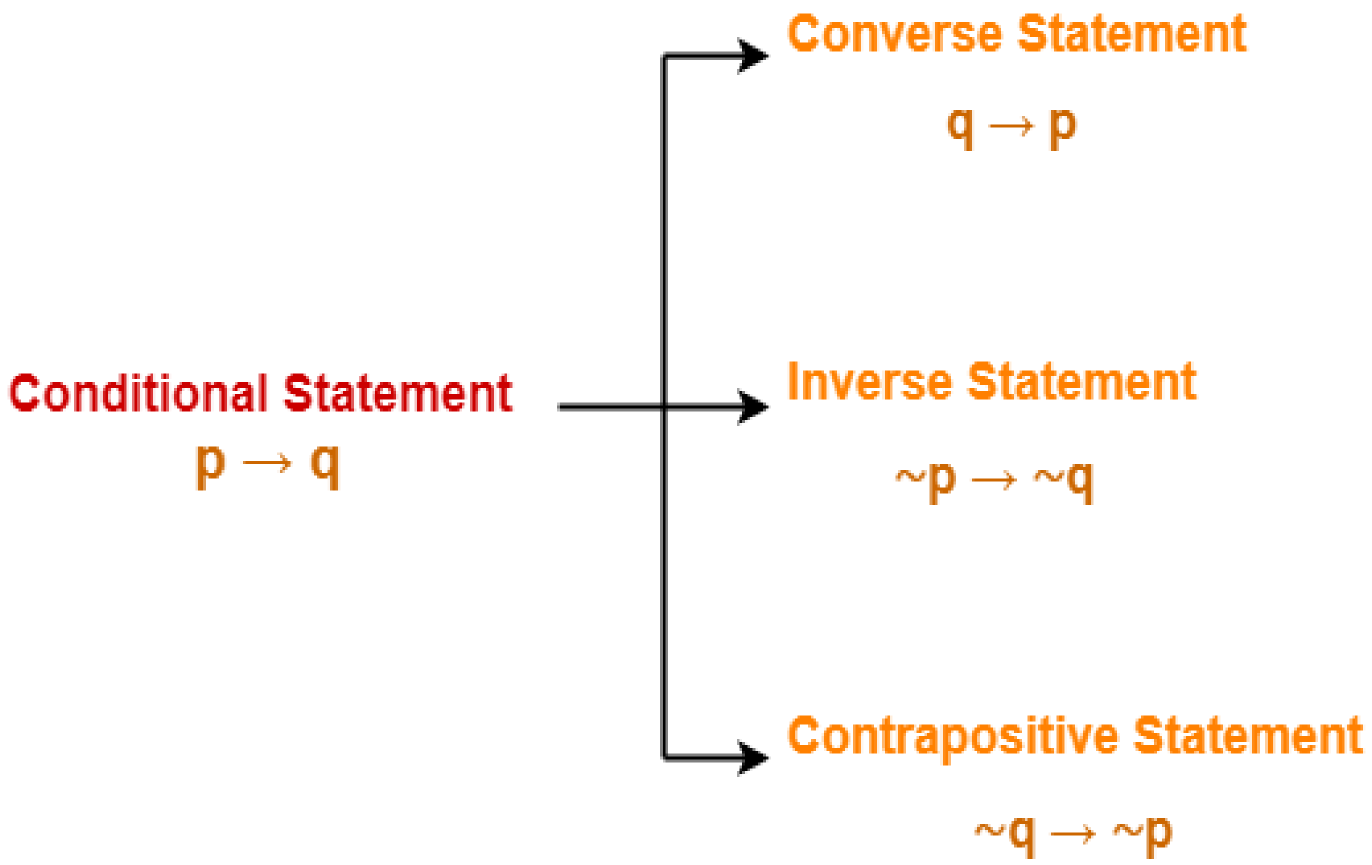
- If two statements are combined by using the logical connective ‘if…then’ then the resulting statement is called a conditional statement.
- If P and Q are two statements forming the implication “if P then Q” then we denotes this implication $P \rightarrow Q$.
- In the implication $P \rightarrow Q$, P is called antecedent or hypothesis
- Q is called consequent or conclusion.

The statement $P \rightarrow Q$ false. is true in all cases except when P is true and Q is

Truth Table

P	Q	$P \to Q$
T	T	T
T	F	F
F	T	T
F	F	T

- Since conditional statement play an essential role in mathematical reasoning a variety of terminology is used to express $P \rightarrow Q$.
- i) If P then Q
- ii) P implies Q
- iii) P only if Q
- iv) Q if P
- v) P is sufficient condition for Q
- vi) Q when P
- vii) Q is necessary for P
- viii) Q follows from P
- ix) if P, Q
- x) Q unless $\neg P$


Conditional Statement
p → q
Converse Statement
q → p
Inverse Statement
~p → ~q
Contrapositive Statement
~q → ~p

Inverse, Converse, and Contra-positive

Implication / if-then (→) is also called a conditional statement. It has two parts −

•Hypothesis, p

•Conclusion, q

As mentioned earlier, it is denoted as p→q.

Example of Conditional Statement − “If you do your homework, you will not be punished.” Here, "you do your homework" is the hypothesis, p, and "you will not be punished" is the conclusion, q.

1.Inverse − An inverse of the conditional statement is the negation of both the hypothesis and the conclusion. If the statement is “If p, then q”, the inverse will be “If not p, then not q”. Thus the inverse of p→q is ¬p→¬q.

Example − The inverse of **“If you do your homework, you will not be punished**” is

“If you do not do your homework, you will be punished.”

2.Converse − The converse of the conditional statement is computed by interchanging the hypothesis and the conclusion. If the statement is “If p, then q”, the converse will be “If q, then p”. The converse of $p \rightarrow q$ is $q \rightarrow p$.

Example − The converse of "If you do your homework, you will not be punished" is

"If you will not be punished, you do your homework”.

3.Contra-positive − The contra-positive of the conditional is computed by interchanging the hypothesis and the conclusion of the inverse statement. If the statement is “If p, then q”, the contra-positive will be “If not q, then not p”. The contra-positive of $p \rightarrow q$ is $\neg q \rightarrow \neg p$.

Example − The Contra-positive of " If you do your homework, you will not be punished” is

"If you are punished, you did not do your homework”.

Example: Let

P: You are good in Mathematics.

Q: You are good in Logic

Then, $P \rightarrow Q$: If you are good in Mathematics then you are good in Logic.

1) Converse: $(Q \rightarrow P)$

   If you are good in Logic then you are good in Mathematics.

2) Contra positive: $\neg Q \rightarrow \neg P$

   If you are not good in Logic then you are not good in Mathematics.

3) Inverse: $(\neg P \rightarrow \neg Q)$

   If you are not good in Mathematics then you are not good in Logic.

Write the converse, inverse and contrapositive of the following statements-

1. If today is Sunday, then it is a holiday.
2. If $5x - 1 = 9$, then $x = 2$.
3. If it rains, then I will stay at home

1. **Converse Statement-** If it is a holiday, then today is Sunday.
- **Inverse Statement-** If today is not Sunday, then it is not a holiday.
- **Contrapositive Statement-** If it is not a holiday, then today is not Sunday.

2. **Converse Statement-** If $x = 2$, then $5x – 1 = 9$.
- **Inverse Statement-** If $5x – 1 \neq 9$, then $x \neq 2$.
- **Contrapositive Statement-** If $x \neq 2$, then $5x – 1 \neq 9$.

3. **Converse Statement-** If I will stay at home, then it rains.
- **Inverse Statement-** If it does not rain, then I will not stay at home.
- **Contrapositive Statement-** If I will not stay at home, then it does not rain.

5. BICONDITIONAL STATEMENT

Biconditional Statement: Let P and Q be propositions. The biconditional statement $P \leftrightarrow Q$ is the proposition "P if and only if Q". The biconditional statement is true when P and Q have same truth values and is false otherwise.

Biconditional statements are also called bi-implications. It is also read as p is necessary and sufficient condition for Q.

The truth table for biconditional statement is as follows.

P	Q	$P \leftrightarrow Q$
T	T	T
T	F	F
F	T	F
F	F	T

- Example : Let P : Ram can take the flight.
- Q : Ram buy a ticket.

- Then $P \leftrightarrow Q$ is the statement. “Ram can take the flight iff Ram buy a ticket”.

Precedence of Logical Operators:

- We can construct compound propositions using the negation operator and the logical operators defined so far. We will generally use parentheses to specify the order in which logical operators in a compound proposition are to be applied.
- In order to avoid an excessive number of parentheses.
- We sometimes adopt an order of precedence for the logical connectives.

The following table displays the precedence levels of the logical operators.

Operator	Precedence
$\neg$	1
$\wedge$	2
$\wedge$	3
$\rightarrow$	4
$\leftrightarrow$	5

DERIVED CONNECTORS

1. NAND: It means negation after ANDing of two statements. Assume p and q be two propositions. NANDing of p and q to be a proposition which is false when both p and q are true, otherwise true. It is denoted by $p \uparrow q$.

Logical NAND		
p	q	$p \uparrow q$
T	T	F
T	F	T
F	T	T
F	F	T

2. NOR or Joint Denial: It means negation after ORing of two statements. Assume p and q be two propositions. NORing of p and q to be a proposition which is true when both p and q are false, otherwise false. It is denoted by $p_{\downarrow}q$.

Logical NOR		
p	q	$p \downarrow q$
T	T	F
T	F	F
F	T	F
F	F	T

3. XOR: Assume p and q be two propositions. XORing of p and q is true if p is true or q is true but not both and vice-versa. It is denoted by **p ⊕ q**.

p	q	p ⊕ q
T	T	F
T	F	T
F	T	T
F	F	F

Example1: Prove that $X \oplus Y \cong (X \wedge {\sim}Y) \vee ({\sim}X \wedge Y)$.

Solution: Construct the truth table for both the propositions.

X	Y	X⊕Y	~Y	~X	X ∧~Y	~X∧Y	(X ∧~Y)∨(~X∧Y)
T	T	F	F	F	F	F	F
T	F	T	T	F	T	F	T
F	T	T	F	T	F	T	T
F	F	F	T	T	F	F	F

As the truth table for both the proposition is the same.

> $X \oplus Y \cong (X \wedge {\sim}Y) \vee ({\sim}X \wedge Y)$. Hence Proved.

Example2: Show that $(p \oplus q) \vee (p \downarrow q)$ is equivalent to $p \uparrow q$.

Solution: Construct the truth table for both the propositions.

p	q	$p \oplus q$	$(p \downarrow q)$	$(p \oplus q) \vee (p \downarrow q)$	$p \uparrow q$
T	T	F	F	F	F
T	F	T	F	T	T
F	T	T	F	T	T
F	F	F	T	T	T

Example: Prove that $p \leftrightarrow q$ is equivalent to $(p \rightarrow q) \wedge (q \rightarrow p)$.

Solution: Construct the truth table for both the propositions:

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

p	q	$p \rightarrow q$	$q \rightarrow p$	$(p \rightarrow q) \wedge (q \rightarrow p)$
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

Since, the truth tables are the same, hence they are logically equivalent. Hence Proved.

Converting English Words to Propositional Logic

Word	Replacement
And	Conjunction ($\wedge$)
Or	Disjunction ($\vee$)
But	And
Whenever	If
When	If
Either p or q	p or q
Neither p nor q	Not p and Not q
p unless q	$\sim q \rightarrow p$
p is necessary but not sufficient for q	$(q \rightarrow p)$ and $\sim(p \rightarrow q)$

Examples

1. If it rains, then I will stay at home.
2. If I will go to Australia, then I will earn more money.
3. He is poor but honest.
4. If a = b and b = c then a = c.
5. Neither it is hot nor cold today.
6. Either today is Sunday or Monday.
7. You will qualify GATE only if you work hard.
8. Presence of cycle in a single instance RAG is a necessary and sufficient condition for deadlock.
9. Presence of cycle in a multi instance RAG is a necessary but not sufficient condition for deadlock.
10. I will dance only if you sing.
11. Neither the red nor the green is available in size 5.

Part-01:

- The given sentence is- “If it rains, then I will stay at home.”
- This sentence is of the form- “If p then q”.

So, the symbolic form is $\mathbf{p} \rightarrow \mathbf{q}$ where-

p : It rains

q : I will stay at home

Part-02:

- The given sentence is- “If I will go to Australia, then I will earn more money.”
- This sentence is of the form- “If p then q”.

So, the symbolic form is $\mathbf{p} \rightarrow \mathbf{q}$ where-

p : I will go to Australia

q : I will earn more money

Part-03:

- The given sentence is- “He is poor but honest.”
- We can replace “but” with “and”.
- Then, the sentence is- “He is poor and honest.”

So, the symbolic form is $\mathbf{p} \wedge \mathbf{q}$ where-

p : He is poor

q : He is honest

Part-04:

- The given sentence is- “If a = b and b = c then a = c.”
- This sentence is of the form- “If p then q”.

So, the symbolic form is $(p \wedge q) \rightarrow r$ where-

p : a = b

q : b = c

r : a = c

Part-05:

- The given sentence is- “Neither it is hot nor cold today.”
- This sentence is of the form- “Neither p nor q”.
- “Neither p nor q” can be re-written as “Not p and Not q”.

So, the symbolic form is $\sim p \wedge \sim q$ where-

p : It is hot today

q : It is cold today

Part-6:

- The given sentence is- “Either today is Sunday or Monday.”
- It can be re-written as- “Today is Sunday or Monday.”

So, the symbolic form is $p \vee q$ where-

p : Today is Sunday

q : Today is Monday

Part-7:

- The given sentence is- “You will qualify GATE only if you work hard.”
- This sentence is of the form- “p only if q”.

So, the symbolic form is $p \rightarrow q$ where-

p : You will qualify GATE

q : You work hard

Part-17:

The given sentence is- “Presence of cycle in a single instance RAG is a necessary and sufficient condition for deadlock.”

This sentence is of the form- “p is necessary and sufficient for q”.

So, the symbolic form is **p ↔ q** where-

p : Presence of cycle in a single instance RAG

q : Presence of deadlock

Part-18:

The given sentence is- “Presence of cycle in a multi instance RAG is a necessary but not sufficient condition for deadlock.”

This sentence is of the form- “p is necessary but not sufficient for q”.

So, the symbolic form is **(q → p) ∧ ~(p → q)** where-

p : Presence of cycle in a multi instance RAG

q : Presence of deadlock

Part-19:

The given sentence is- “I will dance only if you sing.”

This sentence is of the form- “p only if q”.

So, the symbolic form is **p → q** where-

p : I will dance

q : You sing

Part-20:

The given sentence is- “Neither the red nor the green is available in size 5.”

This sentence is of the form- “Neither p nor q”.

“Neither p nor q” can be written as “Not p and Not q”.

So, the symbolic form is **~p ∧ ~q** where-

p : Red is available in size 5

q : Green is available in size 5