Linear Algebra Problems: -

1. $[\,25 \quad 2 \quad -3 \quad -23\,]$

2. $Y_{2 \times 4}$

3. $Y_{2,3}$

4. $\mathbf{u} = [\,-2 \quad 0\,]$, $\mathbf{v} = [\,1.5 \quad 1.5\,]$, $\mathbf{w} = [\,4 \quad 1\,]$

   Hence, $\mathbf{u} + \mathbf{v} = [\,(-2 + 1.5) \quad (0 + 1.5)\,] = [\,-0.5 \quad 1.5\,]$,

   $\mathbf{v} + \mathbf{w} = [\,(1.5 + 4) \quad (1.5 + 1)\,] = [\,5.5 \quad 2.5\,]$,

   $\mathbf{w} + \mathbf{u} = [\,(4 - 2) \quad (1 + 0)\,] = [\,2 \quad 1\,]$,

   $\mathbf{u} + \mathbf{v} + \mathbf{w} = [\,(-2 + 1.5 + 4) \quad (0 + 1.5 + 1)\,] = [\,3.5 \quad 2.5\,]$

5.
   a. Let $\mathbf{x}$ = Jill's Mark I design = 1 kJ / day, $\mathbf{y}$ = Jill's Mark II design = 4 kJ / day, $\mathbf{E_1}$ = Total energy produced by Mark I, $\mathbf{E_2}$ = Total energy produced by Mark II, t_1 = days after Mark II started generating power

      $\mathbf{E_1} = (t_1 + 30) * \mathbf{x}$, $\mathbf{E_2} = t_1 * \mathbf{y}$

      But on the specified day, $\mathbf{E_1} = \mathbf{E_2}$

      Therefore, $(t_1 + 30) * \mathbf{x} = t_1 * y$

      Therefore, $1 * (t_1 + 30) = 4 * t_1$

      Therefore, $3 * t_1 = 30$

Therefore, $t_1 = 10$ days

Hence, the power generated by Mark I and Mark II will by equal by 10th May.

b. $\boldsymbol{E_1} = \boldsymbol{E_2} = (t_1 + 30) = 10 + 30 = 40$ kJ

Hence, $\boldsymbol{E_1} + \boldsymbol{E_2} = 40 + 40 = 80$ kJ

c. If $\boldsymbol{y}$ = Jill's Mark II design = 1 kJ / day ,

$1 * (t_1 + 30) = 1 * t_1$

Thus, there would be no solutions to the problem.

Calculus Problems: -

1. $y = -5x^3$

   Therefore, $dy/dx = -5 * 3 * x^2 = -15x^2$

2. $y = 2x^2 + 2x + 2$

   Therefore, $dy/dx = 2 * 2 * x + 2 = 4x + 2$

3. $y = 10x^5 - 6x^3 - x - 1$

   Therefore, $dy/dx = 10 * 5 * x^4 - 6 * 3 * x^2 - 1 = 50x^4 - 18x^2 - 1$

4. $y = x^2 + 2x + 2$

   Therefore, $dy/dx = 2x + 2$

   At $x = 2$, $dy/dx = 5$

   At $x = -1$, $dy/dx = 0$

5. $y = (2x^2 + 6x)(2x^3 + 5x^2)$

   $dy/dx = (2x^2 + 6x)(2 * 3 * x^2 + 5 * 2 * x) + (2 * 2 * x + 6)(2x^3 + 5x^2)$

   $dy/dx = (2x^2 + 6x)(6x^2 + 10x) + (4x + 6)(2x^3 + 5x^2)$

6. $y = 6x^2 / (2 - x)$

   $dy/dx = ((2 - x) * 6 * 2 * x - 6x^2 * (-1)) / (2 - x)^2$

   $dy/dx = (12x(2 - x) + 6x^2) / (2 - x)^2$

7. $y = (3x + 1)^2$

   $dy/dx = 2 * (3x + 1) * 3 = 6(3x + 1)$

8. $y = (x^2 + 5x)^6$

   $dy/dx = 6 * (x^2 + 5x) * (2x + 5) = 6(2x + 5)(x^2 + 5x)$

9. $y = 1 / ((x^4 + 1)^5 + 7) = ((x^4 + 1)^5 + 7)^{-1}$

   $dy/dx = -1 * ((x^4 + 1)^5 + 7)^{-2} * 5 * (x^4 + 1) * 4 * x^3$

   $dy/dx = -20x^3 (x^4 + 1) / ((x^4 + 1)^5 + 7)^2$

Statistics Short Notes: -

1. Z-scores are a tool for determining outlying data based on data locations on graphs. Z-scores base this information on data distribution and using the standard deviation measurements of data to calculate outlier under the understanding that about 68% of measurements will be within one standard deviation of the mean and about 95% of measurements will be within two standard deviations of the mean.

Z score = (x -mean) / std. Deviation

If the z score of a data point is more than 3, it indicates that the data point is quite different from the other data points. Such a data point can be an outlier.

2. p-value is the probability of obtaining results at least as extreme as the observed results of a statistical hypothesis test, assuming that the null hypothesis is correct. The p-value is used as an alternative to rejection points to provide the smallest level of significance at which the null hypothesis would be rejected. A smaller p-value means that there is stronger evidence in favor of the alternative hypothesis.

P-values are usually found using p-value tables or spreadsheets/statistical software. These calculations are based on the assumed or known probability distribution of the specific statistic being tested. P-values are calculated from the deviation between the observed value and a chosen reference value, given the probability distribution of the statistic, with a greater difference between the two values corresponding to a lower p-value. Mathematically, the p-value is calculated using integral calculus from the area under the probability distribution curve for all values of statistic that are at least as far from the reference value as the observed value is, relative to the total area under the probability distribution curve. In a nutshell, the greater the difference between two observed values, the less likely it is that the difference is due to simple random chance, and this is reflected by a lower p-value.

3. The T-test is a common method for comparing the mean of one group to a value or the mean of one group to another. T-tests are very useful because they usually perform well in the face of minor to moderate departures from normality of the underlying group distributions. In addition to providing the p-value information for the appropriate test, the T-test procedures also provide confidence intervals for means or differences, confidence intervals for the variation, z-tests, power reports, and nonparametric analogs to the tests, such as randomization tests, the quantile (sign) test, the Wilcoxon Signed-Rank Test, the Mann-Whitney test, and the Kolmogorov-Smirnov

test. The test additionally provides a test of assumptions about variation and the distribution.

4. A Confidence Interval is a range of values we are fairly sure our true value lies in. A confidence interval, in statistics, refers to the probability that a population parameter will fall between two set values for a certain proportion of times. Confidence intervals measure the degree of uncertainty or certainty in a sampling method. A confidence interval can take any number of probabilities, with the most common being a 95% or 99% confidence level.

Statisticians use confidence intervals to measure uncertainty. For example, a researcher selects different samples randomly from the same population and computes a confidence interval for each sample. The resulting datasets are all different; some intervals include the true population parameter and others do not.

5. An ANOVA test is a way to find out if survey or experiment results are significant. In other words, they help you to figure out if you need to reject the null hypothesis or accept the alternate hypothesis. Basically, you're testing groups to see if there's a difference between them.

One-way or two-way refers to the number of independent variables (IVs) in your Analysis of Variance test. One-way has one independent variable (with 2 levels). For example: *brand of cereal.* Two-way has two independent variables (it can have multiple levels). For example: *brand of cereal, calories*.

6. The null hypothesis is a statement that you want to test. In general, the null hypothesis is that things are the same as each other, or the same as a theoretical expectation. For example, if you measure the size of the feet of male and female chickens, the null hypothesis could be that the average foot size in male chickens is the same as the average foot size in female chickens. If you count the number of male and female chickens born to a

set of hens, the null hypothesis could be that the ratio of males to females is equal to a theoretical expectation of a 1:1 ratio.

7. The alternative hypothesis is that things are different from each other, or different from a theoretical expectation. For example, one alternative hypothesis would be that male chickens have a different average foot size than female chickens; another would be that the sex ratio is different from 1:1.

Probability Short Notes: -

1. In algebra you probably remember using variables like “x” or “y” which represent an unknown quantity like y = x + 1. You solve for the value of x, and x therefore represents a particular number (or set of numbers, if you’re talking about a function). Then you get to statistics and different kinds of variables are used, including random variables. These variables are still quantities, but unlike “x” or “y” (which are simply just numbers), random variables have distinct characteristics and behaviors.

If you see a lowercase x or y, that’s the kind of variable you’re used to in algebra. It refers to an unknown quantity or quantities. If you see an uppercase X or Y, that’s a random variable and it usually refers to the probability of getting a certain outcome.

Random variables are numerical in the same way that x or y is numerical, except it is attached to a random event. Let’s take rolling a die as an example. It’s a random event, but you can quantify (i.e. give a number to) the outcome. Let’s say you wanted to know how many sixes you get if you roll the die a certain number of times. Your random variable, X could be equal to 1 if you get a six and 0 if you get any other number.

2. A discrete variable can be described as simply a statistical data item whose value is constant and can be determined accurately to a degree of certainty. On the other hand, the continuous variable is the exact opposite because it has a value that keeps changing but what makes it difficult to measure is the fact that it can take an infinite number of values.

The simplest similarity that a discrete variable shares with a continuous variable is that both are variables meaning they have a changing value. A discrete variable has a complete range of values. What this means is that the values within a range to which can be assigned a discrete variable are known and exact. To the contrary, those values that a continuous random variable can take within a specified range are not exact nor complete. A continuous variable is uncountable while a discrete variable has countable values. If you were to plot a graphical representation of a continuous variable, you would end up with connected points forming a line or polygon whereas a discrete variable can be represented graphically using isolated points.

3. The probability distribution of a random variable x describes the probability of each outcome (a probability of 1 meaning that the variable will always take this value and a probability of 0 that it will never be encountered). This function is called probability distribution. More specifically, it is called the probability mass function for a discrete variable and probability density function for a continuous variable.

The probability mass function is the function which describes the probability associated with the random variable x. This function is named P(x) or P(x=x) to avoid confusion. P(x=x) corresponds to the probability that the random variable x take the value x.

Some variables are not discrete. They can take an infinite number of values in a certain range. But we still need to describe the probability associated with outcomes. The equivalent of the probability mass function for a continuous variable is called the probability density function. To get the probability, we need to calculate the area under the curve. The advantage is that it leads to the probabilities according to a certain range (on the x-axis): the area under the curve increases if the range increases.

4. In probability theory, an expected value is the theoretical mean value of a numerical experiment over many repetitions of the experiment. Expected value is a measure of central tendency; a value for which the results will tend to. When a probability distribution is normal, a plurality of the outcomes will be close to the expected value.

Any given random variable contains a wealth of information. It can have many (or infinite) possible outcomes, and each outcome could have different likelihood. The expected value is a way to summarize all this information in a single numerical value.