EES 445: Global Climate Change

Modelling the Climate System

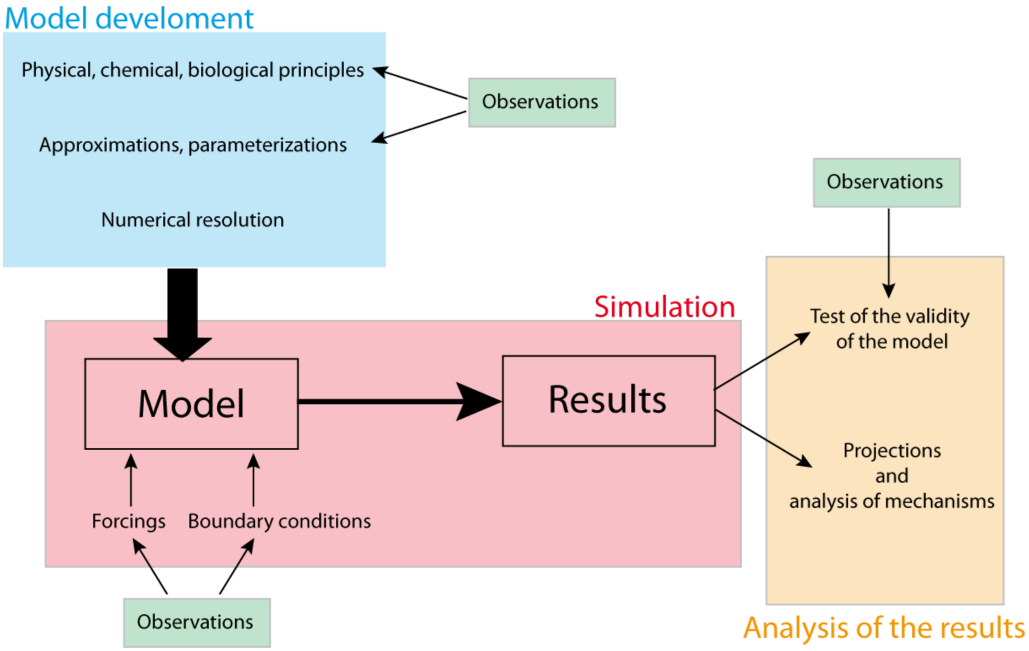
Assignment -1

18376 Om Mahesh Vaknalli

Introduction to Climate Models

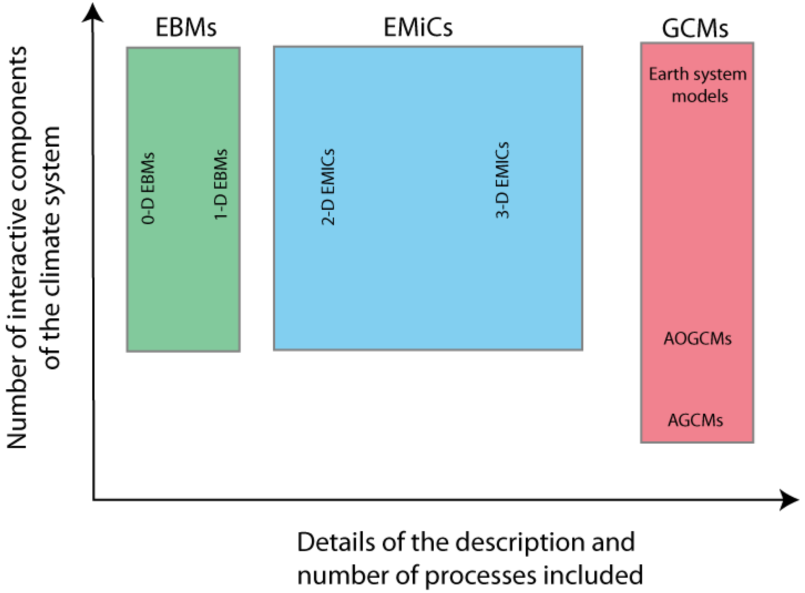
- Climate Models are mathematical representations of climate systems based on physical, biological and chemical principles.The solutions provided by climate models are discrete in space and time: averages over regions the size of which depends on model resolution.

- Schematic representation of different components in a Climate Model:



Types of climate models

- Climate Models can be classified on the basis of:
  - Number of interactive components considered in a model
  - Complexity of processes involved in a model
- Broad classification based on complexity: EBMs, GCMs. EMiCs



- There is no perfect model suitable for all purposes

Model Hierarchy

Energy Balance Models (EBMs):

- Estimate the changes in the climate system from an analysis of the energy budget of the Earth.
- Do not include any explicit spatial dimension, providing only globally averaged values..
- Also known as zero-dimensional EBMs.
- Fundamental equation of EBMs:

Changes in heat storage = absorbed solar radiation - emitted terrestrial radiation

$$C_E \frac{\partial T_s}{\partial t} = \left((1 - \alpha_p) \frac{S_0}{4} - A \uparrow \right)$$

where $A \uparrow = \epsilon \sigma T_s^4 \tau_a$ (Stefan-Boltzman Law)

- 0D-EBMs can be extended to one or two horizontal dimensions in order to take the geographical distribution of temperature on Earth's surface into account:

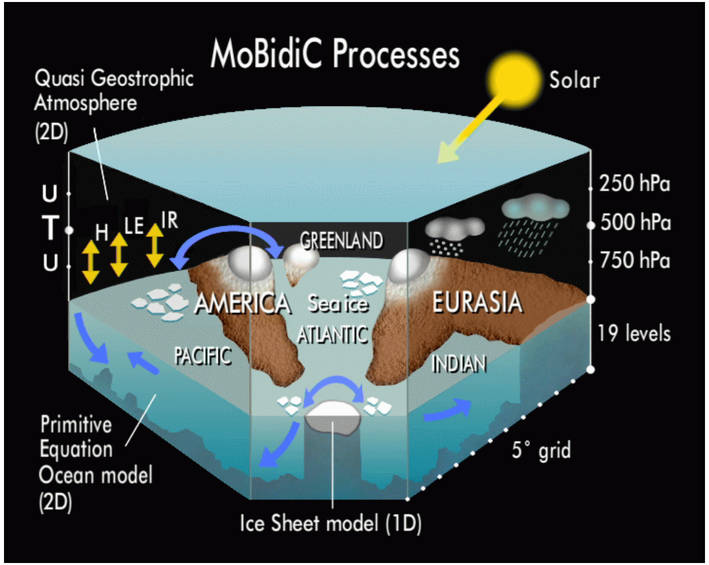
$$C_E \frac{\partial T_{s,i}}{\partial t} = \left((1 - \alpha_p) \frac{S_0}{4} - A \uparrow \right) + \Delta transp$$

where $\Delta transp =$ Heat |input-output|, associated with horizontal transport

Model Hierarchy (Contin.)

Intermediate Complexity Models (EMICs):

- EMICs involve some simplifications, but they always include a representation of the Earth's geography, i.e. they provide more than averages over the whole Earth or large boxes. Secondly, they include many more degrees of freedom than EBMs.
- As a consequence, the parameters of EMICs cannot easily be adjusted to reproduce the observed characteristics of the climate system, as can be done with some simpler models
- The level of approximation involved in the development of the model varies widely between different EMICs.
- Some interactive components, usually includes the atmosphere component, are simplified to increase computation speeds.

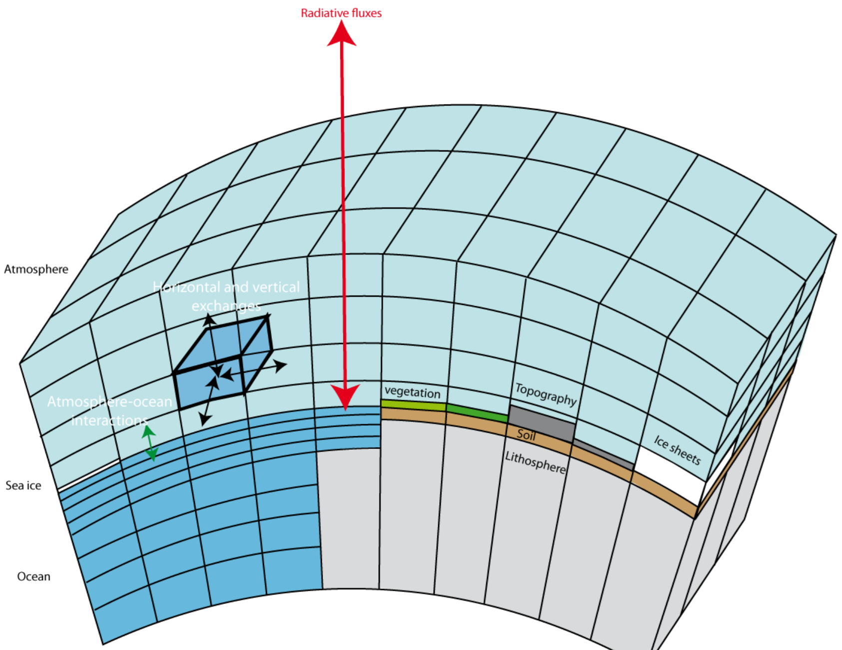


Schematic illustration of the structure of MoBidiC, an EMIC type climate model

Model Hierarchy (Contin.)

General Circulation Models (GCMs):

- General circulation models provide the most precise and complex description of the climate system.
- Currently, their grid resolution is typically of the order of 100 to 200 km.
- Provide much more detailed information on a regional-scale.
- Because of the large number of processes included and their relatively high resolution, GCM simulations require a large amount of computer time.
- As computing power increases, longer simulations with a higher resolution become affordable, providing more regional details than the previous generation of models.



A simplified representation of part of the domain of a GCM, illustrating some important components and processes.

Components of a Climate Model

Atmosphere:

- Set of seven equations with seven variables govern the atmosphere.
- Namely variables, velocity with three components(u,v and w), pressure(p), temperature(T), specific humidity(q), and density.
- (1-3) Newton's second law (momentum balance, i.e. F = ma , force equals mass times acceleration)

$$\frac{d\vec{v}}{dt} = -\frac{1}{\rho}\vec{\nabla}p - \vec{g} + \vec{F}_{fric} - 2\vec{\Omega} \times \vec{v}$$

- In this equation, d /dt is the total derivative, including a transport term,

$$\frac{d}{dt} = \frac{\partial}{\partial t} + \vec{v}.\vec{\nabla}$$

- The continuity equation or the conservation of mass,

$$\frac{\partial \rho}{\partial t} = -\vec{\nabla}.(\rho \vec{v})$$

- The conservation of the mass of water vapour, where E and C are evaporation and condensation respectively,

$$\frac{\partial \rho q}{\partial t} = -\vec{\nabla}.(\rho \vec{v} q) + \rho(E - C)$$

- The first law of thermodynamics (the conservation of energy),

$$Q = C_p \frac{dT}{dt} - \frac{1}{\rho}\frac{dp}{dt}$$

- The equation of state

$$p = \rho R_g T$$

<u>Ocean</u>:

- Mostly same equations as atmosphere, but equation of specific humidity is not required, while a new equation of salinity needs to be added.

- Equation of state is different from atmospheric one, because no simple law for ocean.

- Heating rate is easier to calculate because of only one source of heat, i.e. solar radiation.

- Even easier for salinity because there is no source or sink inside the ocean.

- We use a grid scale model, which works on reference with density level;
  - Isopycnal: Along surfaces of equal density.
  - Diapycnal: Normal to surface of constant density.

Sea ice

- A mixed model of both atmospheric and oceanic model.

- Thermodynamic decay or growth of the ice, depends on exchange with atmosphere and ocean.

- Thermodynamic code of sea-ice model is basically one-dimensional over the vertical, because horizontal scale is much larger than vertical, it can be neglected.

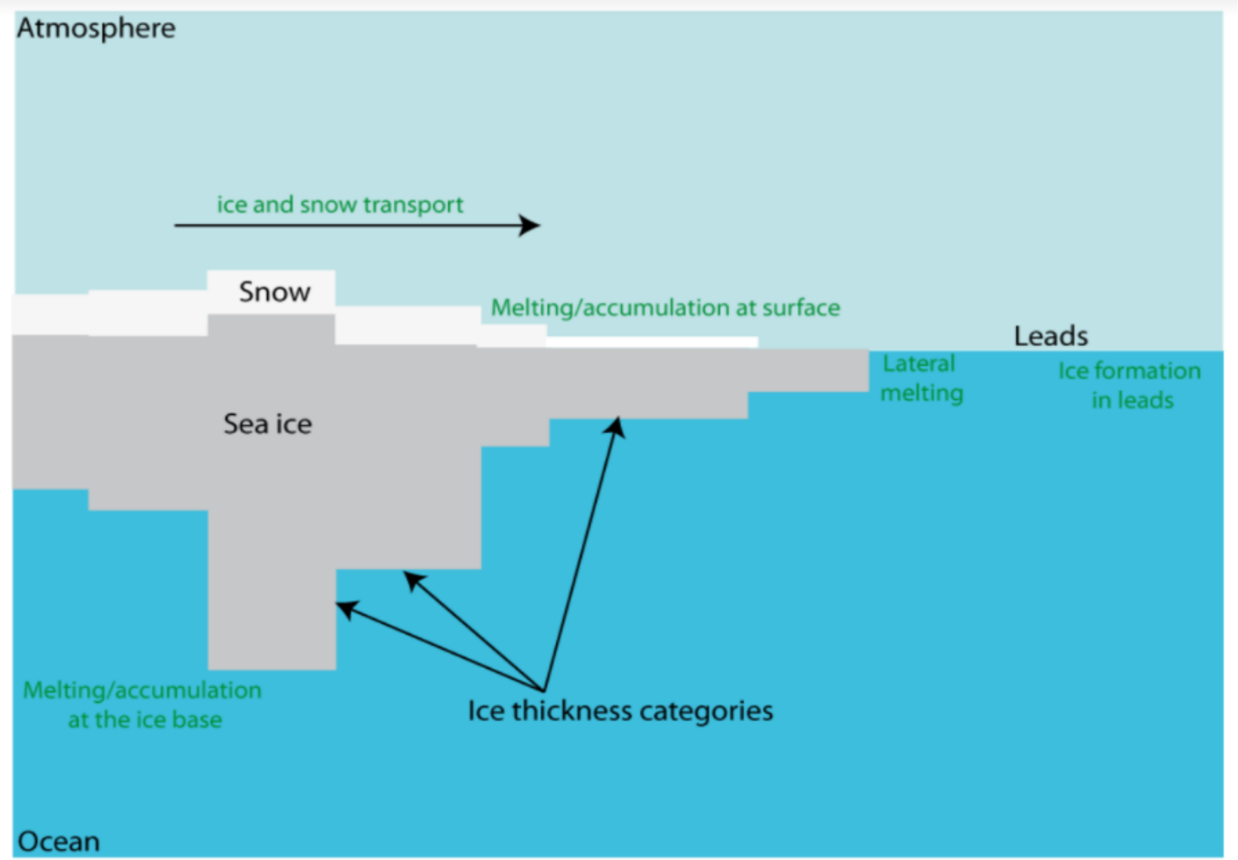
- The corresponding equation for it can be written as:

$$\rho_c c_{pc} \frac{\partial T_c}{\partial t} = k_c \frac{\partial^2 T_c}{\partial z^2}$$

- Where ρc, cpc, and kc are the density, specific heat and thermal conductivity, and Tc is the temperature. The subscript c stands for either ice (i) or snow (s).

- The heat balance at the surface allows the computation of the surface temperature and of the snow or ice melting.

- When studying the large-scale dynamics of sea ice, the ice is modelled as a two-dimensional continuum.



The main processes that have to be taken into account in a sea ice model.

Land surface

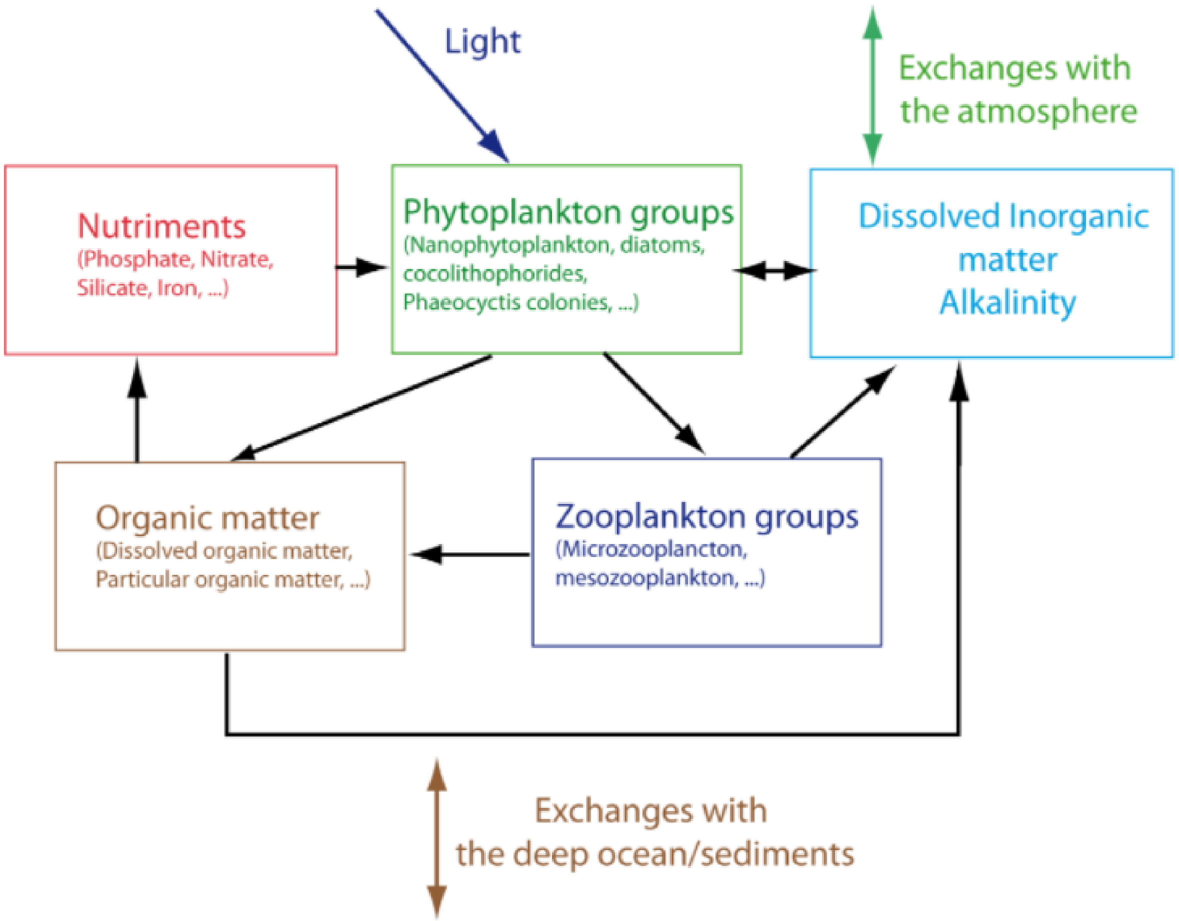
- As with sea ice, horizontal heat conduction and transport in soil can be safely neglected.
- Soil temperature can then be computed from the energy balance at the surface by the following equation:

$$\rho c_p h_{su} \frac{\partial T_s}{\partial t} = (1-\alpha) F_{sol} + F_{IR\downarrow} + F_{IR\uparrow} + F_{SE} + L_f E + F_{cond}$$

- A land surface model also simulates the water content of the soil.

Marine Biogeochemistry

- A simplified scheme representing some of the variables of a biogeochemical model.
- The interactions between the groups are complex as the different types of phytoplankton need different nutrients, are grazed by different types of zooplankton etc.



Ice sheets

- Ice-sheet models can be decomposed into two major components: a dynamic core that computes the flow of the ice and a thermodynamic part that estimates the changes in ice temperatures, snow accumulation, melting, etc.

- The conservation of ice volume can be written as:

$$\frac{\partial H}{\partial t} = -\vec{\nabla}.(\vec{v}_m H) + M_b$$

- Where Vm is the depth-averaged horizontal velocity field and Mb is the mass balance accounting for snow accumulation as well as basal and surface meltings.

- An important element in the mass balance at the surface of the ice sheets is the position of the equilibrium line between the regions where, on a yearly average, snow tends to accumulate and the ablation region.

Coupling between the components - Earth system models

- The interactions between the various components of the system play a crucial role in the dynamics of climate.

- Some of those interactions are quite straightforward to compute from the models state variables, while more sophisticated parameterisations are required for others.

- The technical coupling of the various components to obtain a climate- or Earth-system model brings additional difficulties.

- The numerical codes have generally been developed independently by different groups, using different coding standards, different numerical grids, etc.

Numerical Resolution of Equations

- Most Climate Laws = Partial Differential Equations

- Why to do it ?
  - Save computing time
  - Because exact answer not required
  - Computers grasp simpler concepts faster
  - Some analytical solutions are impossible to find

- Conditions for solvability of numerical solutions:
  - Initial and boundary conditions should be specified
  - They should be interpretable and solvable by a computer
  - Error of approximation of the solution should be acceptable

- Major computational solving methods:
  - Finite Difference Method
    - Upward Scheme / Forward Euler Method
    - Centered Scheme / Leap-Frog Method
    - Implicit Scheme
  - Finite Element Method / Galerkin Method

Finite Difference Method

Upward Scheme / Forward Euler Method

- Discretisation of the PDE

- $\dfrac{du}{dt} = A\cos(t) \quad \rightarrow \quad \dfrac{U^{n+1} - U^{n}}{\Delta t} = A\cos(n\Delta t)$

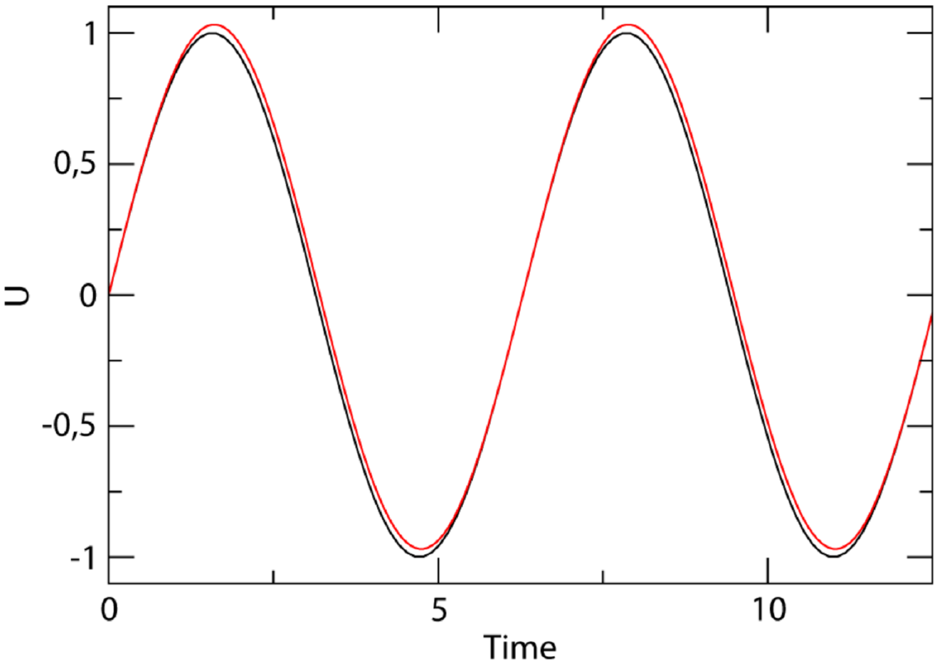
Centered Scheme / Leap-Frog Method

- $\dfrac{U^{n+1} - U^{n-1}}{2\Delta t} = F(U^{n})$

Implicit Scheme

- RHS = $F(U^{n}, U^{n+1})$

- F(U^{n+1}) only $\rightarrow$ Fully Implicit / Backward Scheme

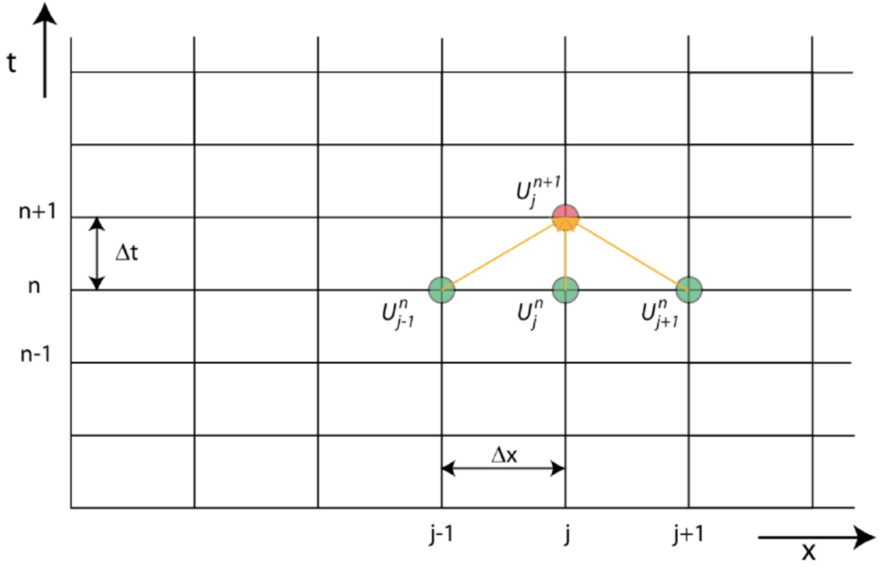


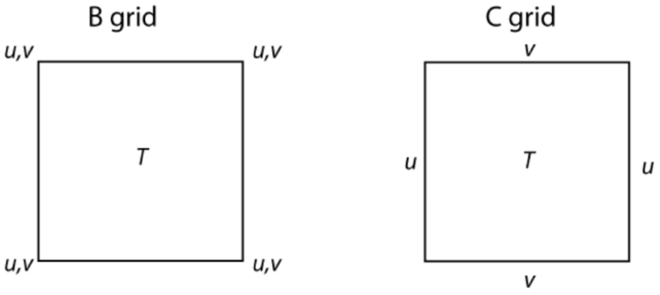
Finite Difference Method

- Conditions for useability of the methods:
  - As Δt, Δx → 0, FDE → PDE
    - $U^{n+1} = U^n + \frac{du}{dt}\Delta t + \frac{1}{2}\frac{d^2u}{dt^2}\Delta t^2 + \text{higher order terms} \quad \rightarrow \quad \frac{U^{n+1} - U^n}{\Delta t} = \frac{du}{dt} + \frac{1}{2}\frac{d^2u}{dt^2}\Delta t + \text{higher order terms}$
  - U(nΔt) → u(t) as Δt → 0
    - Convergence of the solutions = Computational Stability
  - Adequately time demanding

Finite Difference Method: Multi-Variable Discretisation

- Diffusion Equation: $\frac{\partial u}{\partial t} = k \frac{\partial^2 u}{\partial x^2}$
- Discretized Equation: $\frac{U_j^{n+1} - U_j^n}{\Delta t} = k \frac{U_{j+1}^n - 2U_j^n + U_{j-1}^n}{\Delta x^2}$
- Computationally stable if: $k \frac{\Delta t}{\Delta x^2} \leq \frac{1}{2}$
- Classification of grids:
  - Grid type A: For Uniequational problems
  - Grid type B: Multivariate/Multi-equational
    - Scalars at centre, vectors at vertices
  - Grid type C: Multivariate/Multi-equational
    - Scalars at centre, vectors at edges





Finite Element Method / Galerkin Method

- U(x,t) = $\int A_k(t)\varphi_k(x)$ dk $\rightarrow$ $U(x,t) = \sum_{k=1}^{K} A_k(t)\varphi_k(x)$

- Derivation of Basis Functions: Fourier Series, Spherical Harmonics

- Advantages:
  - Explicitly and easily ensures the conservation of some important properties of the original equation system
  - Space derivative of U(x,t) can be analytically computed from d ϕk(x)/dx without any additional approximation.

- Comparison and combination of the methods

Verification, Validation, Testing

- Difficulty in designing computer climate models → Primary scientific objectives must be kept in mind

- Verification: Numerical model adequately solves the equations of the physical model
  - Checking for coding errors
  - Checking accuracy of numerical methods in order to solve for the model equations
    - Comparing high idealised cases of numerical solutions with their analytical counterparts

- Validation: Determining whether the model accurately represents reality
  - Comparing model results with observations under the same conditions (visually, statistically)
  - For a single variable field:

$$RMS = \sqrt{\frac{1}{n}\sum_{k=1}^{n}(T^k_{s,\mathrm{mod}} - T^k_{s,obs})^2}$$

  - Calibration /Tuning of a model: Adjusting disagreeing parameters between model and observations
  - Robustness of a model: Agreeable results obtained from more than one observation sites for a single model

Verification, Validation, Testing

- Testing:
  - Checking if the model can reasonably simulate climate of recent decades?
  - Comparing long term averages of various variables with observations on a common grid
  - Checking the ability of the model to reproduce the observed climate variability across time scales

- Inter-model Comparisons
  - Doubling of atmospheric CO_2
  - Freshwater hosing in the Atlantic