Parsing

Jaruloj Chongstitvatana
Department of Mathematics and Computer Science
Chulalongkorn University

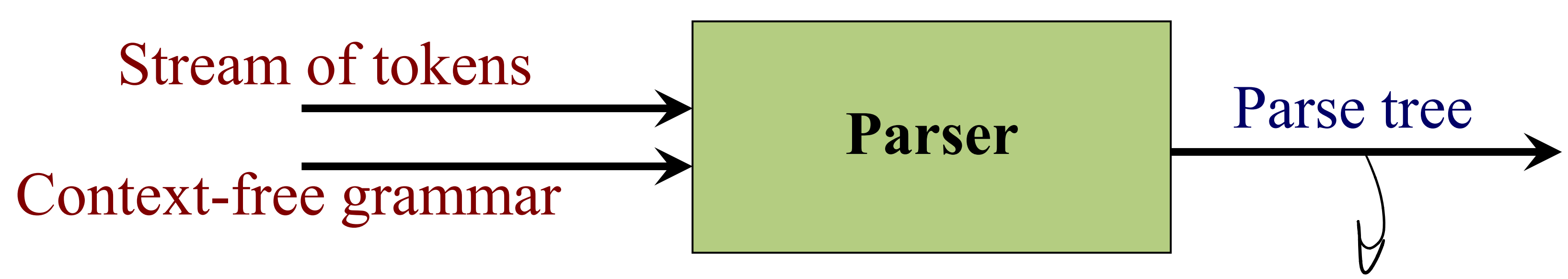
Outline

มีข้อจำกัดในภาษา จริง, ใช้ได้ มีข้อจำกัดน้อยกว่า

- Top-down parsing
  - Recursive-descent parsing
  - LL(1) parsing
    - LL(1) parsing algorithm
    - First and follow sets
    - Constructing LL(1) parsing table
    - Error recovery
- Bottom-up parsing
  - Shift-reduce parsers
  - LR(0) parsing
    - LR(0) items
    - Finite automata of items
    - LR(0) parsing algorithm
    - LR(0) grammar
  - SLR(1) parsing
    - SLR(1) parsing algorithm
    - SLR(1) grammar
    - Parsing conflict

Introduction

- Parsing is a process that constructs a syntactic structure (i.e. parse tree) from the stream of tokens.
- We already learn how to describe the syntactic structure of a language using (context-free) grammar.
- So, a parser only need to do this?



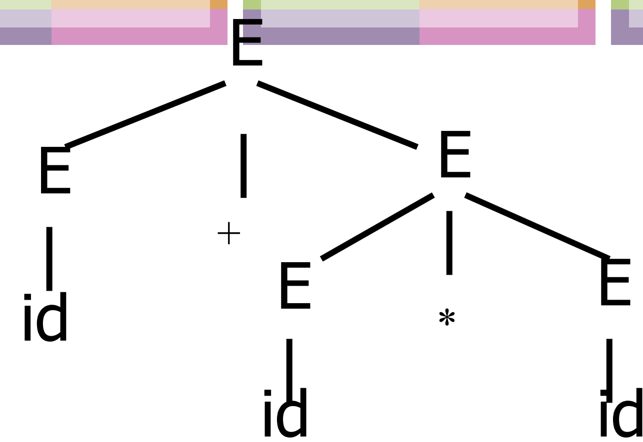
สำเร็จ หรือไม่สำเร็จ

Top–Down Parsing	**Bottom–Up Parsing**
A parse tree is created from root to leaves	A parse tree is created from leaves to root
The traversal of parse trees is a preorder traversal	The traversal of parse trees is a reversal of postorder traversal
Tracing leftmost derivation	Tracing rightmost derivation
Two types: • Backtracking parser • Predictive parser	

Try different structures and backtrack if it does not matched the input

Guess the structure of the parse tree from the next input

Parse Trees and Derivations



Top-down parsing

$$
\begin{aligned}
E &\Rightarrow E + E \\
&\Rightarrow id + E \\
&\Rightarrow id + E * E \\
&\Rightarrow id + id * E \\
&\Rightarrow id + id * id
\end{aligned}
$$

Top-down Parsing

- What does a parser need to decide?
  - Which production rule is to be used at each point of time ?
- How to guess?
- What is the guess based on?
  - What is the next token?
    - Reserved word if, open parentheses, etc.
  - What is the structure to be built?
    - If statement, expression, etc.

Top-down Parsing

- Why is it difficult?
  - Cannot decide until later
    - Next token: **if**   Structure to be built: St
    - St → MatchedSt | UnmatchedSt
    - UnmatchedSt →

      if (E) St| **if** (E) MatchedSt **else** UnmatchedSt
    - MatchedSt → **if** (E) MatchedSt **else** MatchedSt |...
  - Production with empty string
    - Next token: **id**   Structure to be built: par
    - par → parList | λ
    - parList → exp , parList | exp

Recursive-Descent

- Write one procedure for each set of productions with the same nonterminal in the LHS
- Each procedure recognizes a structure described by a nonterminal.
- A procedure calls other procedures if it need to recognize other structures.
- A procedure calls *match* procedure if it need to recognize a terminal.

Recursive-Descent: Example

แก้ grammar ให้ต้องอยู่ในรูปแบบ (เพราะเกิด left recursion ต้องแก้ก่อน)

E → E O F | F
O → + | -
F → (E) | id

E ::= F {O F}
O ::= + | -
F ::= (E) | id

```
procedure F
{   switch token
    {     case (:  match('(');
                   E;
                   match(')');
          case id: match(id);
          default:  error;
    }
}
```

```
procedure E
{          E; O; F; }
```

- For this grammar:
  - We cannot decide which rule to use for E, and
  - If we choose E → E O F, it leads to infinitely recursive loops.
- Rewrite the grammar into EBNF

```
procedure E
{   F;
    while (token=+ or token=-)
    {     O; F;   }
}
```

Match procedure

```
procedure match(expTok)
{   if (token==expTok)
    then    getToken
    else    error
}
```

- The token is not consumed until **`getToken`** is executed.

Problems in Recursive-Descent

- Difficult to convert grammars into EBNF
- Cannot decide which production to use at each point
- Cannot decide when to use λ-production $A \rightarrow \lambda$

LL(1) Parsing

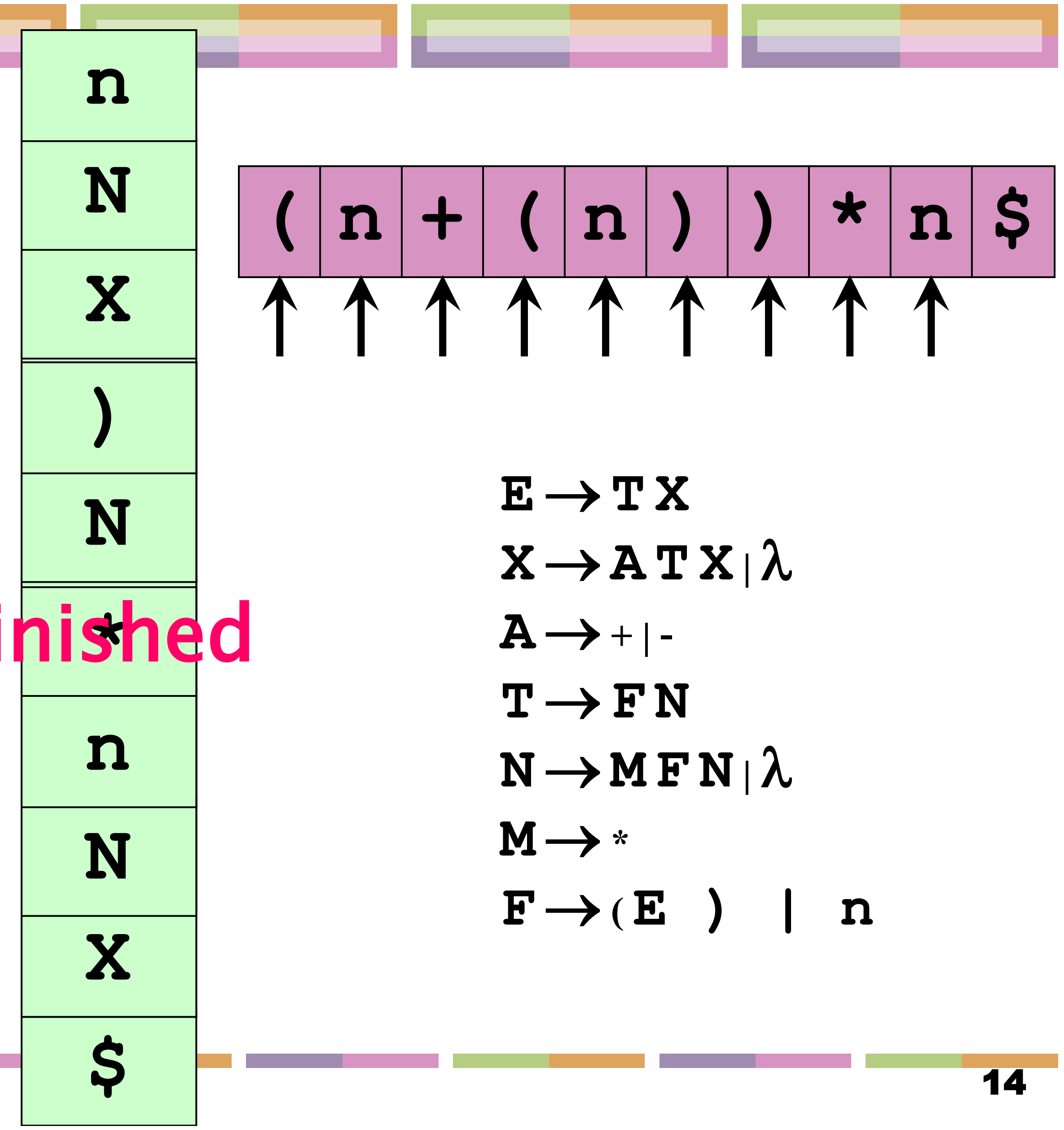
- LL(1)
  - Read input from (**L**) left to right
  - Simulate (**L**) leftmost derivation
  - **1** lookahead symbol
- Use stack to simulate leftmost derivation
  - Part of sentential form produced in the leftmost derivation is stored in the stack.
  - Top of stack is the leftmost nonterminal symbol in the fragment of sentential form.

Concept of LL(1) Parsing

- Simulate leftmost derivation of the input.
- Keep part of sentential form in the stack.
- If the symbol on the top of stack is a terminal, try to match it with the next input token and pop it out of stack.
- If the symbol on the top of stack is a nonterminal X, replace it with Y if we have a production rule $X \rightarrow Y$.
  - Which production will be chosen, if there are both $X \rightarrow Y$ and $X \rightarrow Z$?

Example of LL(1) Parsing

E ⇒ TX
⇒ FNX
⇒ (E)NX
⇒ (TX)NX
⇒ (FNX)NX
⇒ (nNX)NX
⇒ (nX)NX
⇒ (nATX)NX
⇒ (n+TX)NX
⇒ (n+FNX)NX
⇒ (n+(E)NX)NX
⇒ (n+(TX)NX)NX
⇒ (n+(FNX)NX)NX
⇒ (n+(nNX)NX)NX
⇒ (n+(nX)NX)NX
⇒ (n+(n)NX)NX
⇒ (n+(n)X)NX
⇒ (n+(n))NX
⇒ (n+(n))MFNX
⇒ (n+(n))*FNX
⇒ (n+(n))*nNX
⇒ (n+(n))*nX
⇒ (n+(n))*n



Finished

E → T X
X → A T X | λ
A → + | -
T → F N
N → M F N | λ
M → *
F → (E) | n

LL(1) Parsing Algorithm

Push the start symbol into the stack

WHILE stack is not empty ($ is not on top of stack) and the stream of tokens is not empty (the next input token is not $)

SWITCH (Top of stack, next token)

CASE (terminal a, a):

Pop stack; Get next token

CASE (nonterminal A, terminal a):

IF the parsing table entry M[A, a] is not empty THEN

Get $A \rightarrow X_1 X_2 \ldots X_n$ from the parsing table entry M[A, a] Pop stack;

Push $X_n \ldots X_2 X_1$ into stack in that order

ELSE Error

CASE ($,$): Accept

OTHER: Error

LL(1) Parsing Table

If the nonterminal N is on the top of stack and the next token is t, which production rule to use?

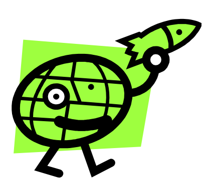
- Choose a rule $N \to X$ such that
  - $X \Rightarrow^* tY$ or
  - $X \Rightarrow^* \lambda$ and $S \Rightarrow^* WNtY$

เมื่อมีหลายทางเลือก เราสามารถ [illegible] ได้ 2 แบบ

t
Y
Q

X
t
Y

t	...	...	...



First Set

- Let X be λ or be in V or T.
- First(X) is the set of the first terminal in any sentential form derived from X.
  - If X is a terminal or λ, then First(X) ={X}.
  - If X is a nonterminal and $X \rightarrow X_1 X_2 \ldots X_n$ is a rule, then
    - First(X_1) -{λ} is a subset of First(X)
    - First(X_i)-{λ} is a subset of First(X) if for all $j<i$ First(X_j) contains {λ}
    - λ is in First(X) if for all $j \leq n$ First(X_j)contains λ

ต้องจบด้วย term ตัวแรกสุด

Examples of First Set

exp → exp addop term | term

addop → + | -

term → term mulop factor | factor

mulop → *

factor → (exp) | num

First(addop) = {+, -}

First(mulop) = {*}

First(factor) = {(, num}

First(term) = {(, num}

First(exp) = {(, num}

st → ifst | other

ifst → if (exp) st elsepart

elsepart → else st | λ

exp → 0 | 1

First(exp) = {0,1}

First(elsepart) = {else, λ}

First(ifst) = {if}

First(st) = {if, other}

Algorithm for finding First(A)

For all terminals a, First(a) = {a}

For all nonterminals A, First(A) := { }

While there are changes to any First(A)

For each rule $A \to X_1 X_2 \dots X_n$

For each X_i in $\{X_1, X_2, \dots, X_n\}$

If for all j<i First(X_j) contains λ,

Then

add First(X_i)-{λ} to First(A)

If λ is in First(X_1), First(X_2), ..., and First(X_n)

Then add λ to First(A)

If A is a terminal or λ, then First(A) = {A}.

If A is a nonterminal, then for each rule $A \to X_1 X_2 \dots X_n$, First(A) contains First(X_1) - {λ}.

If also for some i<n, First(X_1), First(X_2), ..., and First(X_i) contain λ, then First(A) contains First(X_{i+1})-{λ}.

If First(X_1), First(X_2), ..., and First(X_n) contain λ, then First(A) also contains λ.

Finding First Set: An Example

exp → term exp′

exp′ → addop term exp′ | λ

addop → + | -

term → factor term′

term′ → mulop factor term′ | λ

mulop → *

factor → (exp) | num

	First
exp	
exp′	+ - λ
addop	+ -
term	(num
term′	λ
mulop	*
factor	(num

Follow Set

- Let \$ denote the end of input tokens
- If A is the start symbol, then \$ is in Follow(A).
- If there is a rule $B \rightarrow X\ A\ Y$, then First(Y) - {λ} is in Follow(A).
- If there is production $B \rightarrow X\ A\ Y$ and λ is in First(Y), then Follow(A) contains Follow(B).

Algorithm for Finding Follow(A)

Follow(S) = {\$}
FOR each A in V-{S}
    Follow(A)={}
WHILE change is made to some Follow sets
    FOR each production $A \rightarrow X_1\ X_2 \ldots X_n$,
        FOR each nonterminal X_i
            Add First($X_{i+1}\ X_{i+2} \ldots X_n$)-{$\lambda$} into Follow($X_i$).
            (NOTE: If i=n, $X_{i+1}\ X_{i+2} \ldots X_n = \lambda$)
        IF λ is in First($X_{i+1}\ X_{i+2} \ldots X_n$) THEN
            Add Follow(A) to Follow(X_i)

If A is the start symbol, then \$ is in Follow(A).

If there is a rule $A \rightarrow Y\ X\ Z$, then First(Z) - {λ} is in Follow(X).

If there is production $B \rightarrow X\ A\ Y$ and λ is in First(Y), then Follow(A) contains Follow(B).

start symbol ($) | Follow ดูว่าตัวแสดงครบ

แล้วสุดท้ายเป็น (λ) | ในบทต่าง ขนแปลงก่อน ทำสีไ

Finding Follow Set: An Example

exp → term exp'

exp' → addop term exp' | λ

addop → + | -

term → factor term'

term' → mulop factor term' | λ

mulop → *

factor → (exp) | num

	First	Follow
exp	(num	$)
exp'	+ - λ	$)
addop	+ -	(num
term	(num	+ - $)
term'	* λ	+ - $)
mulop	*	(num
factor	(num	+ - $)

Constructing LL(1) Parsing Tables

FOR each nonterminal A and a production A $\rightarrow$ X

  FOR each token a in First(X)

    A $\rightarrow$ X is in M(A, a)

    IF λ is in First(X) THEN

      FOR each element a in Follow(A)

        Add A $\rightarrow$ X to M(A, a)

Example: Constructing LL(1) Parsing Table

ดูให้เป็น

	First	Follow
exp	{(, num}	{$,)}
exp'	{+,-, λ}	{$,)}
addop	{+,-}	{(,num}
term	{(,num}	{+,-,),$}
term'	{*, λ}	{+,-,),$}
mulop	{*}	{(,num}
factor	{(, num}	{*,+,-,),$}

ถ้า λ ดู follow(factor) follow(term')

1 exp → term exp'
2 exp' → addop term exp'
3 exp' → λ
4 addop → +
5 addop → -
6 term → factor term'
7 term' → mulop factor term'
8 term' → λ
9 mulop → *
10 factor → (exp)
11 factor → num

	(	)	+	-	*	n	$
exp	1					1	
exp'		3	2	2			3
addop			4	5			
term	6					6	
term'		8	8	8	7		8
mulop					9		
factor	10					11	

1 exp → term exp'
2 exp' → addop term exp'
3 exp' → λ
4 addop → +
5 addop → -
6 term → factor term'
7 term' → mulop factor term'
8 term' → λ
9 mulop → *
10 factor → (exp)
11 factor → num

exp'

LL(1) Grammar

- A grammar is an LL(1) grammar if its LL(1) parsing table has at most one production in each table entry.

LL(1) Parsing Table for non-LL(1) Grammar

1 exp → exp addop term
2 exp → term
3 term → term mulop factor
4 term → factor
5 factor → (exp)
6 factor → num
7 addop → +
8 addop → -
9 mulop → *

เกิดความกำกวม แต่ต้องแก้ที่ grammar

	(	)	+	-	*	num	$
exp	1,2					1,2	
term	3,4					3,4	
factor	5					6	
addop			7	8			
mulop					9		

First(exp) = { (, num }
First(term) = { (, num }
First(factor) = { (, num }
First(addop) = { +, - }
First(mulop) = { * }

Causes of Non-LL(1) Grammar

- What causes grammar being non-LL(1)?
  - Left-recursion ตัวแก้
  - Left factor

Left Recursion

- Immediate left recursion
  - $A \rightarrow A\ X \mid Y$ $\quad$ $A = Y\ X^*$
  - $A \rightarrow A\ X_1 \mid A\ X_2 \mid \ldots \mid A\ X_n \mid Y_1 \mid Y_2 \mid \ldots \mid Y_m$

$A=\{Y_1, Y_2, \ldots, Y_m\}\ \{X_1, X_2, \ldots, X_n\}^*$

- Can be removed very easily
  - $A \rightarrow Y\ A',\ A' \rightarrow X\ A' \mid \lambda$
  - $A \rightarrow Y_1\ A' \mid Y_2\ A' \mid \ldots \mid Y_m\ A',$ $A' \rightarrow X_1\ A' \mid X_2\ A' \mid \ldots \mid X_n\ A' \mid \lambda$
- General left recursion
  - $A \Rightarrow X \Rightarrow^* A\ Y$
- Can be removed when there is no empty-string production and no cycle in the grammar

Removal of Immediate Left Recursion

$exp \rightarrow exp + term \mid exp - term \mid term$

$term \rightarrow term * factor \mid factor$

$factor \rightarrow (exp) \mid num$

- Remove left recursion

$exp \rightarrow term\ exp'$ $\qquad$ $exp = term\ (\pm\ term)^*$

$exp' \rightarrow + term\ exp' \mid - term\ exp' \mid \lambda$

$term \rightarrow factor\ term'$ $\qquad$ $term = factor\ (*\ factor)^*$

$term' \rightarrow * factor\ term' \mid \lambda$

$factor \rightarrow (exp) \mid num$

General Left Recursion

- Bad News!
  - Can only be removed when there is no empty-string production and no cycle in the grammar.
- Good News!!!!
  - Never seen in grammars of any programming languages

Left Factoring

- Left factor causes non-LL(1)
  - Given $A \to X\ Y \mid X\ Z$. Both $A \to X\ Y$ and $A \to X\ Z$ can be chosen when A is on top of stack and a token in First(X) is the next token.

$A \to X\ Y \mid X\ Z$

can be left-factored as

$A \to X\ A'$ and $A' \to Y \mid Z$

Example of Left Factor

ifSt $\rightarrow$ **if (** exp **)** st **else** st | **if (** exp **)** st

can be left-factored as

ifSt $\rightarrow$ **if (** exp **)** st elsePart

elsePart $\rightarrow$ **else** st | λ

seq $\rightarrow$ st ; seq | st

can be left-factored as

seq $\rightarrow$ st seq'

seq' $\rightarrow$; seq | λ

Outline

- Top-down parsing
  - Recursive-descent parsing
  - LL(1) parsing
    - LL(1) parsing algorithm
    - First and follow sets
    - Constructing LL(1) parsing table
    - Error recovery
- Bottom-up parsing
  - Shift-reduce parsers
  - LR(0) parsing
    - LR(0) items
    - Finite automata of items
    - LR(0) parsing algorithm
    - LR(0) grammar
  - SLR(1) parsing
    - SLR(1) parsing algorithm
    - SLR(1) grammar
    - Parsing conflict

Bottom-up Parsing

- Use explicit stack to perform a parse
- Simulate rightmost derivation (R) from left (L) to right, thus called LR parsing
- More powerful than top-down parsing
  - Left recursion does not cause problem
- Two actions
  - Shift: take next input token into the stack
  - Reduce: replace a string B on top of stack by a nonterminal A, given a production A $\rightarrow$ B

Example of Shift-reduce Parsing

- Grammar

  $S' \rightarrow S$

  $S \rightarrow (S)S \mid \lambda$

- Parsing actions

Stack	Input	Action
\$	(( )) \$	
\$ (	( )) \$	
\$ ((	)) \$	
\$ ((S	)) \$	
\$ ((S)	) \$	
\$ ((S) S	) \$	
\$ (S	) \$	
\$ (S)	\$	
\$ (S) S	\$	
\$ S	\$	

- Reverse of
- rightmost derivation
- from left to right

1 $\Rightarrow$ **(( ))**

2 $\Rightarrow$ **(( ))**

3 $\Rightarrow$ **(( ))**

4 $\Rightarrow$ **((S))**

5 $\Rightarrow$ **((S))**

6 $\Rightarrow$ **((S) S)**

7 $\Rightarrow$ **(S)**

8 $\Rightarrow$ **(S)**

9 $\Rightarrow$ **(S) S**

10 S′ $\Rightarrow$ **S**

Example of Shift-reduce Parsing

- Grammar

  S′ → S

  S → (S)S | λ

- Parsing actions

Stack	Input	Action
$	(( )) $	shift
$ (	( )) $	shift
$ ((	)) $	reduce S → λ
$ ((S	)) $	shift
$ ((S)	) $	reduce S → λ
$ ((S) S	) $	reduce S → (S) S
$ (S	) $	shift
$ (S)	$	reduce S → λ
$ (S) S	$	reduce S → (S) S
$ S	$	accept

Viable prefix

1	⇒ **(( ))**
2	⇒ **(( ))**
3	⇒ **(( ))**
4	⇒ **((S))**
5	⇒ **((S))**
6	⇒ **((S) S)**
7	⇒ **(S)**
8	⇒ **(S)**
9	⇒ **(S) S**
10 S′	⇒ **S**

handle

Terminologies

- Right sentential form
  - sentential form in a rightmost derivation
- Viable prefix
  - sequence of symbols on the parsing stack
- Handle
  - right sentential form + position where reduction can be performed + production used for reduction
- LR(0) item
  - production with distinguished position in its RHS

- Right sentential form
  - (S) S
  - ((S) S)
- Viable prefix
  - (S) S, (S), (S, (
  - ((S) S, ((S), ((S , ((, (
- Handle
  - (S) S. with $S \rightarrow \lambda$
  - (S) S . with $S \rightarrow \lambda$
  - ((S) S .) with $S \rightarrow (S) S$
- LR(0) item
  - $S \rightarrow (S) S.$
  - $S \rightarrow (S) . S$
  - $S \rightarrow (S .) S$
  - $S \rightarrow (. S) S$
  - $S \rightarrow . (S) S$

Shift-reduce parsers

- There are two possible actions:
  - shift and reduce
- Parsing is completed when
  - the input stream is empty and
  - the stack contains only the start symbol
- The grammar must be *augmented*
  - a new start symbol S′ is added
  - a production S′ $\rightarrow$ S is added
    - To make sure that parsing is finished when S′ is on top of stack because S′ never appears on the RHS of any production.

LR(0) parsing

- Keep track of what is left to be done in the parsing process by using finite automata of items
  - An item $A \rightarrow w\ .\ B\ y$ means:
    - $A \rightarrow w\ B\ y$ might be used for the reduction in the future,
    - at the time, we know we already construct w in the parsing process,
    - if B is constructed next, we get the new item $A \rightarrow w\ B\ .\ Y$

LR(0) items

- LR(0) item
  - production with a distinguished position in the RHS
- Initial Item
  - Item with the distinguished position on the leftmost of the production
- Complete Item
  - Item with the distinguished position on the rightmost of the production
- Closure Item of x
  - Item x together with items which can be reached from x via λ-transition
- Kernel Item
  - Original item, not including closure items

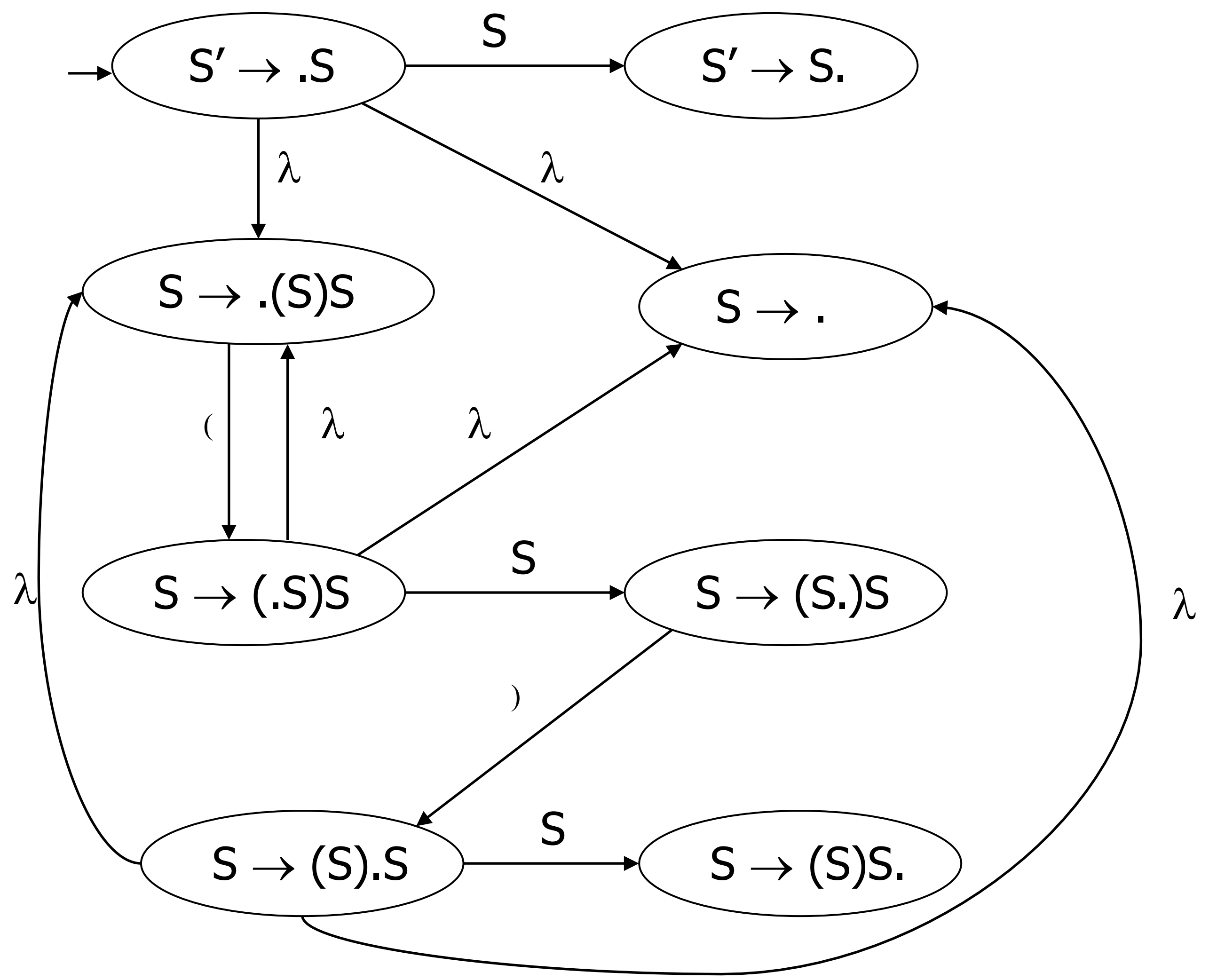
Finite automata of items

Grammar:

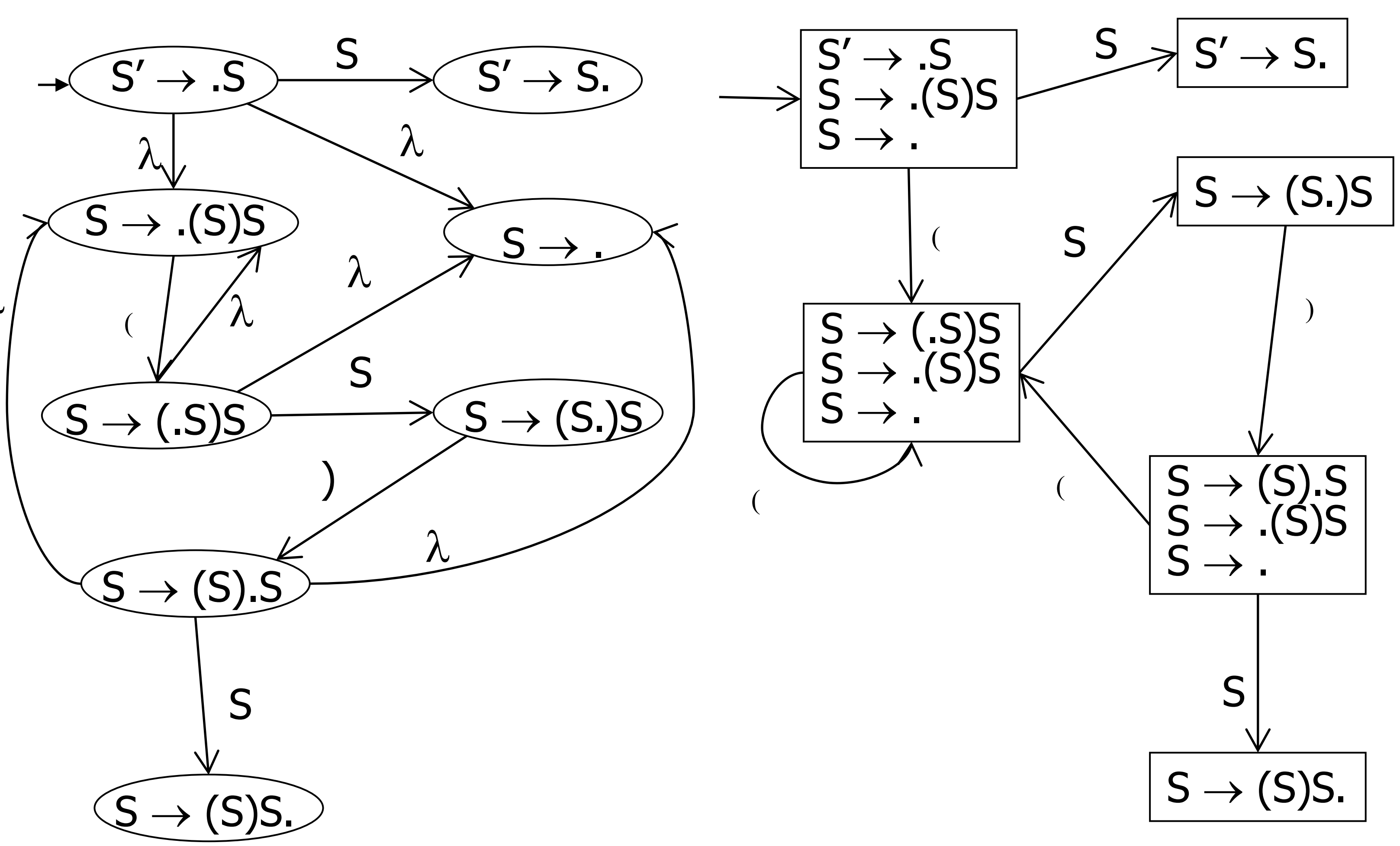
- $S' \to S$
- $S \to (S)S$
- $S \to \lambda$

Items:

- $S' \to .S$
- $S' \to S.$
- $S \to .(S)S$
- $S \to (.S)S$
- $S \to (S.)S$
- $S \to (S).S$
- $S \to (S)S.$
- $S \to .$



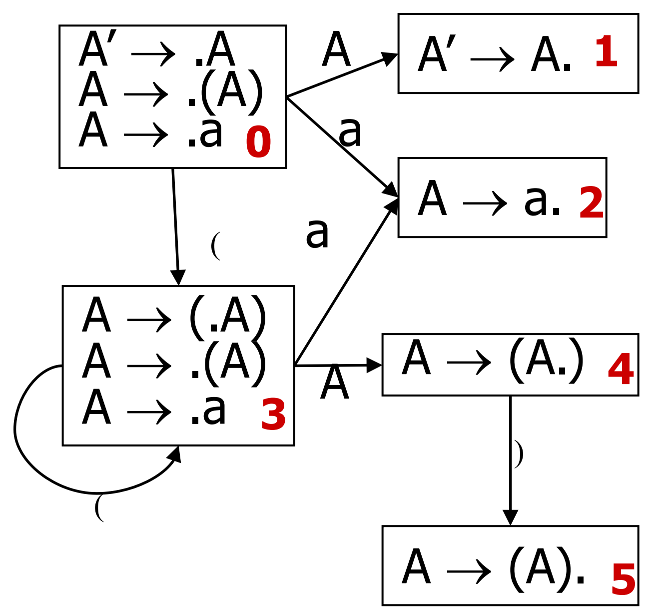
DFA of LR(0) Items



LR(0) parsing algorithm

Item in state	token	Action
A-> x.By where B is terminal	B	shift B and push state s containing A -> xB.y
A-> x.By where B is terminal	not B	error
A -> x.	-	reduce with A -> x (i.e. pop x, backup to the state s on top of stack) and push A with new state d(s,A)
S′ -> S.	none	accept
S′ -> S.	any	error

LR(0) Parsing Table



State	Action	Rule	(	a	)	A
0	shift		3	2		1
1	reduce	A′ -> A				
2	reduce	A -> a				
3	shift		3	2		4
4	shift				5	
5	reduce	A -> (A)				

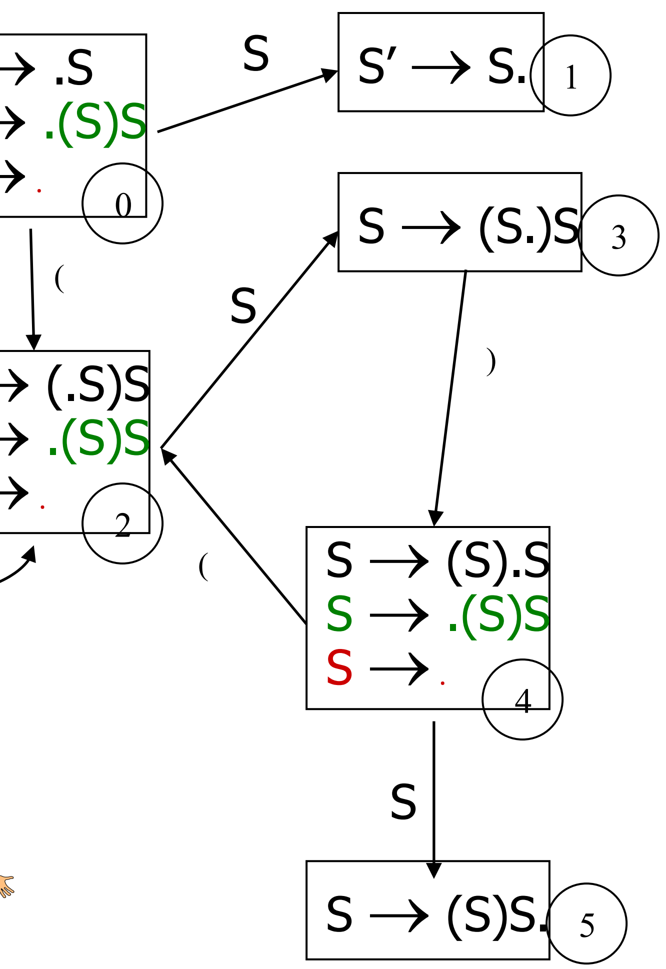
Example of LR(0) Parsing

State	Action	Rule	(	a	)	A
0	shift		3	2		1
1	reduce	A' -> A				
2	reduce	A -> a				
3	shift		3	2		4
4	shift				5	
5	reduce	A -> (A)				

Stack	Input	Action
$0	((a)) $	shift
$0(3	(a)) $	shift
$0(3(3	a)) $	shift
$0(3(3a2	)) $	reduce
$0(3(3A4	)) $	shift
$0(3(3A4)5	) $	reduce
$0(3A4	) $	shift
$0(3A4)5	$	reduce
$0A1	$	accept

Non-LR(0)Grammar

- Conflict
  - Shift-reduce conflict
    - A state contains a complete item A → x. and a shift item A → x.By
  - Reduce-reduce conflict
    - A state contains more than one complete items.
- A grammar is a LR(0) grammar if there is no conflict in the grammar.



SLR(1) parsing

- Simple LR with 1 lookahead symbol
- Examine the next token before deciding to shift or reduce
  - If the next token is the token expected in an item, then it can be shifted into the stack.
  - If a complete item $A \rightarrow x.$ is constructed and the next token is in Follow(A), then reduction can be done using $A \rightarrow x.$
  - Otherwise, error occurs.
- Can avoid conflict

SLR(1) parsing algorithm

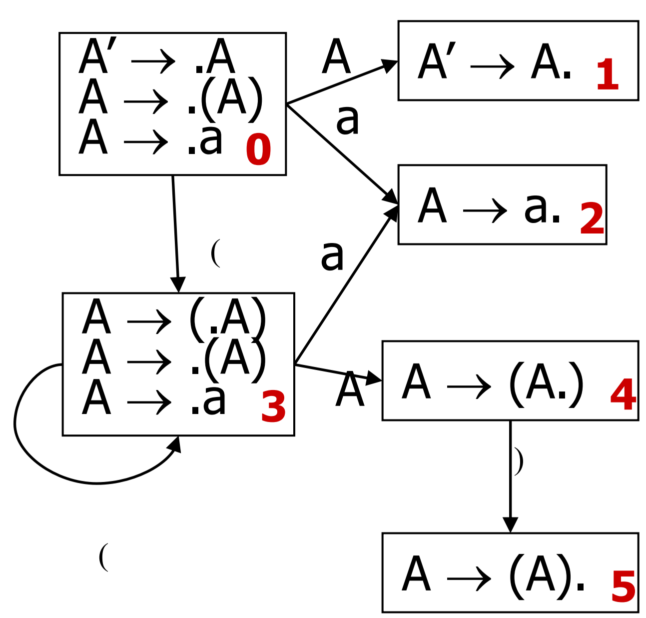
Item in state	token	Action
A-> x.By (B is terminal)	B	shift B and push state s containing A -> xB.y
A-> x.By (B is terminal)	not B	error
A -> x.	in Follow(A)	reduce with A -> x (i.e. pop x, backup to the state s on top of stack) and push A with new state d(s,A)
A -> x.	not in Follow(A)	error
S’ -> S.	none	accept
S’ -> S.	any	error

SLR(1) grammar

- Conflict
  - Shift-reduce conflict
    - A state contains a shift item $A \rightarrow x.Wy$ such that W is a terminal and a complete item $B \rightarrow z.$ such that W is in Follow(B).
  - Reduce-reduce conflict
    - A state contains more than one complete item with some common Follow set.
- A grammar is an SLR(1) grammar if there is no conflict in the grammar.

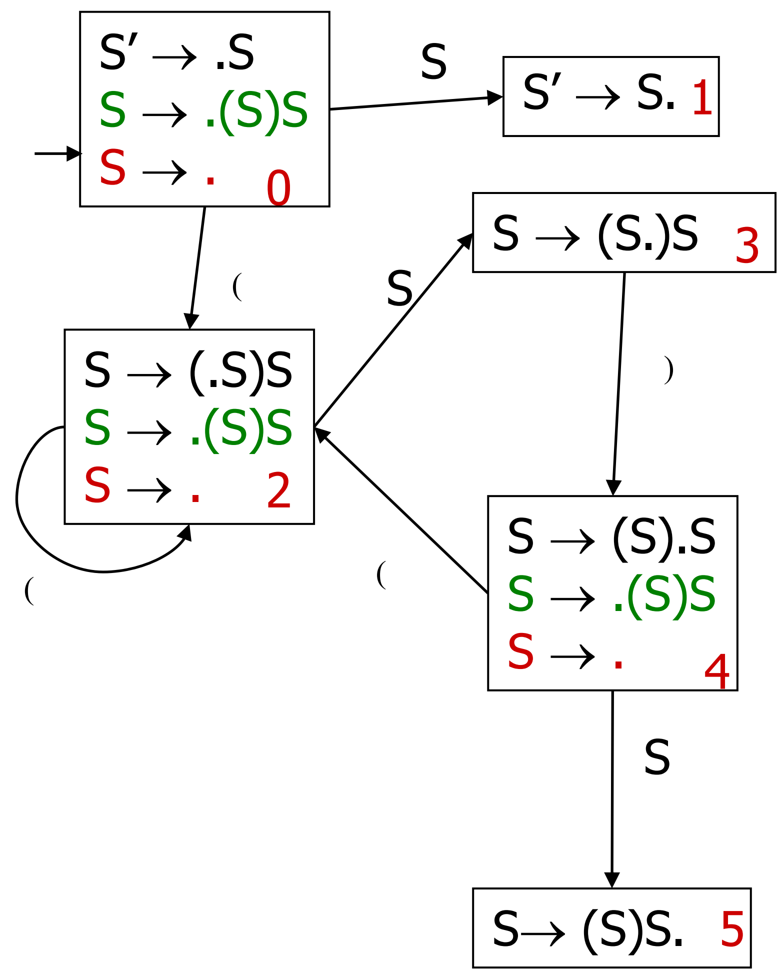
SLR(1) Parsing Table

A → (A) | a



State	(	a	)	$	A
0	S3	S2			1
1				AC	
2			R2		
3	S3	S2			4
4			S5		
5			R1		

SLR(1) Grammar not LR(0)



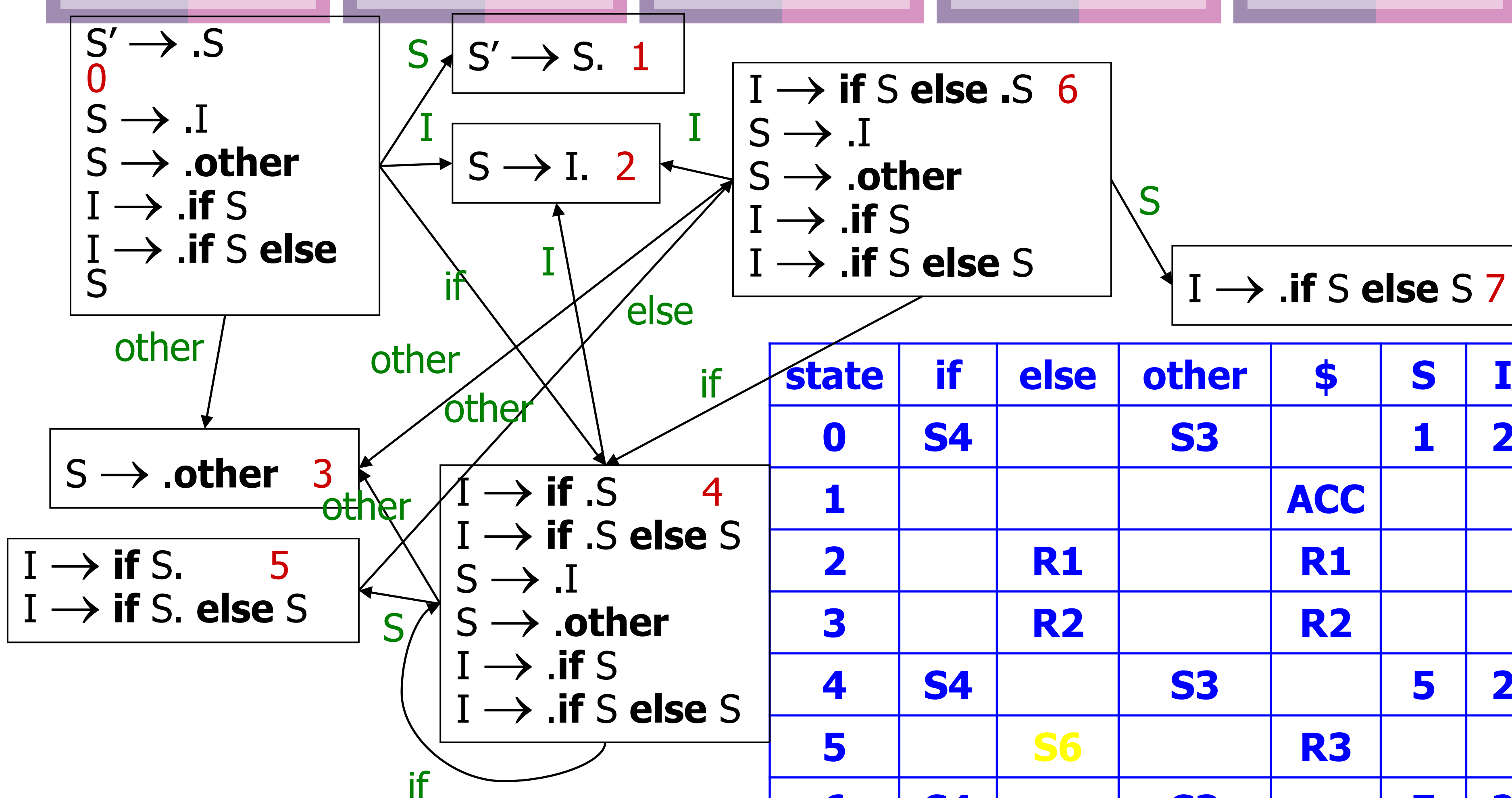
$S \rightarrow (S)S \mid \lambda$

State	(	)	$	S
0	S2	R2	R2	1
1			AC	
2	S2	R2	R2	3
3		S4		
4	S2	R2	R2	5
5		R1	R1	

Disambiguating Rules for Parsing Conflict

- Shift-reduce conflict
  - Prefer shift over reduce
    - In case of nested if statements, preferring shift over reduce implies most closely nested rule for dangling else
- Reduce-reduce conflict
  - Error in design

Dangling Else



state	if	else	other	$	S	I
0	S4		S3		1	2
1				ACC		
2		R1		R1		
3		R2		R2		
4	S4		S3		5	2
5		S6		R3		
6	S4		S3		7	2
7		R4		R4		

End