

1)

$$a) \quad \mathbb{P}(X=x \mid x > 0) = \frac{\lambda^x}{x! (e^\lambda - 1)} \quad , \quad x=1, 2, \dots$$

$$\text{versemblanza: } \prod_{i=1}^n \frac{\lambda^{x_i}}{x_i! (e^\lambda - 1)} = \frac{\lambda^{\sum x_i}}{\prod (x_i!) \cdot (e^\lambda - 1)^n}$$

$$\text{log-versemblanza: } \log \left(\frac{\lambda^{\sum x_i}}{\prod (x_i!) \cdot (e^\lambda - 1)^n} \right) = \log(\lambda)^{\sum x_i} - \log(\prod (x_i!) \cdot (e^\lambda - 1)^n) = \sum x_i \log(\lambda) - (\sum \log(x_i!) + \log(e^\lambda - 1)^n) =$$

$$= \sum x_i \log(\lambda) - \sum \log(x_i!) - \log(e^\lambda - 1)^n =$$

$$= \boxed{\sum x_i \log(\lambda) - \sum \log(x_i!) - n \log(e^\lambda - 1)}$$

$$\text{Fisher score: } \frac{d \ell(\lambda | \mathcal{X})}{d\lambda} = \sum x_i \cdot \left(\frac{1}{\lambda} \right) - 0 - n \cdot \left(\frac{e^\lambda}{e^\lambda - 1} \right) =$$

$$= \boxed{\frac{\sum x_i}{\lambda} - \frac{n e^\lambda}{e^\lambda - 1}}$$