Universitat Autònoma de Barcelona

Facultat de Ciències

Seminari 3: Series de Fourier

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Índex

1 Exercici 3.2.4

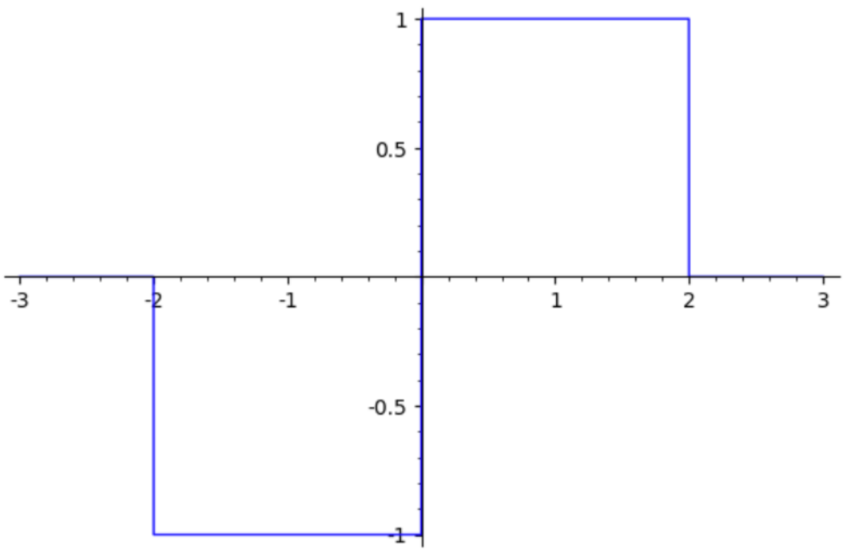
Feu el mateix amb la funció $P_T, a(x)(0 < 2a < T)$ que val $sign(x)$ a (a, a) i zero als intervals $[T/2, a]$ i $[a, T/2]$.

(a) **Dibuixar P_T, a (doneu valors concrets de T i a)**

Per dibuixar P_T, a, farem us de la comanda *piecewise*.

```
var('x,T,a',domain = 'real')
var('n',domain = 'integer')
assume(a>0)
assume(T>2*a)

a = 2
T = 6
P = piecewise(([-T/2,-a],0),((-a,0),-1),([0,a],1),((a,T/2),0)],var=X)
plot(P,(x,-T/2,T/2))
```

Output:



(b) **Trobar a_0.**

```
a0 = (2/T)*integral(1-abs(x)/a,(x,-T/2,T/2)).simplify_full()
show(a0)
```

Output:

Executant aquest codi, el *show* ens mostra el valor d' a_0 que és $\frac{1}{2}$.

(c) **Trobar els coeficients de Fourier a_n i b_n. Calcular c_n.**

```
an=2/T*(integral(-1*cos(2*pi*n*x/T),(x,-a,0))+
    integral(1*cos(2*pi*n*x/T),(x,0,a))).simplify_full()
show(an)

bn=2/T*(integral(-1*sin(2*pi*n*x/T),(x,-a,0))+
    integral(1*sin(2*pi*n*x/T),(x,0,a))).simplify_full()
show(bn)

cn = 1/2*(an-I*bn)
show(cn)
```

Output:

$$a_n = 0$$

$$b_n = -\frac{2(cos(\frac{3}{2}\pi) - 1)}{\pi n}$$

$$c_n = \frac{i(cos(\frac{3}{2}\pi) - 1)}{\pi n}$$

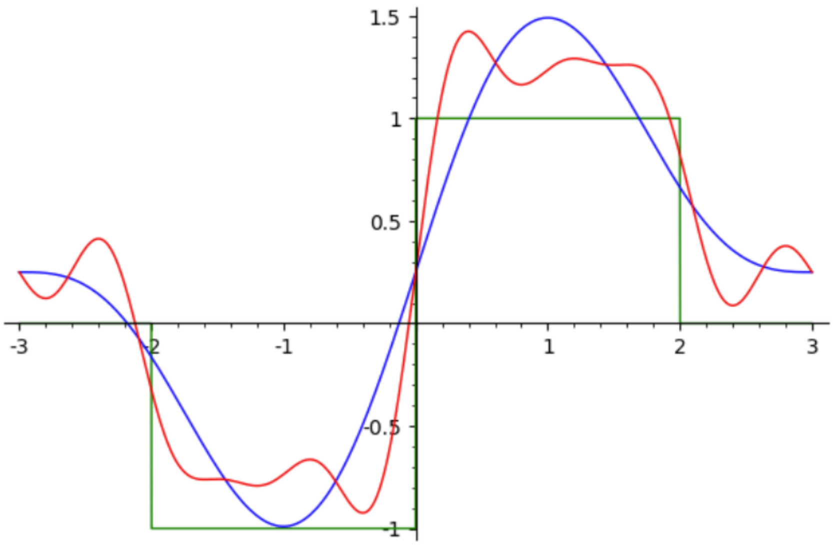
(d) **Dibuixar el polinomis de Fourier $s_7(x)$ i $s_3(x)$ i comparar amb la gràfica de $P_{T,a}$.**

```
s3=a0/2+sum(bn(n)*sin(2*pi*n*x/T) for n in range(1,4))
s7=a0/2+sum(bn(n)*sin(2*pi*n*x/T) for n in range(1,8))
Q = s3.plot(x,-T/2,T/2)+s7.plot(x,-T/2,T/2, color = 'red')
W = plot(P,(x,-T/2,T/2), color = 'green')
show(W+Q)
```

Output:

$s_3 = \frac{sin(6x}{3\pi} + \frac{sin(4x}{2\pi} + \frac{sin(2x}{\pi} + \frac{1}{2}$

$s_7 = 3\frac{sin(\frac{7}{3}\pi \cdot x}{7\pi} + 3\frac{sin(\frac{5}{3}\pi x}{5\pi} + 3\frac{sin(\frac{4}{3}\pi x}{4\pi} + 3\frac{sin(\frac{2}{3}\pi x}{2\pi} + 3\frac{sin(\frac{1}{3}\pi x}{1\pi} + \frac{1}{4}$



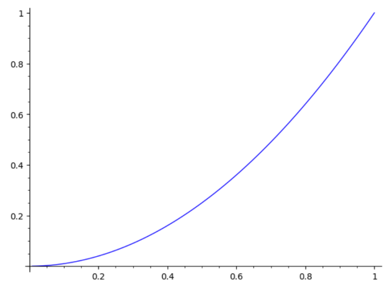
2 Exercici 3.3.6

Feu el mateix que a l'exercici anterior amb la funció $f(x) = x^2$ quan $x \in [0, 2\pi]$.

(a) **Definiu aquest funció en Sage amb la comanda piecewise.**

```
var('x')
P = piecewise([([0,2*pi],x^2)],var=x)
plot(P)
```

Output:



(b) **Trobeu els a_n, b_n quan n va de 0 a 10.**

```
an=[P.fourier_series_cosine_coefficient(n) for n in range(11)]
bn=[P.fourier_series_sine_coefficient(n) for n in range(11)]
show(an,bn)
```

Output:

	0	1	2	3	4	5	6	7	8	9	10
a	$\frac{8}{3}\pi^2$	4	1	$\frac{4}{9}$	$\frac{1}{4}$	$\frac{4}{25}$	$\frac{1}{9}$	$\frac{4}{49}$	$\frac{1}{16}$	$\frac{4}{81}$	$\frac{1}{25}$
b	0	-4π	-2π	$-\frac{4}{3}\pi$	$-\pi$	$-\frac{4}{5}\pi$	$-\frac{2}{3}\pi$	$-\frac{4}{7}\pi$	$-\frac{1}{2}\pi$	$-\frac{4}{9}\pi$	$-\frac{2}{5}\pi$

(c) **Trobeu els polinomis trigonomètric d'ordre 5 i d'ordre 10.**

```
T = 2*pi
s5=sum(an[n]*cos(2*pi*n*x/T) +bn[n]*sin(2*pi*n*x/T) for n in range(0,5))
show(s5)

s10=sum(an[n]*cos(2*pi*n*x/T) +bn[n]*sin(2*pi*n*x/T) for n in range(0,10))
show(s10)
```

Output:

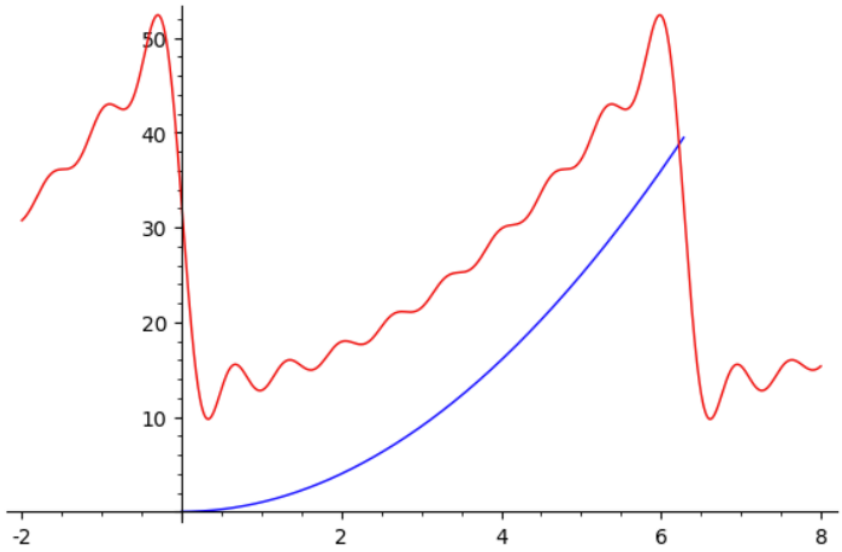
$$\begin{aligned} s_5 =& \frac{8}{3}\pi^2 - \pi sin(4x - \frac{4}{3}\pi sin(3x) - 2\pi sin(2x) - 4\pi sin(x) + \frac{1}{4}cos(4x) + \frac{4}{9}cos(3x) \\ &+ cos(2x) + 4cos(x) \end{aligned}$$

$$\begin{aligned} s_{10} =& \frac{8}{3}\pi^2 - \frac{4}{9}\pi sin(9x) - \frac{1}{2}\pi sin(8x) - \frac{4}{7}\pi sin(7x) - \frac{2}{3}\pi sin(6x) - \frac{4}{5}\pi sin(5x) - \pi sin(4x) \\ &- \frac{4}{3}\pi sin(3x) - 2\pi sin(2x) - 4\pi sin(x) + \frac{4}{81}cos(9x) + \frac{1}{16}cos(8x) + \frac{4}{49}cos(7x) \\ &+ \frac{1}{9}cos(6x) + \frac{4}{25}cos(5x) + \frac{1}{4}cos(4x) + \frac{4}{9}cos(3x) + cos(2x) + 4cos(x) \end{aligned}$$

(d) **Feu un gràfic que compari les gràfiques de T i s_10. S'observa fenomen de Gibbs?**

```
P1 = plot(P, (x,0,2*pi))
P2 = plot(s10,(x,-2,8), color = 'red')
show(P1+P2)
```

Output:

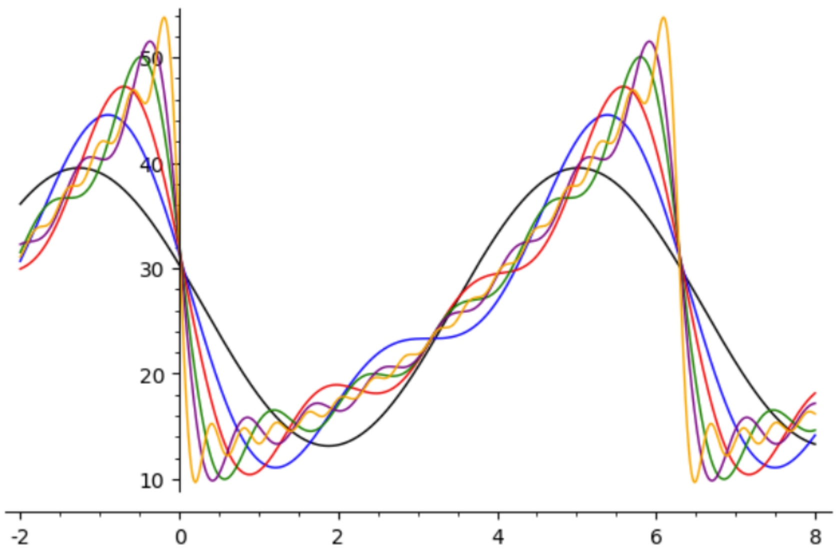


Com és pot observar al gràfic, la funció s_{10} (sèrie de Fourier d'ordre 10) pateix el fenòmen de *Gibbs* (fent que als extrems de la "periodicitat" vagi al punt mig).

(e) **Dibuixeu, en un sol gràfic, els polinomis s_n per n = 1, 2, 3, 5, 7, 15.**

```
n_list = [1, 2, 3, 5, 7, 15]
s_list = []
plot_list = []

an=[P.fourier_series_cosine_coefficient(n) for n in range(16)]
bn=[P.fourier_series_sine_coefficient(n) for n in range(16)]

for i in n_list:
    s_list.append(sum(an[n]*cos(2*pi*n*x/T) \
                         + bn[n]*sin(2*pi*n*x/T) for n in range(0,i+1)))

color_list = ['black','blue','red','green','purple','orange']

for i,j in zip(s_list,color_list):
    plot_list.append(plot(i,(x,-2,8), color = j))

show(sum(plot_list))
```

Output:

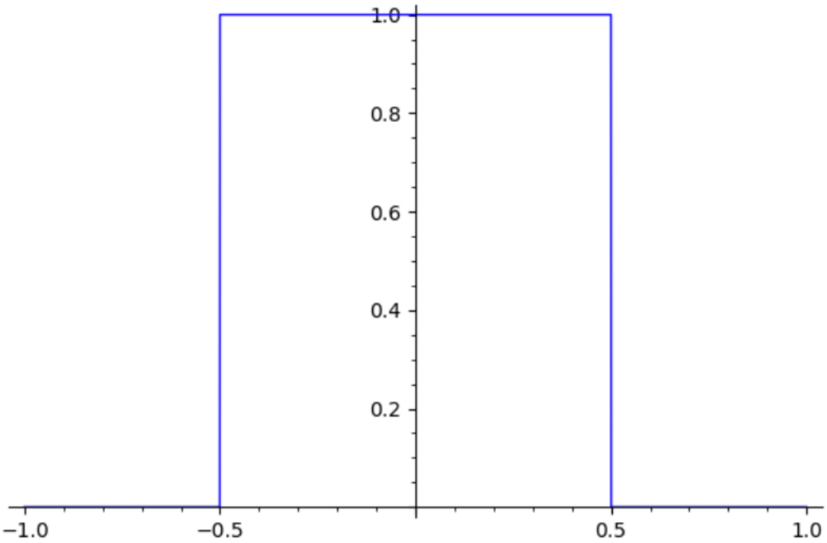


3 Exercici 3.7.11

(a) Reescaleu els coeficients i poseu $C_n = Tc_n$. Dibuixeu els punts $\{(n/T, C_n) : |n/T| \leq 8\}$ quan $T = 2, 4, 16, 30$ (és el dibuix de l'espectre amb signe).

```
var('x')
T = 2
P = piecewise([([-T/2,-1/2],0),((-1/2,1/2),1),((1/2,T/2),0)],var=x)
plot(P)
```

Output:




```
var('n','t')

an=2/t*(integral(cos(2*pi*n*x/t),(x,-1/2,1/2))).simplify_full()

bn=2/t*(integral(sin(2*pi*n*x/t),(x,-1/2,1/2))).simplify_full()

cn = 1/2*(an-I*bn)

show(an)
show(bn)
show(cn)
```

Output:

$$a_n = \frac{2sin(\frac{\pi n}{t})}{\pi n}$$

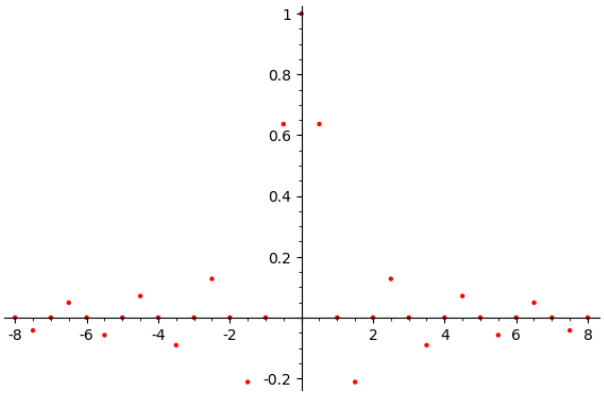
$$b_n = 0$$

$$c_n = \frac{sin(\frac{\pi n}{t})}{\pi n}$$

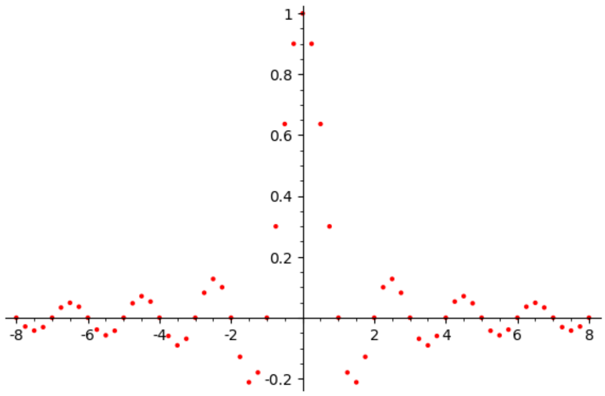
Com que $b_n = 0$, es calcula c_n com $\frac{1}{2} \cdot a_n$.

```
fac = 1
for T in [2, 4, 16, 30]:
    cn=1/T*(integral(cos(2*pi*n*x/T),(x,-1/2,1/2))).simplify_full()

    r1 = range(-8*T*fac,0)
    r2 = range(1,8*T*fac+1)
    p = [(n/fac/T,cn(n/fac)*T) for n in r1]
    p += [(n/fac/T,cn(n/fac)*T) for n in r2]
    show(points(p,color='red', pointsize = 10) )
```

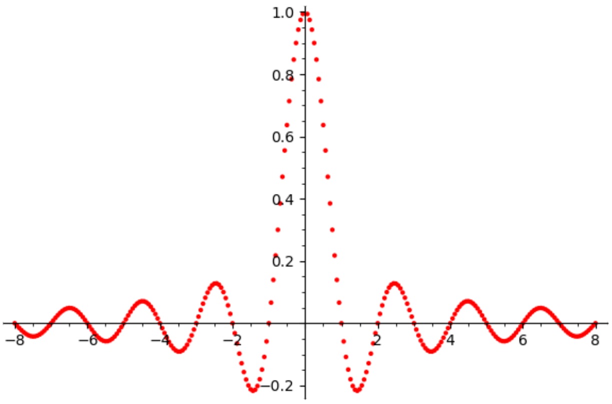
Output:



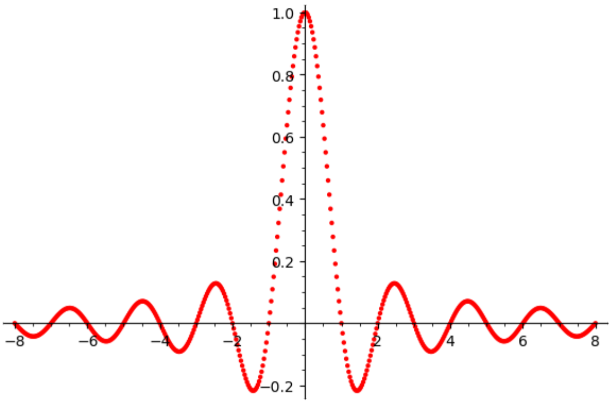
(a) $T = 2$



(b) $T = 4$



(c) $T = 16$



(d) $T = 30$