AIMS – Cameroon
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AIMS CAMEROON SEMINAR
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On the convergence of derivatives of Bernstein Approximation

By

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Abstract:

This work is concerned with sequences of polynomials named after their creator S. N. Bernstein. Given a function f on [0, 1], we can define the Bernstein polynomial. Thus there is a sequence of Bernstein polynomials corresponding to each function f . If f is continuous on [0, 1], its sequence of Bernstein polynomials converges uniformly to f on [0, 1], thus giving a constructive proof of Weierstrass's Theorem. One might wonder why Bernstein created "new"polynomials for this purpose, instead of using polynomials that were already known to mathematics. Taylor polynomials or interpolating polynomials are not appropriate. For even setting aside questions of convergence, Taylor polynomials are applicable only to functions that are infinitely differentiable, and not to all continuous functions. We will consider how Bernstein discovered his polynomials, and extend it to a class of function continuous to any compact [a,b] and study its uniform convergence as well as the uniform convergence of the derivative (based on additional assumptions). We will also see that although the convergence of the Bernstein polynomials is slow, they have compensating "shape-preserving" properties. For example, the Bernstein polynomial of a convex function is itself convex.

Keywords: Bernstein polynomials, Taylor polynomials, uniform convergence, convex function.