CMSI 485 – Classwork 3

Instructions:
This worksheet will not only provide you with practice problems for your upcoming exam, but will add to your deep understanding of the mechanics of many probabilistic reasoning systems.

- Provide answers to each of the following questions and write your responses in the blanks. If you are expected to show your work in arriving at a particular solution, space will be provided for you.
- Place the names of your group members below:

Group Members:

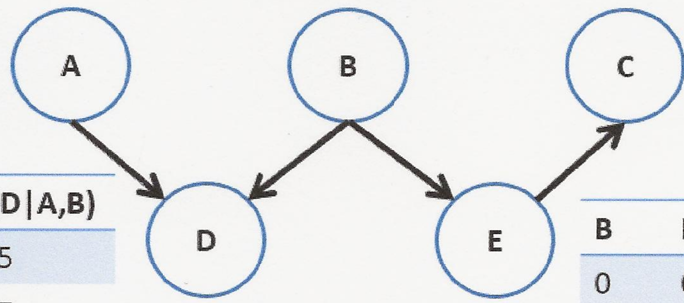
1. Ona Igbinedion
2. Thomas Kelly
3. Raul Rodriguez
4. ____________

Problem 1 - Bayesian Network Exact Inference

Consider the following Bayesian Network and use it to answer the questions that follow.

B	P(B)
0	0.6

A	P(A)
0	0.8



E	C	P(C|E)
0	0	0.4
1	0	0.2

A	B	D	P(D|A,B)
0	0	0	0.5
0	1	0	0.7
1	0	0	0.8
1	1	0	0.1

B	E	P(E|B)
0	0	0.9
1	0	0.4

While examining Exact Inference in Bayesian Networks, we saw some methods for *simplifying* queries and the resulting computations that can yield large performance improvements when implemented. E.g., variables whose CPTs never affect the query outcome can be ignored.

1.1. For each of the following queries, determine *which variables' CPTs* will at all affect the answer to the query. Justify your responses in the boxes that follow. Hint: in your justification, express the query in terms of: $P(Q|e)=\alpha\sum P(\ldots)$ with $\alpha=\frac{1}{P(e)}$

1.1.1. $P(A|B=b,D=d)$

CPTs Used: yes A yes B no C yes D no E

Justification:

$P(A|B=b,D=d) = \alpha\, P(A)\, P(B=b)\, P(D=d\,|\,A,B=b)$

1.1.2. $P(E|D=d)$

CPTs Used: yes A yes B no C yes D yes E

Justification:

$$P(E|D=d) = \alpha \sum_{B} \sum_{A} P(E|B=b)\, P(B)\, (P(D=d|A=a, B=d)\, P(A=a)$$

$$= \alpha \sum_{B} P(E|B=b) \cdot P(B=b) \sum_{A} P(D=d|A=a, B=b) \cdot P(A=a)$$

1.2. Using the Bayesian Network on the previous page, find the solutions to the following.

1.2.1. $P(A=0 \vee B=1, D=1)$ *(Box your answer once finished)*

$$A=0: \overset{0.8}{P(A=0)}\ \overset{0.4}{P(B=1)} \cdot \overset{0.3}{P(D=1|A=0, B=1)}$$

$$A=1: \overset{0.2}{P(A=1)} \cdot \overset{0.4}{P(B=1)} \cdot \overset{0.9}{P(D=1|A=1, B=1)}$$

$$\alpha = \frac{1}{(0.096 + 0.072)} = 5.952$$

$$P(A=0|B=1, D=1) = \frac{0.096}{5.95}$$

$$P(A=0|B=1, D=1) = \boxed{0.0161}$$

1.2.2. $P(B=1 \vee A=0, D=1, E=0, C=1)$ *(Box your answer once finished)*

$$P(B=1|A=0, D=1, E=0, C=1)$$

$$B=1: \overset{0.4}{P(B=1)} \cdot \overset{0.8}{P(A=0)} \cdot \overset{0.3}{P(D=1|A=0, B=1)} \cdot \overset{0.4}{P(E=0|B=1)}$$

$$\cdot \overset{0.6}{P(C=1|E=0)} = 0.02304$$

$$B=0: \overset{0.6}{P(B=0)} \cdot \overset{0.8}{P(A=0)} \cdot \overset{0.5}{P(D=1|A=0, B=0)}\, \overset{0.9}{P(E=0|B=0)}$$

$$\cdot \overset{0.6}{P(C=1|E=0)} = 0.1296$$

$$\alpha = \frac{1}{0.02304 + 0.1296} = 6.551$$

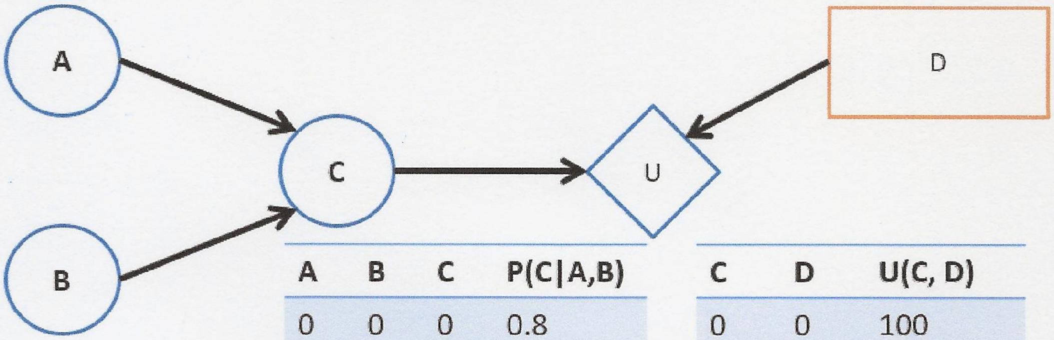
$$P(B=1|A=0, D=1, E=0, C=1) = \frac{0.02304}{6.551} = \boxed{0.003517}$$

Problem 2 - Decision Networks & MEU

Use the following Decision Network with chance nodes A, B, C, decision node $D \in \{0,1\}$, and utility node U to answer the questions that follow.

A	P(A)
0	0.2

B	P(B)
0	0.5



A	B	C	P(C|A,B)
0	0	0	0.8
0	1	0	0.6
1	0	0	0.1
1	1	0	0.5

C	D	U(C, D)
0	0	100
0	1	20
1	0	40
1	1	80

2.1. Find the $MEU(B=0)$ (i.e., the Maximum Expected Utility with evidence $B=0$).

MEU(B=0)

$\sum_{a\in A}\sum_{c\in C}\sum_{d\in D} P(B=0)\cdot P(A=a)\cdot P(C=c \mid A=a, B=b)\cdot U(C=c, D=d)$

$P(B=0)\sum_{a\in A} P(A=a)\sum_{c\in C} P(C=c \mid A=a, B=0)\sum_{d\in D} U(C=c, D=d)$

$\cdot\, P(A \overset{0.2}{=} 0)\cdot P(C=0 \mid A=0, B=0)\, U(C=0, D=0) = 16$

0.2 0.8 100

$P(A=0)\, P(C=0 \mid A=0, B=0)\, U(C=0, D=1) = 3.2$

$P(A \underset{0.2}{=} 0)\, P(C \overset{0.8}{=} 1 \mid A=0, B=0)\, U(C \overset{20}{=} 1, D=0) = 1.6$

$P(A \underset{0.2}{=} 0)\, P(C \overset{0.2}{=} 1 \mid A=0, B=0)\, U(C \overset{40}{=} 1, D=1) = 3.2$

$P(A \underset{0.2}{=} 0)\, P(C \overset{0.2}{=} 1 \mid A=1, B=0)\, U(C \overset{80}{=} 0, D=0) = 8$

$P(A \underset{0.1}{=} 1)\, P(C \overset{0.2}{=} 0 \mid A=1, B=0)\, U(C \overset{100}{=} 0, D=1) = 1.6$

$P(A \underset{0.1}{=} 1)\, P(C \overset{0.1}{=} 0 \mid A=1, B=0)\, U(C \overset{20}{=} 1, D=0) = 28.8$

$P(A \underset{0.1}{=} 1)\, P(C \overset{0.1}{=} 1 \mid A=1, B=0)\, U(C \overset{40}{=} 1, D=1) = 57.6$

$P(A \underset{0.1}{=} 1)\, P(C = 1 \mid A=1,$ 0.9 80

$\boxed{MEU(B=0) = 65.6}$

2.2. Given your computations above, what decision should your agent make by MEU?

The agent should choose action D=1

Problem 3 – Value of Perfect Information

Using the network and your answer from the previous problem, we're going to compute the Value of Perfect Information (VPI) of knowing the state of variable A when $B=0$ is given. Let's do so step-by-step:

3.1. Find $MEU(A=0, B=0)$.

$MEU(A=0, B=0$

$\sum_{c \in C} \sum_{d \in D} P(C=c \mid A=0, B=0) \cdot U(C=c, D=d$

$D=0$ $\quad P(C=0 \mid \overset{0.8}{A}=0, B=0) \cdot U(C\overset{100}{=}0, D=0) +$
$P(C=1 \mid \overset{0.2}{A}=0, B=0 \quad U(C\overset{49}{=}1, D=0) = 88$

$D=1$ $\quad P(C\overset{0.8}{=}0 \mid A=0, B=0) \cdot U(C\overset{20}{=}0, D=1) +$
$P(C=1 \mid \overset{0.2}{A}=0, B=0) \cdot U(C=1\overset{80}{,} D=1) = 32$

$\boxed{MEU(A=0, B=0) = 88}$

3.2. Using the your answer to ~~4~~3.1 and knowledge that $MEU(A=1, B=0)=74$ (freebee!) find $MEU(A, B=0)$.

$MEU(A, B=0) = 88$ when $A=0$ $D=1$

$88 \cdot P(A=0 \mid B=0) + 74 \cdot P(A=1 \mid B=0)$

$P(A=0 \mid B=0) = \frac{P(A=0, B=0)}{P(B=0)} = \frac{P(A=0)\,\cancel{P(B=0)}}{\cancel{P(B=0)}}$

$\boxed{MEU = 88 \cdot 0.2 + 74 \cdot 0.8 = 76.8}$

3.3. Compute the $VPI(A \mid B=0)$.

$VPI(A \mid B=0) =$

$MEU(A, B=0) - MEU(B=0) =$

$76.8 - 65.6 = 11.2$

$\boxed{VPI(A \mid B=0) = 11.2}$

3.4. If the utility scores represent dollar amounts, what would be a fair price for A when $B=0$?

$11.20