Differential equation

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1 Differential equation

The given equation is $y' = xy^2 + 3xy, y(0) = 3$, which can be solved as separable variables:

$\frac{dy}{y^2+3y} = xdx$
$\int(\frac{dy}{y^2+3y}) = \int xdx$
$\frac{log_e \frac{y}{y+3}}{3} = \frac{x^2}{2} + C_1$
$y = -\frac{3e^{(3(C_1+\frac{x^2}{2}))}}{e^{(3(C_1+\frac{x^2}{2}))}-1}$ is the general solution.
Solve IVP:
$C_1 = \frac{log_e 2}{3}$
Find exact solution:
$\frac{log_e \frac{y}{y+3}}{3} = \frac{x^2}{2} + \frac{log_e 2}{3}$
$y = -\frac{3e^{\frac{3x^2}{2}}}{e^{\frac{3x^2}{2}}-2}$ is our exact solution.

The plot of the exact solution on the given interval $x \in [0, 5.5]$ you can find on Figure 1.

As you can see the behaviour is asymptotic. Since that I should manually handle this case. Neither of numerical methods are able to do it. To improve this, I separated the whole interval into two. From initial position to asymptot, which was find from exact solution.

All numerical methods are written in python3 in Jupyter Notebook with usage of Bokeh to draw plots. The reference to the source code of methods' implementation and their explanation is **here**. The common structure is number of functions written in different cells for convenience.

For the given equation all three methods coped with task with good approximation at step equaled to 0.01 already. However, near to asymptot the behaviour of the approximation is too unpredictable and make the main part of errors for all methods and steps. If you decrease the step, error decrease too. It means that these methods are appropriate for given task.

Below you can see plots of all required solutions with their global truncation error at step equaled 0.1 and 0.01.

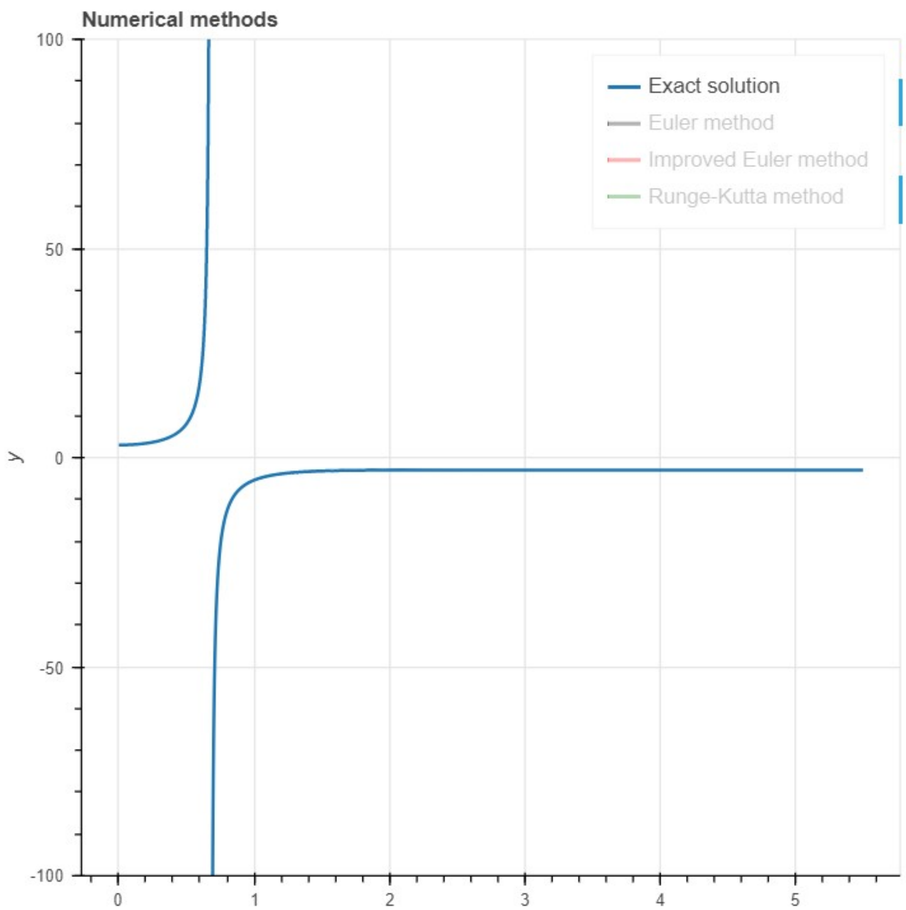

Figure 1: Exact solution.

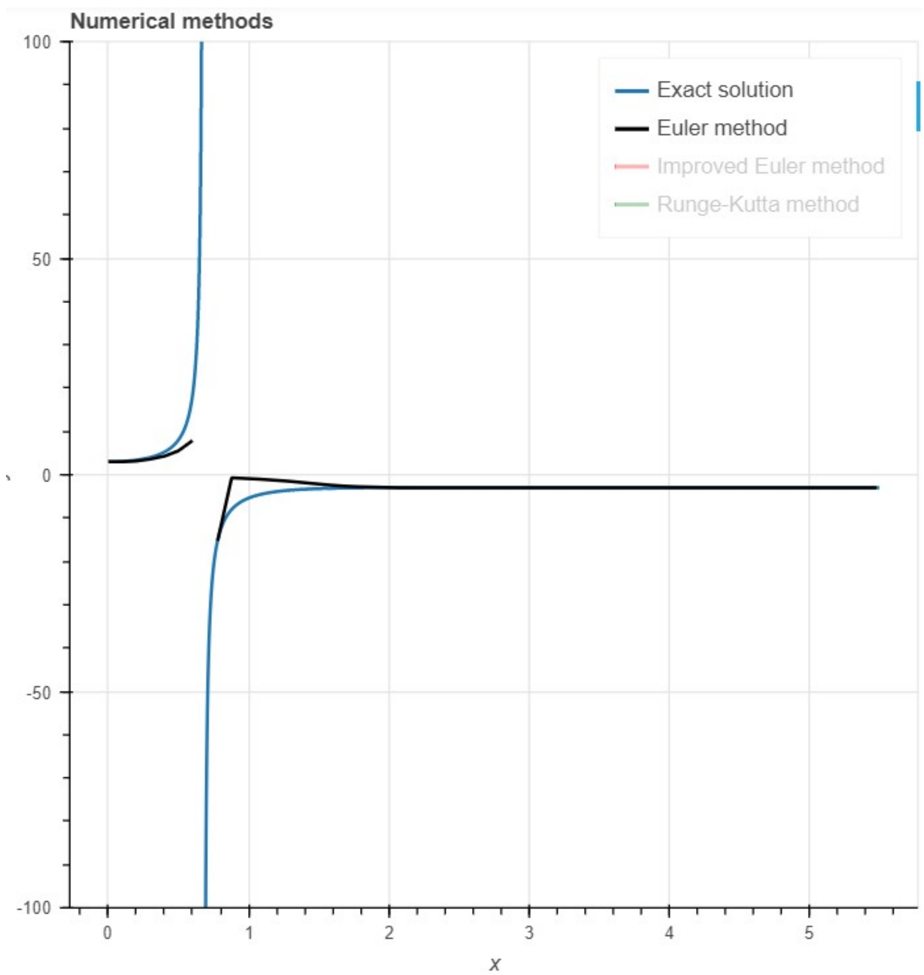

Figure 2: Euler method with h = 0.1.

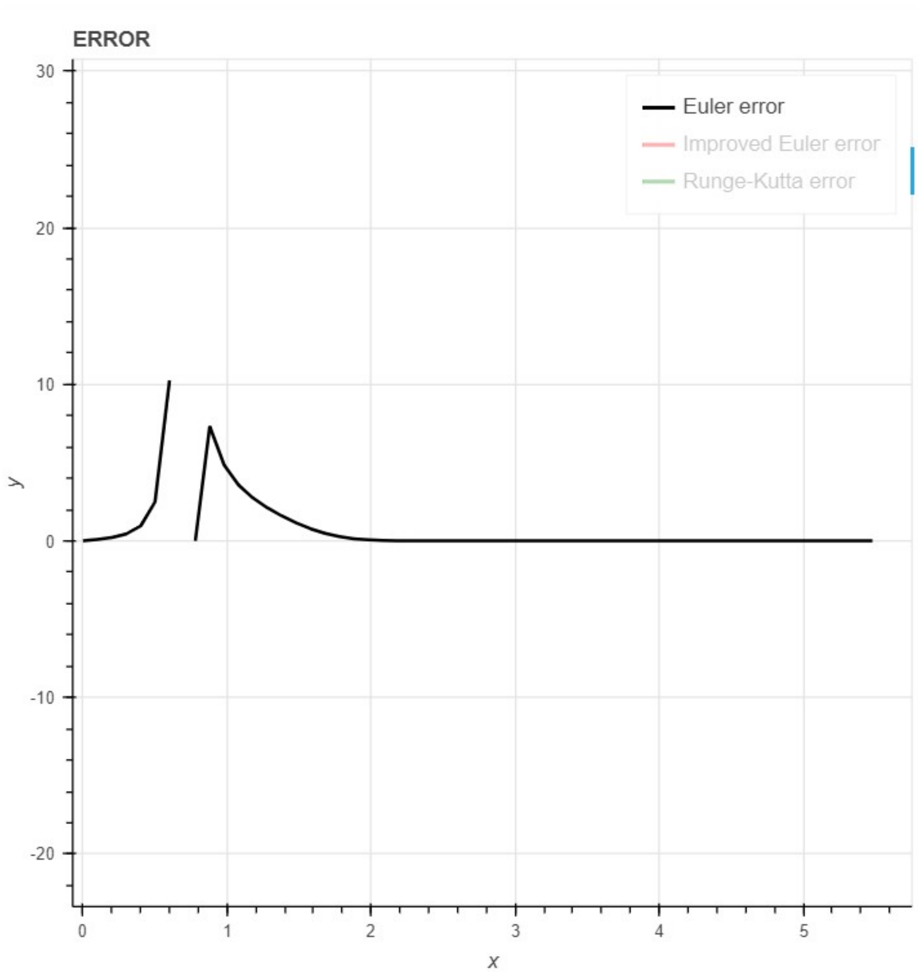

Figure 3: Euler method global truncation error with h = 0.1.

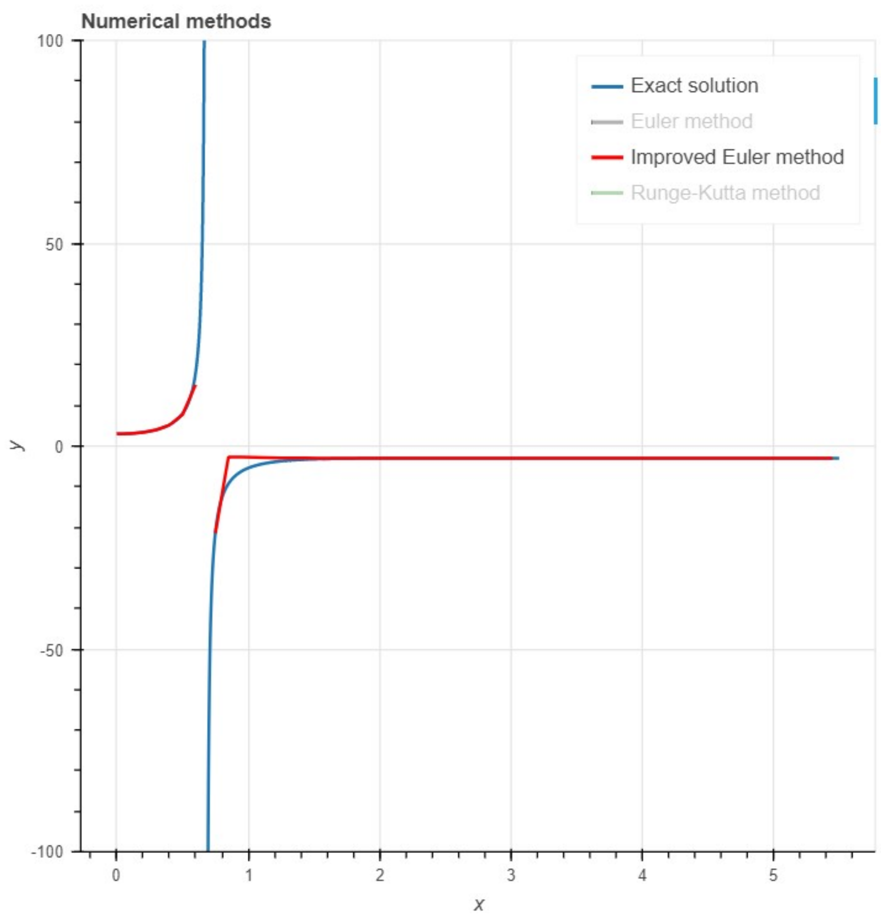

Figure 4: Improved Euler method with h = 0.1.

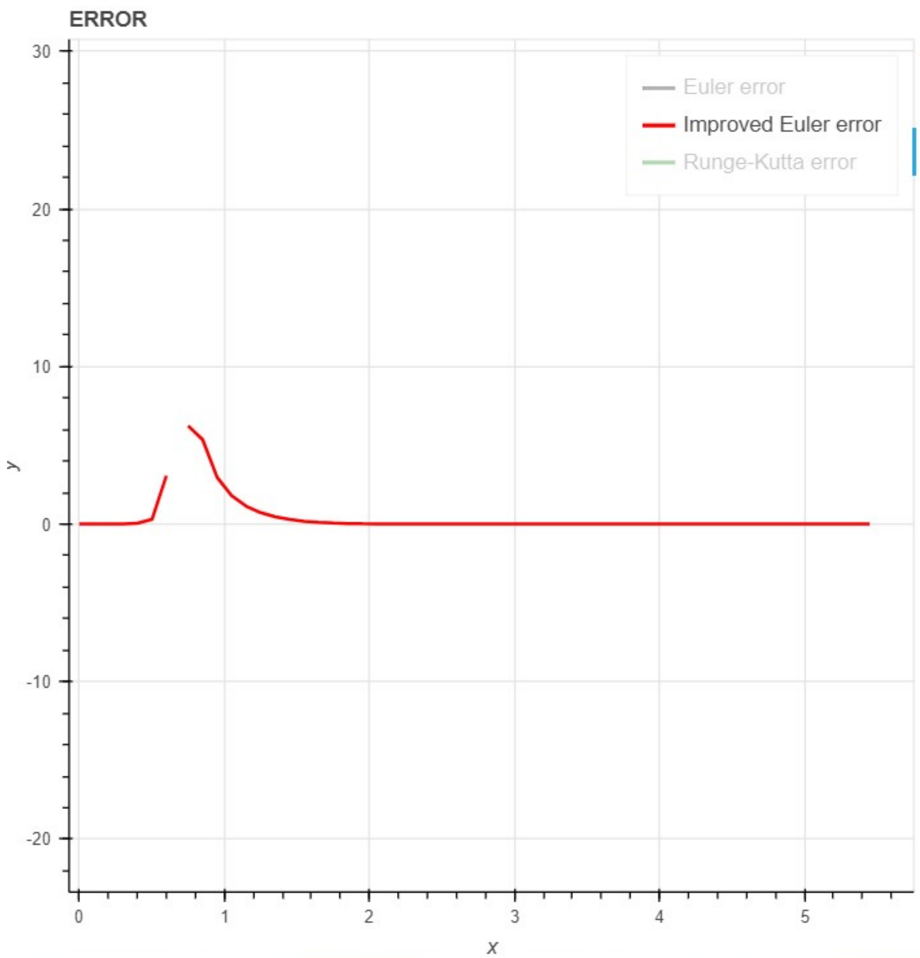

Figure 5: Improved Euler method with global truncation error h = 0.1.

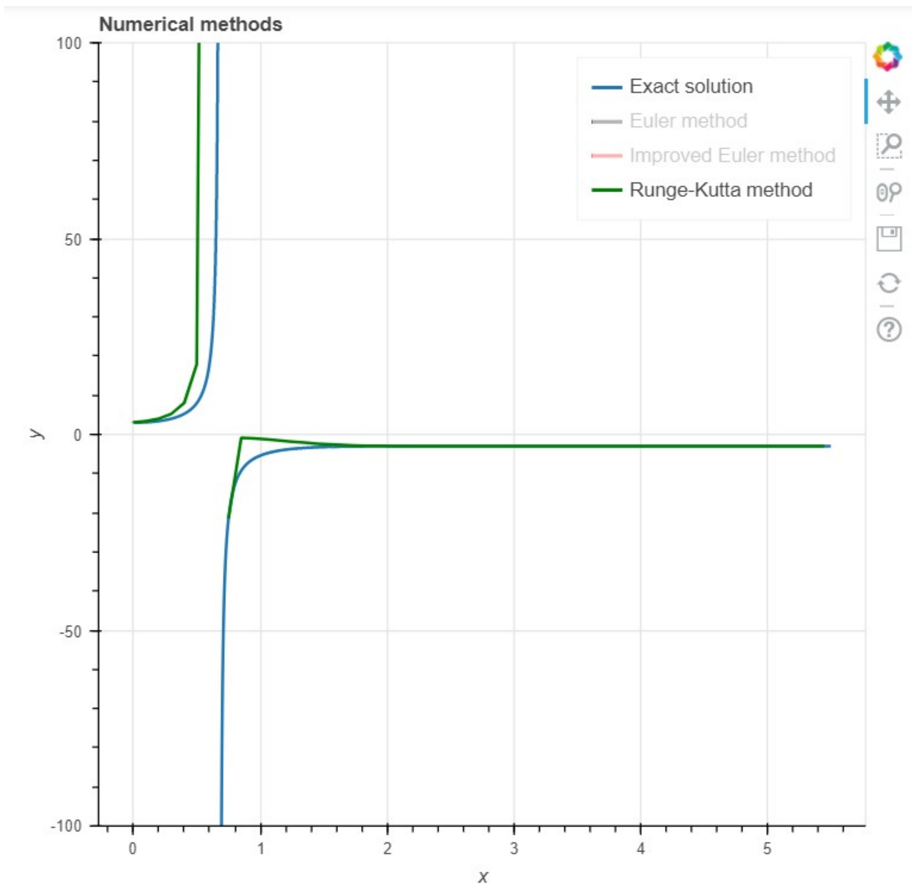

Figure 6: Runge-Kutta method with h = 0.1.

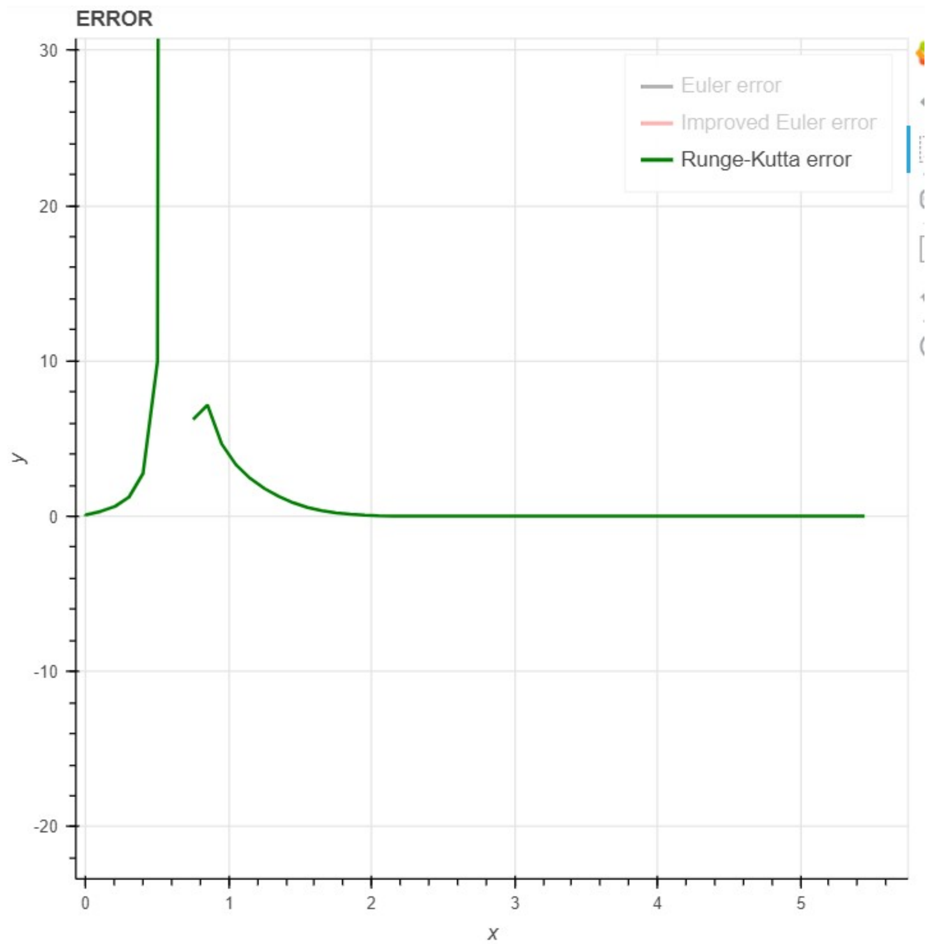

Figure 7: Runge-Kutta method global truncation error with h = 0.1.

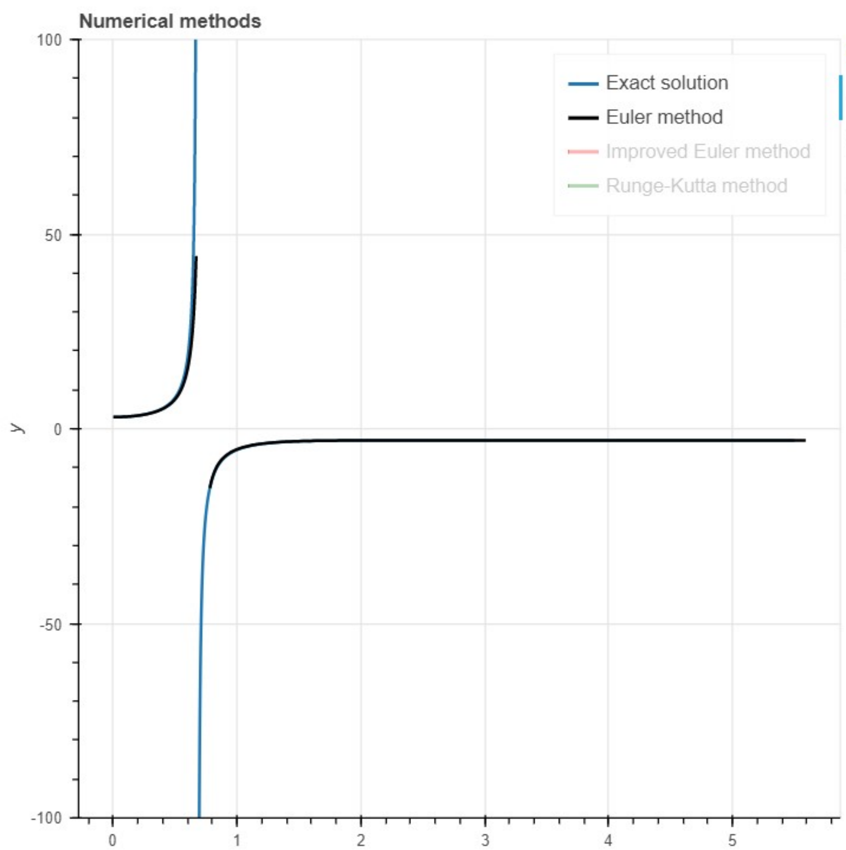

Figure 8: Euler method with h = 0.01.

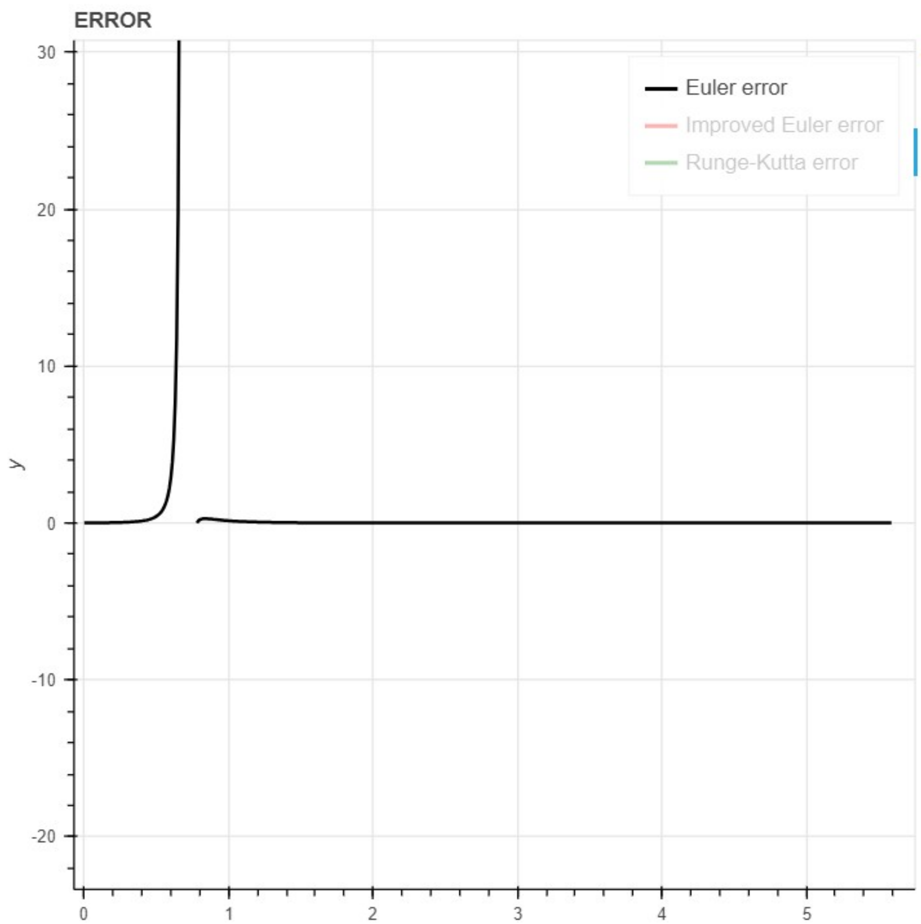

Figure 9: Euler method global truncation error with h = 0.01.

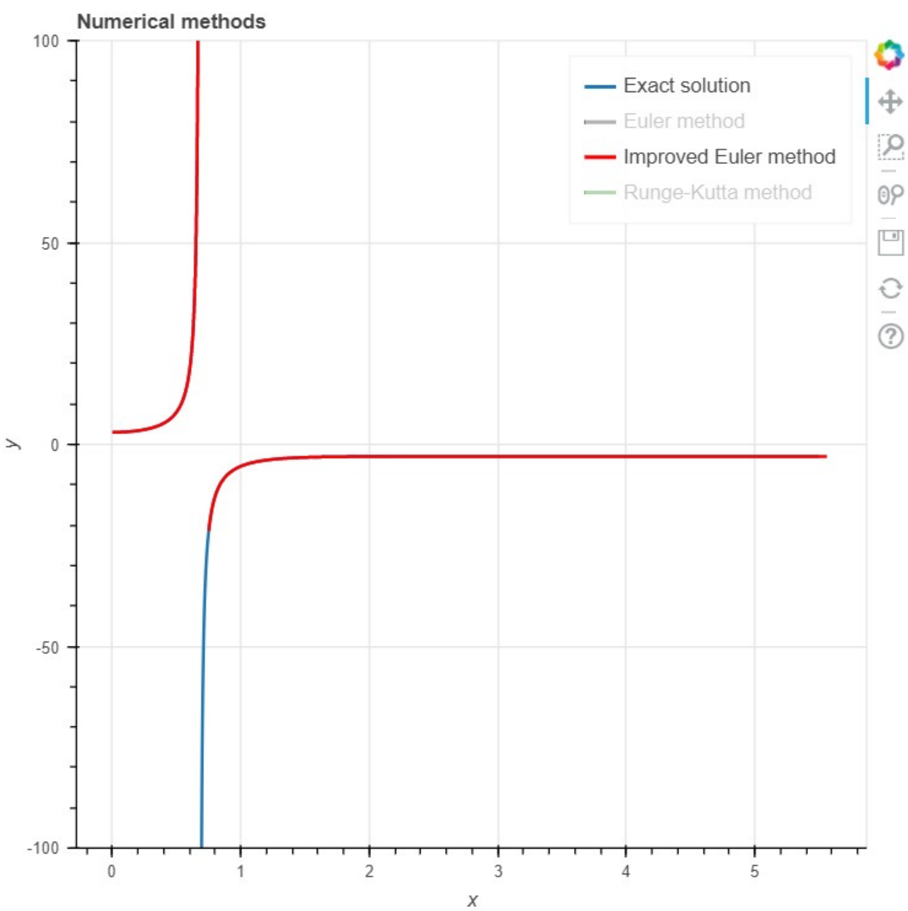

Figure 10: Improved Euler method with h = 0.01.

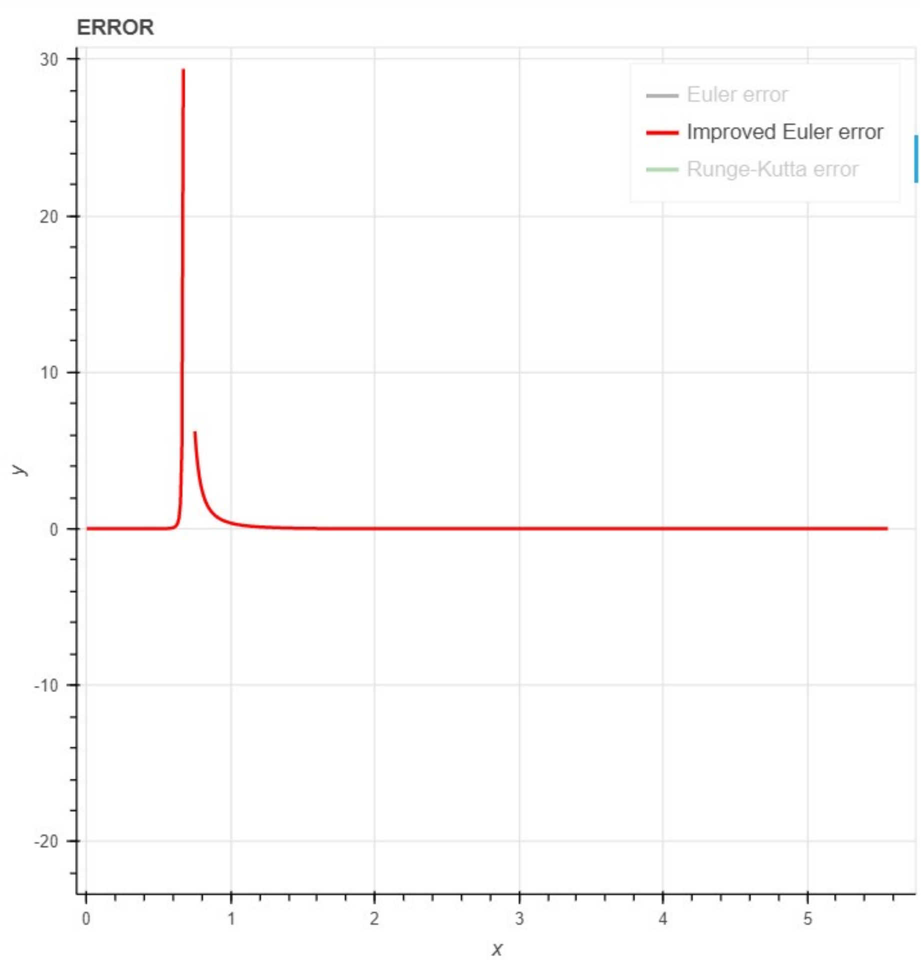

Figure 11: Improved Euler method with global truncation error h = 0.01.

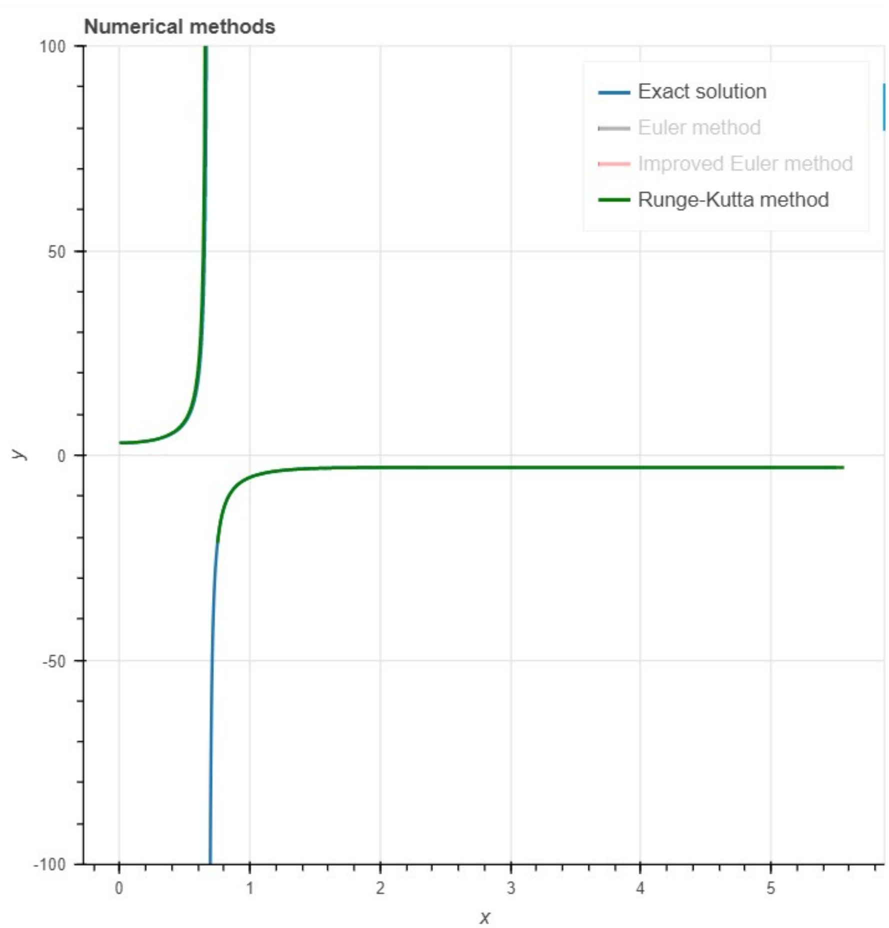

Figure 12: Runge-Kutta method with h = 0.01.

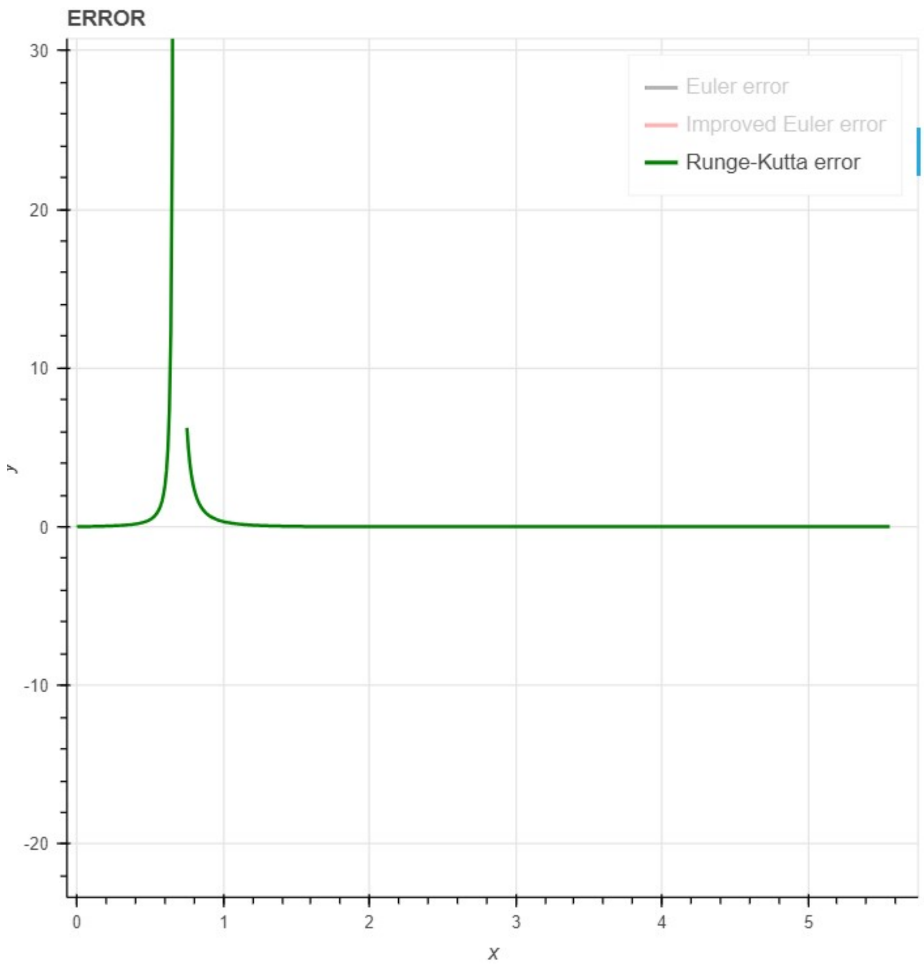

Figure 13: Runge-Kutta method global truncation error with h = 0.01.