

# Computational Graph and Linear Regression

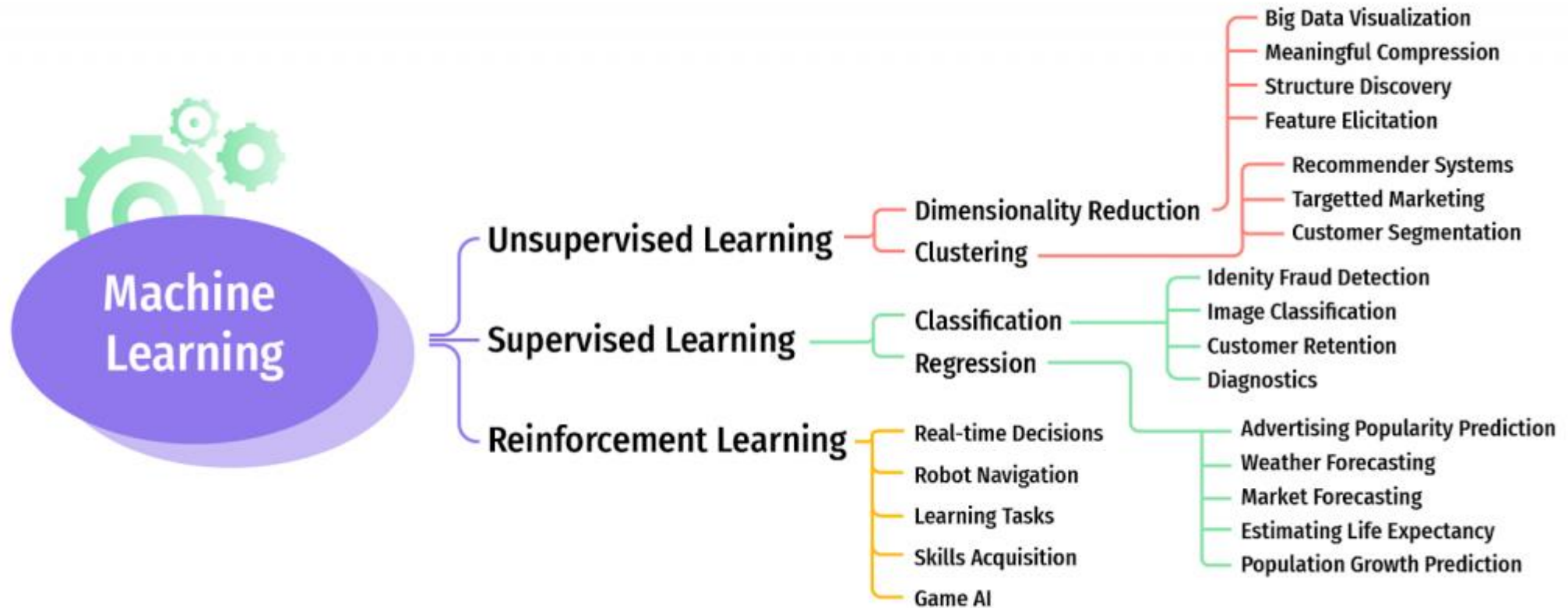
Quang-Vinh Dinh  
Ph.D. in Computer Science

# Outline

- Machine Learning
- Derivative/Gradient
- Linear Regression
- Computational Graph
- Generalized formulae

# Machine Learning

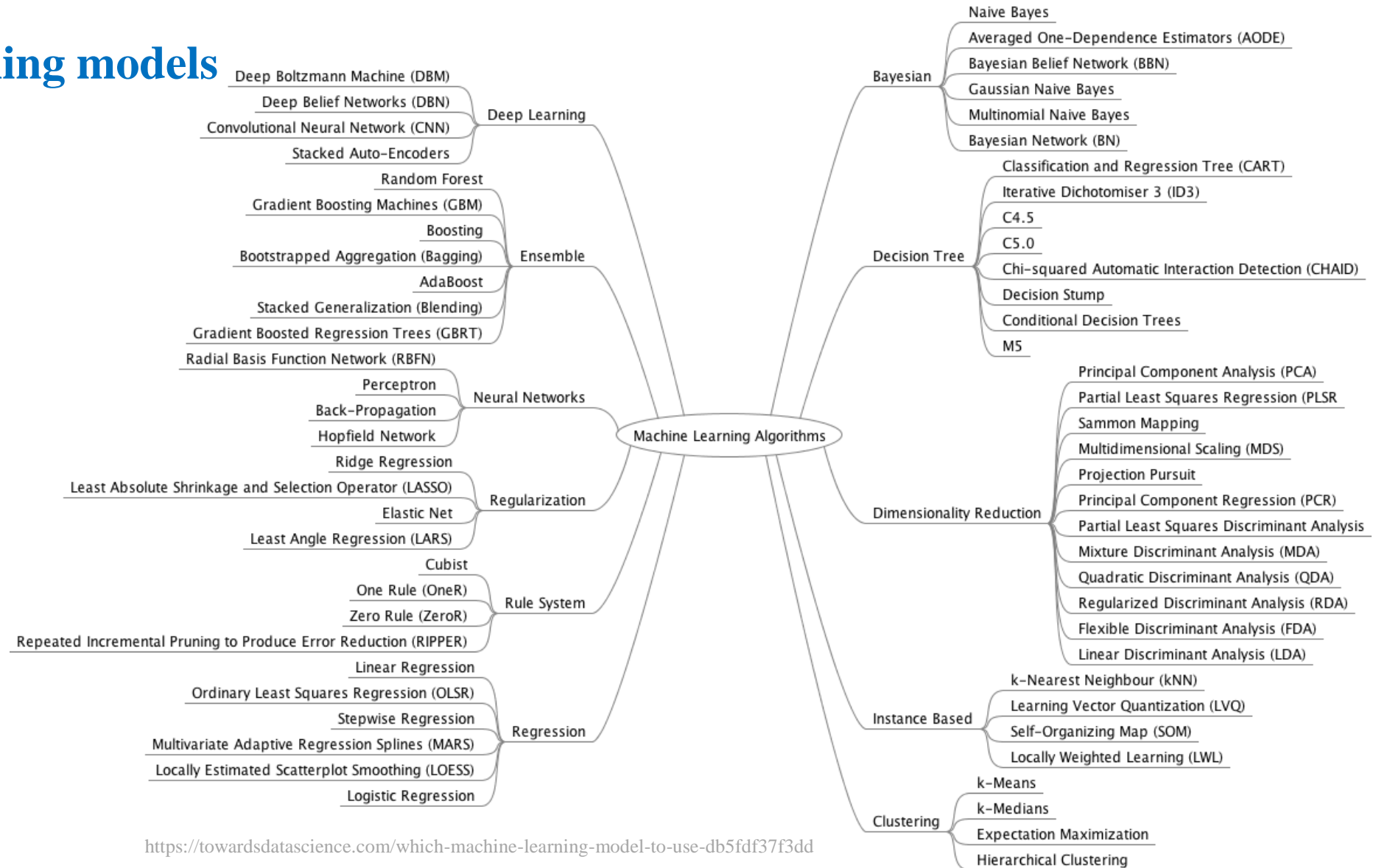
## ❖ Overview



<https://idapgroup.com/blog/types-of-machine-learning-out-there/>

# Machine Learning

## Machine learning models



# Machine Learning

## ❖ Supervised learning

Input and output  
data is provided

■ Training data

■ Cats

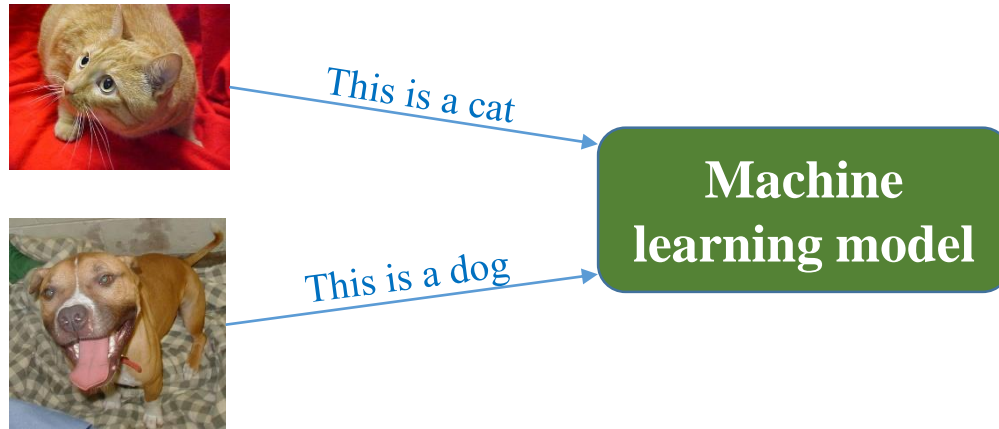
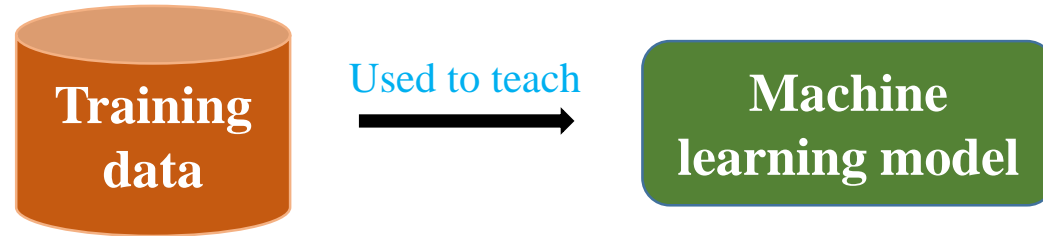
■ Dogs



From Cat-Dog dataset

# Machine Learning

## ❖ Supervised learning

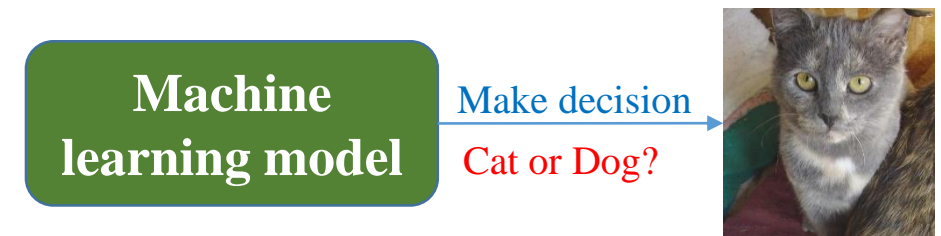


**Training phase**

From Cat-Dog dataset



**Testing data ( $\neq$  training data)**



**Testing phase**



# Machine Learning

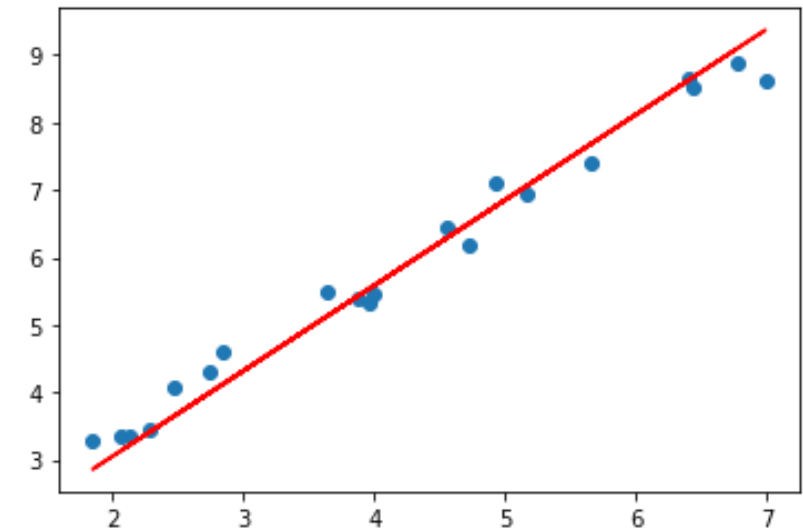
## ❖ Supervised learning

### ❖ Regression (prediction)

Linear regression models ← Linear equations

Linear equation =  $w_1x_1 + w_2x_2 + \dots + w_nx_n + b$

where  $\mathbf{w}$  is a weight vector  
and  $\mathbf{x}$  is feature vector



# Machine Learning

- ❖ Supervised learning
  - ❖ Linear Regression: Data processing

	Feature	Label	
	area	price	
	6.7	9.1	
	4.6	5.9	
	3.5	4.6	
	5.5	6.7	

House price data

$$\text{Model: } y = w_1 x_1 + b$$

$$\text{price} = a * \text{area} + b$$

Features			Label
TV	↕ Radio	↕ Newspaper	↕ Sales
230.1	37.8	69.2	22.1
44.5	39.3	45.1	10.4
17.2	45.9	69.3	12
151.5	41.3	58.5	16.5
180.8	10.8	58.4	17.9

Advertising data

$$\text{Model: } y = w_1 x_1 + w_2 x_2 + w_3 x_3 + b$$

$$\text{Sale} = w_1 * TV + w_2 * Radio + w_3 * Newspaper + b$$



# Machine Learning

## ❖ Supervised learning

### ❖ Linear Regression: Data processing

Boston House  
Price Data

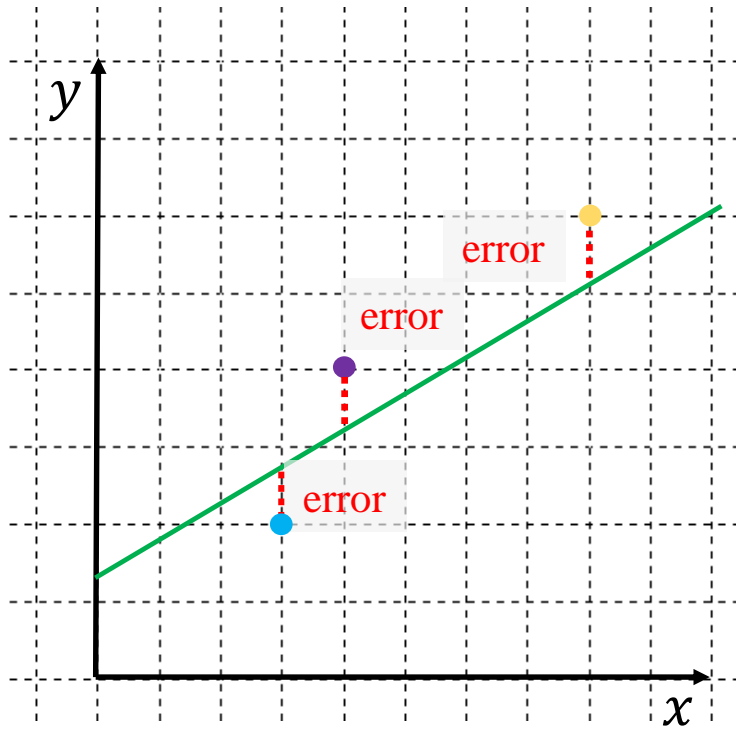
Features														Label													
crim	↕	zn	↕	indus	↕	chas	↕	nox	↕	rm	↕	age	↕	dis	↕	rad	↕	tax	↕	ptratio	↕	black	↕	lstat	↕	medv	↕
0.00632		18		2.31		0		0.538		6.575		65.2		4.09		1		296		15.3		396.9		4.98		24	
0.02731		0		7.07		0		0.469		6.421		78.9		4.9671		2		242		17.8		396.9		9.14		21.6	
0.03237		0		2.18		0		0.458		6.998		45.8		6.0622		3		222		18.7		394.63		2.94		33.4	
0.06905		0		2.18		0		0.458		7.147		54.2		6.0622		3		222		18.7		396.9		5.33		36.2	
0.08829		12.5		7.87		0		0.524		6.012		66.6		5.5605		5		311		15.2		395.6		12.43		22.9	

$$\text{medv} = w_1 * x_1 + \dots + w_{13} * x_{13} + b$$

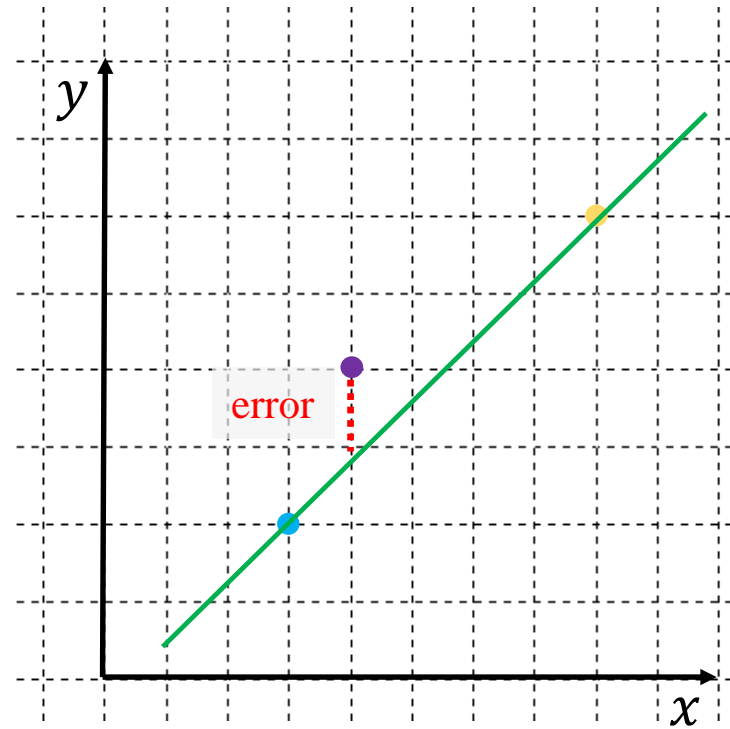
# Machine Learning

## ❖ Linear regression

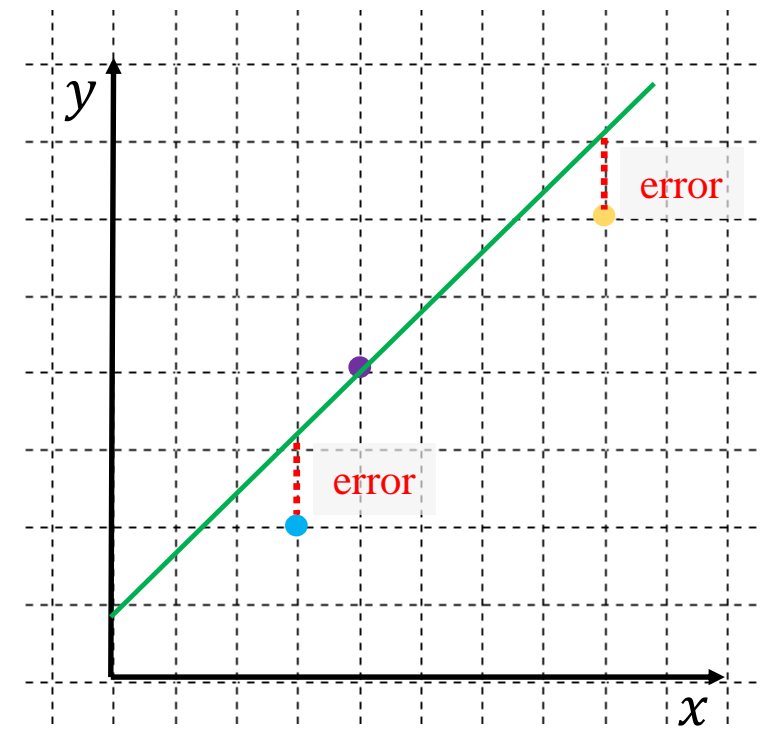
❖ Aim to fit data



$$y = a_1x + b_1$$



$$y = a_2x + b_2$$



$$y = a_3x + b_3$$

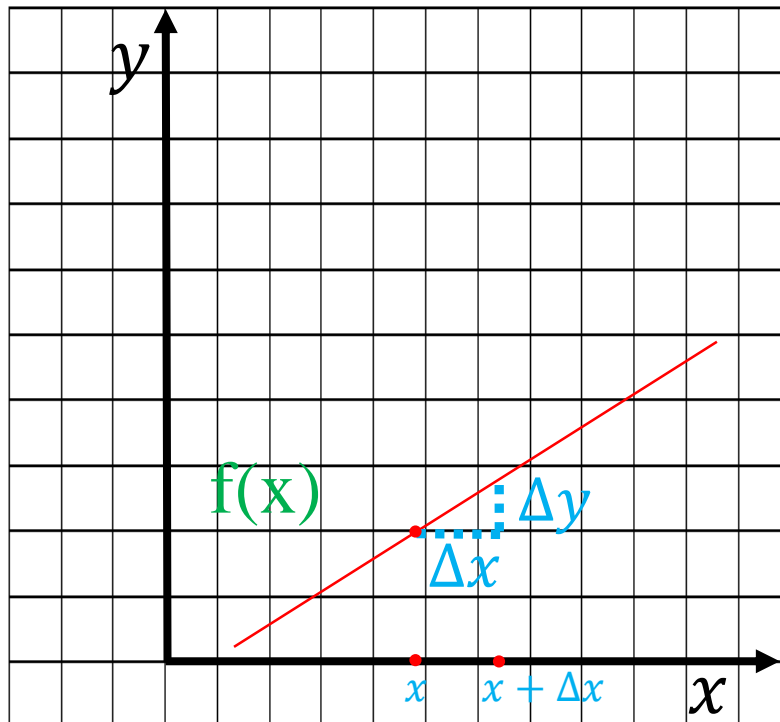
How to find  $w$  and  $b$  that have the smallest error?

# Outline

- Machine Learning
- Derivative/Gradient
- Linear Regression
- Computational Graph
- Generalized formulae

# Derivative/Gradient

## ❖ A cue to optimize a function



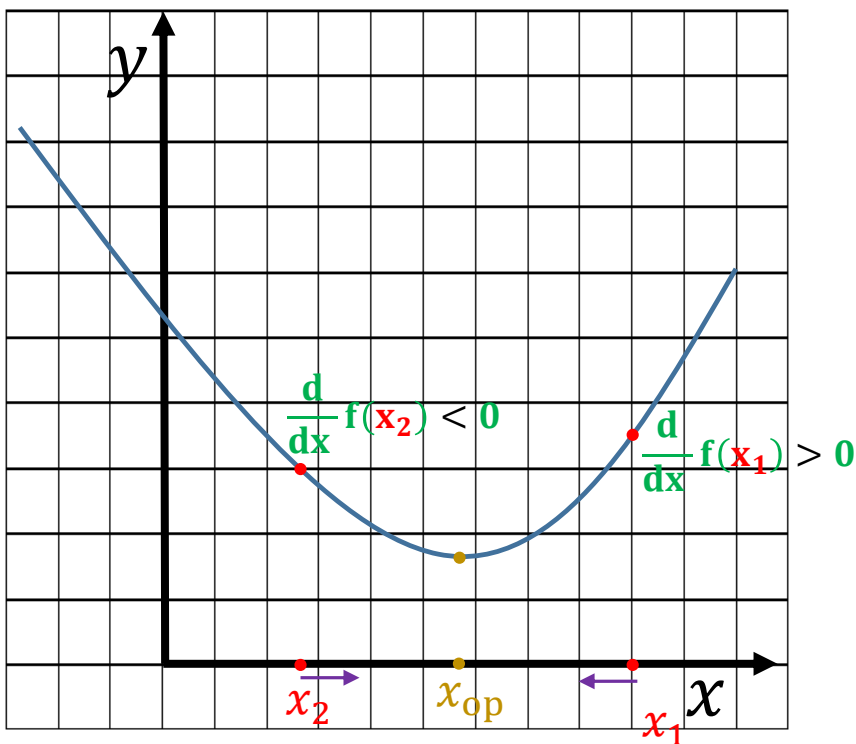
Đạo hàm =  $\frac{\text{Thay đổi theo } y}{\text{Thay đổi theo } x} = \frac{\Delta y}{\Delta x}$

$$\frac{d}{dx} f(x) = \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

$\Delta x$  cần tiến về 0 để  
đường tiếp tuyến tiến  
về hàm  $f(x)$  trong vùng  
lân cận tại  $x$

# Derivative/Gradient

## ❖ A cue to optimize a function



Quan sát:  $x_{op}$  ở vị trí ngược hướng đạo hàm tại  $x_1$  và  $x_2$

Cách xử lý việc di chuyển ngược hướng đạo hàm cho  $x_1$  và  $x_2$  (để tìm  $x_{op}$ ) khác nhau hình thành các thuật toán tối ưu hóa khác nhau

Cách cập nhật giá trị  $x$  đơn giản

$$x = x - \eta \frac{d}{dx} f(x)$$

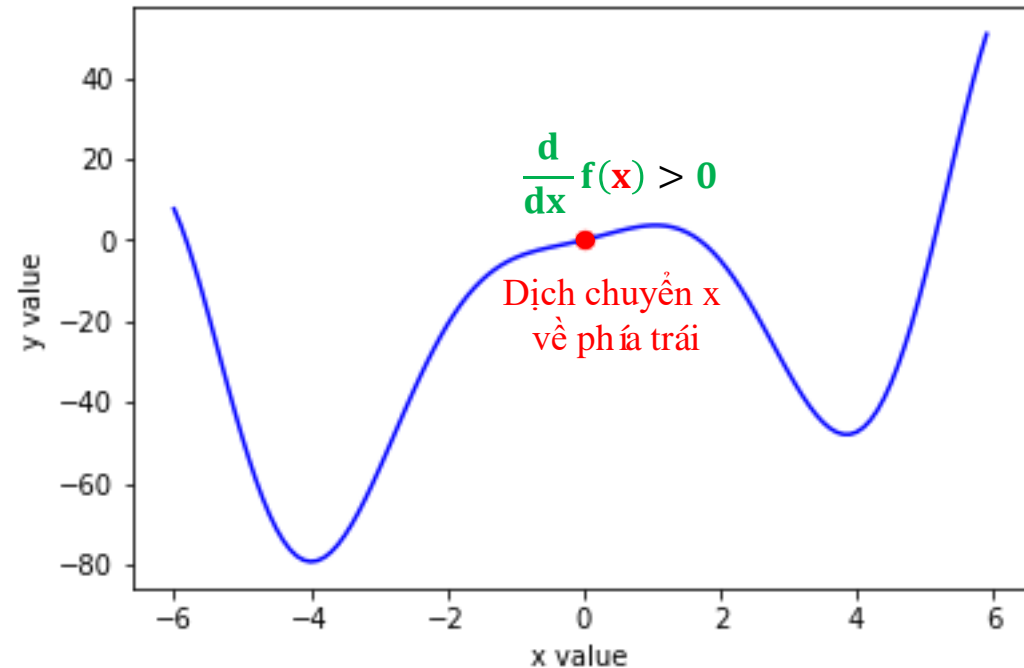
Đạo hàm tại  $x$

Trọng số

# Derivative/Gradient

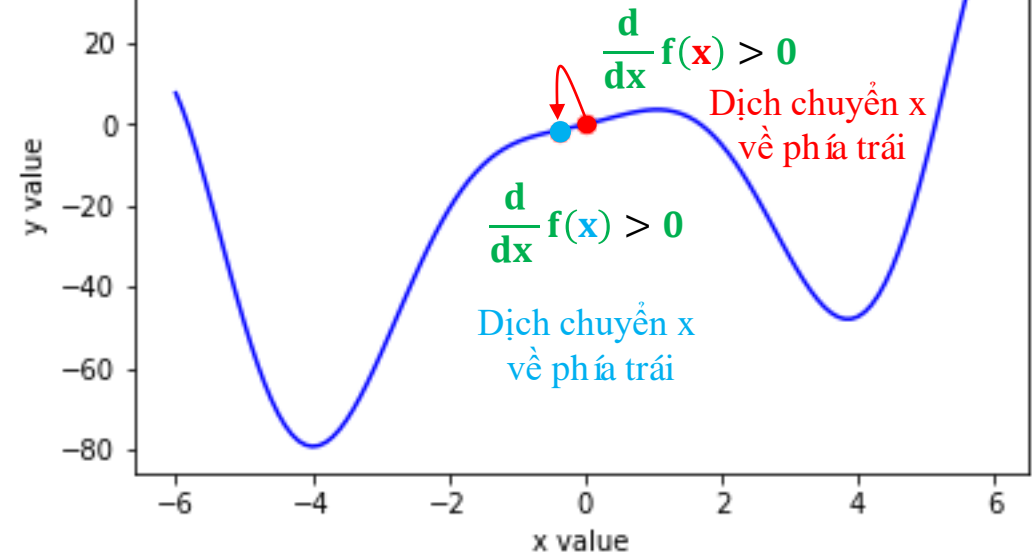
❖ A cue to optimize a function

Khởi tạo giá trị x

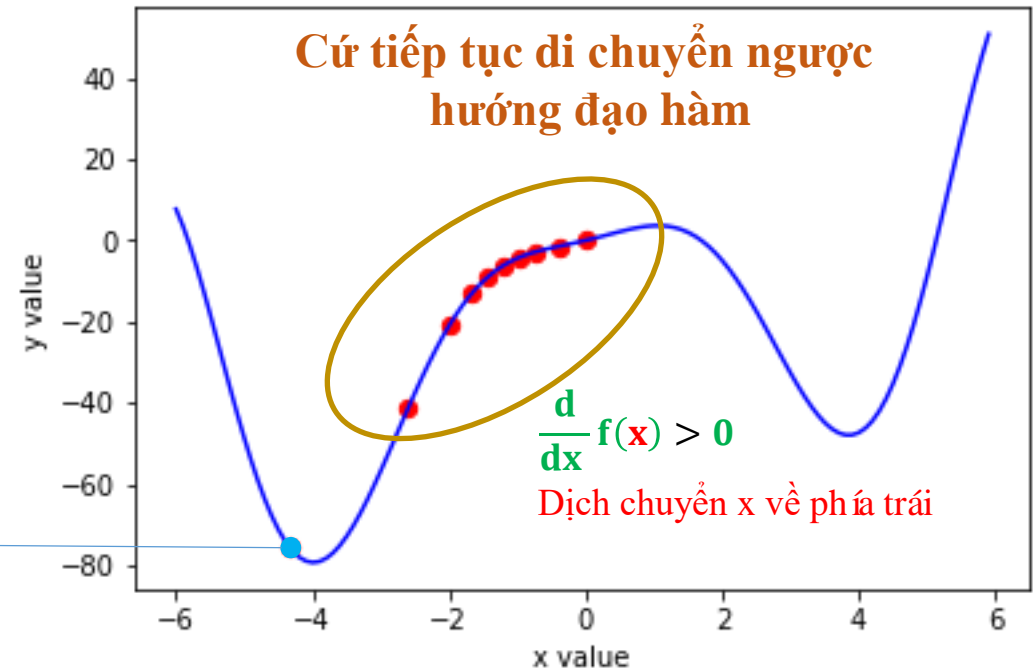


$\frac{d}{dx} f(x) < 0$   
Dịch chuyển x về phía phải

Di chuyển x ngược hướng đạo hàm



Cứ tiếp tục di chuyển ngược hướng đạo hàm



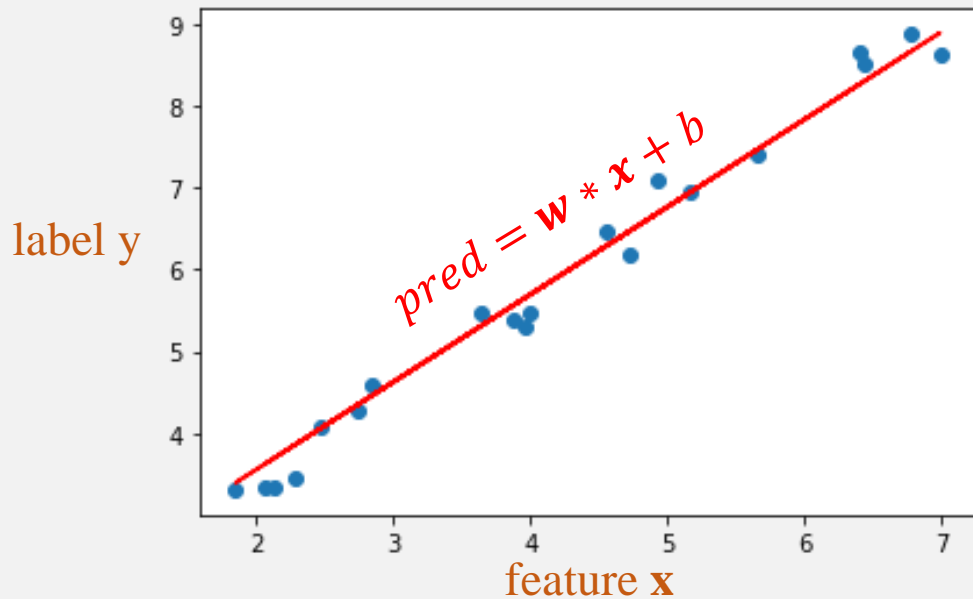
# Outline

- Machine Learning
- Derivative/Gradient
- Linear Regression
- Computational Graph
- Generalized formulae



# Linear Regression

Model the relationship between  
feature  $x$  and label  $y$



Using a linear equation to fit data

Samples  $(x, y)$  are given in advance

Linear equation

$$o = w_1x_1 + w_2x_2 + \dots + w_nx_n + b$$

where  $o$  is a predicted value,

$w_1, w_2, \dots, w_n$  and  $b$  are parameters

and  $\mathbf{x} = [x_1 \ x_2 \ \dots \ x_n]^T$  is feature vector.

Error (loss) computation

**Idea:** compare predicted values  $o$  and label values  $y$

Squared loss

$$L(\mathbf{w}, b) = (o - y)^2$$

How to find optimal  $w$  and  $b$ ?

# Linear Regression

## Linear equation

$$o = w_1x_1 + w_2x_2 + \dots + w_nx_n + b$$

where  $o$  is a predicted value,  
 $w_1, w_2, \dots, w_n$  and  $b$  are parameters  
and  $\mathbf{x} = [x_1 \ x_2 \ \dots \ x_n]^T$  is feature vector.

## Error (loss) computation

**Idea:** compare predicted values  $o$  and label values  $y$   
Squared loss

$$L(\mathbf{w}, b) = (o - y)^2$$

## How to find optimal $\mathbf{w}$ and $b$ ?

Use gradient descent to minimize the loss function

Tính đạo hàm

$$\frac{\partial L}{\partial w_j} = \frac{\partial L}{\partial o} \frac{\partial o}{\partial w_j} = 2x_j(o - y)$$

$$\frac{\partial L}{\partial b} = \frac{\partial L}{\partial o} \frac{\partial o}{\partial b} = 2(o - y)$$

Cập nhật tham số

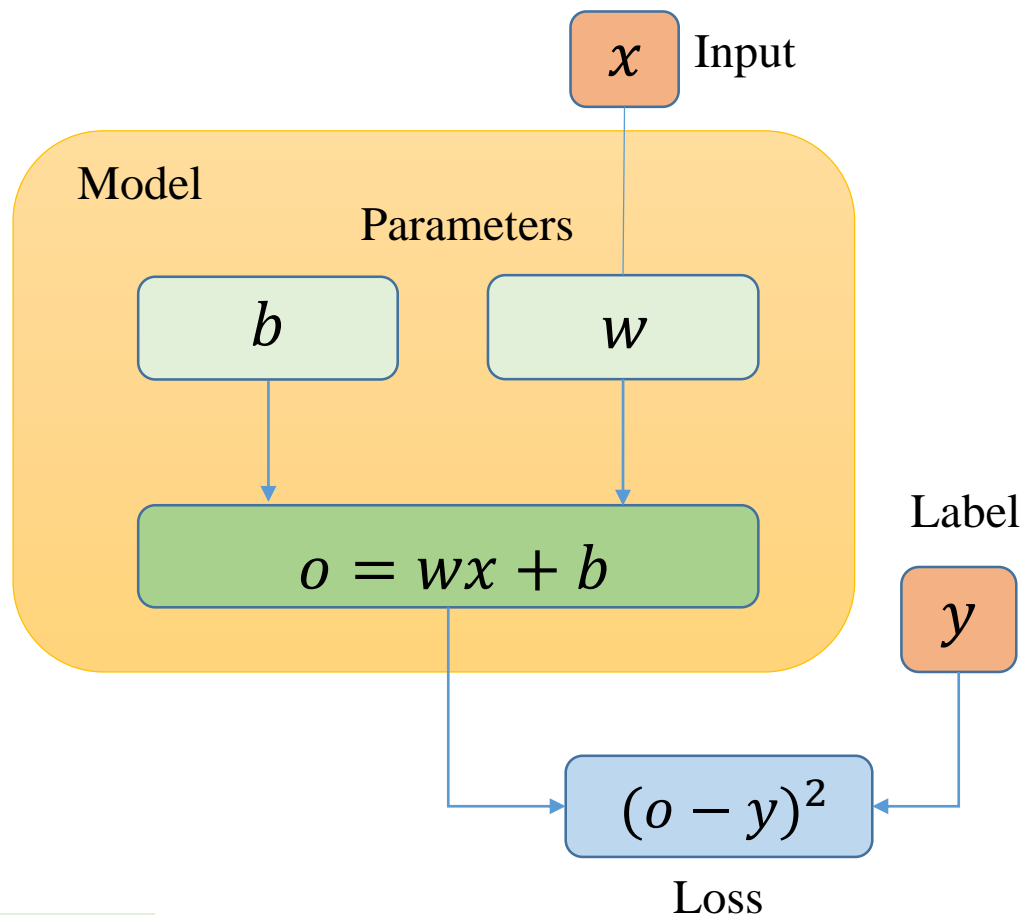
$$w_j = w_j - \eta L'_{w_j}$$

$$b = b - \eta L'_b$$

$\eta$  is learning rate

# Linear Regression

## Diagram



## Cheat sheet

Tính output  $o$

$$o = wx + b$$

Tính Loss

$$L = (o - y)^2$$

Tính đạo hàm

$$L'_{w_j} = 2x_j(o - y)$$

$$L'_b = 2(o - y)$$

Cập nhật tham số

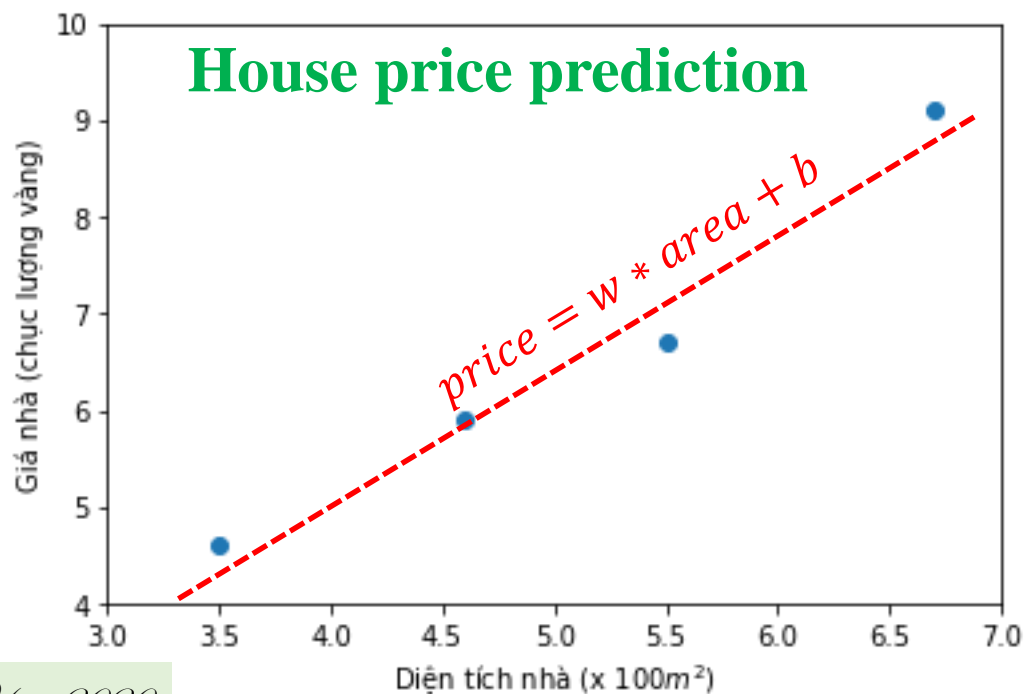
$$w_j = w_j - \eta L'_{w_j}$$

$$b = b - \eta L'_b$$

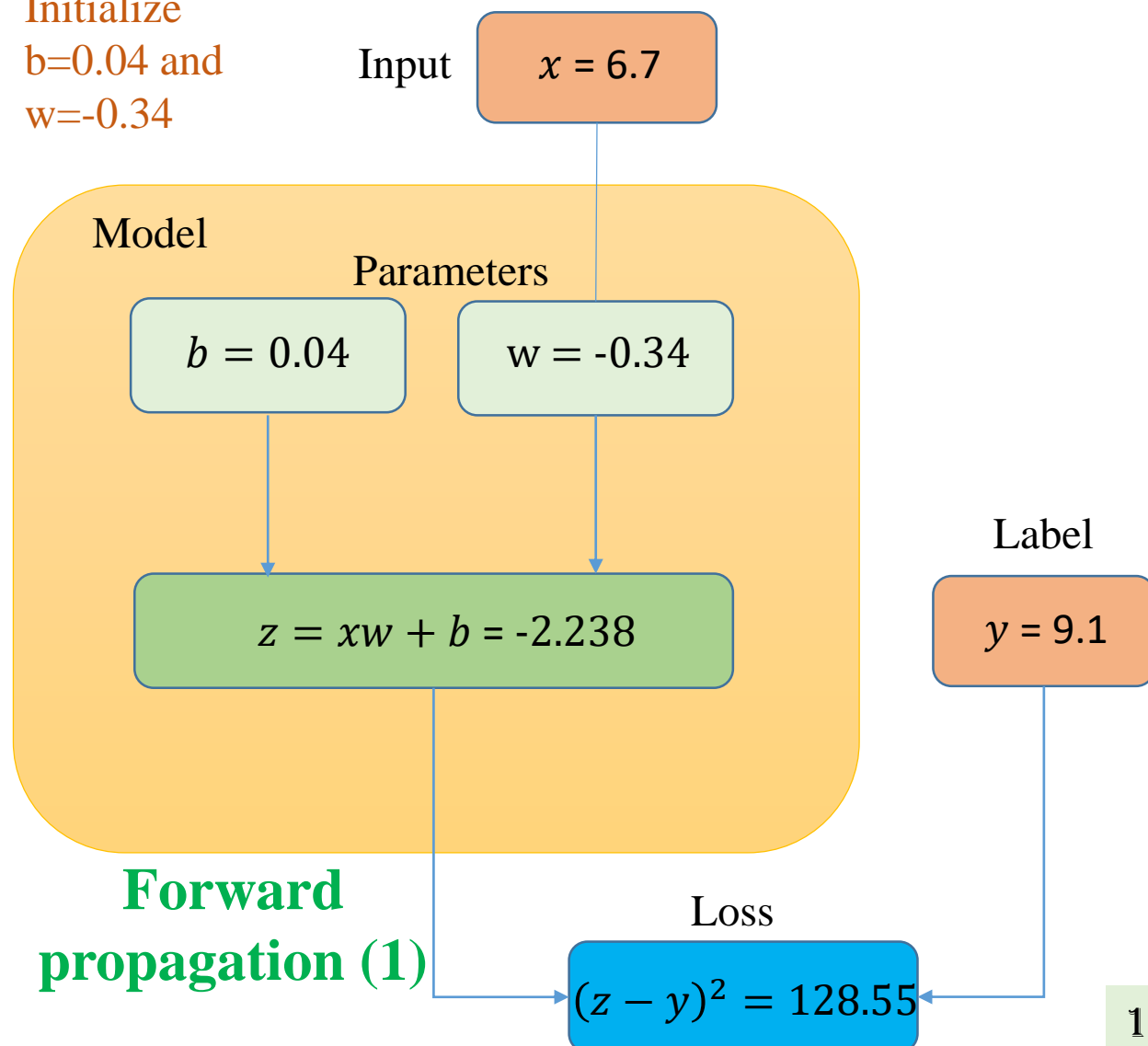
# Linear Regression

Given  
sample  
data

	area	price
	6.7	9.1
	4.6	5.9
	3.5	4.6
	5.5	6.7



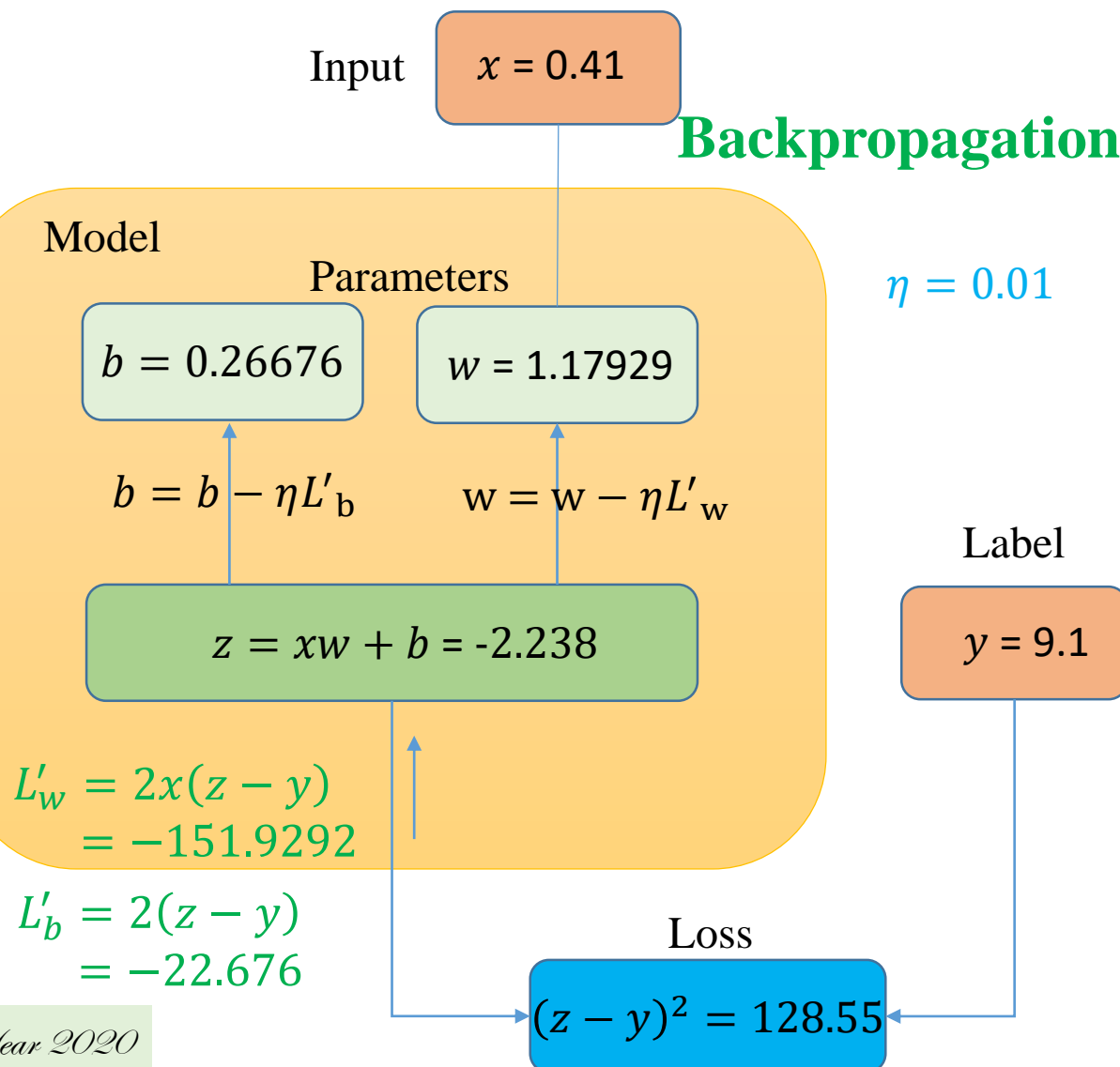
Initialize  
 $b=0.04$  and  
 $w=-0.34$



# Linear Regression

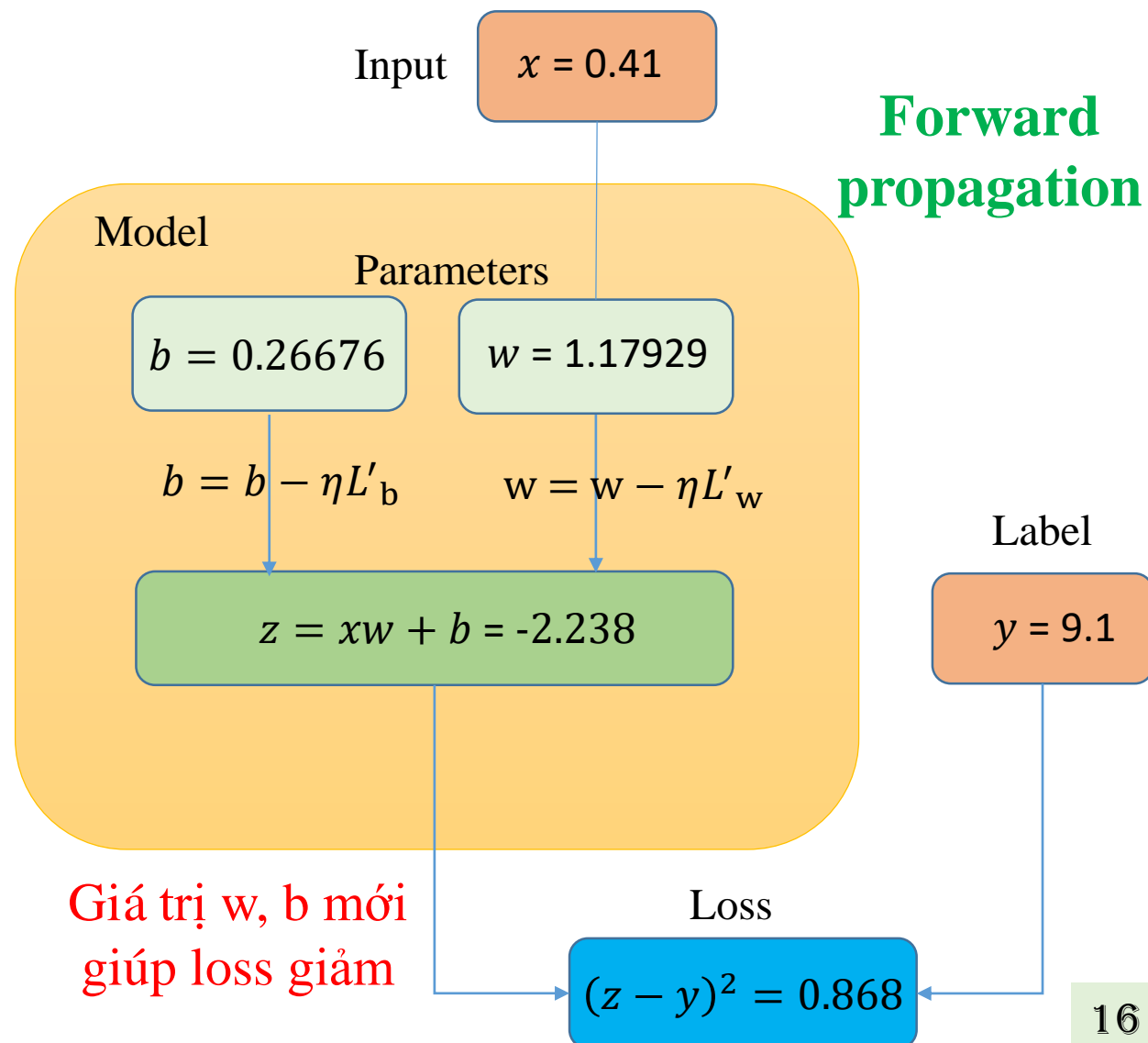
## Backpropagation

$\eta = 0.01$



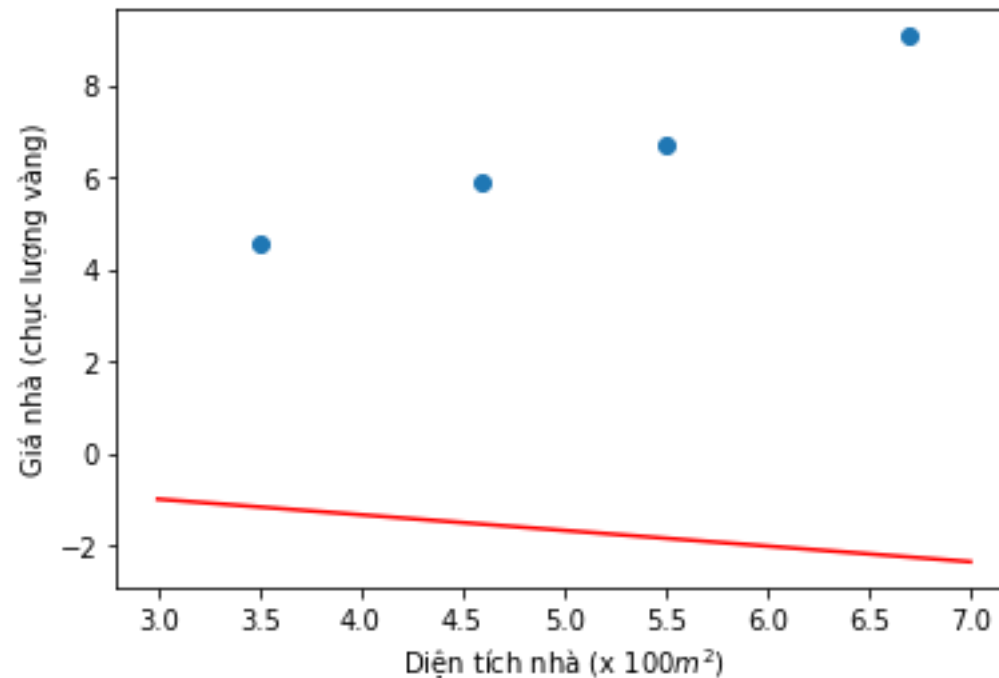
## Forward propagation

Giá trị  $w, b$  mới  
giúp loss giảm



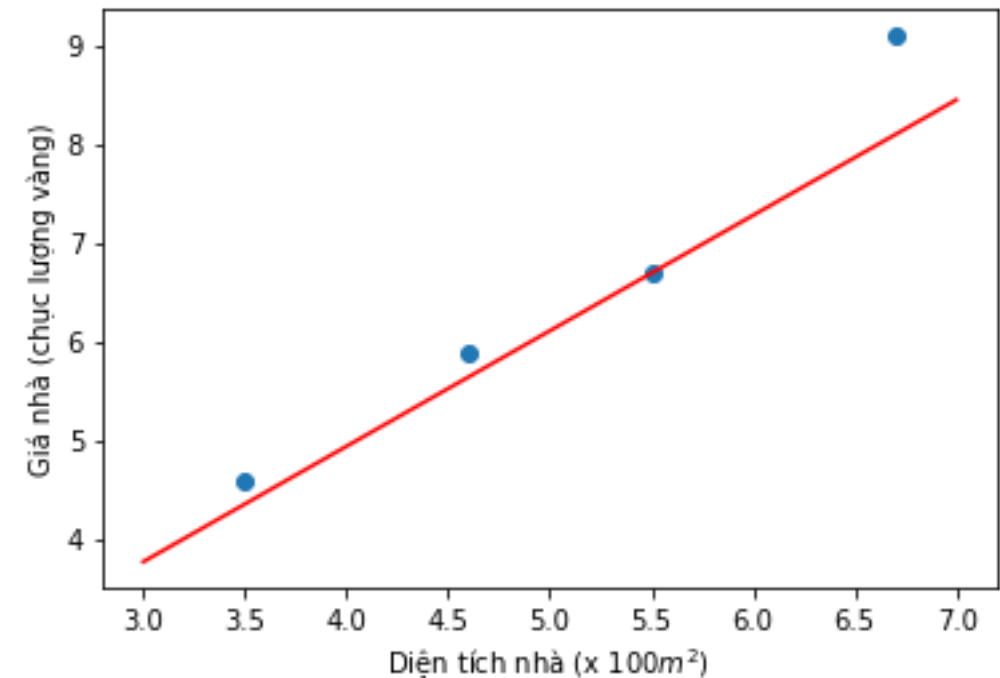
# Linear Regression

## Model prediction before and after the first update



$w = -0.34$        $b = 0.04$        $L = 128.55$

**Before updating**



$w = 1.179292$        $b = 0.26676$        $L = 0.868$

**After updating**

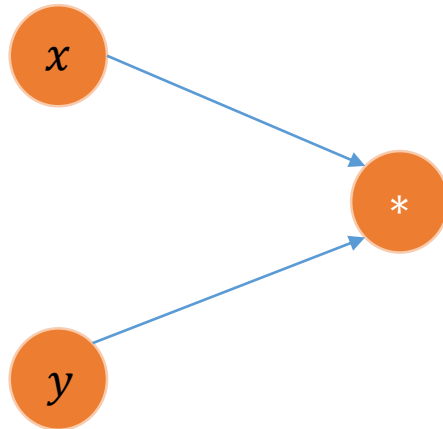
# Outline

- Machine Learning
- Derivative/Gradient
- Linear Regression
- Computational Graph
- Generalized formulae



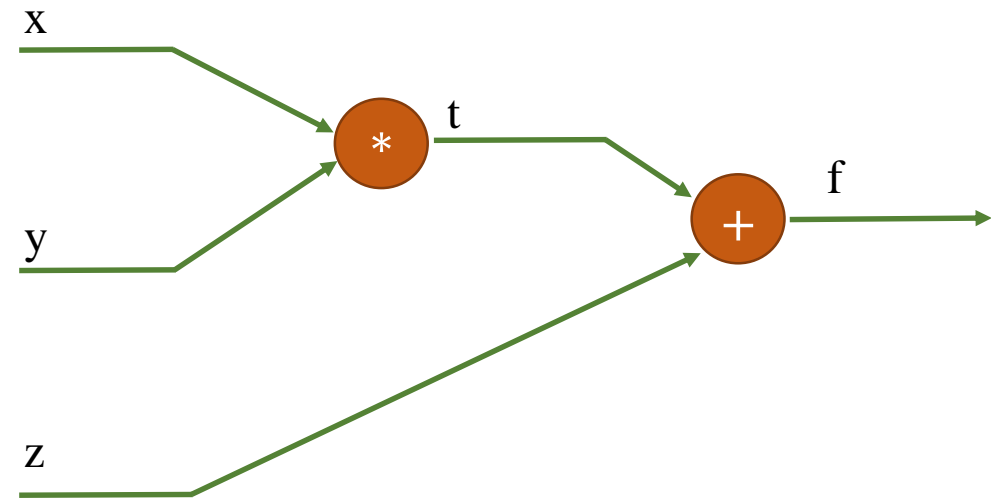
# Computational graph

- ❖ A directed graph
- ❖ Nodes represent variables or operations



- ❖ Construct computational graph for  $f(x, y, z) = x * y + z$
- ❖ Rewrite  $f(x, y, z)$  as

$$f(t, z) = t + z \quad \text{where} \quad t = x * y$$



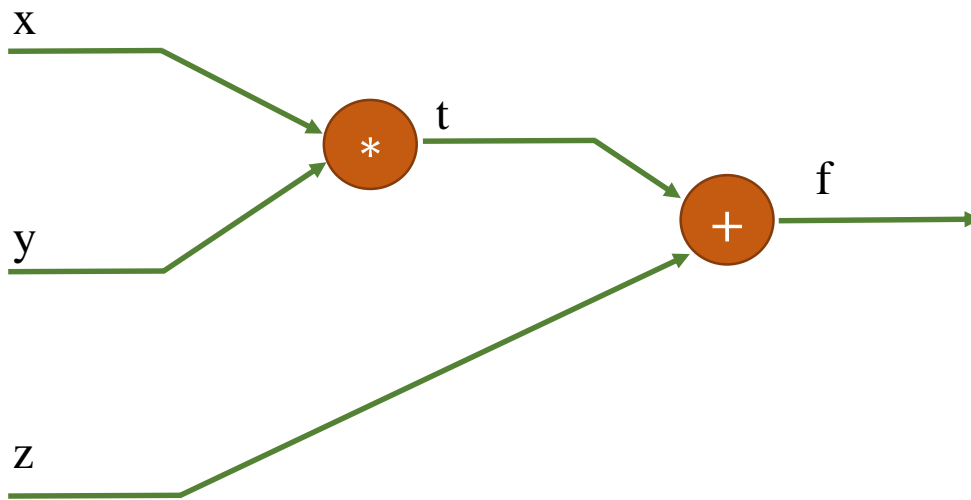
# Computational graph

❖ Construct computational graph for

$$f(x, y, z) = x * y + z$$

❖ Rewrite  $f(x, y, z)$  as

$$f(t, z) = t + z \quad \text{where} \quad t = x * y$$



Partial derivative

$$\frac{\partial t}{\partial x} = y$$

$$\frac{\partial t}{\partial y} = x$$

$$\frac{\partial f}{\partial z} = 1$$

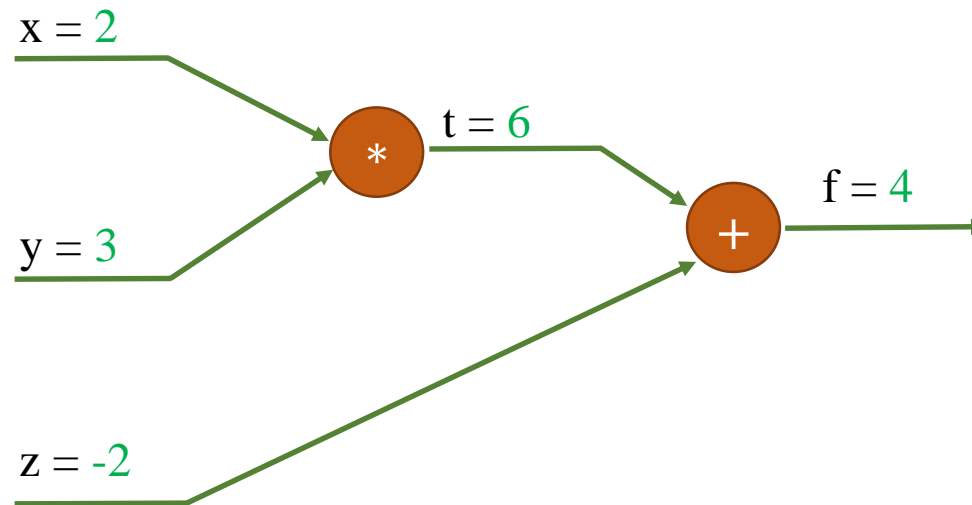
$$\frac{\partial f}{\partial t} = 1$$

$$\frac{\partial f}{\partial x} = y$$

$$\frac{\partial f}{\partial y} = x$$

# Computational graph

❖ Compute  $f(x, y, z)$  với  $x = 2$ ,  $y = 3$  và  $z = -2$ .



Partial derivative

$$\frac{\partial t}{\partial x} = y$$

$$\frac{\partial t}{\partial y} = x$$

$$\frac{\partial f}{\partial z} = 1$$

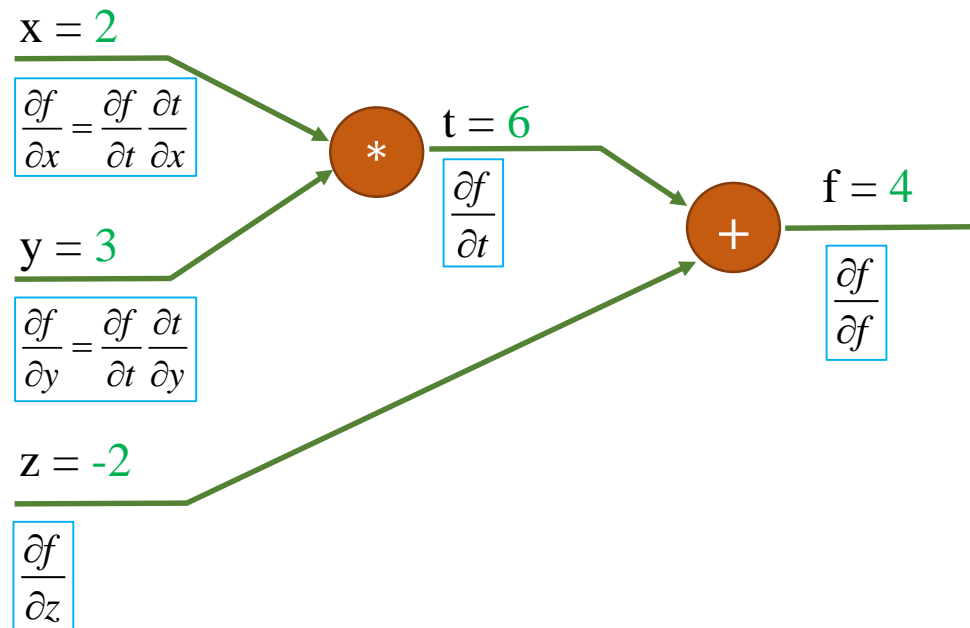
$$\frac{\partial f}{\partial t} = 1$$

$$\frac{\partial f}{\partial x} = y$$

$$\frac{\partial f}{\partial y} = x$$

# Computational graph

❖ Compute  $f(x, y, z)$  với  $x = 2$ ,  $y = 3$  và  $z = -2$ .



Partial derivative

$$\frac{\partial t}{\partial x} = y$$

$$\frac{\partial t}{\partial y} = x$$

$$\frac{\partial f}{\partial z} = 1$$

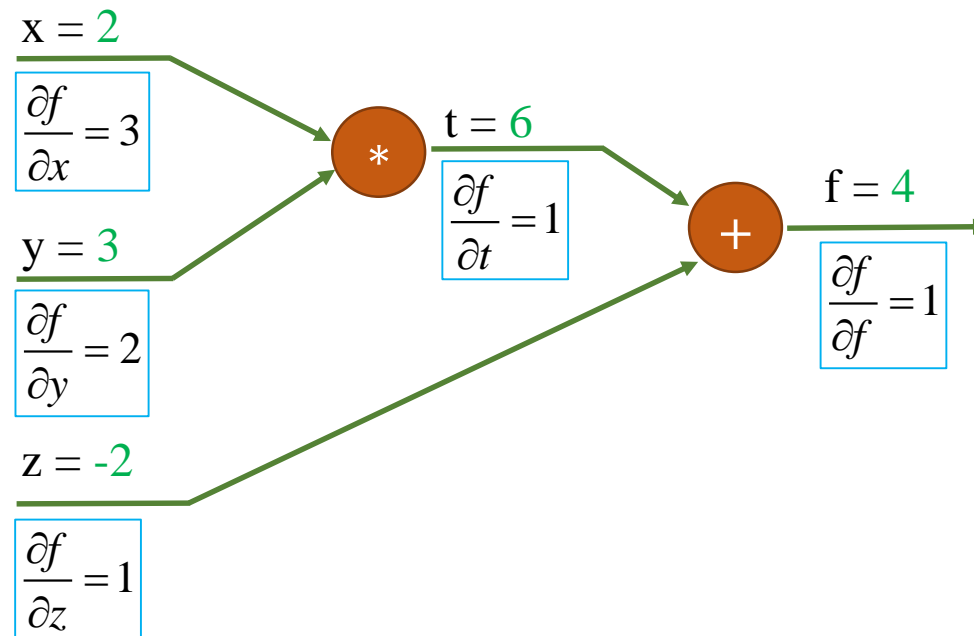
$$\frac{\partial f}{\partial t} = 1$$

$$\frac{\partial f}{\partial x} = y$$

$$\frac{\partial f}{\partial y} = x$$

# Computational graph

❖ Compute  $f(x, y, z)$  với  $x = 2$ ,  $y = 3$  và  $z = -2$ .



Partial derivative

$$\frac{\partial t}{\partial x} = y$$

$$\frac{\partial t}{\partial y} = x$$

$$\frac{\partial f}{\partial z} = 1$$

$$\frac{\partial f}{\partial t} = 1$$

$$\frac{\partial f}{\partial x} = y$$

$$\frac{\partial f}{\partial y} = x$$

# Outline

- Machine Learning
- Derivative/Gradient
- Linear Regression
- Computational Graph
  - 1-sample training
- Generalized formulae

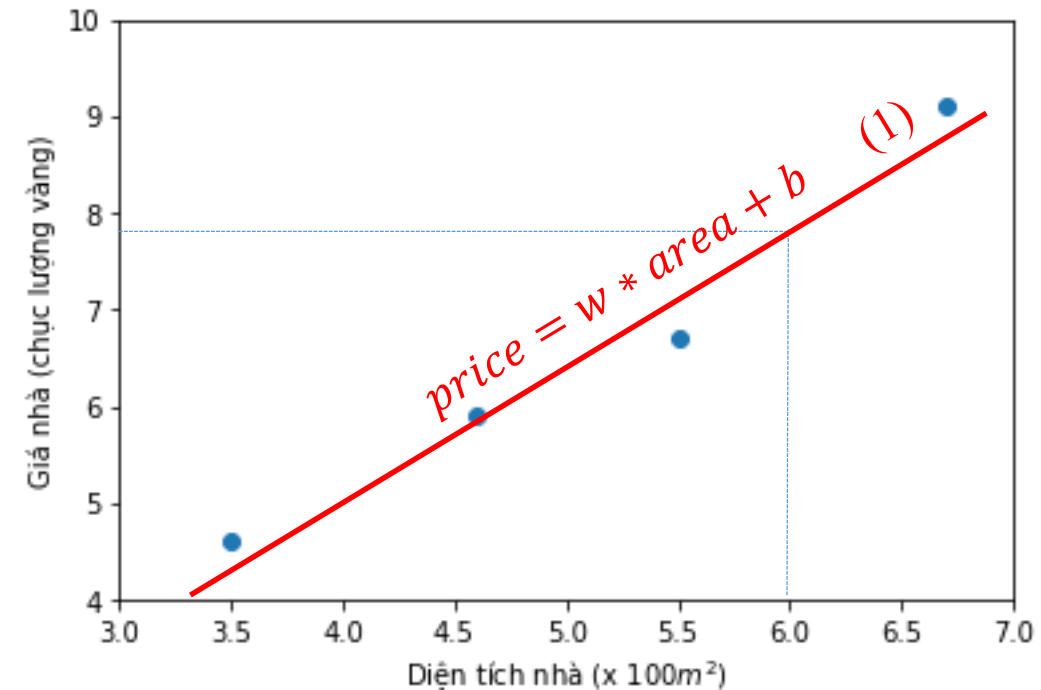
# Computational graph

## ❖ House price predictions

### ❖ How much for a 600- $m^2$ house?

	area	price	
	6.7	9.1	
	4.6	5.9	
	3.5	4.6	
	5.5	6.7	

Given sample data



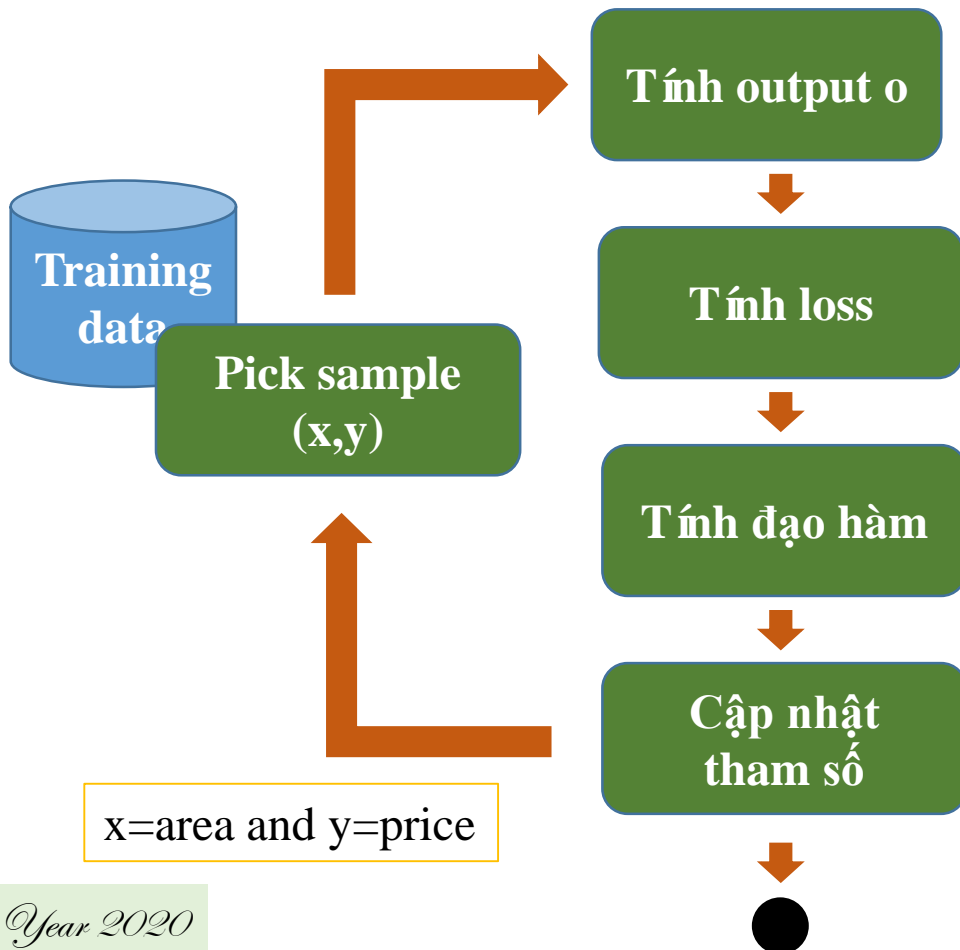
How to compute (1) using computational graph



# Computational graph

## ❖ House price prediction

### ❖ One-sample training



1) Pick a sample  $(x, y)$  from training data

2) Tính output  $o$

$$o = wx + b$$

3) Tính loss

$$L = (o - y)^2$$

4) Tính đạo hàm

$$L'_w = 2x(o - y)$$

$$L'_b = 2(o - y)$$

5) Cập nhật tham số

$$w = w - \eta L'_w$$

$$b = b - \eta L'_b$$

Learning rate  $\eta$

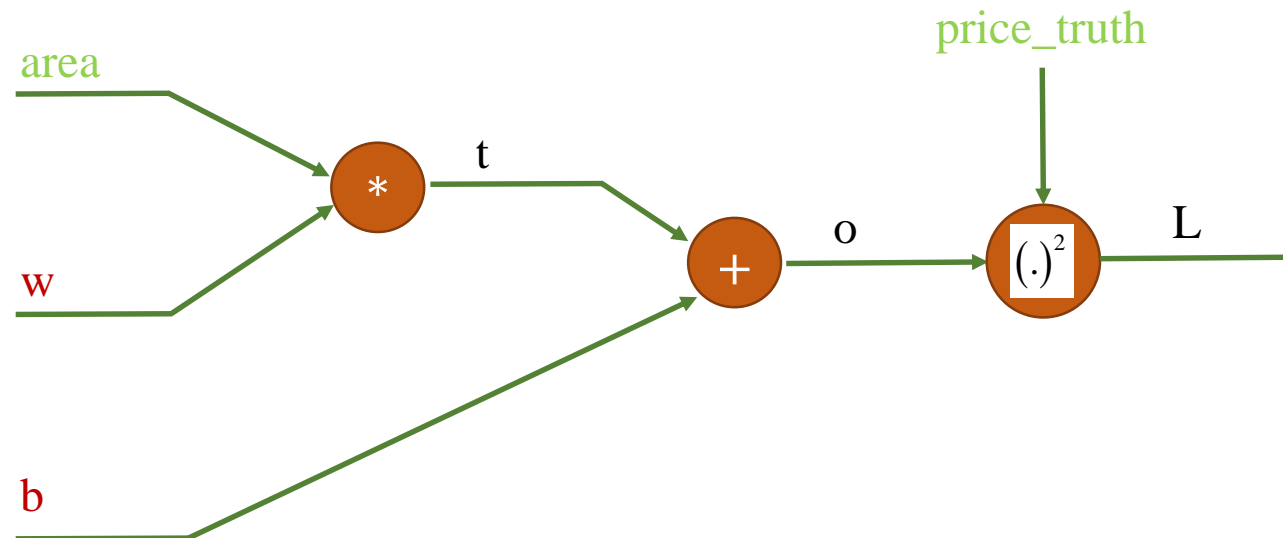
# Computational graph

## ❖ House price prediction

### ❖ One-sample training

$$price = o = w * area + b$$

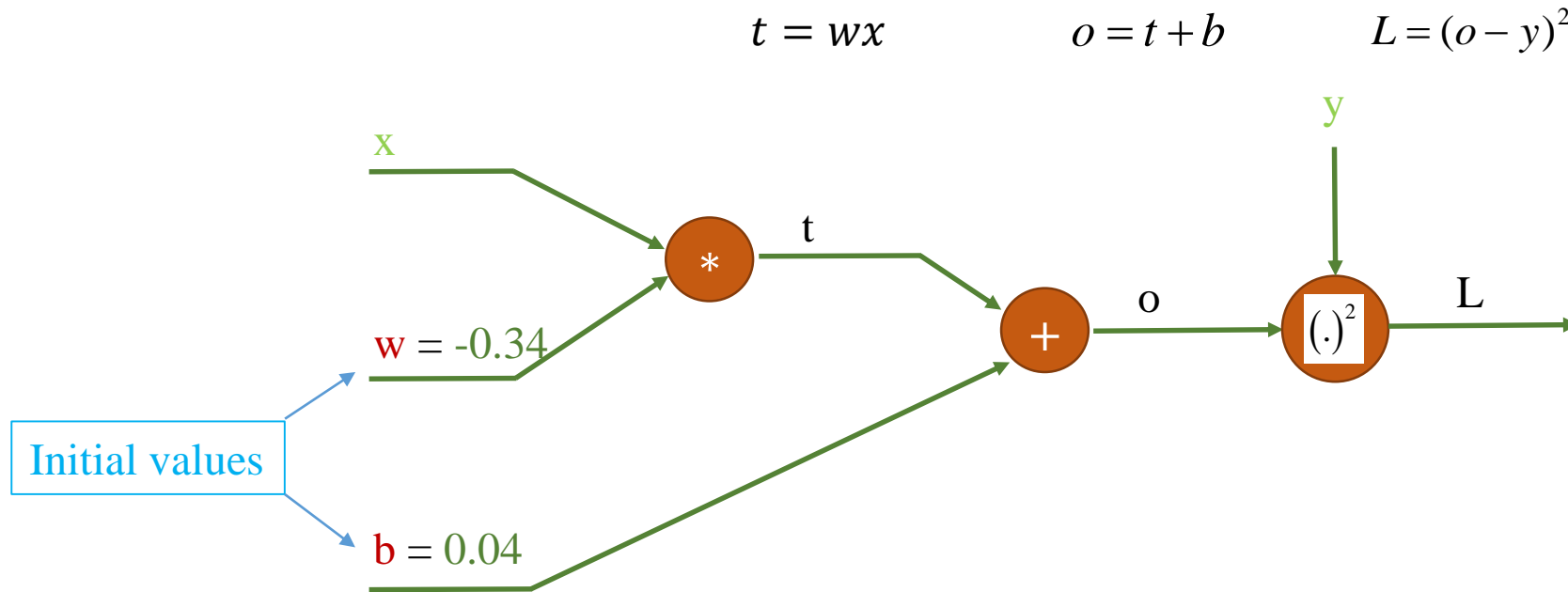
$$t = w * area$$



# Computational graph

## ❖ House price prediction

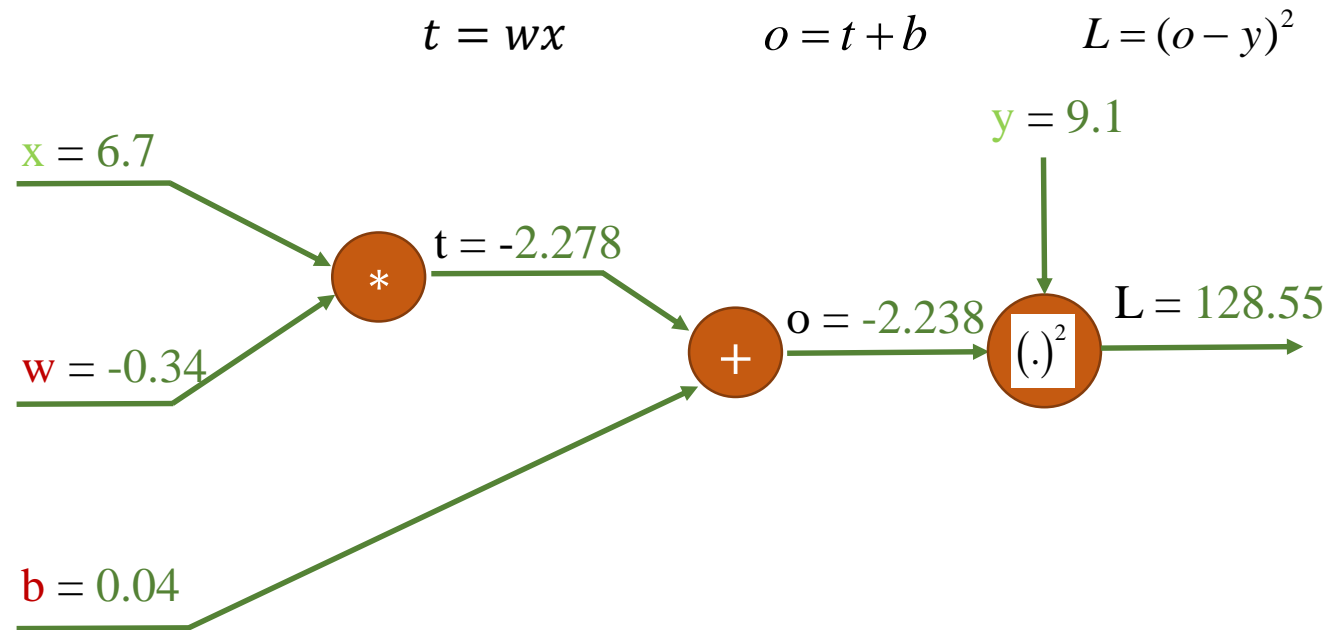
### ❖ One-sample training



# Computational graph

## ❖ House price prediction

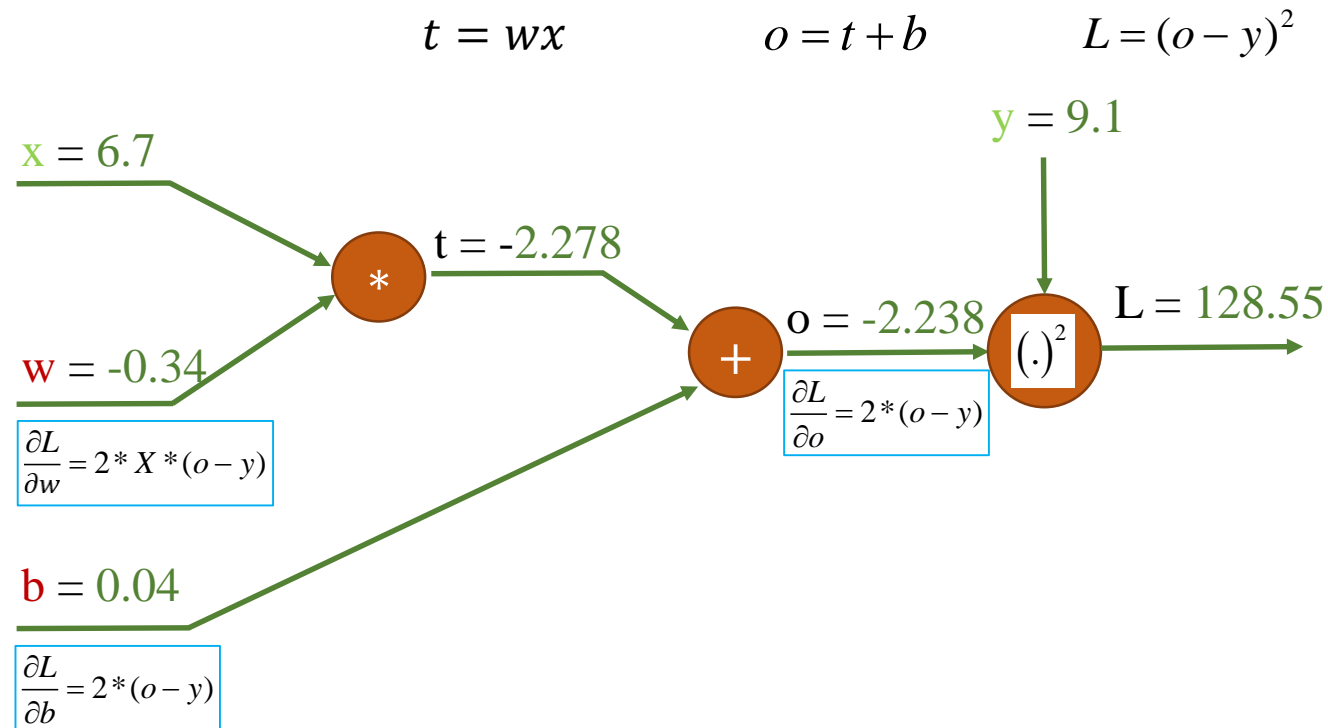
### ❖ One-sample training



# Computational graph

## ❖ House price prediction

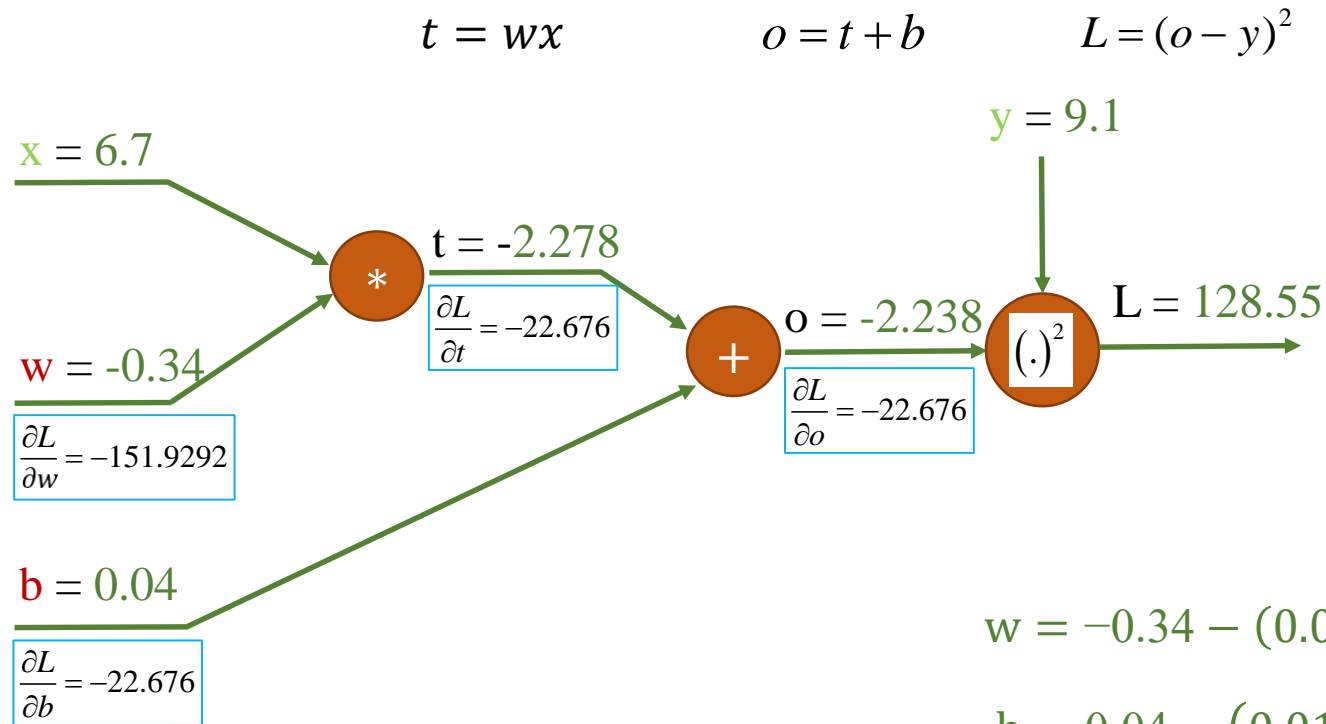
### ❖ One-sample training



# Computational graph

## ❖ House price prediction

### ❖ One-sample training



Cách cập nhật a và b

$$w = w - \eta * \frac{\partial L}{\partial w}$$

$$b = b - \eta * \frac{\partial L}{\partial b}$$

Learning rate  $\eta = 0.01$

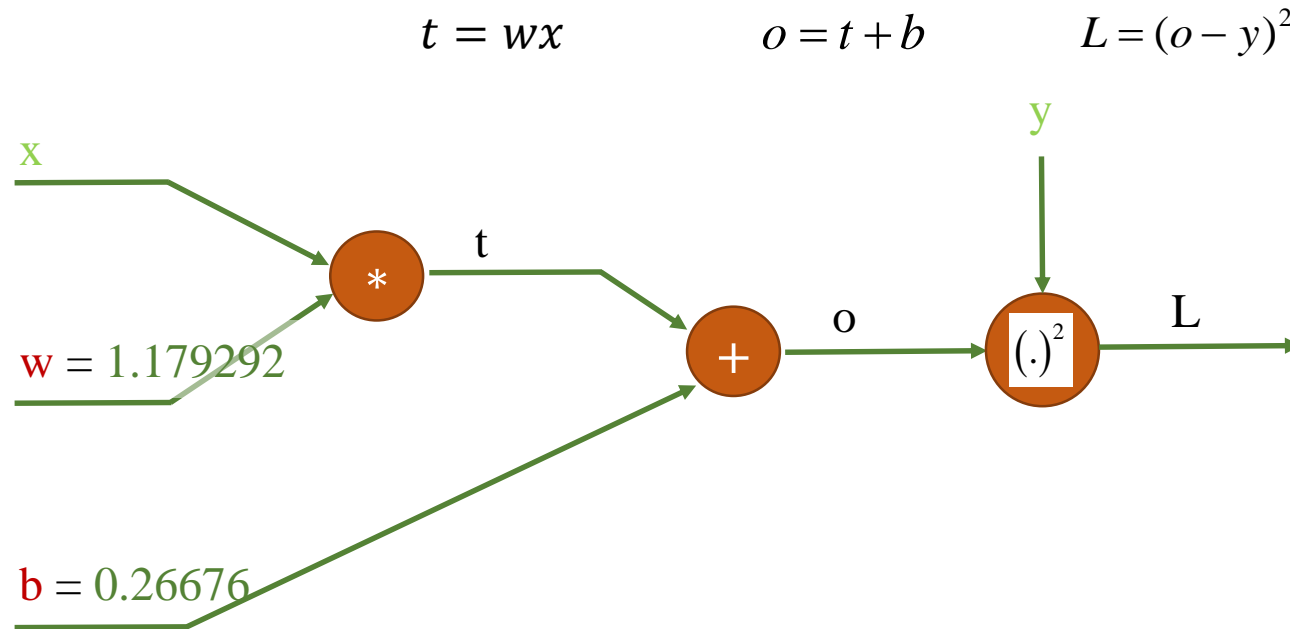
$$w = -0.34 - (0.01 * (-151.9)) = 1.179$$

$$b = 0.04 - (0.01 * (-22.67)) = 0.266$$

# Computational graph

## ❖ House price prediction

### ❖ One-sample training

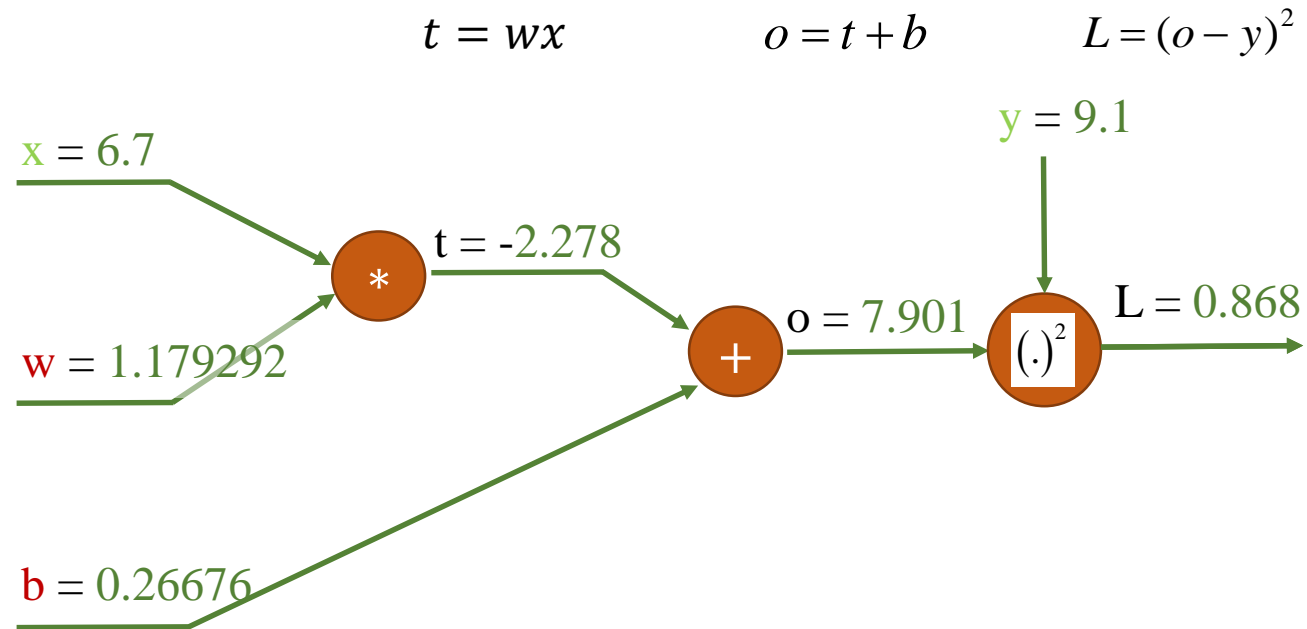




# Computational graph

## ❖ House price prediction

### ❖ One-sample training



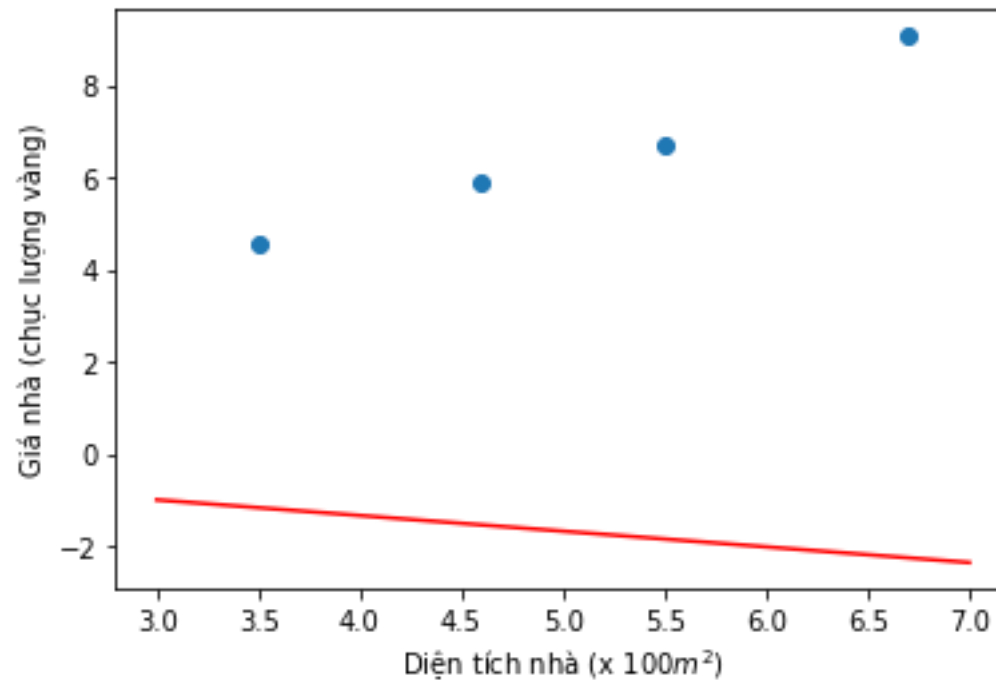
previous  $L = 128.55$

**Updated  $a$  and  $b$  values help to reduce the  $L$  value**

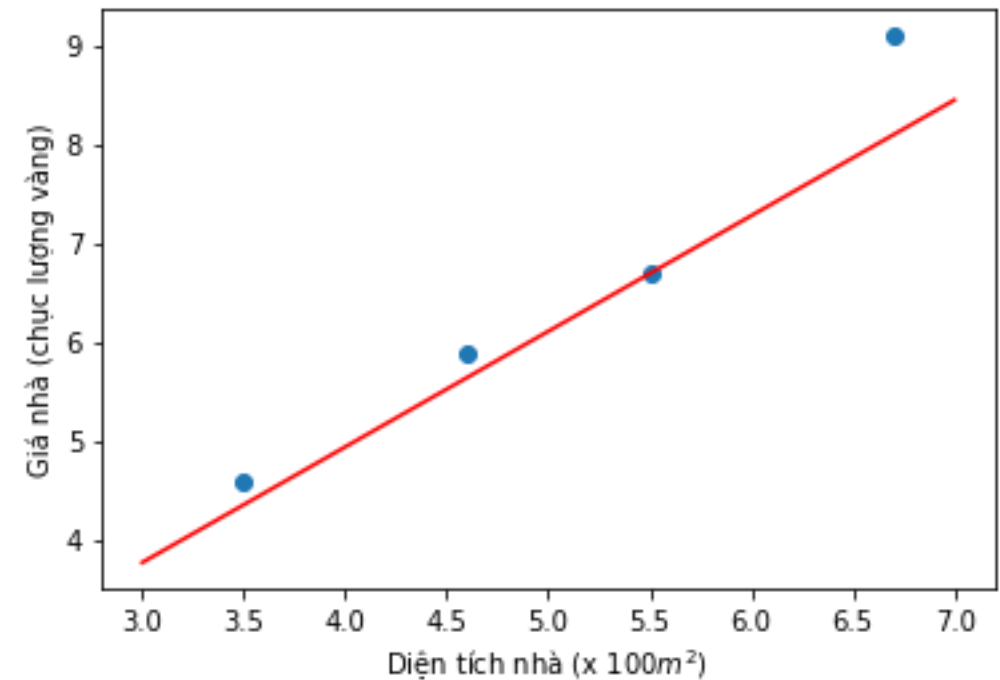
# Computational graph

## ❖ House price prediction

### ❖ One-sample training



$w = -0.34$        $b = 0.04$        $L = 128.55$

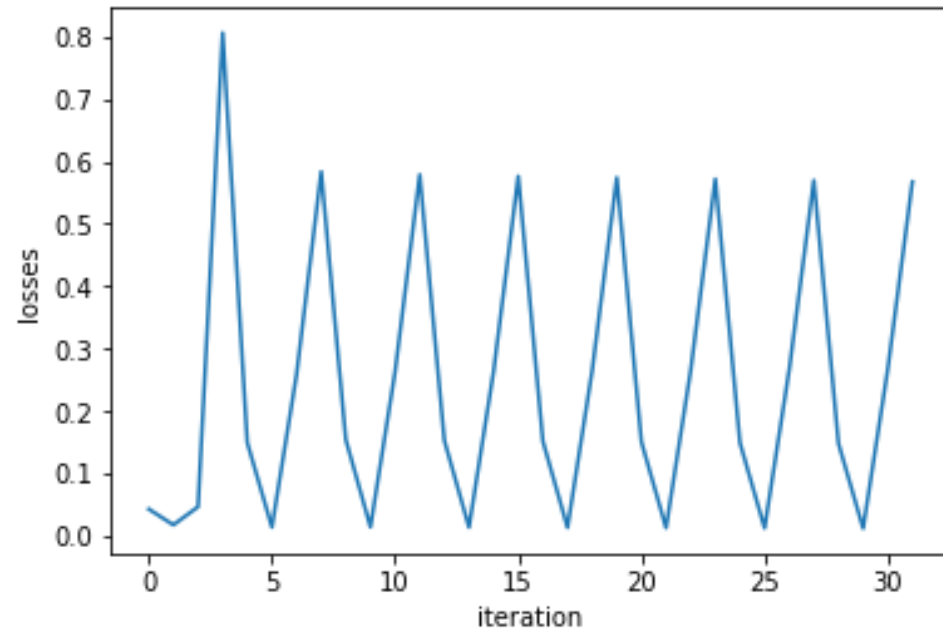


$w = 1.179292$        $b = 0.26676$        $L = 0.868$

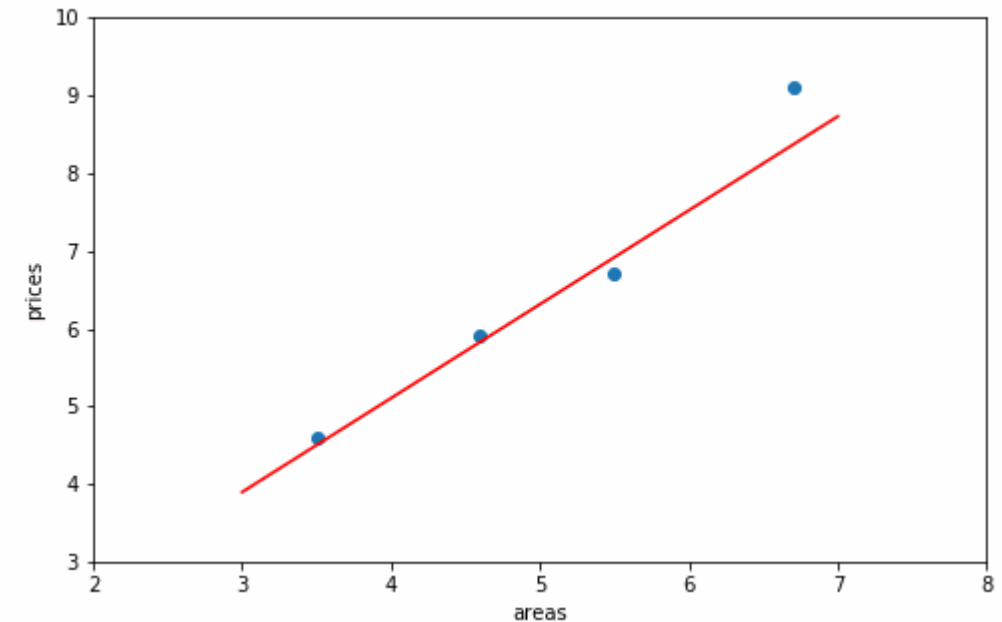
# Computational graph

## ❖ House price prediction

### ❖ One-sample training



Losses for 30 iterations



Model updating for different iterations

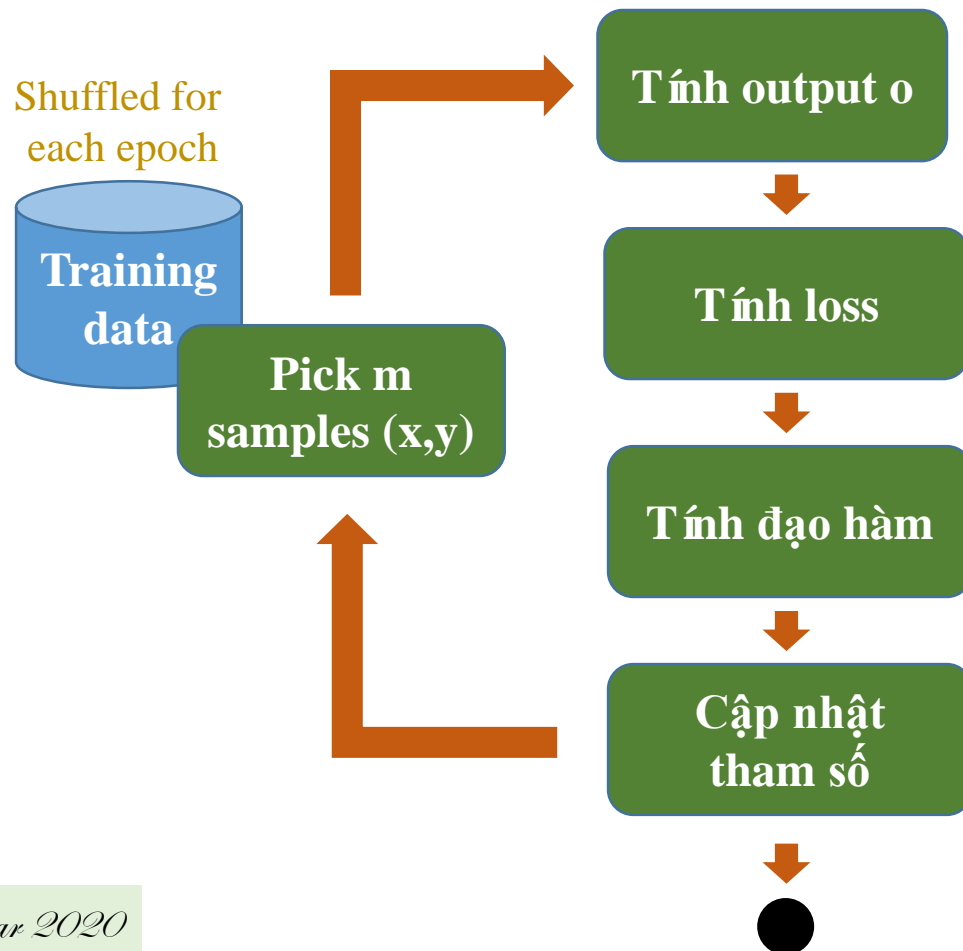
# Outline

- Machine Learning
- Derivative/Gradient
- Linear Regression
- Computational Graph
  - 1-sample training
  - m-sample training
- Generalized formulae

# Computational graph

## ❖ House price prediction

### ❖ m-sample training ( $1 < m < N$ )



1) Pick  $m$  samples  $(x^{(i)}, y^{(i)})$  from training data

2) Tính output  $o_i$

$$o^{(i)} = wx^{(i)} + b \quad \text{for } 0 \leq i < m$$

3) Tính loss

$$L^{(i)} = (o^{(i)} - y^{(i)})^2 \quad \text{for } 0 \leq i < m$$

4) Tính đạo hàm

$$\begin{aligned} L'_w{}^{(i)} &= 2x(o^{(i)} - y^{(i)}) \\ L'_b{}^{(i)} &= 2(o^{(i)} - y^{(i)}) \end{aligned} \quad \text{for } 0 \leq i < m$$

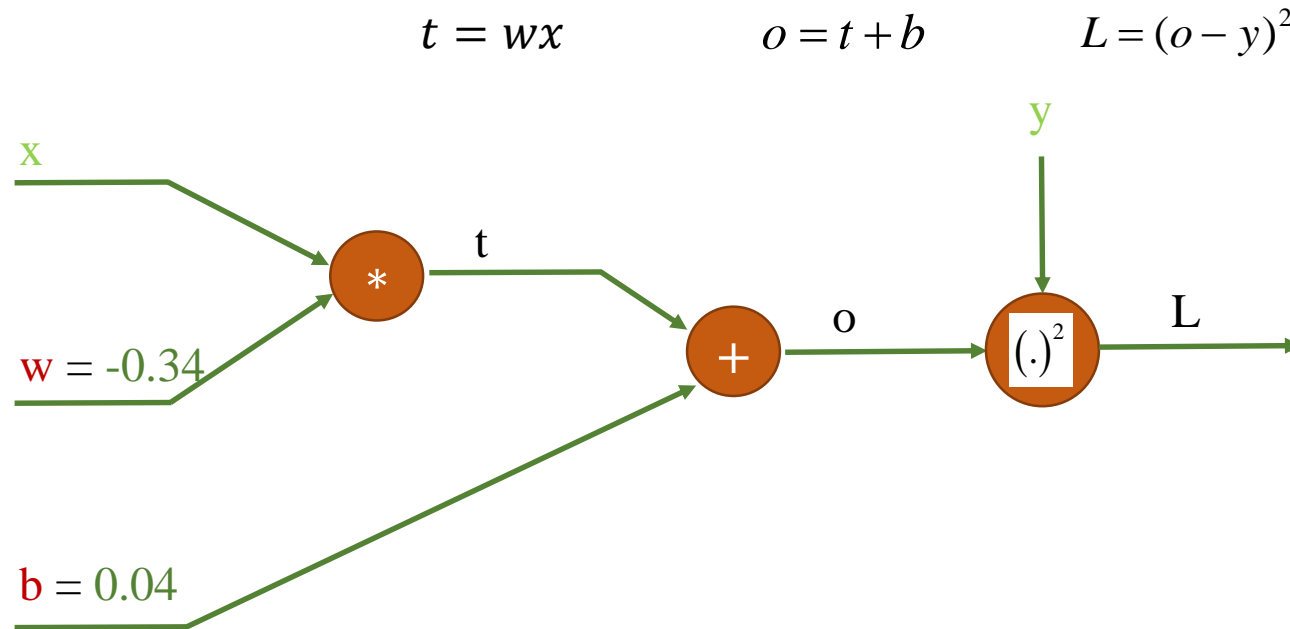
5) Cập nhật tham số

$$\begin{aligned} w &= w - \eta \frac{\sum_i L'_w{}^{(i)}}{m} \\ b &= b - \eta \frac{\sum_i L'_b{}^{(i)}}{m} \end{aligned} \quad \text{Learning rate } \eta$$

# Computational graph

## ❖ House price prediction

### ❖ m-sample training ( $1 < m < N$ )



# Computational graph

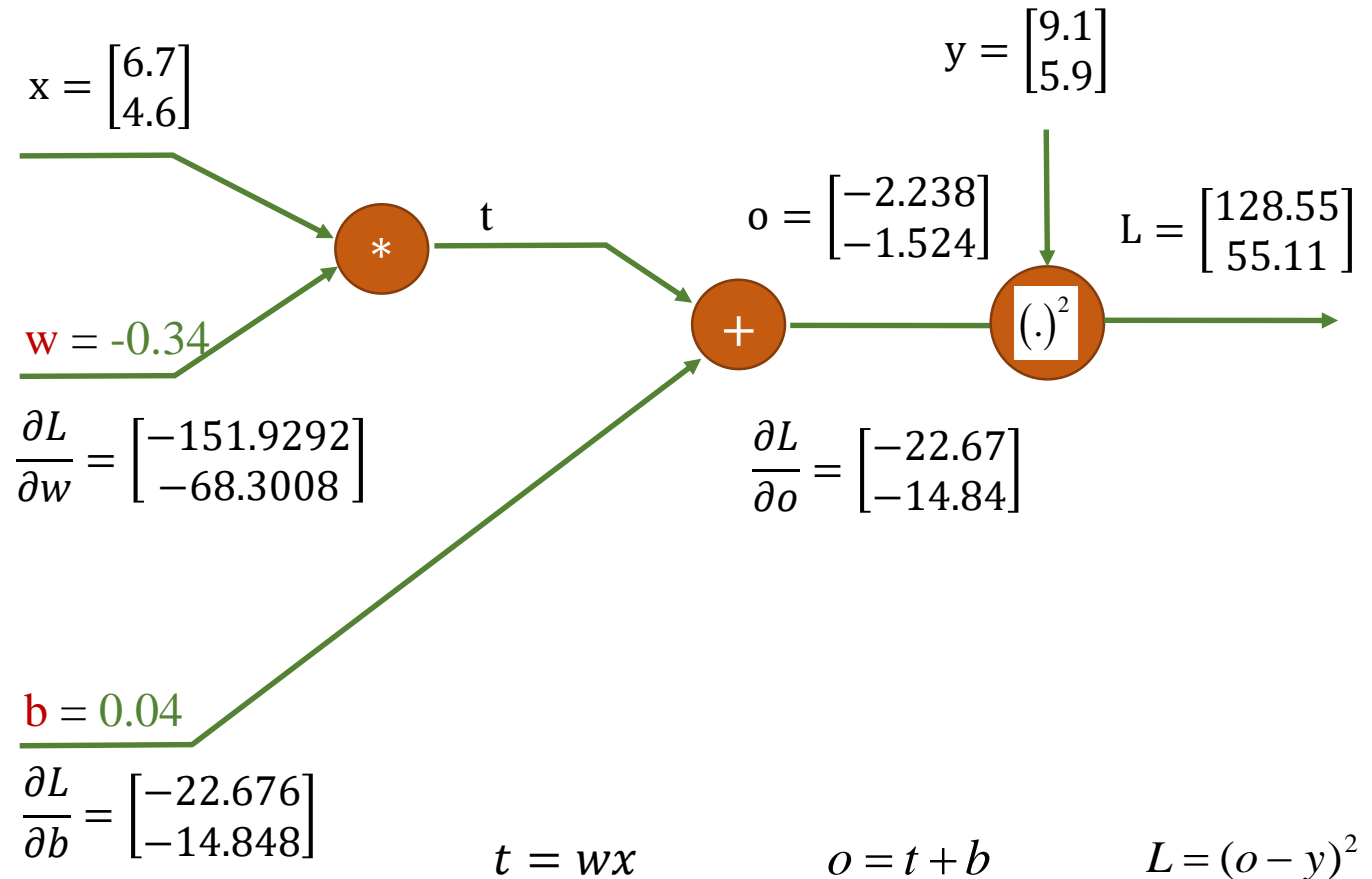
## ❖ House price prediction

### ❖ m-sample training ( $1 < m < N$ )

$m = 2$

$$\frac{\text{sum}(\frac{\partial L}{\partial w})}{m} = -110.115$$

$$\frac{\text{sum}(\frac{\partial L}{\partial b})}{m} = -18.762$$



# Computational graph

## ❖ House price prediction

### ❖ m-sample training ( $1 < m < N$ )

#### Cách cập nhật a và b

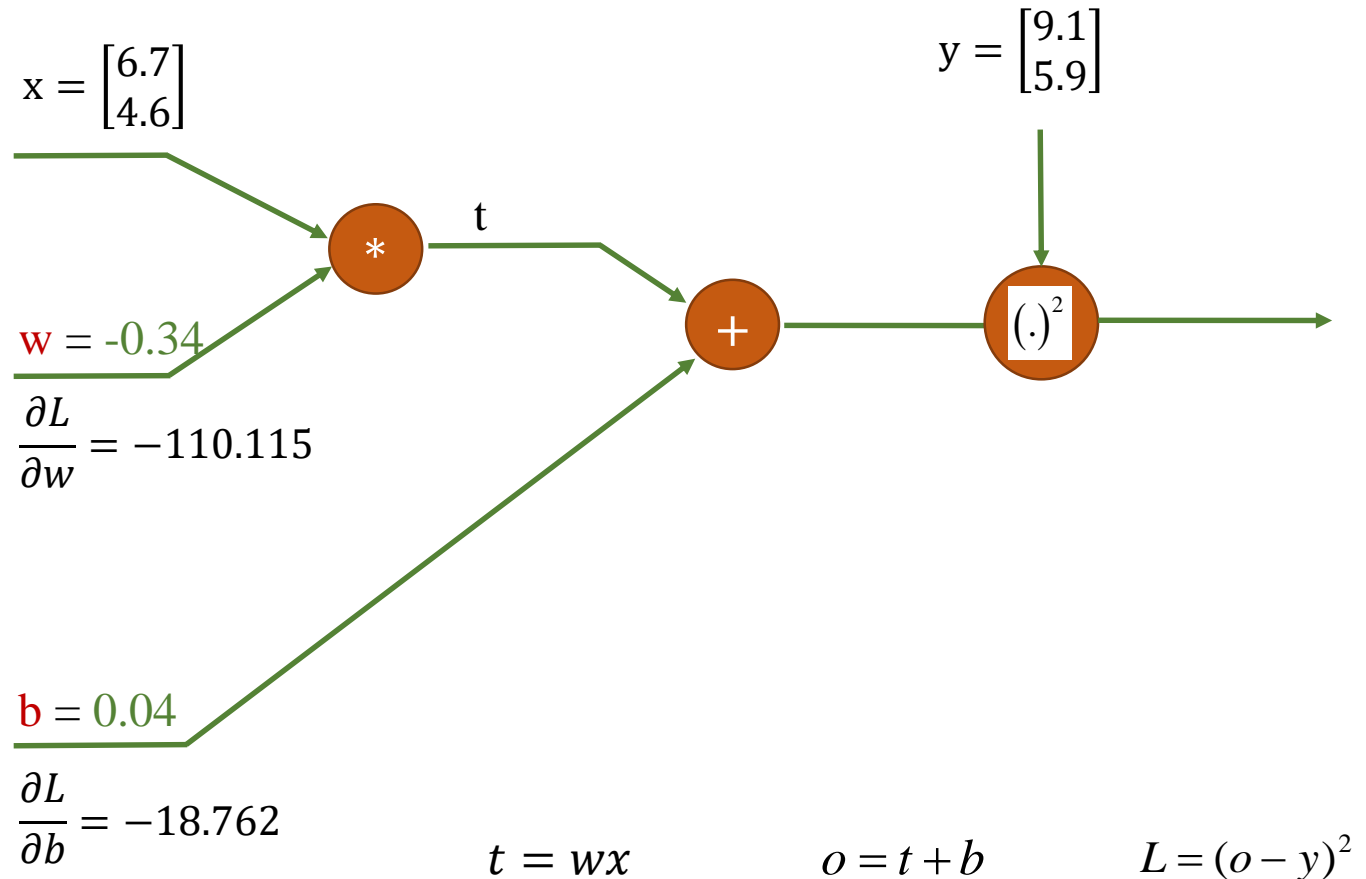
$$w = w - \eta * \frac{\partial L}{\partial w}$$

$$b = b - \eta * \frac{\partial L}{\partial b}$$

Learning rate  $\eta = 0.01$

$$w = -0.34 - (0.01 * (-110.115)) = 0.761$$

$$b = 0.04 - (0.01 * (-18.762)) = 0.227$$

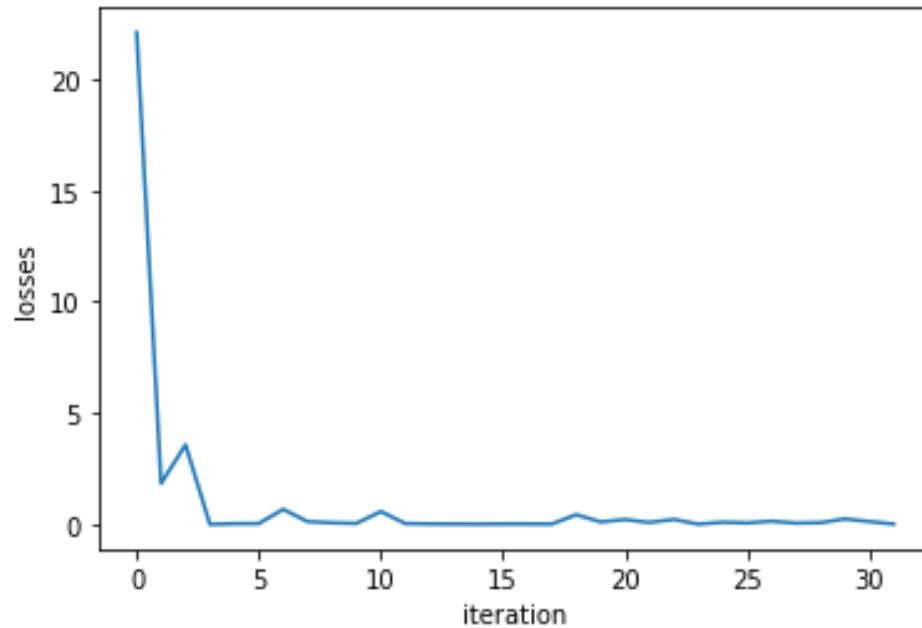




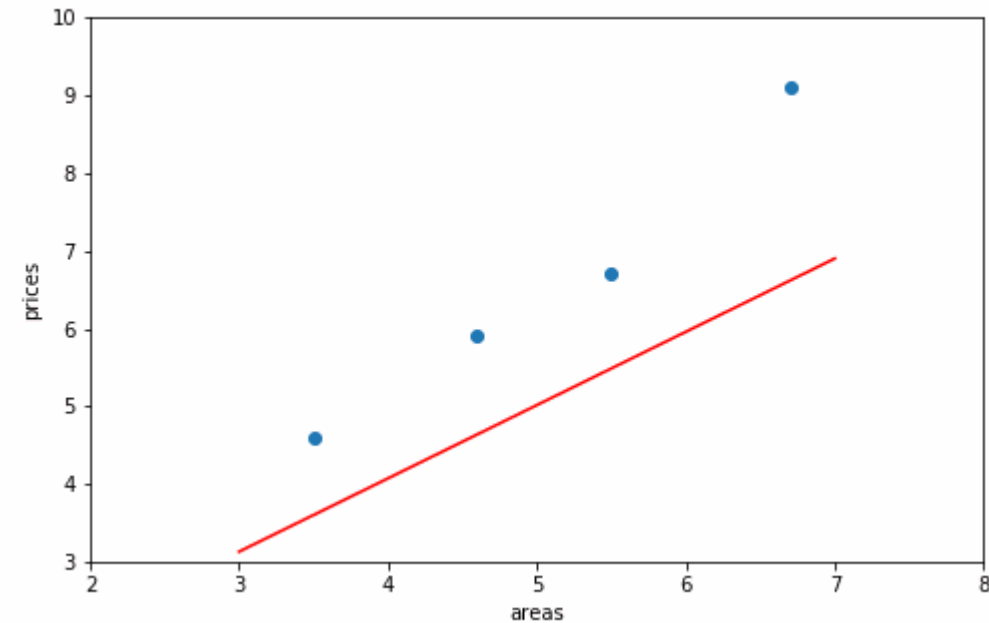
# Computational graph

## ❖ House price prediction

### ❖ m-sample training ( $1 < m < N$ )



Losses for 30 iterations



Model updating for different iterations

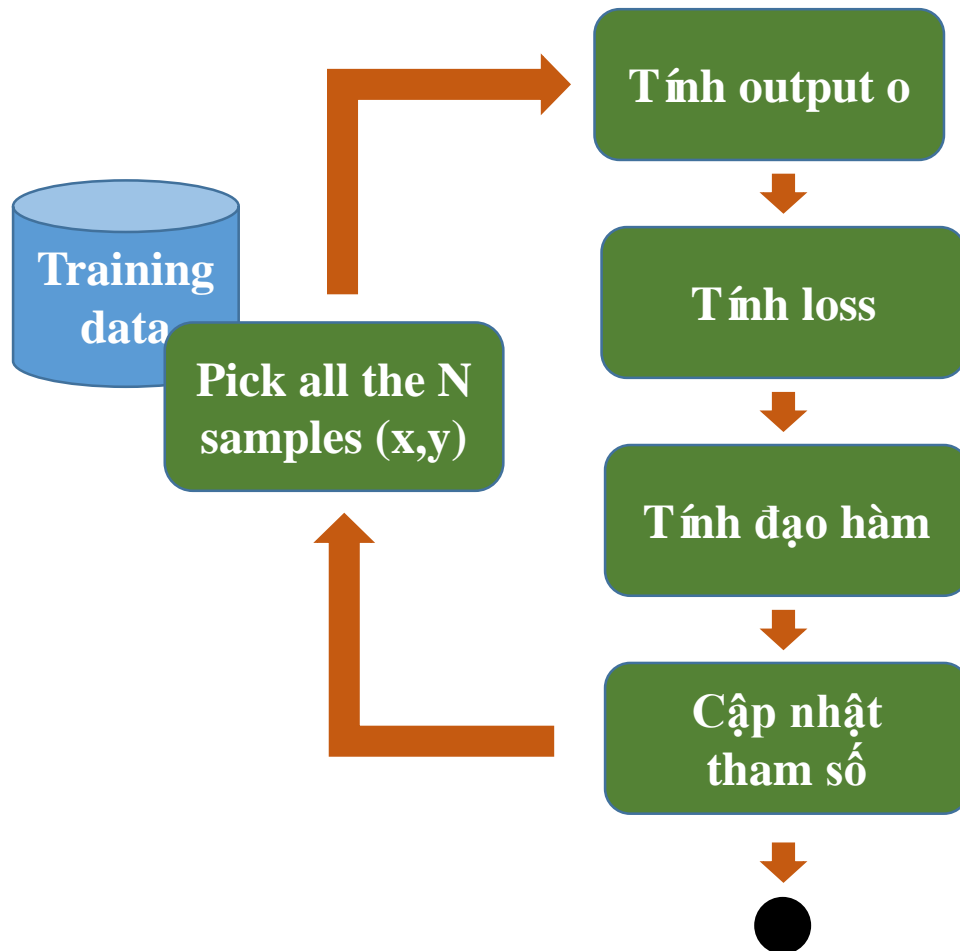
# Outline

- Machine Learning
- Derivative/Gradient
- Linear Regression
- Computational Graph
  - 1-sample training
  - m-sample training
  - N-sample training
- Generalized formulae

# Computational graph

## ❖ House price prediction

### ❖ N-sample training



1) Pick all the N samples  $(x^{(i)}, y^{(i)})$  from training data

2) Tính output  $o_i$

$$o^{(i)} = wx^{(i)} + b \quad \text{for } 0 \leq i < N$$

3) Tính loss

$$L^{(i)} = (o^{(i)} - y^{(i)})^2 \quad \text{for } 0 \leq i < N$$

4) Tính đạo hàm

$$L'_w{}^{(i)} = 2x(o^{(i)} - y^{(i)})$$
$$L'_b{}^{(i)} = 2(o^{(i)} - y^{(i)}) \quad \text{for } 0 \leq i < N$$

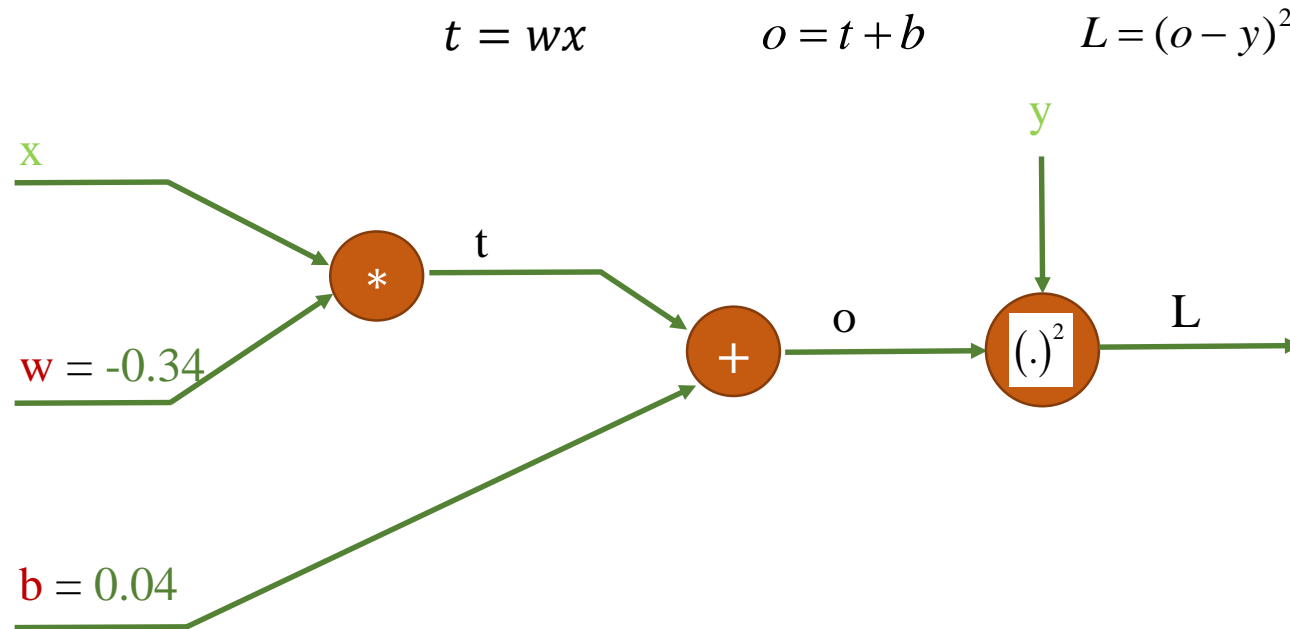
5) Cập nhật tham số

$$w = w - \eta \frac{\sum_i L'_w{}^{(i)}}{N}$$
$$b = b - \eta \frac{\sum_i L'_b{}^{(i)}}{N} \quad \text{Learning rate } \eta$$

# Computational graph

## ❖ House price prediction

### ❖ N-sample training



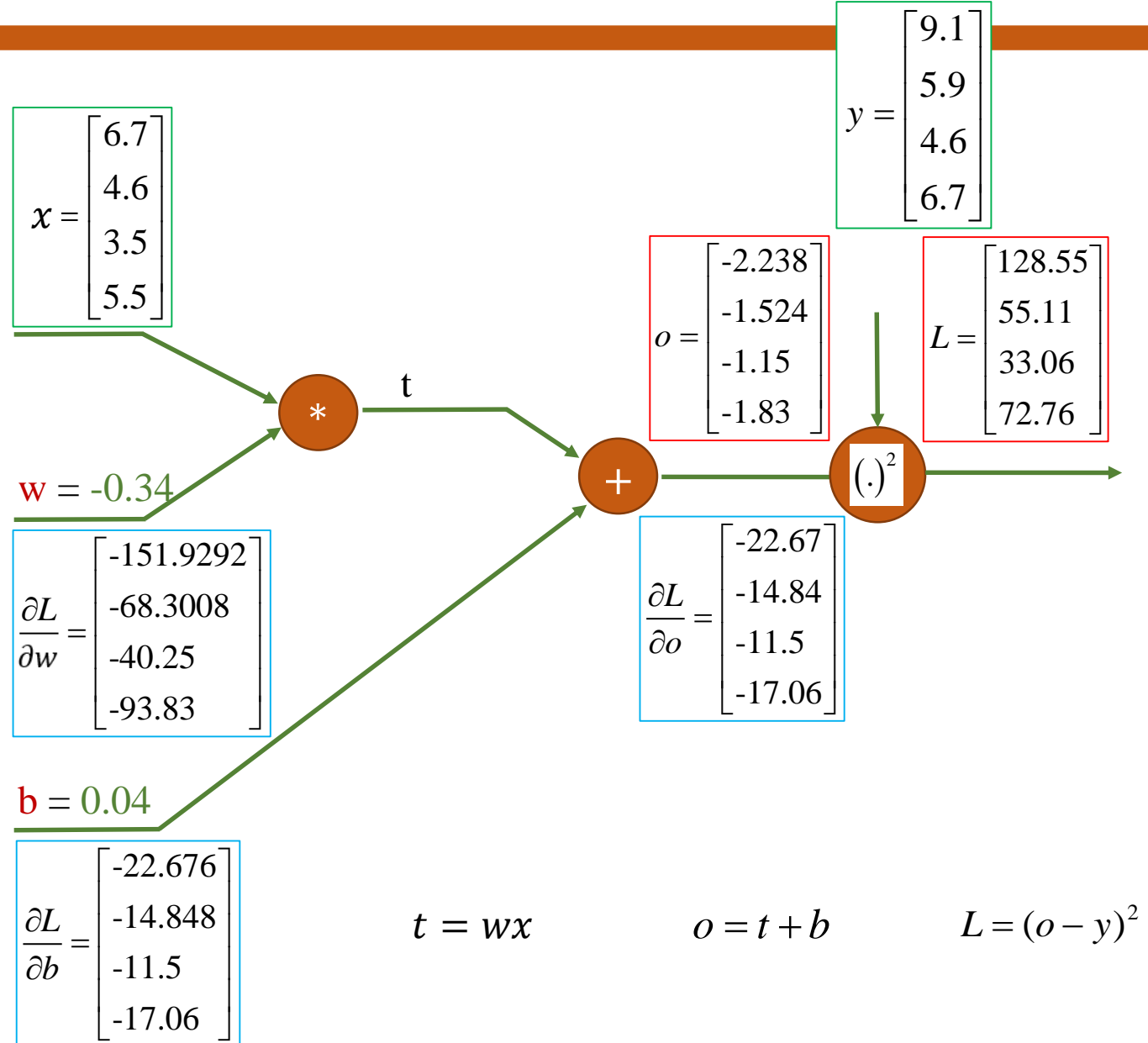
# Computational graph

## ❖ House price prediction

### ❖ N-sample training

$$\frac{\text{sum}(\frac{\partial L}{\partial w})}{4} = -88.5775$$

$$\frac{\text{sum}(\frac{\partial L}{\partial b})}{4} = -16.521$$



# Computational graph

## ❖ House price prediction

### ❖ N-sample training

#### Cách cập nhật a và b

$$w = w - \eta * \frac{\partial L}{\partial w}$$

$$b = b - \eta * \frac{\partial L}{\partial b}$$

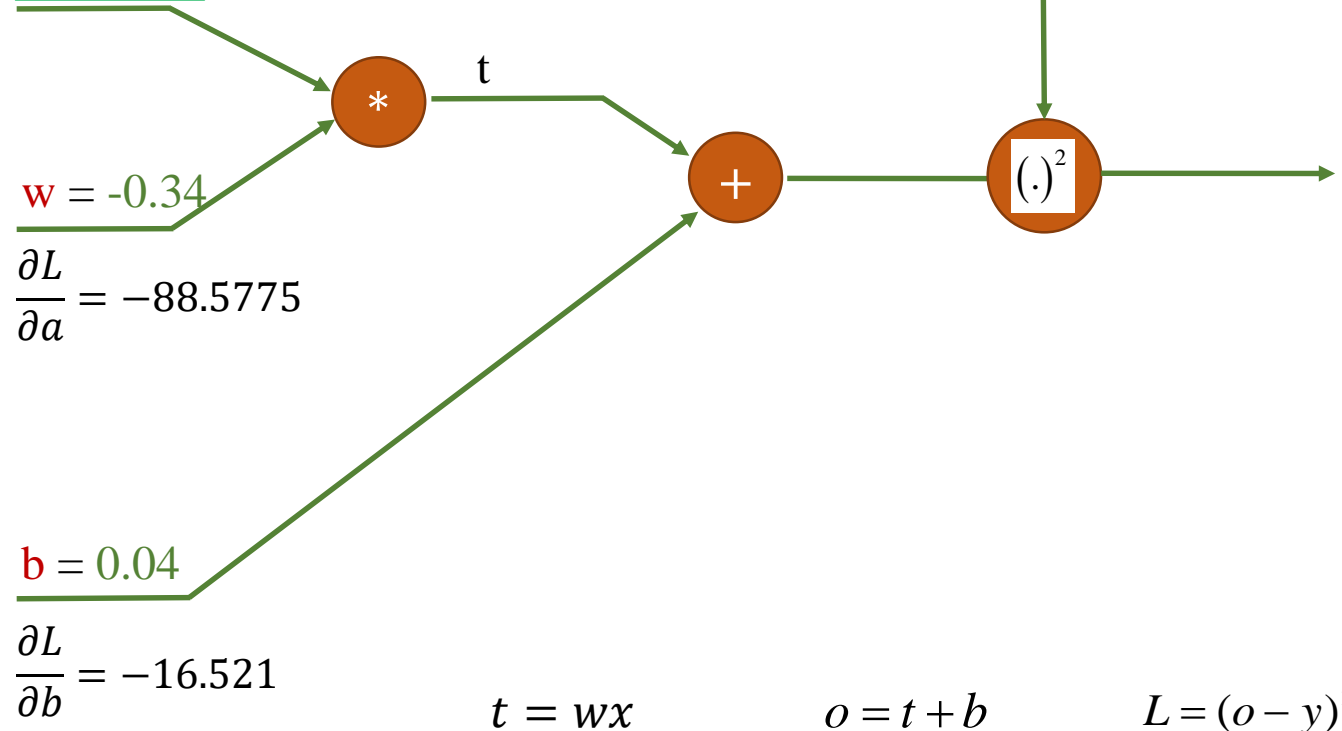
Learning rate  $\eta = 0.01$

$$w = -0.34 - (0.01 * (-88.5775)) = 0.54$$

$$b = 0.04 - (0.01 * (-16.521)) = 0.205$$

$$x = \begin{bmatrix} 6.7 \\ 4.6 \\ 3.5 \\ 5.5 \end{bmatrix}$$

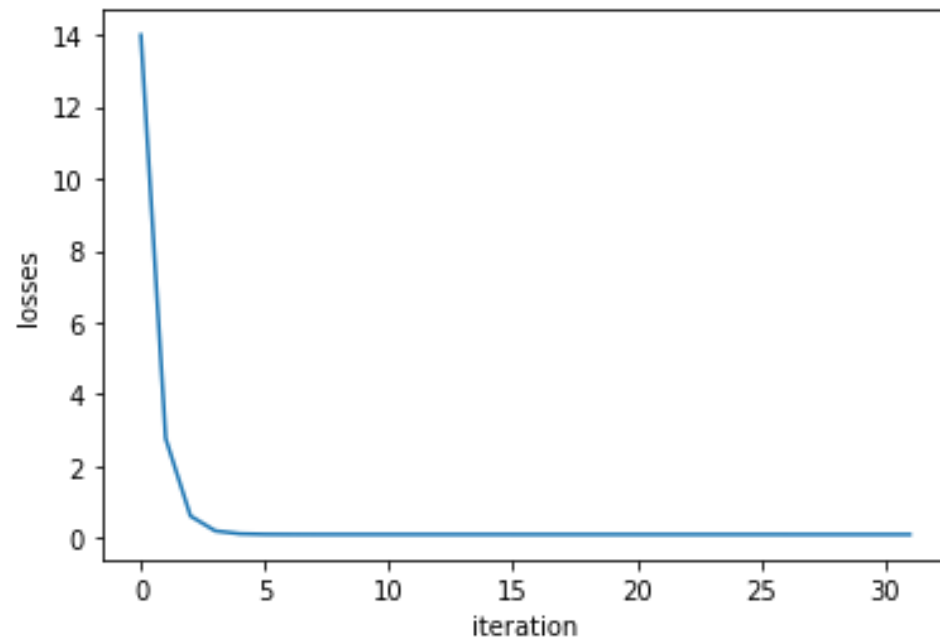
$$y = \begin{bmatrix} 9.1 \\ 5.9 \\ 4.6 \\ 6.7 \end{bmatrix}$$



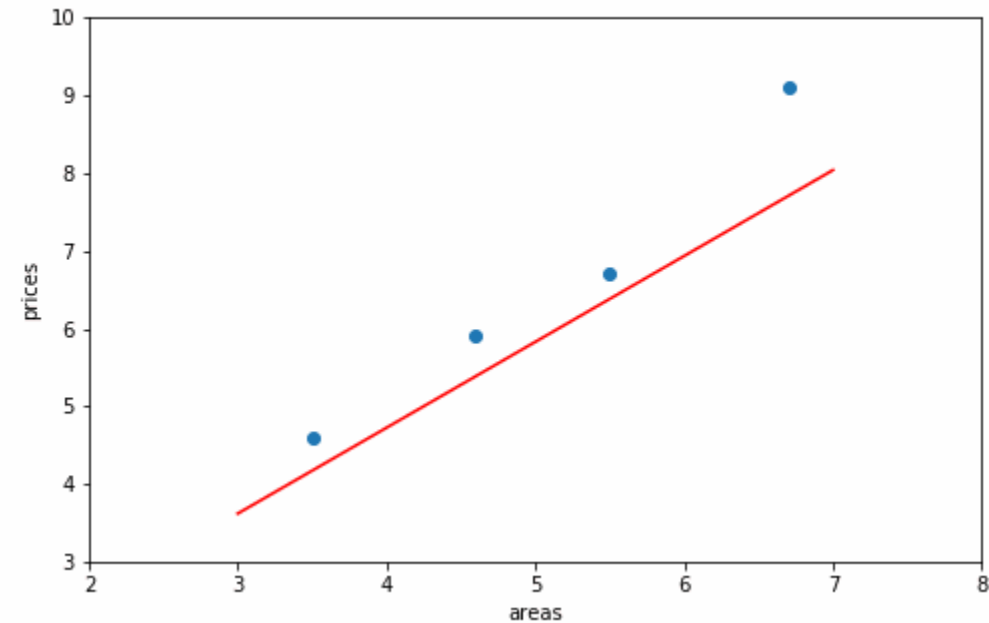
# Computational graph

## ❖ House price prediction

### ❖ N-sample training



Losses for 30 iterations



Model updating for different iterations

# Outline

- Machine Learning
- Derivative/Gradient
- Linear Regression
- Computational Graph
- Generalized formulae



# Linear Regression

## ❖ Generalized formula

House  
price data

	Feature	Label	
	area	price	
	6.7	9.1	
	4.6	5.9	
	3.5	4.6	
	5.5	6.7	

Model

$$\text{price} = w * \text{area} + b$$

$$y = wx + b$$

Model (vectorization)

$$y = \theta^T x \quad \text{where} \quad \theta^T = [b \quad w]^T$$

$$x = [x_0 \quad \text{area}]^T$$

$$x_0 = 1$$

Features			Label
TV	Radio	Newspaper	Sales
230.1	37.8	69.2	22.1
44.5	39.3	45.1	10.4
17.2	45.9	69.3	12
151.5	41.3	58.5	16.5
180.8	10.8	58.4	17.9

Advertising  
data

Model

$$\text{Sale} = w_1 * TV + w_2 * \text{Radio} + w_3 * \text{Newspaper} + b$$

$$y = w_1 x_1 + w_2 x_2 + w_3 x_3 + b$$

Model (vectorization)

$$y = \theta^T x \quad \text{where} \quad \theta^T = [b \quad w_1 \quad w_2 \quad w_3]^T$$

$$x = [x_0 \quad TV \quad \text{Radio} \quad \text{Newspaper}]^T$$

$$x_0 = 1$$

# Linear Regression

## ❖ Generalized formula

Boston House  
Price Data

Features														Label
crim	zn	indus	chas	nox	rm	age	dis	rad	tax	ptratio	black	lstat	medv	
0.00632	18	2.31	0	0.538	6.575	65.2	4.09	1	296	15.3	396.9	4.98	24	
0.02731	0	7.07	0	0.469	6.421	78.9	4.9671	2	242	17.8	396.9	9.14	21.6	
0.03237	0	2.18	0	0.458	6.998	45.8	6.0622	3	222	18.7	394.63	2.94	33.4	
0.06905	0	2.18	0	0.458	7.147	54.2	6.0622	3	222	18.7	396.9	5.33	36.2	
0.08829	12.5	7.87	0	0.524	6.012	66.6	5.5605	5	311	15.2	395.6	12.43	22.9	

### Model

$$\text{medv} = w_1 * x_1 + \dots + w_{13} * x_{13} + b$$

### Model (vectorization)

$$y = \boldsymbol{\theta}^T \mathbf{x} \quad \text{where} \quad \boldsymbol{\theta}^T = [b \quad w_1 \quad \dots \quad w_{13}]^T$$

$$\mathbf{x} = [x_0 \quad x_1 \quad \dots \quad x_{13}]^T$$

$$x_0 = 1$$

# Linear Regression (1-sample)

1) Pick a sample  $(x, y)$  from training data

2) Tính output  $o$

$$o = wx + b$$

3) Tính loss

$$L = (o - y)^2$$

4) Tính đạo hàm

$$L'_w = 2x(o - y)$$

$$L'_b = 2(o - y)$$

5) Cập nhật tham số

$$w = w - \eta L'_w$$

$$b = b - \eta L'_b$$

$\eta$  is learning rate

1) Pick a sample  $(x, y)$  from training data

2) Tính output  $o$

$$o = \theta^T x$$

3) Tính loss

$$L = (o - y)^2$$

4) Tính đạo hàm

$$L'_\theta = 2x(o - y)$$

5) Cập nhật tham số

$$\theta = \theta - \eta L'_\theta$$

$\eta$  is learning rate

# Linear Regression (m-samples)

1) Pick m samples  $(x^{(i)}, y^{(i)})$  from training data

1.1) Tính output  $o^{(i)}$

$$o^{(i)} = wx^{(i)} + b \quad \text{for } 0 \leq i < m$$

1.2) Tính loss

$$L^{(i)} = (o^{(i)} - y^{(i)})^2 \quad \text{for } 0 \leq i < m$$

1.3) Tính đạo hàm

$$\begin{aligned} L'_w{}^{(i)} &= 2x(o^{(i)} - y^{(i)}) \\ L'_b{}^{(i)} &= 2(o^{(i)} - y^{(i)}) \end{aligned} \quad \text{for } 0 \leq i < m$$

2) Cập nhật tham số

$$\begin{aligned} w &= w - \eta \frac{\sum_i L'_w{}^{(i)}}{m} \\ b &= b - \eta \frac{\sum_i L'_b{}^{(i)}}{m} \end{aligned} \quad \eta \text{ is learning rate}$$

**Friendly version**

1) Pick m samples  $(x^{(i)}, y^{(i)})$  from training data

1.1) Tính output  $o^{(i)}$

$$o^{(i)} = \theta^T x^{(i)} \quad \text{for } 0 \leq i < m$$

1.2) Tính loss

$$L^{(i)} = (o^{(i)} - y^{(i)})^2 \quad \text{for } 0 \leq i < m$$

1.3) Tính đạo hàm

$$L'_\theta{}^{(i)} = 2x(o^{(i)} - y^{(i)}) \quad \text{for } 0 \leq i < m$$

2) Cập nhật tham số

$$\theta = \theta - \eta \frac{\sum_i L'_\theta{}^{(i)}}{m} \quad \eta \text{ is learning rate}$$

**Generalized formula**

# Linear Regression (N-samples)

1) Pick all the N samples from training data

2) Tính output  $o^{(i)}$

$$o^{(i)} = wx^{(i)} + b \quad \text{for } 0 \leq i < N$$

3) Tính loss

$$L^{(i)} = (o^{(i)} - y^{(i)})^2 \quad \text{for } 0 \leq i < N$$

4) Tính đạo hàm

$$\begin{aligned} L'_w{}^{(i)} &= 2x(o^{(i)} - y^{(i)}) \\ L'_b{}^{(i)} &= 2(o^{(i)} - y^{(i)}) \end{aligned} \quad \text{for } 0 \leq i < N$$

5) Cập nhật tham số

$$\begin{aligned} w &= w - \eta \frac{\sum_i L'_w{}^{(i)}}{N} \\ b &= b - \eta \frac{\sum_i L'_b{}^{(i)}}{N} \end{aligned} \quad \eta \text{ is learning rate}$$

**Friendly version**

1) Pick all the N samples from training data

2) Tính output  $o^{(i)}$

$$o^{(i)} = \boldsymbol{\theta}^T \mathbf{x}^{(i)} \quad \text{for } 0 \leq i < N$$

3) Tính loss

$$L^{(i)} = (o^{(i)} - y^{(i)})^2 \quad \text{for } 0 \leq i < N$$

4) Tính đạo hàm

$$L'_\theta{}^{(i)} = 2\mathbf{x}(o^{(i)} - y^{(i)}) \quad \text{for } 0 \leq i < N$$

5) Cập nhật tham số

$$\boldsymbol{\theta} = \boldsymbol{\theta} - \eta \frac{\sum_i L'_\theta{}^{(i)}}{N} \quad \eta \text{ is learning rate}$$

**Generalized formula**

# Linear Regression

## ❖ House price prediction

### Demo

```
Python 3.7.3 (default, Apr 24 2019, 15:29:51) [MSC v.1915 64 bit (AMD64)] ::  
Type "help", "copyright", "credits" or "license" for more information.  
>>>  
>>>  
>>>  
>>>  
>>>  
>>>  
>>>  
>>>  
>>> for epoch in range(n_epochs):  
...     sum_of_losses = 0  
...     gradients = np.zeros((2,1))  
...  
...     for index in range(4):  
...         xi = X_b[index:index+1]  
...         yi = y[index:index+1]
```

# Linear Regression

## ❖ Advertising-based sale prediction

### Demo

```
Python 3.7.3 (default, Apr 24 2019, 15:29:51) [MSC v.1915 64 bit (AMD64)] ::  
Type "help", "copyright", "credits" or "license" for more information.  
>>>  
>>>  
>>>  
>>>  
>>>  
>>>  
>>>  
>>>  
>>> for epoch in range(n_epochs):  
...     sum_of_losses = 0  
...     gradients = np.zeros((2,1))  
...  
...     for index in range(4):  
...         xi = X_b[index:index+1]  
...         yi = y[index:index+1]
```

# Linear Regression

## ❖ House price prediction

### ❖ Boston data

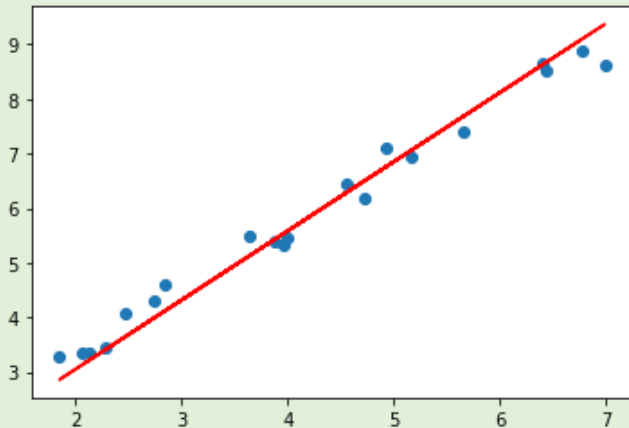
### Demo

```
Python 3.7.3 (default, Apr 24 2019, 15:29:51) [MSC v.1915 64 bit (AMD64)] ::  
Type "help", "copyright", "credits" or "license" for more information.  
>>>  
>>>  
>>>  
>>>  
>>>  
>>>  
>>>  
>>> for epoch in range(n_epochs):  
...     sum_of_losses = 0  
...     gradients = np.zeros((2,1))  
...  
...     for index in range(4):  
...         xi = X_b[index:index+1]  
...         yi = y[index:index+1]
```

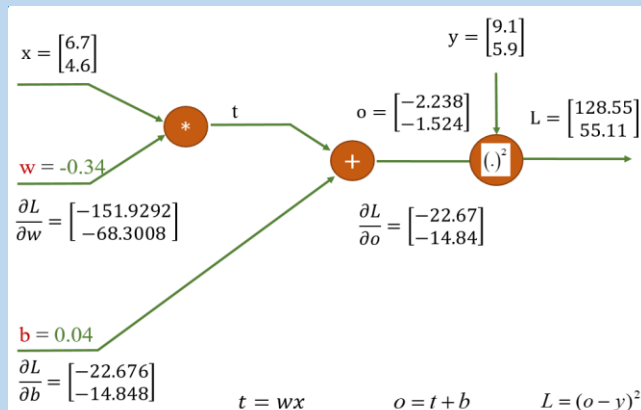


# Summary

## Linear regression



## Computational graph



## Formulae cheat sheet

1) Pick a sample  $(x, y)$  from training data

2) Tính output  $o$

$$o = \theta^T x$$

3) Tính loss

$$L = (o - y)^2$$

4) Tính đạo hàm

$$L'_\theta = 2x(o - y)$$

5) Cập nhật tham số

$$\theta = \theta - \eta L'_\theta$$

$\eta$  is learning rate

## Training procedure

