AI VIETNAM
AI Insight Course

Computational Graph and Linear Regression (Draft)

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Ph.D. in Computer Science

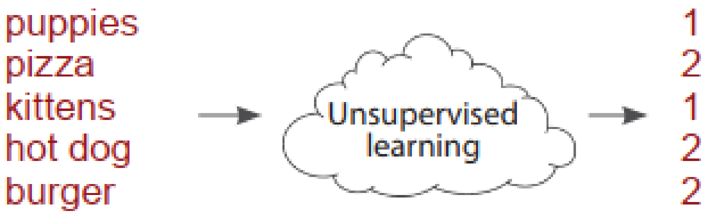
Year 2020

Outline

- Machine Learning
- Derivative/Gradient
- Linear Regression
- Computational Graph
- Summary

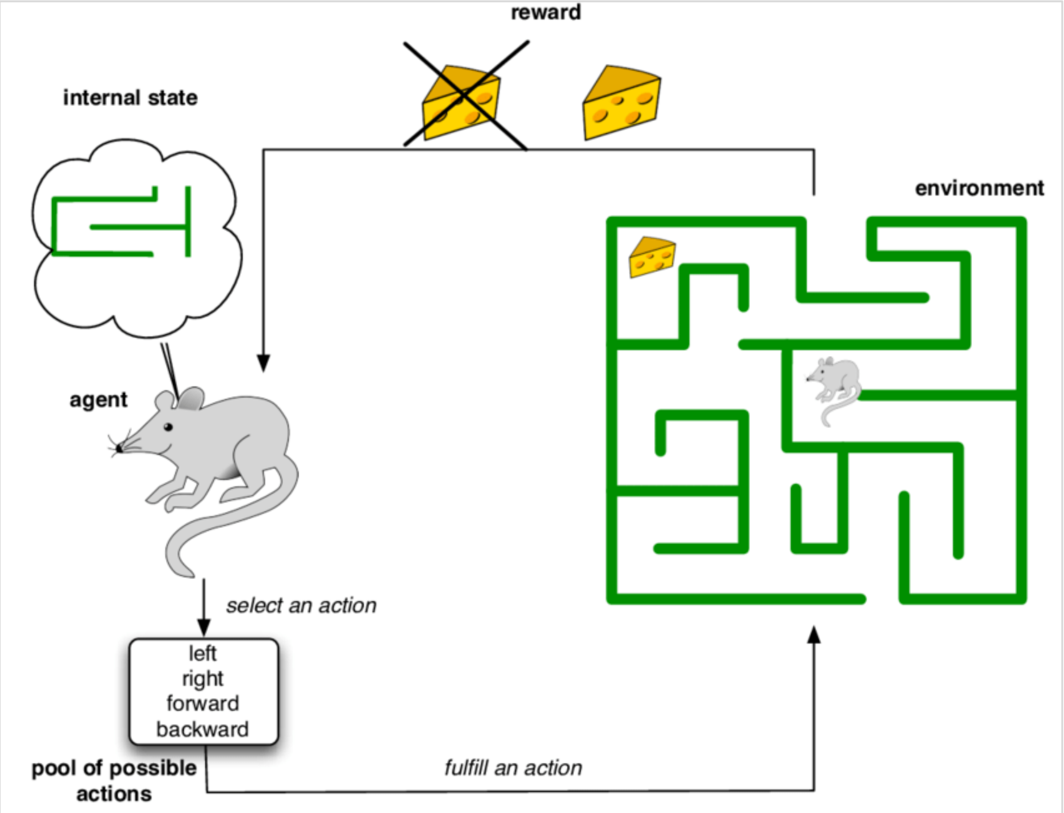
Machine Learning

❖ Definition



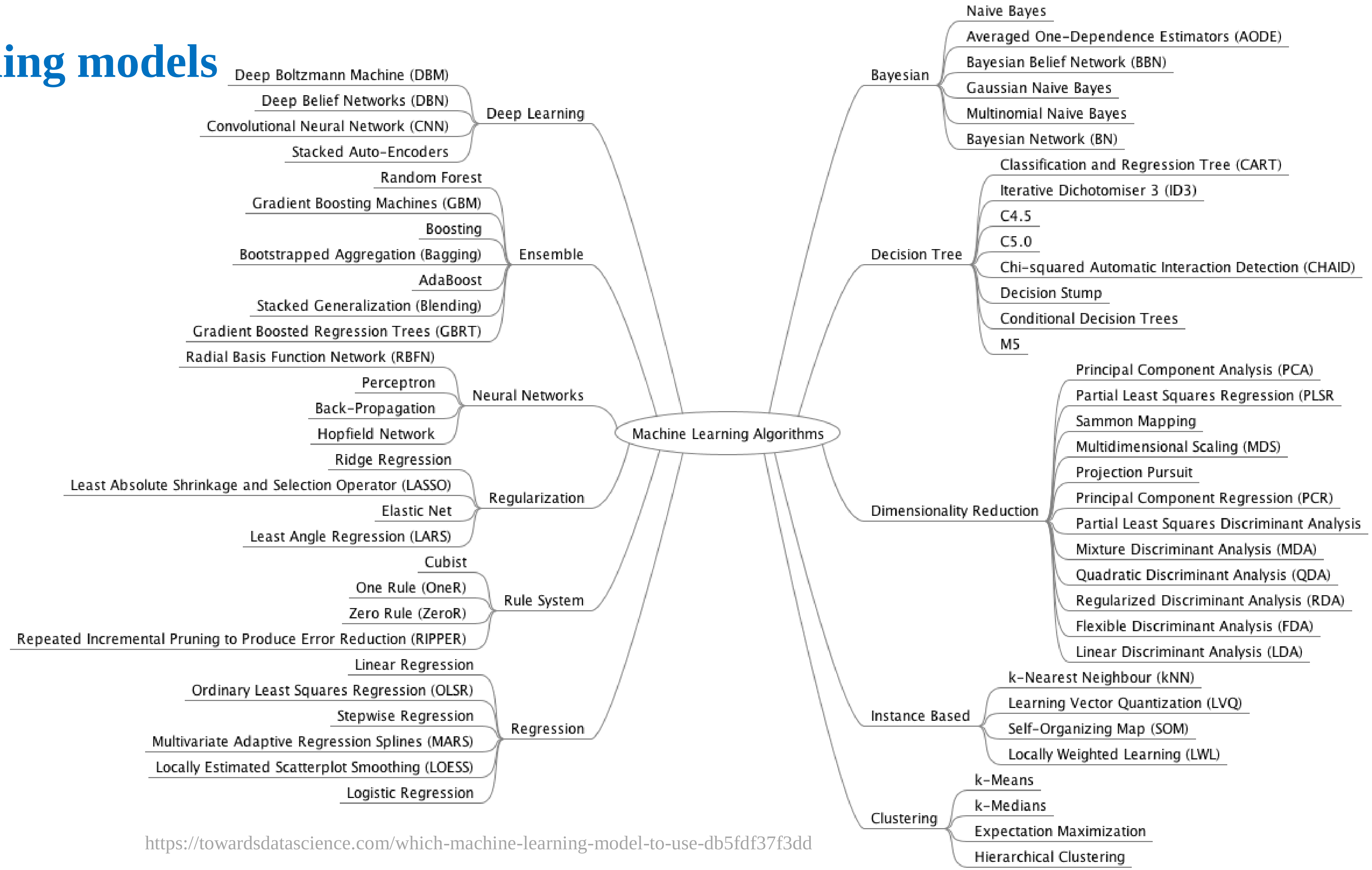
What is machine learning?

> "A field of study that gives computers the ability to learn without being explicitly programmed."
>
> —Attributed to Arthur Samuel



Machine Learning

Machine learning models



https://towardsdatascience.com/which-machine-learning-model-to-use-db5fdf37f3dd

Machine Learning

❖ Supervised learning

Input and output data is provided

Training data

Cats

Dogs

From Cat-Dog dataset

Machine Learning

❖ Supervised learning

Training data → Used to teach → Machine learning model

This is a cat

This is a dog

Machine learning model

Training phase

From Cat-Dog dataset

Testing data (≠ training data)

Machine learning model → Make decision → Cat or Dog?

Testing phase

Machine Learning

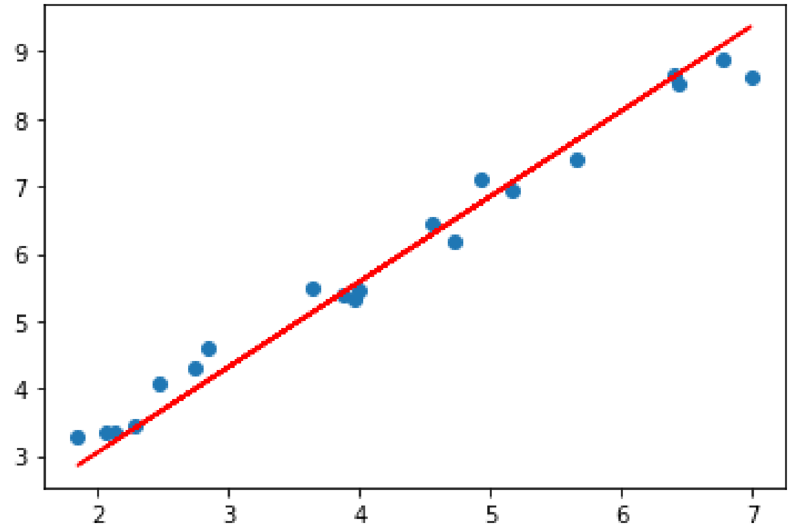
❖ Supervised learning

❖Regression (prediction)

Linear regression models ← Linear equations

Linear equation = $w_1x_1 + w_2x_2 + \cdots + w_nx_n + b$

where $\boldsymbol{w}$ is a weight vector
and $\boldsymbol{x}$ is feature vector



Machine Learning

❖ Supervised learning

❖ Linear Regression: Data processing

Feature	Label
area	price
6.7	9.1
4.6	5.9
3.5	4.6
5.5	6.7

House price data

Model: y = $w_1 x_1 + b$

price = a $* \, area + b$

Features			Label
TV	Radio	Newspaper	Sales
230.1	37.8	69.2	22.1
44.5	39.3	45.1	10.4
17.2	45.9	69.3	12
151.5	41.3	58.5	16.5
180.8	10.8	58.4	17.9

Advertising data

Model: y = $w_1 x_1 + w_2 x_2 + w_3 x_3 + b$

Sale = $w_1 * TV + w_2 * Radio + w_3 * Newspaper + b$

Machine Learning

❖ Supervised learning

❖ Linear Regression: Data processing

Boston House Price Data

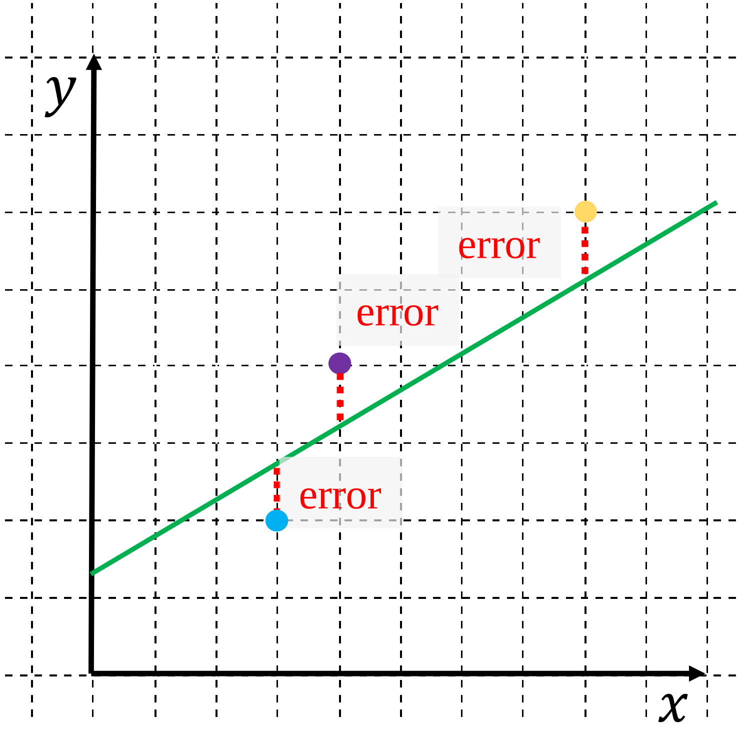
	Features												Label
crim	zn	indus	chas	nox	rm	age	dis	rad	tax	ptratio	black	lstat	medv
0.00632	18	2.31	0	0.538	6.575	65.2	4.09	1	296	15.3	396.9	4.98	24
0.02731	0	7.07	0	0.469	6.421	78.9	4.9671	2	242	17.8	396.9	9.14	21.6
0.03237	0	2.18	0	0.458	6.998	45.8	6.0622	3	222	18.7	394.63	2.94	33.4
0.06905	0	2.18	0	0.458	7.147	54.2	6.0622	3	222	18.7	396.9	5.33	36.2
0.08829	12.5	7.87	0	0.524	6.012	66.6	5.5605	5	311	15.2	395.6	12.43	22.9

$$\text{medv} = w_1 * x_1 + \cdots + w_{13} * x_{13} + b$$

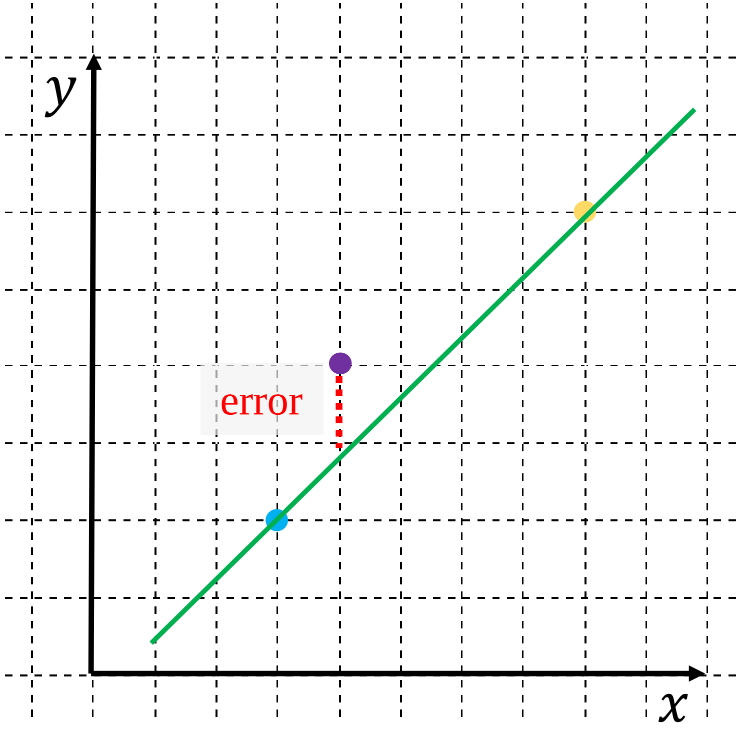
Machine Learning

❖ Linear regression

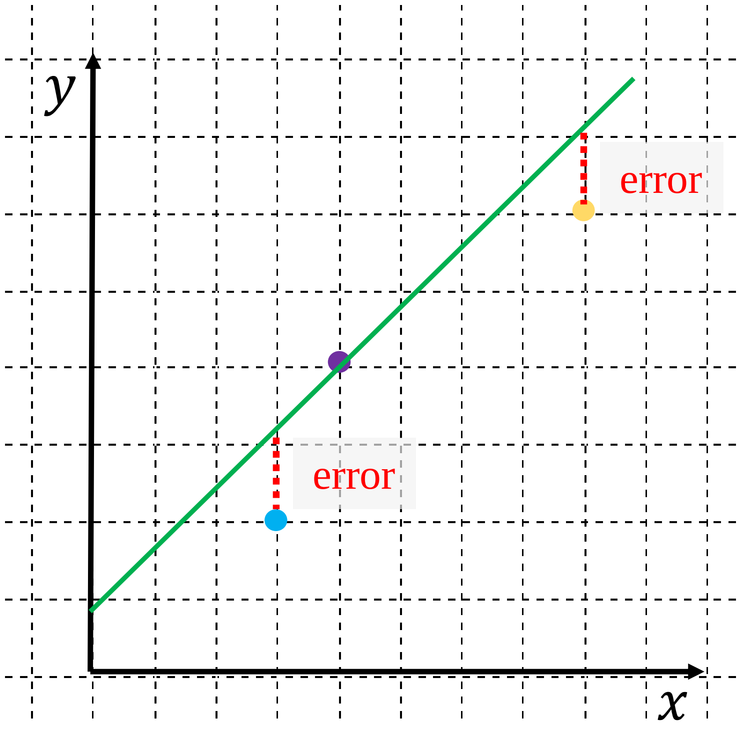
❖ Aim to fit data



$y = a_1 x + b_1$



$y = a_2 x + b_2$



$y = a_3 x + b_3$

Find w and b that have the smallest error. How?

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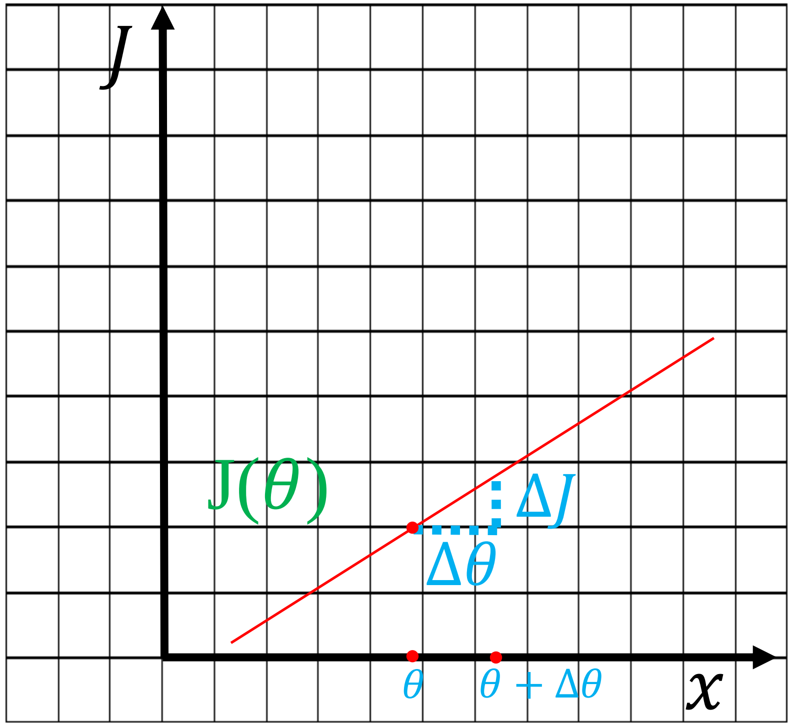
Derivative/Gradient

❖ A cue to optimize a function



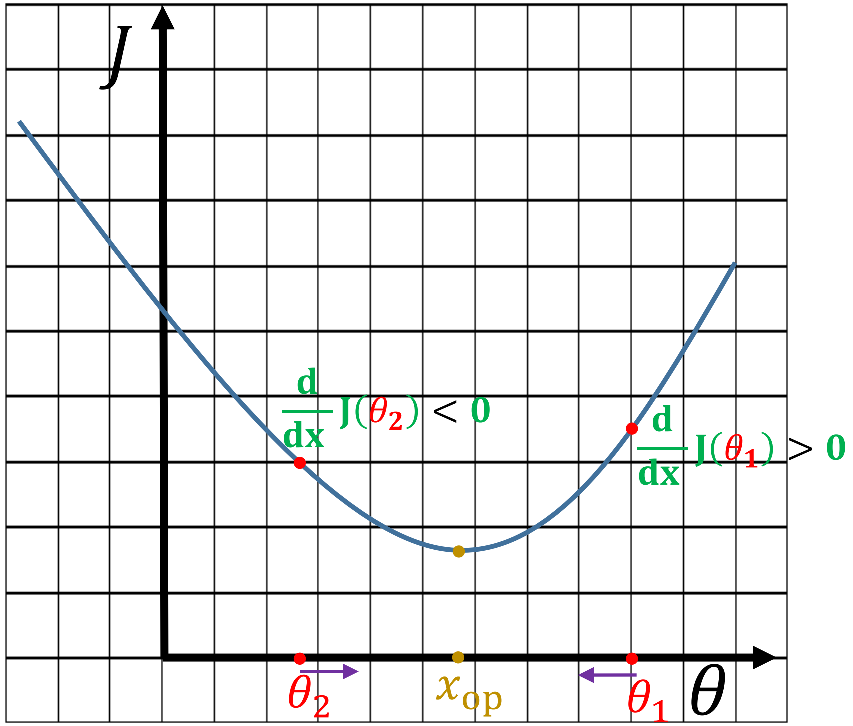
$$\text{Đạo hàm} = \frac{\textit{Thay đổi theo } J}{\textit{Thay đổi theo } \theta} = \frac{\Delta J}{\Delta \theta}$$

$$\frac{d}{d\theta}J(\theta) = \underbrace{\lim_{\Delta\theta \to 0}} \frac{J(\theta + \Delta\theta) - J(\theta)}{\Delta\theta}$$

$\Delta\theta$ cần tiến về 0 để đường tiếp tuyến tiến về hàm J(θ) trong vùng lân cận tại θ

Derivative/Gradient

❖ A cue to optimize a function



Quan sát: θ_{op} ở vị trí ngược hướng đạo hàm tại θ_1 và θ_2

Cách xử lý việc di chuyển ngược hướng đạo hàm cho θ_1 và θ_2 (để tìm θ_{op}) khác nhau hình thành các thuật toán tối ưu hóa khác nhau

Cách cập nhật giá trị x đơn giản

$$\theta = \theta - \eta \frac{\mathbf{d}}{\mathbf{dx}} \mathbf{J}(\theta)$$

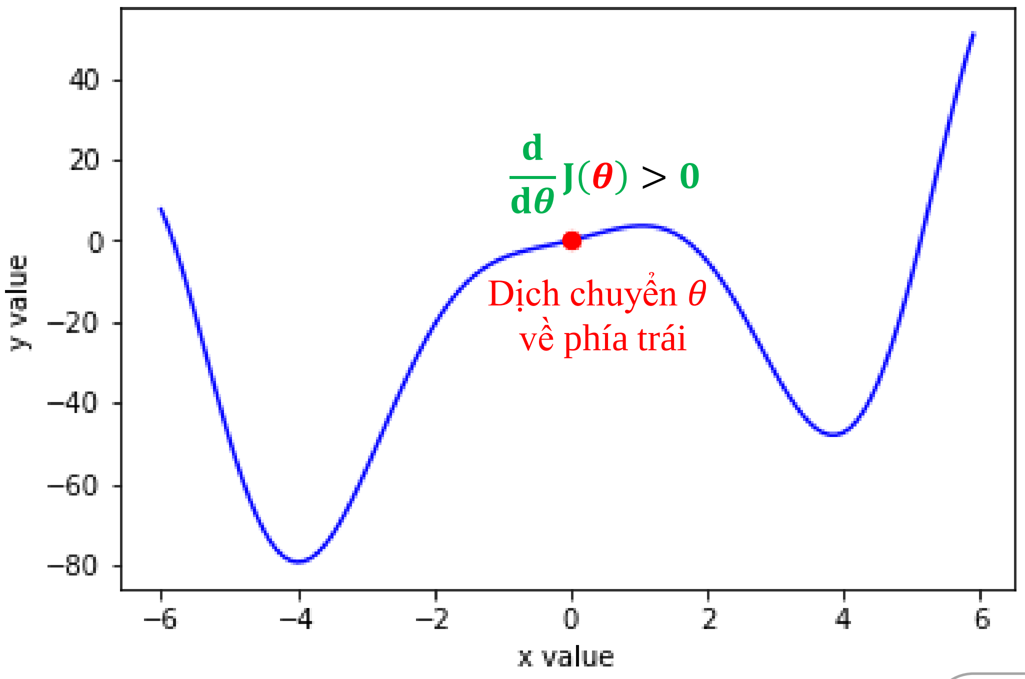
Đạo hàm tại θ

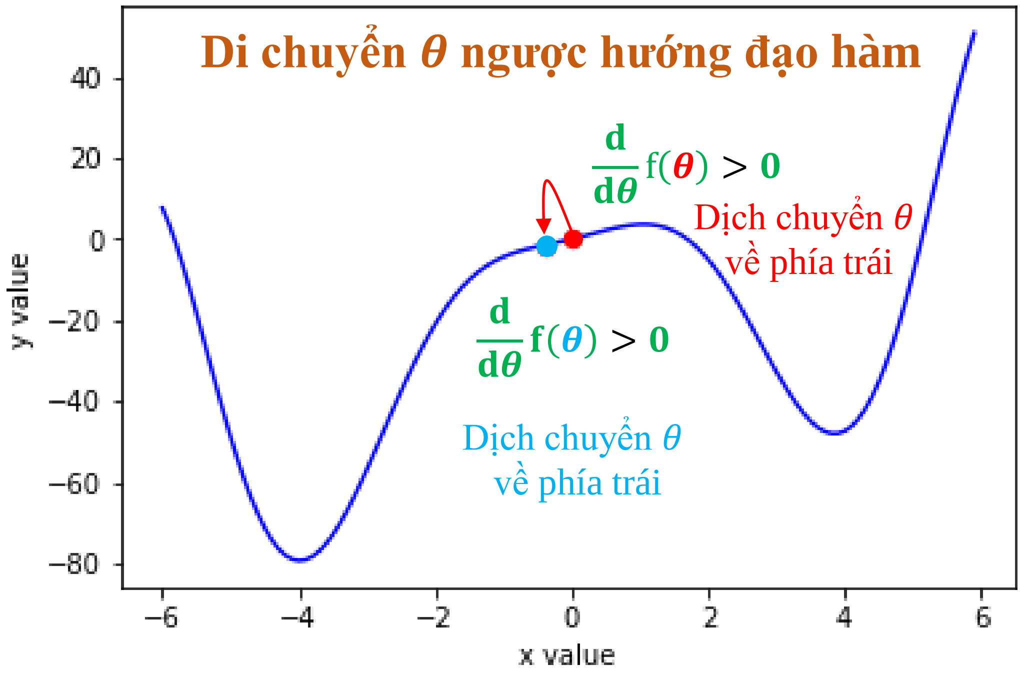
Trọng số

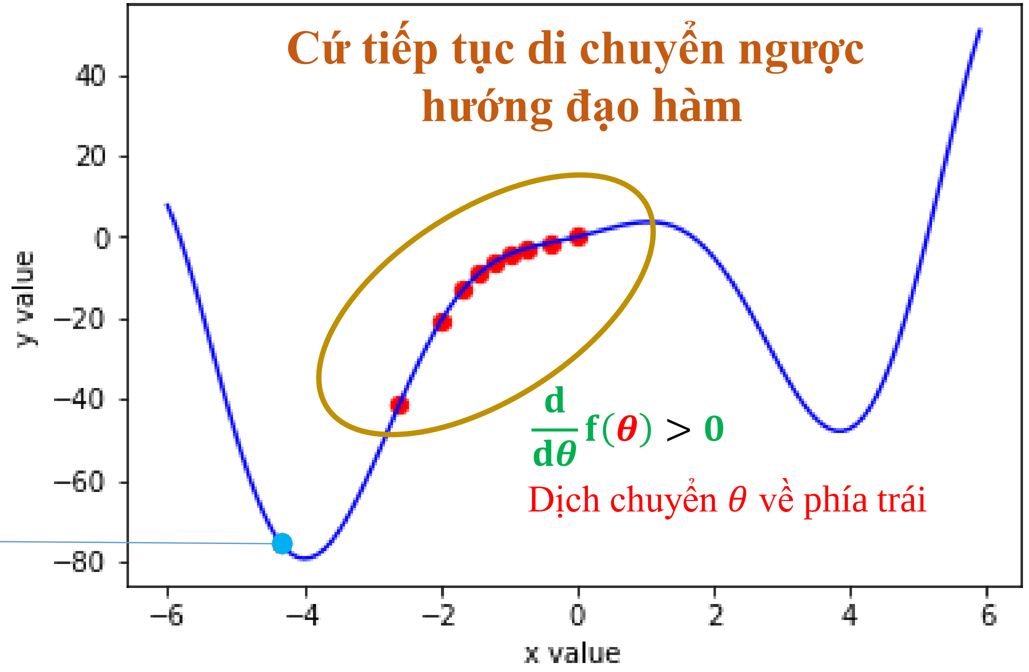
Derivative/Gradient

❖ **A cue to optimize a function**

Khởi tạo giá trị θ







$\frac{d}{d\theta}f(\theta) < 0$

Dịch chuyển θ về phía phải

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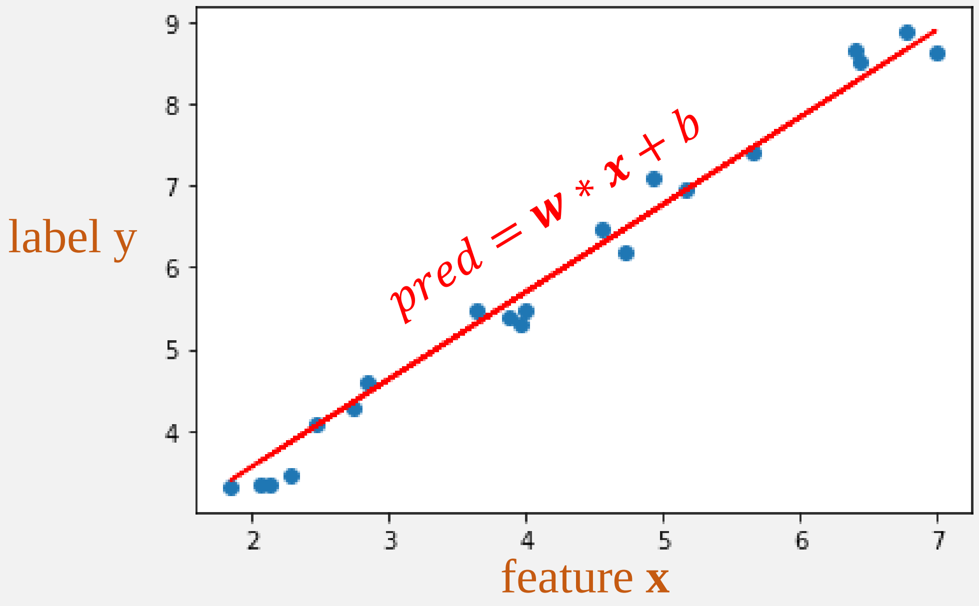
Linear Regression

Model the relationship between feature x and label y



Using a linear equation to fit data

Samples (x, y) are given in advance

Linear equation

$$o = w_1 x_1 + w_2 x_2 + \cdots + w_n x_n + b$$

where o is a predicted value,
$w_1, w_2, \ldots, w_n$ and b are parameters
and $\boldsymbol{x} = [x_1 \ x_2 \ \ldots \ x_n]^T$ is feature vector.

Error (loss) computation

Idea: compare predicted values o and label values y

Squared loss

$$\mathrm{L}(\mathbf{w}, \mathrm{b}) = (o - y)^2$$

How to find optimal w and b?

Linear Regression

Linear equation

$$o = w_1x_1 + w_2x_2 + \cdots + w_nx_n + b$$

where o is a predicted value,
$w_1, w_2, \dots, w_n$ and b are parameters
and $\boldsymbol{x} = [x_1 \;\; x_2 \;\; \dots \;\; x_n]^T$ is feature vector.

Error (loss) computation

Idea: compare predicted values o and label values y

Squared loss

$$\mathrm{L}(\mathbf{w}, \mathrm{b}) = (o - y)^2$$

How to find optimal **w** and b?

Use gradient descent to minimize the loss function

Tính đạo hàm

$$\frac{\partial L}{\partial w_j} = \frac{\partial L}{\partial o}\frac{\partial o}{\partial w_j} = 2x_j(o - y)$$

$$\frac{\partial L}{\partial b} = \frac{\partial L}{\partial o}\frac{\partial o}{\partial b} = 2(o - y)$$

Cập nhật tham số

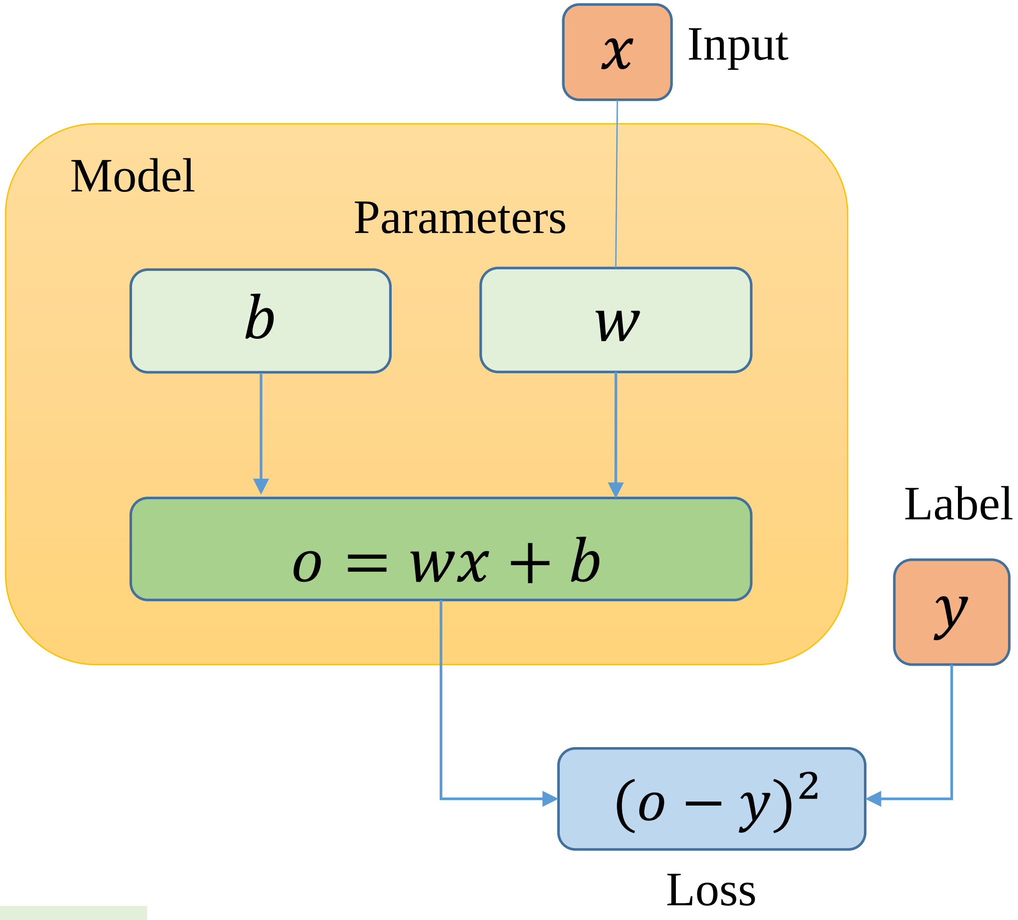
$$w_j = w_j - \eta L'_{w_j}$$

$$b = b - \eta L'_b$$

η is learning rate

Linear Regression

Diagram



Cheat sheet

Tính output o

$$o = \boldsymbol{wx} + b$$

Tính Loss

$$L = (o-y)^2$$

Tính đạo hàm

$$L'_{w_j} = 2x_j(o-y)$$

$$L'_b = 2(o-y)$$

Cập nhật tham số

$$w_j = w_j - \eta L'_{w_j}$$

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Linear Regression

Cheat sheet

Tính output o

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$$L'_b = 2(o - y)$$

Cập nhật tham số

$$w_j = w_j - \eta L'_{w_j}$$

$$b = b - \eta L'_b$$

```
# forward
def predict(x,w,b):
    return x*w + b

# compute gradient
def gradient(z,y,x):
    dw = 2*x*(z-y)
    db = 2*(z-y)

    return (dw, db)

# update weights
def update_weight(w,b,n,dw,db):
    w_new = w - n*dw
    b_new = b - n*db

    return (w_new, b_new)
```

Linear Regression

```python
# forward
def predict(x,w,b):
    return x*w + b

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def gradient(z,y,x):
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# update weights
def update_weight(w,b,n,dw,db):
    w_new = w - n*dw
    b_new = b - n*db

    return (w_new, b_new)
```

```python
# test with a sample
x = 6.7
y = 9.1

# init weight
b = 0.04
w = -0.34
n = 0.01

# predict z
z = predict(x,w,b)
print('z: ', z)

# compute loss
loss = (z-y)*(z-y)
print('Loss: ', loss)

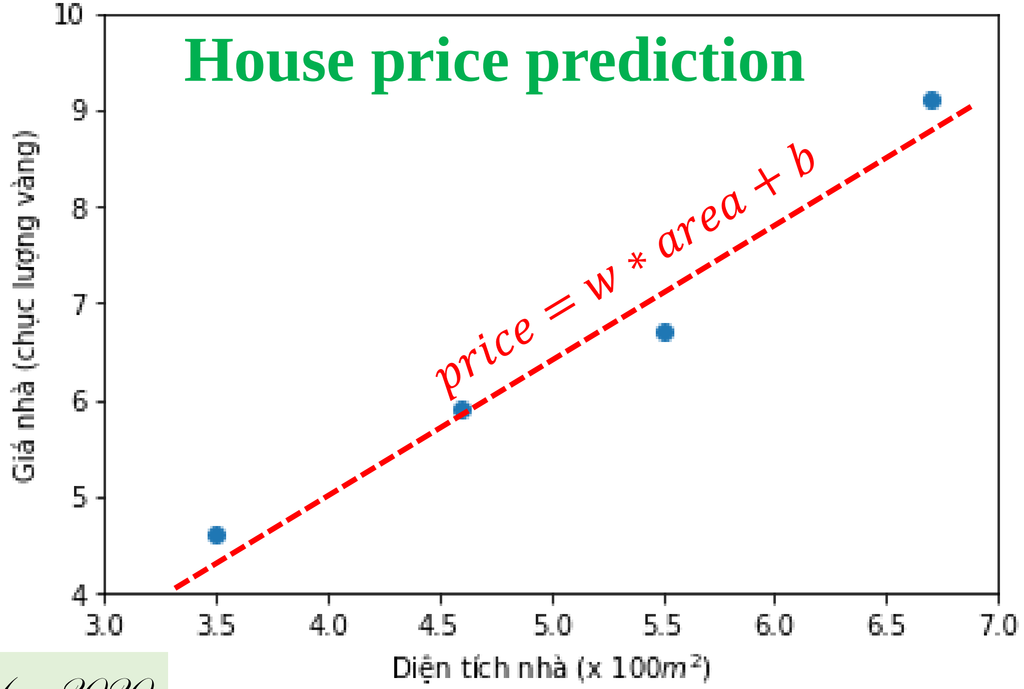
# compute gradient
(dw, db) = gradient(z,y,x)
print('dw: ', dw)
print('db: ', db)

# update weights
(w_new, b_new) = update_weight(w,b,n,dw,db)
print('w_new: ', w_new)
print('b_new: ', b_new)
```

Linear Regression

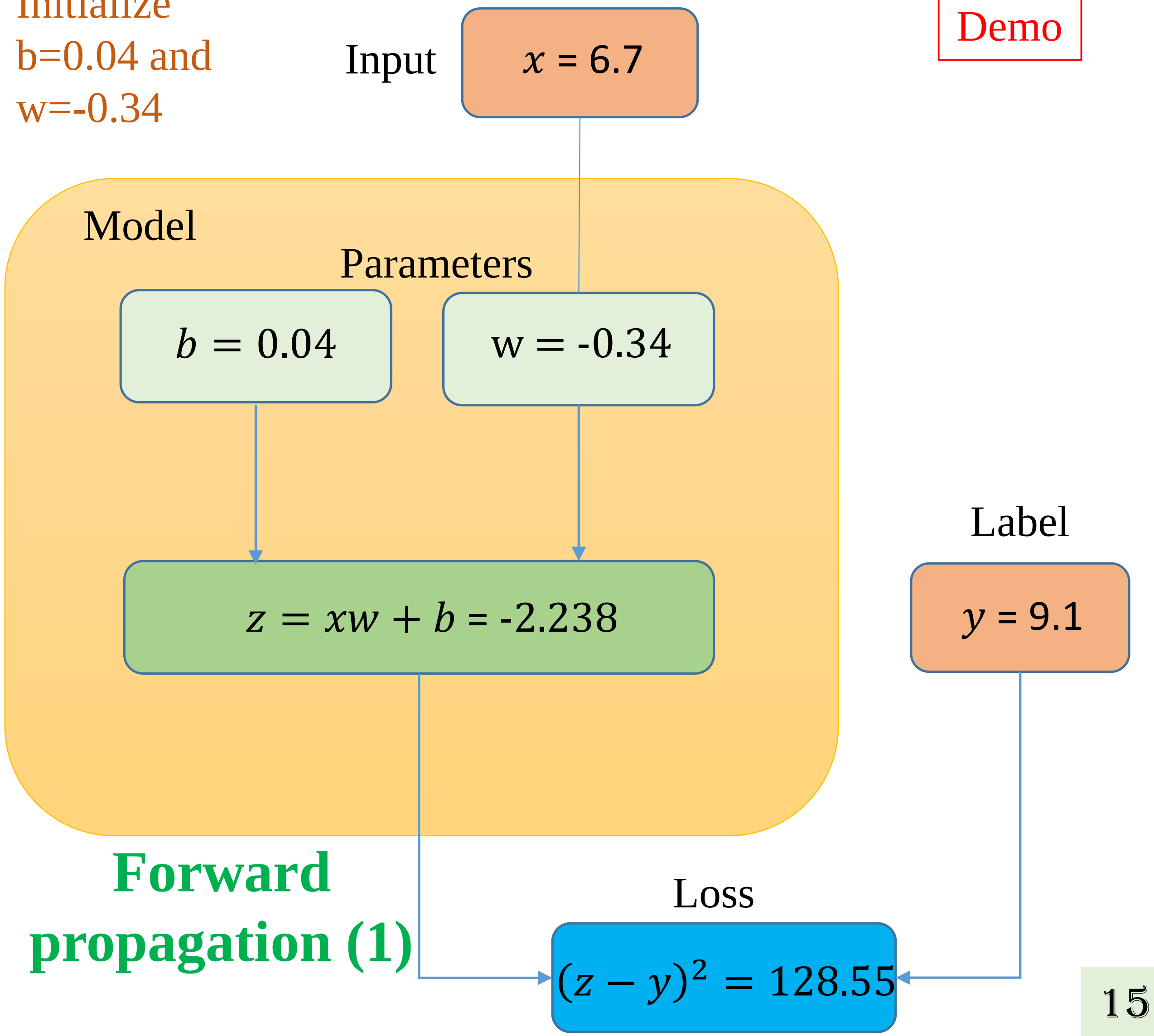
Given sample data

	area	price
	6.7	9.1
	4.6	5.9
	3.5	4.6
	5.5	6.7



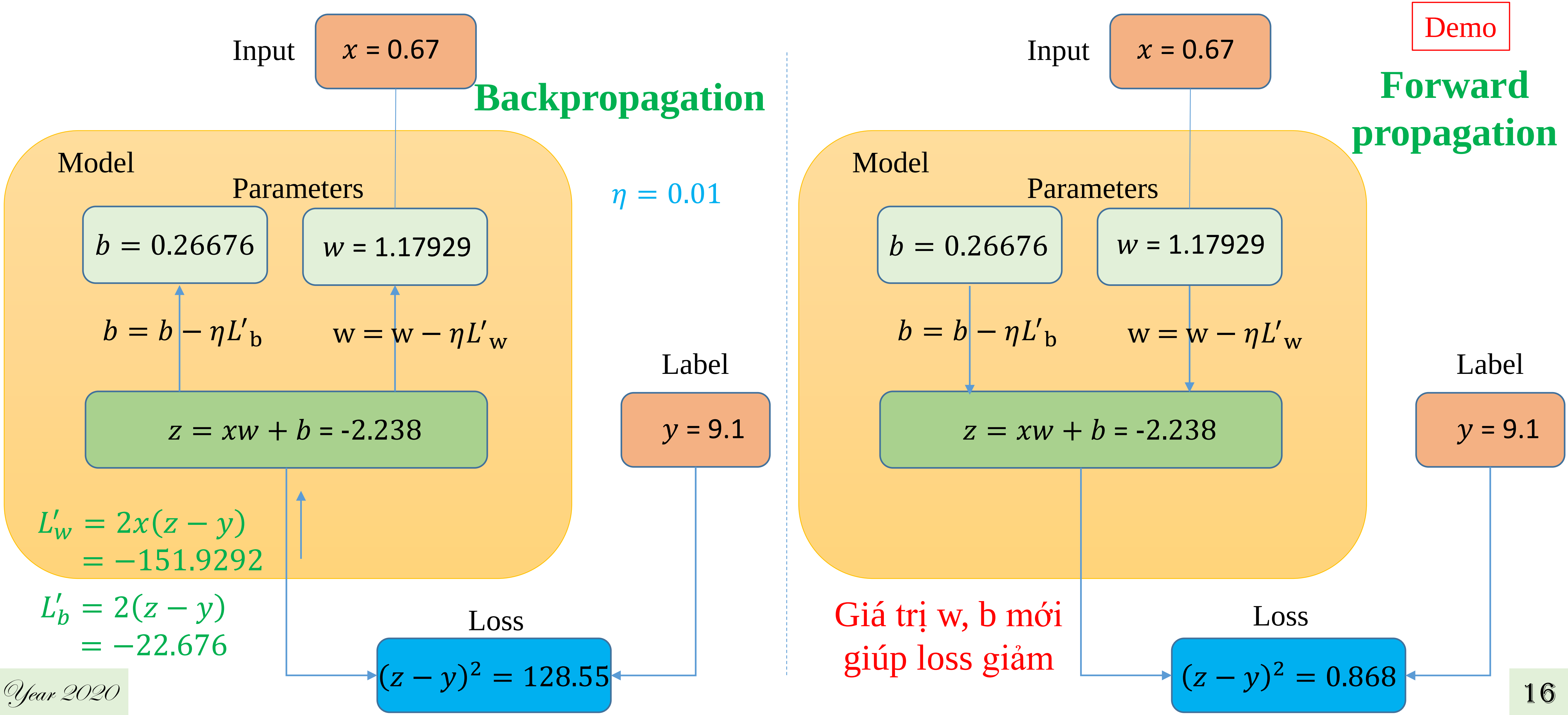
Initialize b=0.04 and w=-0.34

Demo



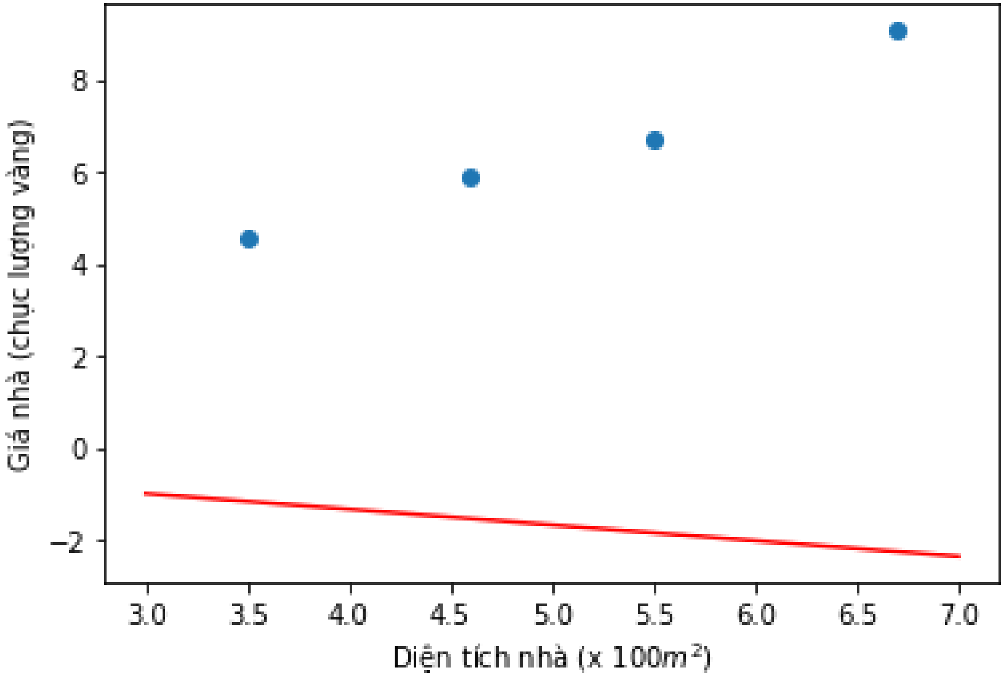
Forward propagation (1)

Linear Regression



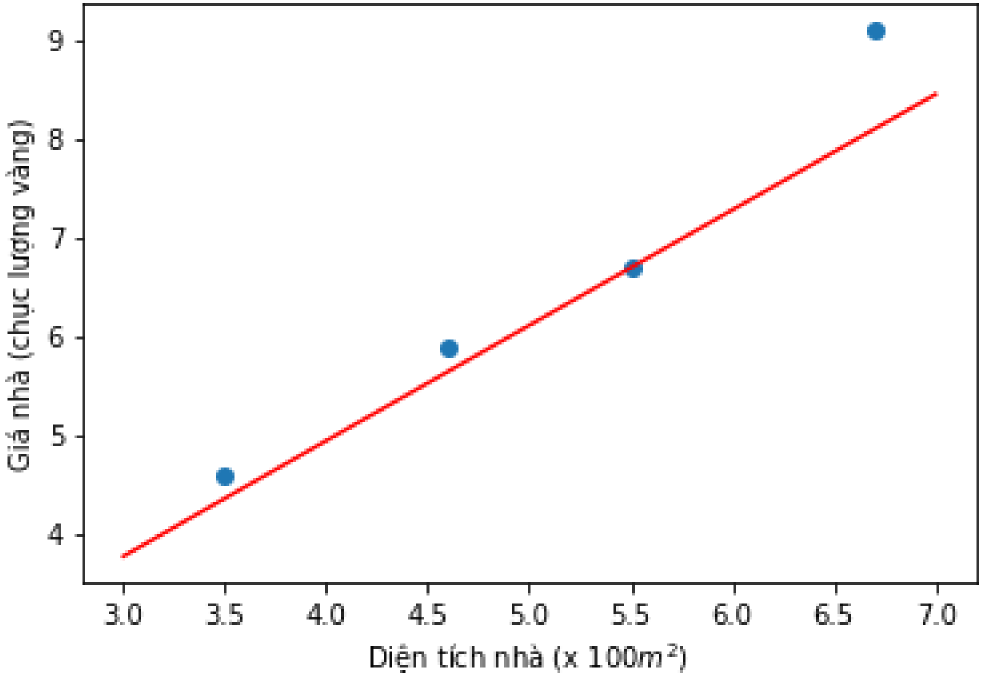
Linear Regression

Model prediction before and after the first update



w = -0.34 b = 0.04 L = 128.55

Before updating



w = 1.179292 b = 0.26676 L = 0.868

After updating

Linear Regression: Vectorization

1) Pick a sample (x, y) from training data

2) Tính output o

$$o = wx + b$$

3) Tính loss

$$L = (o - y)^2$$

4) Tính đạo hàm

$$L'_w = 2x(o - y)$$
$$L'_b = 2(o - y)$$

5) Cập nhật tham số

$$w = w - \eta L'_w$$
$$b = b - \eta L'_b$$

η is learning rate

Friendly version

1) Pick a sample (x, y) from training data

2) Tính output o

$$o = \boldsymbol{\theta}^T \boldsymbol{x}$$

3) Tính loss

$$L = (o - y)^2$$

4) Tính đạo hàm

$$L'_{\boldsymbol{\theta}} = 2\boldsymbol{x}(o - y)$$

5) Cập nhật tham số

$$\boldsymbol{\theta} = \boldsymbol{\theta} - \eta L'_{\boldsymbol{\theta}}$$

η is learning rate

Generalized formula

Linear Regression: Vectorization

1) Pick a sample (x, y) from training data

2) Tính output o

$$o = wx + b$$

3) Tính loss

$$L = (o - y)^2$$

4) Tính đạo hàm

$$L'_w = 2x(o - y)$$
$$L'_b = 2(o - y)$$

5) Cập nhật tham số

$$w = w - \eta L'_w$$
$$b = b - \eta L'_b$$

η is learning rate

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def predict(x,w,b):
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# compute gradient
def gradient(z,y,x):
    dw = 2*x*(z-y)
    db = 2*(z-y)

    return (dw, db)

# update weights
def update_weight(w,b,n,dw,db):
    w_new = w - n*dw
    b_new = b - n*db

    return (w_new, b_new)
```


Friendly version

Linear Regression: Vectorization

1) Pick a sample (x, y) from training data

2) Tính output o

$$o = \boldsymbol{\theta}^T \boldsymbol{x}$$

3) Tính loss

$$L = (o - y)^2$$

4) Tính đạo hàm

$$L'_{\boldsymbol{\theta}} = 2\boldsymbol{x}(o - y)$$

5) Cập nhật tham số

$$\boldsymbol{\theta} = \boldsymbol{\theta} - \eta L'_{\boldsymbol{\theta}}$$

η is learning rate

$$\underset{\mathbf{v}}{\begin{bmatrix} 1 \\ 2 \end{bmatrix}} \bullet \underset{\mathbf{w}}{\begin{bmatrix} 2 \\ 3 \end{bmatrix}} = \underset{\text{result}}{\begin{bmatrix} 8 \end{bmatrix}}$$

```python
# aivietnam.ai
import numpy as np

v = np.array([1, 2])
w = np.array([2, 3])

# Tính inner product giữa v và w
print('method 1 \n', v.dot(w))
print('method 2 \n', np.dot(v, w))
```

```
method 1
8

method 2
8
```

Generalized formula

Linear Regression: Vectorization

1) Pick a sample (x, y) from training data

2) Tính output o

$$o = \boldsymbol{\theta}^T \boldsymbol{x}$$

3) Tính loss

$$L = (o - y)^2$$

4) Tính đạo hàm

$$L'_{\boldsymbol{\theta}} = 2\boldsymbol{x}(o - y)$$

5) Cập nhật tham số

$$\boldsymbol{\theta} = \boldsymbol{\theta} - \eta L'_{\boldsymbol{\theta}}$$

η is learning rate

data x		data y		result
1		5		5
2	*	6	=	12
3		7		21
4		8		32

```
# aivietnam.ai
import numpy as np

x = np.array([1,2,3,4])
y = np.array([5,6,7,8])

print('data x \n', x)
print('data y \n', y)

# Tích các phần tử tương ứng giữa x và y
print('method 1 \n', x*y)
print('method 2 \n', np.multiply(x, y))
```

Generalized formula

Linear Regression: Vectorization

1) Pick a sample (x, y) from training data

2) Tính output o

$$o = \boldsymbol{\theta}^T \boldsymbol{x}$$

3) Tính loss

$$L = (o - y)^2$$

4) Tính đạo hàm

$$L'_{\boldsymbol{\theta}} = 2\boldsymbol{x}(o - y)$$

5) Cập nhật tham số

$$\boldsymbol{\theta} = \boldsymbol{\theta} - \eta L'_{\boldsymbol{\theta}}$$

η is learning rate

```python
import numpy as np

# forward
def predict(x,theta):
    return x.dot(theta)

# compute gradient
def gradient(z,y,x):
    dtheta = 2*x*(z-y)

    return dtheta

# update weights
def update_weight(theta,n,dtheta):
    dtheta_new = theta - n*dtheta

    return dtheta_new

```

Generalized formula

Linear Regression: Vectorization

```python
import numpy as np

# forward
def predict(x,theta):
    return x.dot(theta)

# compute gradient
def gradient(z,y,x):
    dtheta = 2*x*(z-y)

    return dtheta

# update weights
def update_weight(theta,n,dtheta):
    dtheta_new = theta - n*dtheta

    return dtheta_new

# test with a sample
x = np.array([6.7, 1])
y = np.array([9.1])

# init weight
n = 0.01
# thetas = [w, b]
theta = np.array([-0.34, 0.04])


# predict z
z = predict(x, theta)
print('Input data: ', x)
print('Theta: ', theta)
print('z: ', z)

# compute loss
loss = (z-y)*(z-y)
print('Loss: ', loss)

# compute gradient
dtheta = gradient(z,y,x)
print('dtheta: ', dtheta)

# update weights
theta_new = update_weight(theta,n,dtheta)
print('theta_new: ', theta_new)
```

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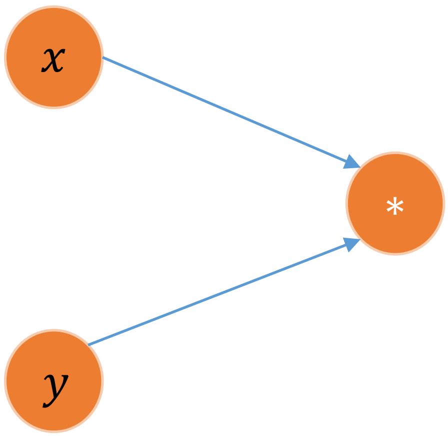
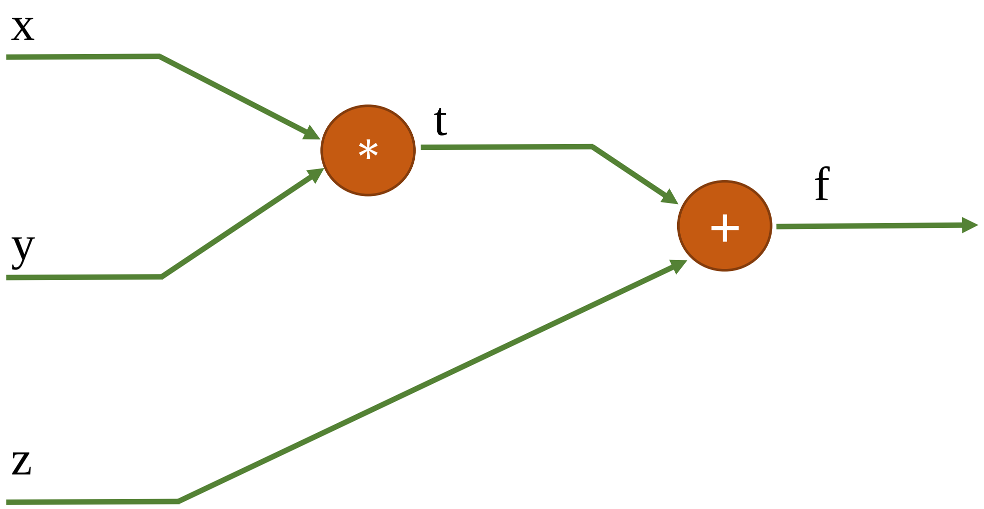
Computational graph

- ❖ A directed graph
- ❖ Nodes represent variables or operations



- ❖ Construct computational graph for $f(x, y, z) = x * y + z$
- ❖ Rewrite $f(x, y, z)$ as

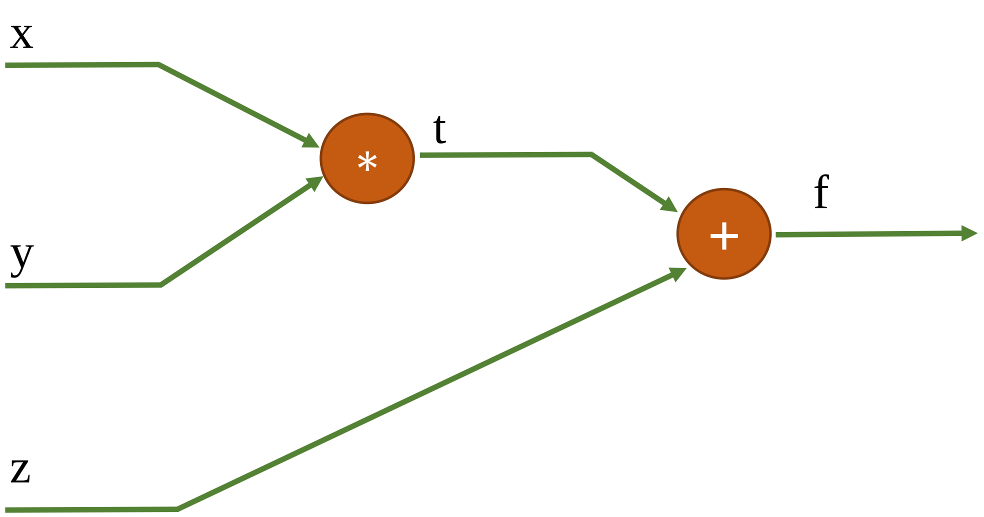
$f(t, z) = t + z$ where $t = x * y$



Computational graph

- **Construct computational graph for** $f(x, y, z) = x * y + z$
- **Rewrite** $f(x, y, z)$ **as**

$f(t, z) = t + z$ where $t = x * y$



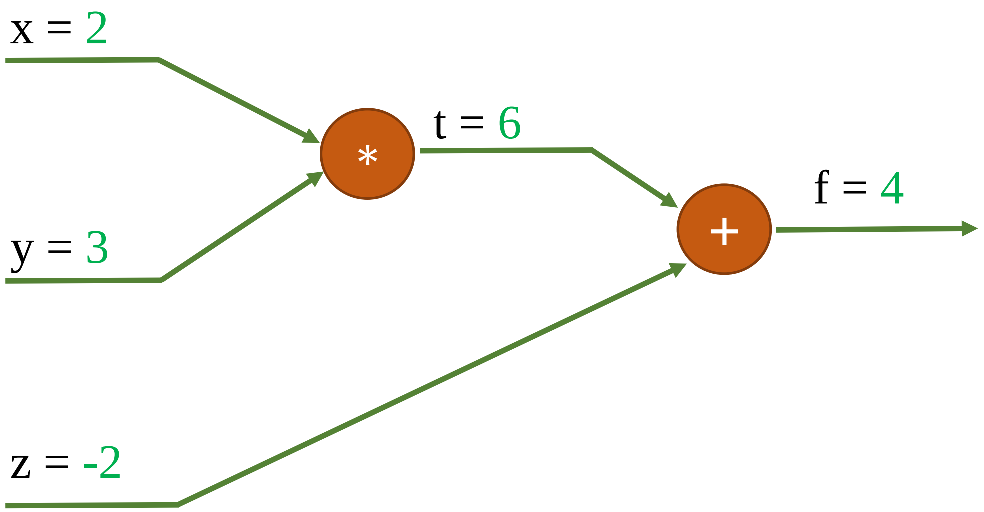
Partial derivative

$$\frac{\partial t}{\partial x} = y \qquad \frac{\partial f}{\partial z} = 1 \qquad \frac{\partial f}{\partial x} = y$$

$$\frac{\partial t}{\partial y} = x \qquad \frac{\partial f}{\partial t} = 1 \qquad \frac{\partial f}{\partial y} = x$$

Computational graph

❖ **Compute** $f(x, y, z)$ **với** $x = 2, y = 3$ **và** $z = -2$.



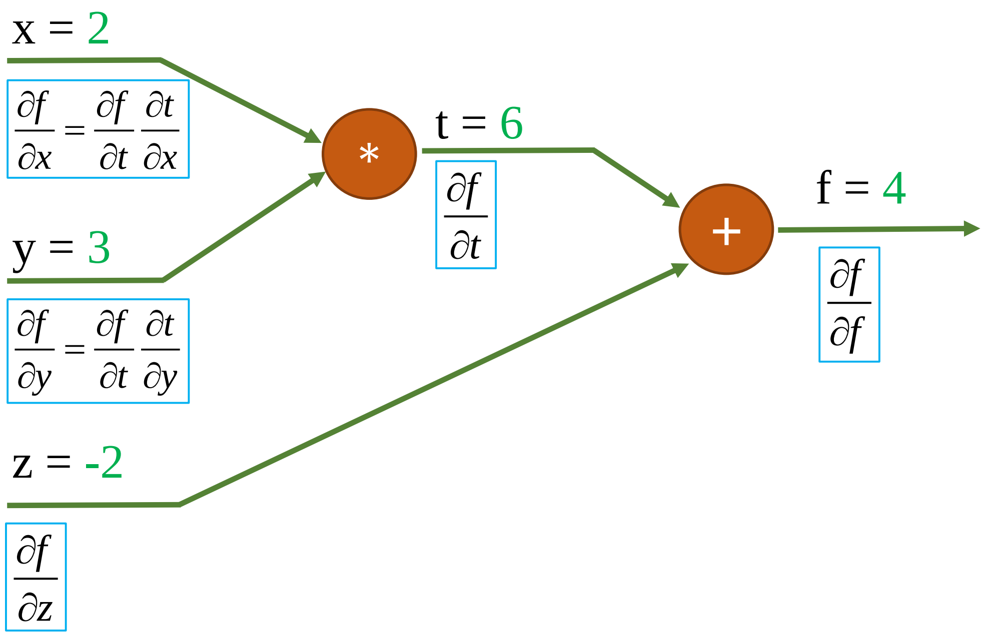
Partial derivative

$$\frac{\partial t}{\partial x} = y \qquad \frac{\partial f}{\partial z} = 1 \qquad \frac{\partial f}{\partial x} = y$$

$$\frac{\partial t}{\partial y} = x \qquad \frac{\partial f}{\partial t} = 1 \qquad \frac{\partial f}{\partial y} = x$$

Computational graph

❖ **Compute** $f(x, y, z)$ **với** $x = 2$, $y = 3$ **và** $z = -2$.



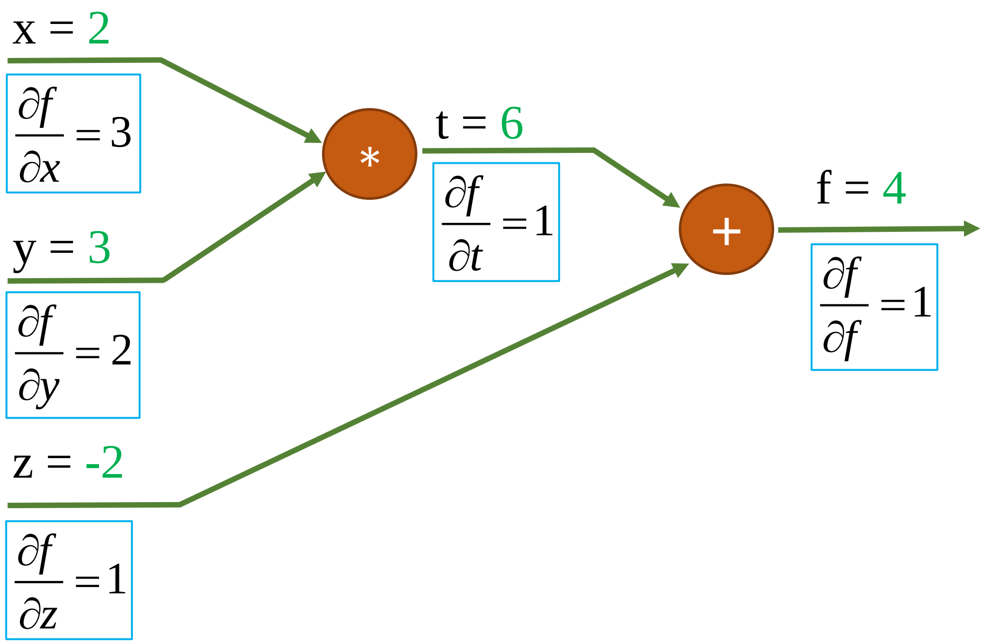
Partial derivative

$$\frac{\partial t}{\partial x} = y \qquad \frac{\partial f}{\partial z} = 1 \qquad \frac{\partial f}{\partial x} = y$$

$$\frac{\partial t}{\partial y} = x \qquad \frac{\partial f}{\partial t} = 1 \qquad \frac{\partial f}{\partial y} = x$$

Computational graph

❖ **Compute** $f(x, y, z)$ **với** $x = 2$, $y = 3$ **và** $z = -2$.



Partial derivative

$$\frac{\partial t}{\partial x} = y \qquad \frac{\partial f}{\partial z} = 1 \qquad \frac{\partial f}{\partial x} = y$$

$$\frac{\partial t}{\partial y} = x \qquad \frac{\partial f}{\partial t} = 1 \qquad \frac{\partial f}{\partial y} = x$$

Outline

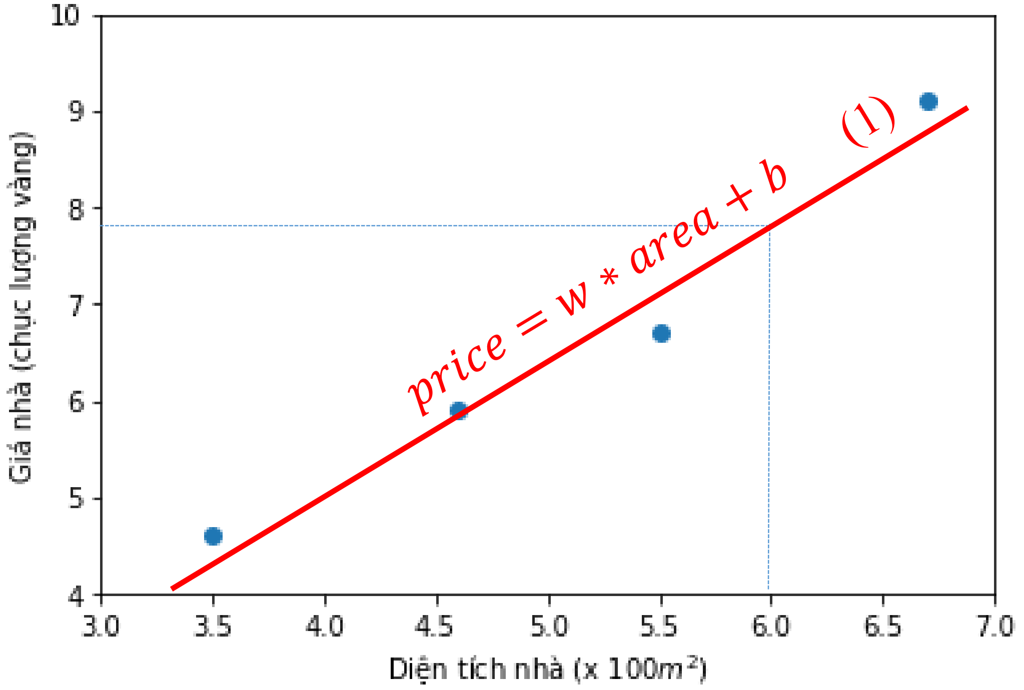
- Machine Learning
- Derivative/Gradient
- Linear Regression
- Computational Graph
    - 1-sample training
- Summary

Computational graph

- **House price predictions**
  - **How much for a 600-m^2 house?**

	area	price	
	6.7	9.1	
	4.6	5.9	
	3.5	4.6	
	5.5	6.7	

Given sample data

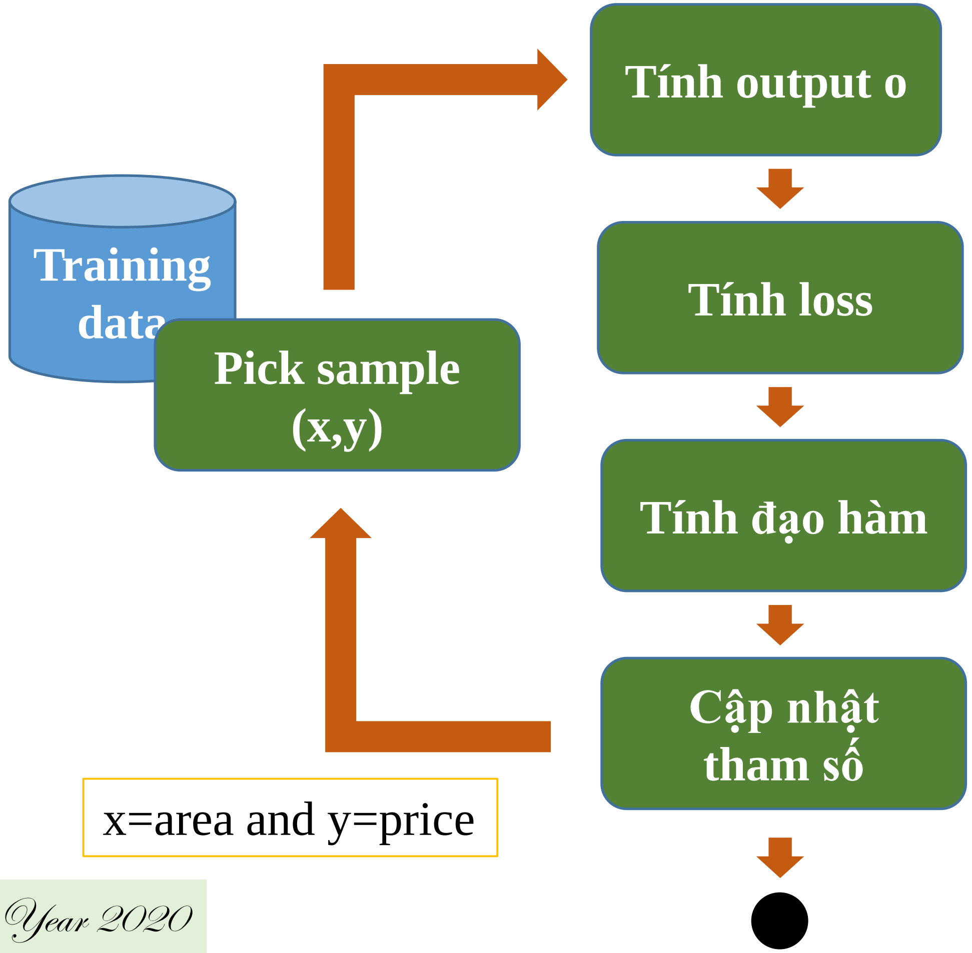


How to compute (1) using computational graph

Computational graph

❖ House price prediction

❖ One-sample training



1) Pick a sample (x, y) from training data

2) Tính output o

$$o = wx + b$$

3) Tính loss

$$L = (o - y)^2$$

4) Tính đạo hàm

$$L'_w = 2x(o - y)$$
$$L'_b = 2(o - y)$$

5) Cập nhật tham số

$$w = w - \eta L'_w$$
$$b = b - \eta L'_b$$

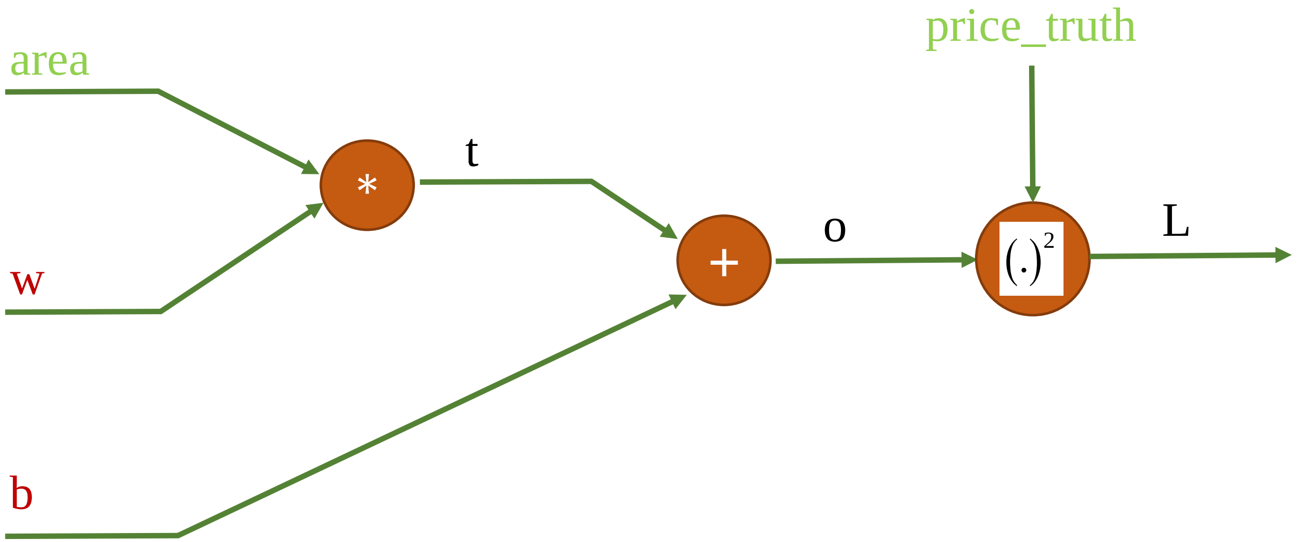
Learning rate η

Computational graph

- **House price prediction**
  - **One-sample training**

$$price = w * area + b$$

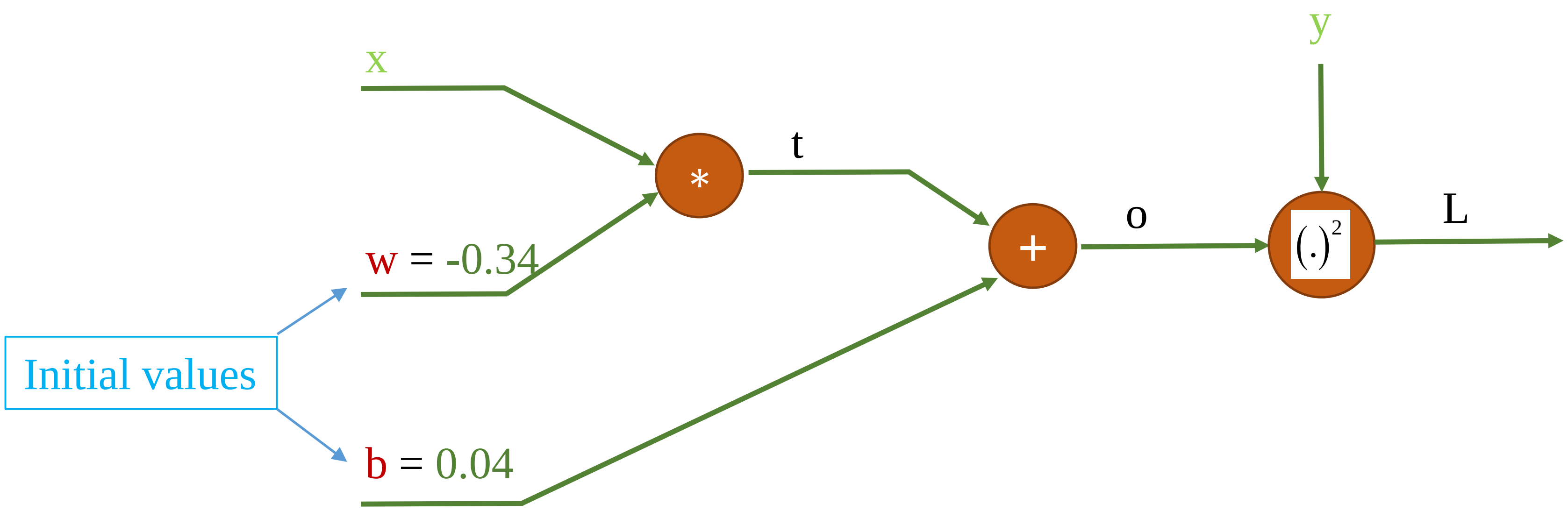
$$t = w * area$$



Computational graph

- **House price prediction**
  - **One-sample training**

$t = wx$ $\quad o = t + b$ $\quad L = (o - y)^2$

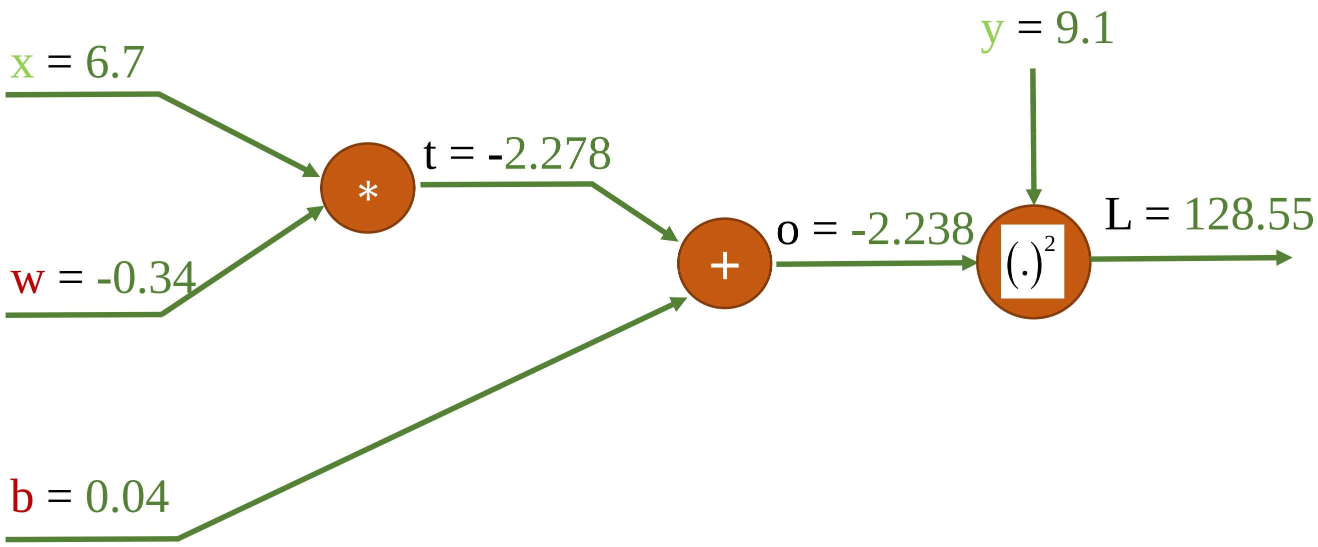


Computational graph

Feature	Label
area	price
6.7	9.1
4.6	5.9
3.5	4.6
5.5	6.7

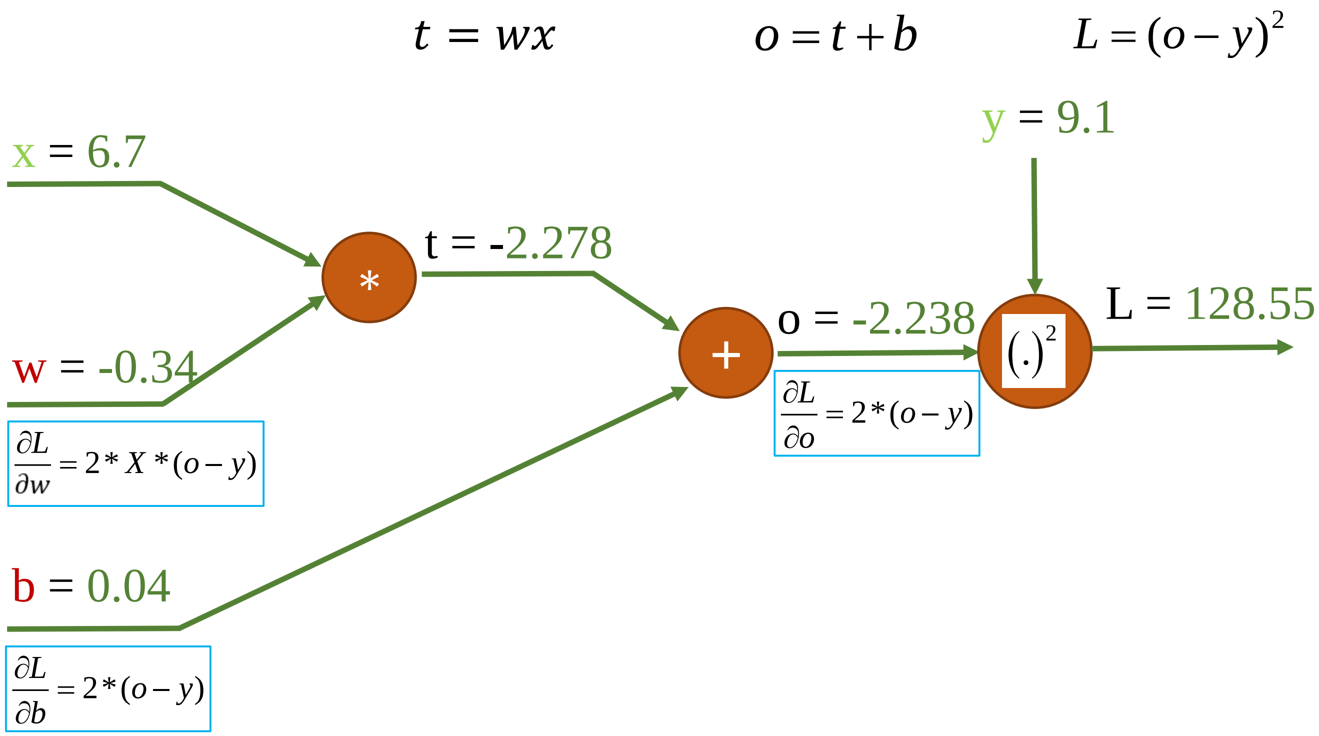
- House price prediction
  - One-sample training

$t = wx$ $\quad o = t + b$ $\quad L = (o - y)^2$



Computational graph

- **House price prediction**
  - **One-sample training**

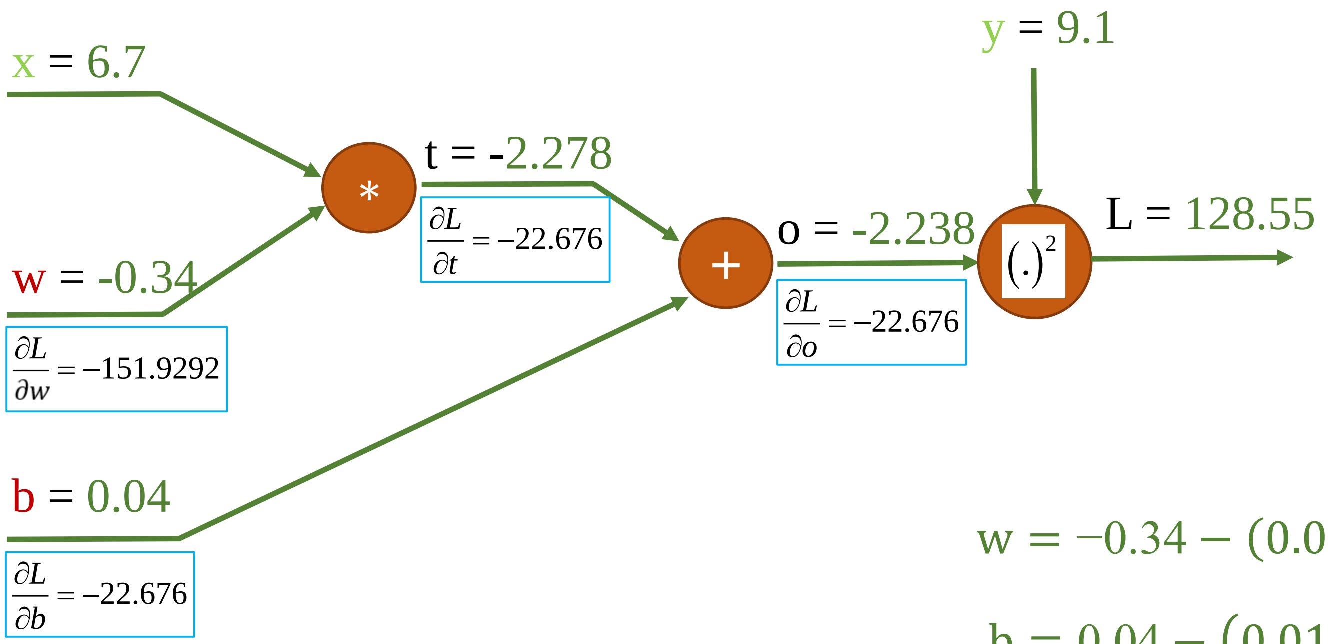


Computational graph

❖ House price prediction

❖ One-sample training

$t = wx$ $\quad o = t + b$ $\quad L = (o - y)^2$



Cách cập nhật a và b

$$w = w - \eta * \frac{\partial L}{\partial w}$$

$$b = b - \eta * \frac{\partial L}{\partial b}$$

Learning rate $\eta = 0.01$

$$w = -0.34 - (0.01 * (-151.9)) = 1.179$$

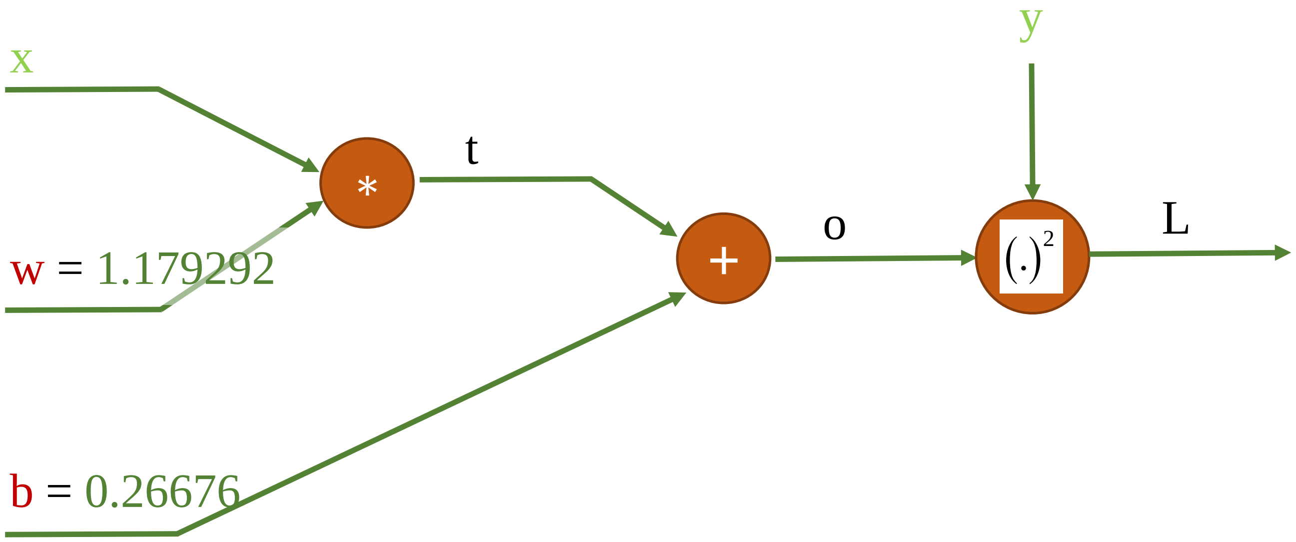
$$b = 0.04 - (0.01 * (-22.67)) = 0.266$$

Computational graph

❖ House price prediction

❖ One-sample training

$$t = wx \qquad o = t + b \qquad L = (o - y)^2$$



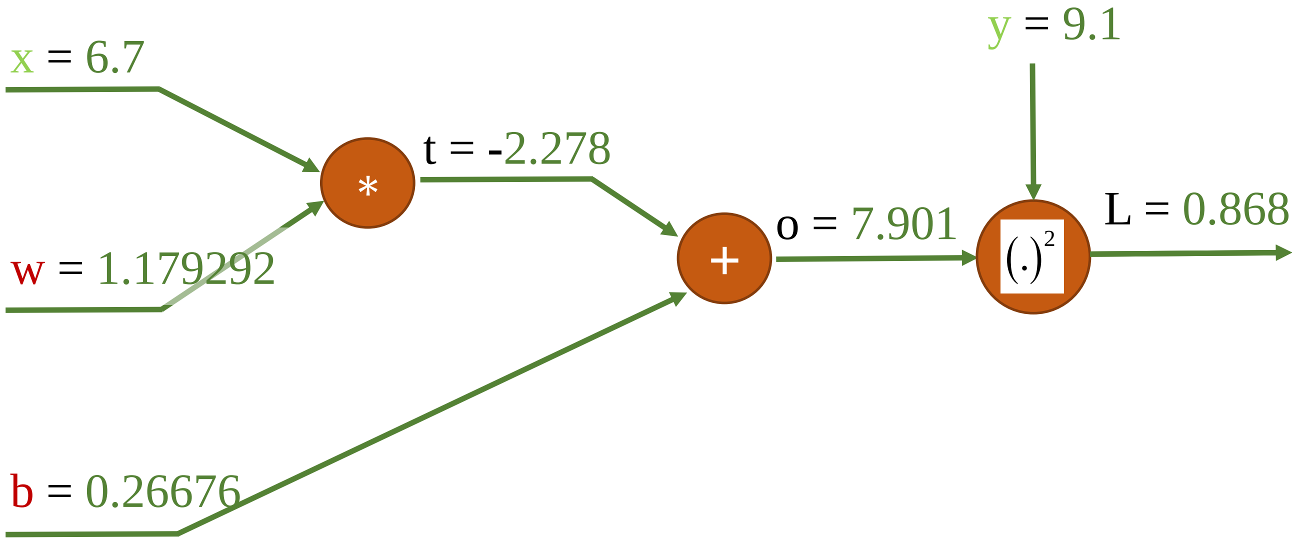
Computational graph

Feature	Label
area	price
6.7	9.1
4.6	5.9
3.5	4.6
5.5	6.7

❖ House price prediction

❖ One-sample training

$t = wx \qquad o = t + b \qquad L = (o - y)^2$

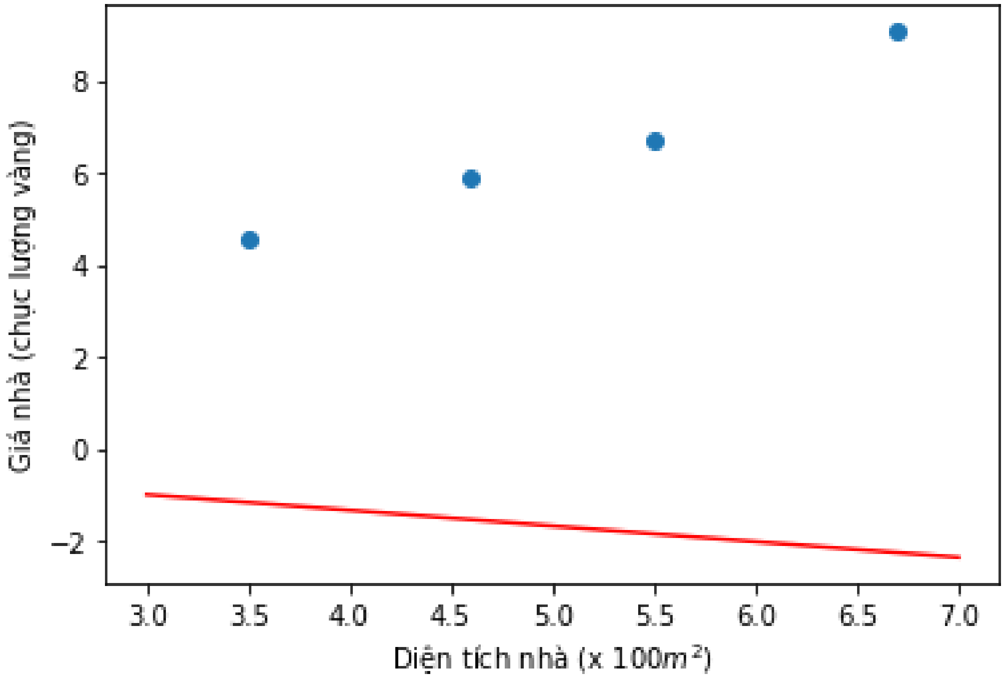


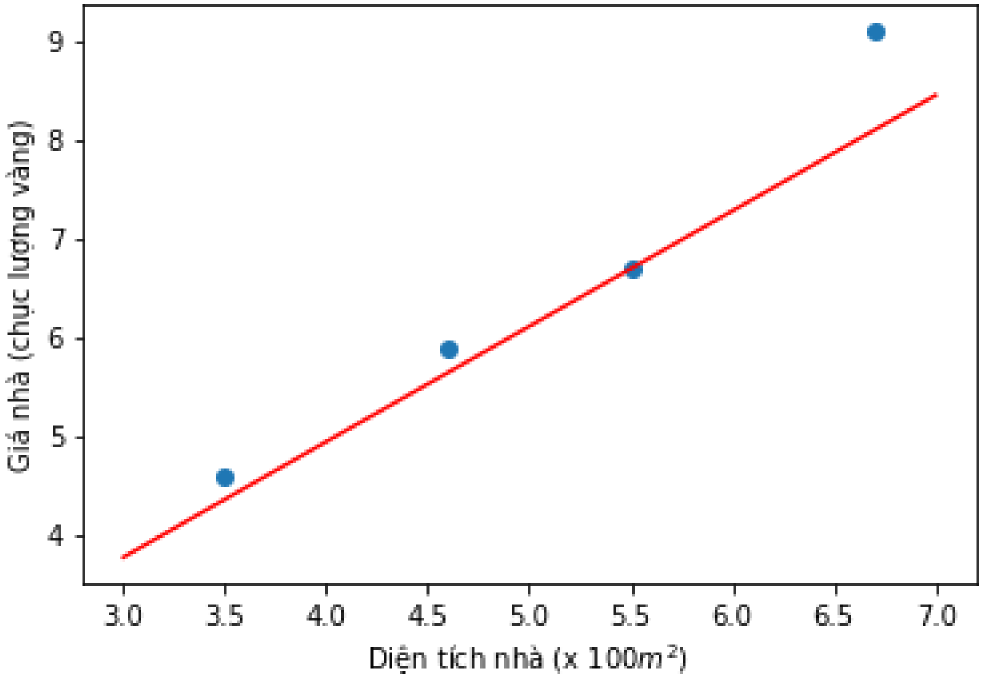
previous L = 128.55

Updated a and b values help to reduce the L value

Computational graph

- **House price prediction**
  - **One-sample training**



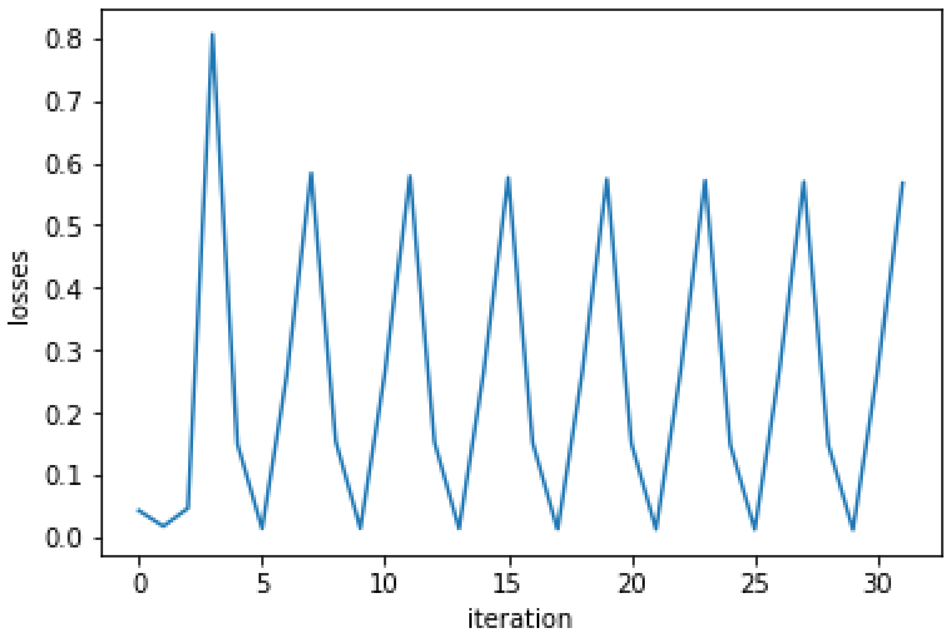


w = -0.34   b = 0.04   L = 128.55

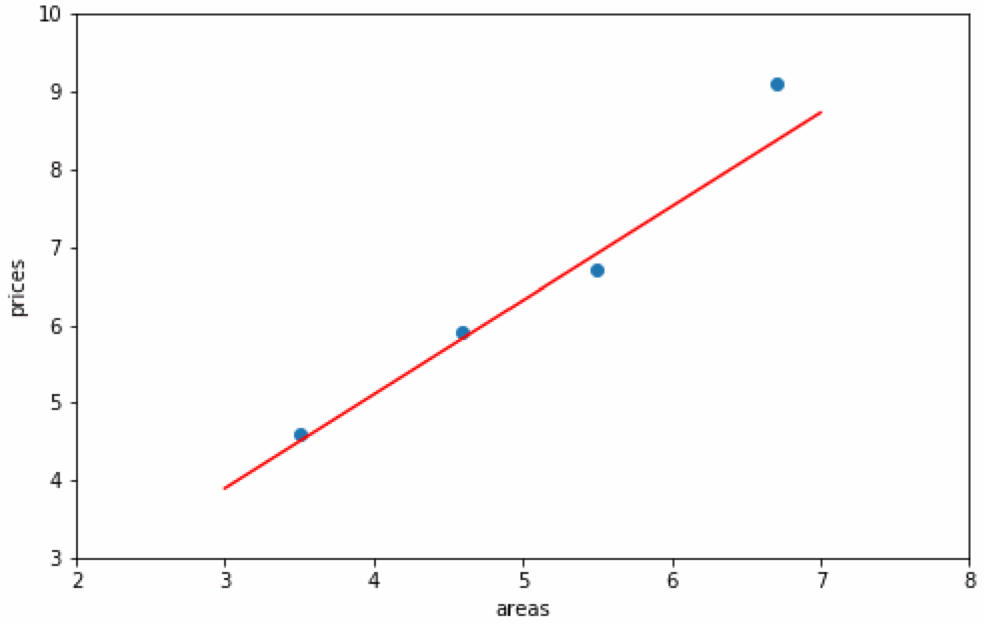
w = 1.179292   b = 0.26676   L = 0.868

Computational graph

- ❖ **House price prediction**
  - ❖ **One-sample training**



Losses for 30 iterations



Model updating for different iterations

Linear Regression (1-sample)

1) Pick a sample (x, y) from training data

2) Tính output o

$$o = wx + b$$

3) Tính loss

$$L = (o - y)^2$$

4) Tính đạo hàm

$$L'_w = 2x(o - y)$$

$$L'_b = 2(o - y)$$

5) Cập nhật tham số

$$w = w - \eta L'_w$$

$$b = b - \eta L'_b$$

η is learning rate

Friendly version

1) Pick a sample (x, y) from training data

2) Tính output o

$$o = \boldsymbol{\theta}^T \boldsymbol{x}$$

3) Tính loss

$$L = (o - y)^2$$

4) Tính đạo hàm

$$L'_{\boldsymbol{\theta}} = 2\boldsymbol{x}(o - y)$$

5) Cập nhật tham số

$$\boldsymbol{\theta} = \boldsymbol{\theta} - \eta L'_{\boldsymbol{\theta}}$$

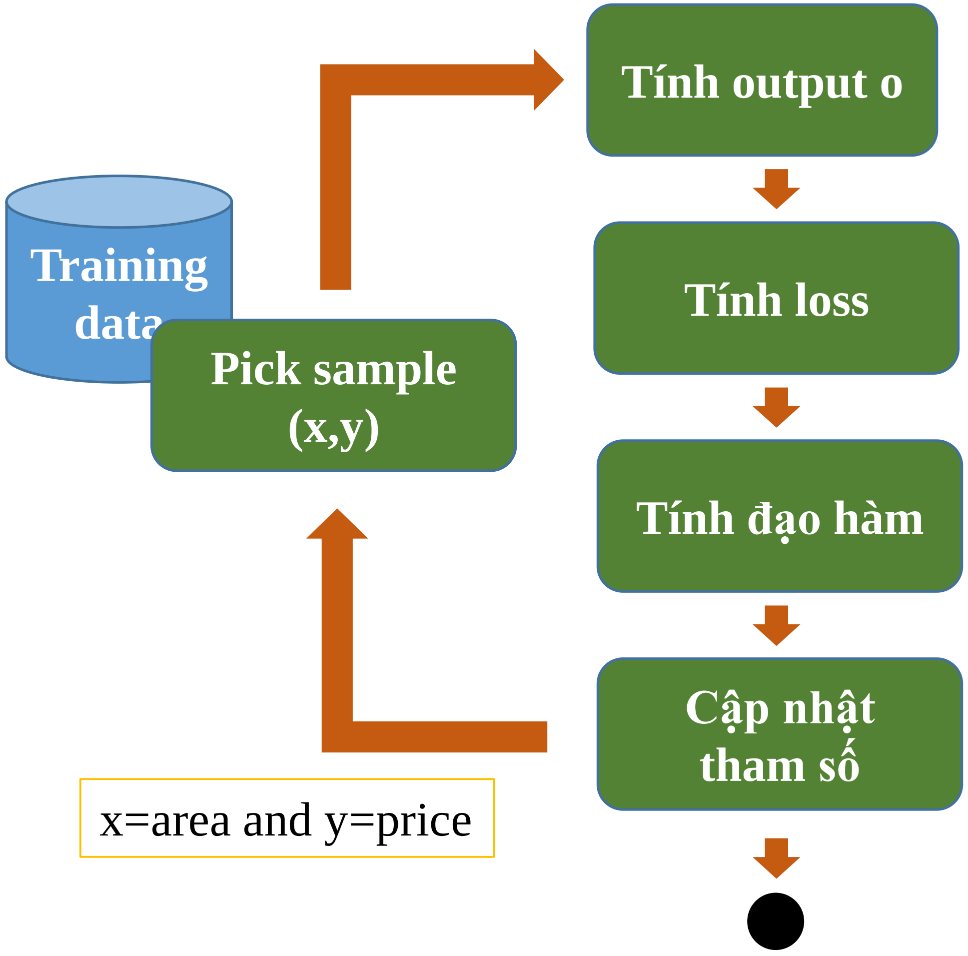
η is learning rate

Generalized formula

Computational graph

❖ House price prediction

❖ One-sample training: implementation




```python
# Naive implementaion

# load data
from numpy import genfromtxt
import matplotlib.pyplot as plt

data = genfromtxt('data.csv', delimiter=',')
areas  = list(data[:,0])
prices = list(data[:,1])
data_size = len(areas)

print('areas: ', areas)
print('prices: ', prices)
print('data_size: ', data_size)

plt.scatter(areas, prices)
plt.xlabel('Diện tích nhà (x 100$m^2$)')
plt.ylabel('Giá nhà (chục lượng vàng)')
plt.xlim(3,7)
plt.ylim(4,10)
plt.show()
```

Computational graph

❖ House price prediction

❖ One-sample training: implementation

1) Pick a sample (x, y) from training data

2) Tính output o

$$o = wx + b$$

3) Tính loss

$$L = (o - y)^2$$

4) Tính đạo hàm

$$L'_w = 2x(o - y)$$

$$L'_b = 2(o - y)$$

5) Cập nhật tham số

η is learning rate

$$w = w - \eta L'_w$$

$$b = b - \eta L'_b$$

```python
# forward
def predict(x,w,b):
    return x*w + b

# compute gradient
def gradient(z,y,x):
    dw = 2*x*(z-y)
    db = 2*(z-y)

    return (dw, db)

# update weights
def update_weight(w,b,n,dw,db):
    w_new = w - n*dw
    b_new = b - n*db

    return (w_new, b_new)
```

Computational graph

❖ House price prediction

❖ One-sample training: implementation

1) Pick a sample (x, y) from training data

2) Tính output o

$$o = wx + b$$

3) Tính loss

$$L = (o - y)^2$$

4) Tính đạo hàm

$$L'_w = 2x(o - y)$$

$$L'_b = 2(o - y)$$

5) Cập nhật tham số

$$w = w - \eta L'_w$$

$$b = b - \eta L'_b$$

η is learning rate

```python
# init weights
b = 0.04
w = -0.34
n = 0.01

# how long
epoch_max = 10

for epoch in range(epoch_max):
    for i in range(data_size):
        # get a sample

        # predict z

        # compute loss

        # compute gradient

        # update weights

```

Computational graph

- **House price prediction**
  - **One-sample training: implementation**

1) Pick a sample (x, y) from training data

2) Tính output o

$$o = wx + b$$

3) Tính loss

$$L = (o - y)^2$$

4) Tính đạo hàm

$$L'_w = 2x(o - y)$$

$$L'_b = 2(o - y)$$

5) Cập nhật tham số

η is learning rate

$$w = w - \eta L'_w$$

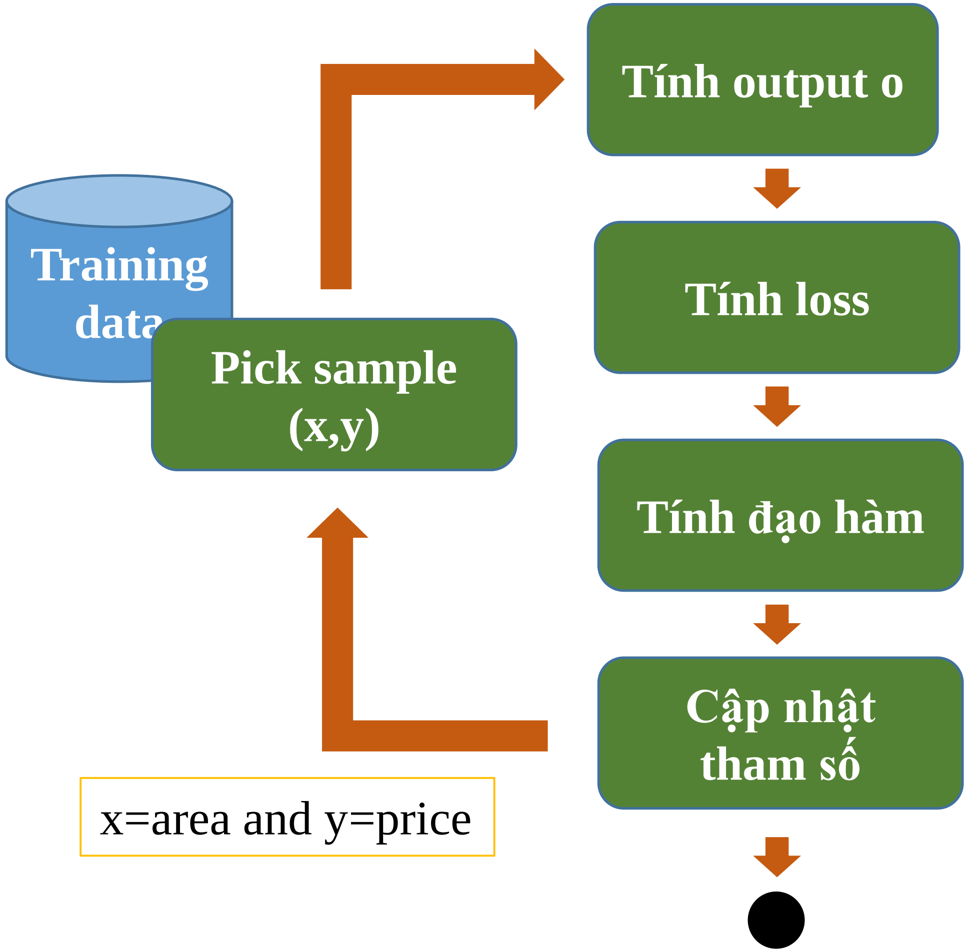
$$b = b - \eta L'_b$$

```python
# init weights
b = 0.04
w = -0.34
n = 0.01

# how long
epoch_max = 10

for epoch in range(epoch_max):
    for i in range(data_size):
        # get a sample
        x = areas[i]
        y = prices[i]
        print('sample: ', x, y)

        # predict z
        z = predict(x,w,b)
        print('z: ', z)

        # compute loss
        loss = (z-y)*(z-y)
        print('Loss: ', loss)

        # compute gradient
        (dw, db) = gradient(z,y,x)
        print('dw: ', dw)
        print('db: ', db)

        # update weights
        (w, b) = update_weight(w,b,n,dw,db)
        print('w_new: ', w)
        print('b_new: ', b)
        print('\n\n')
```

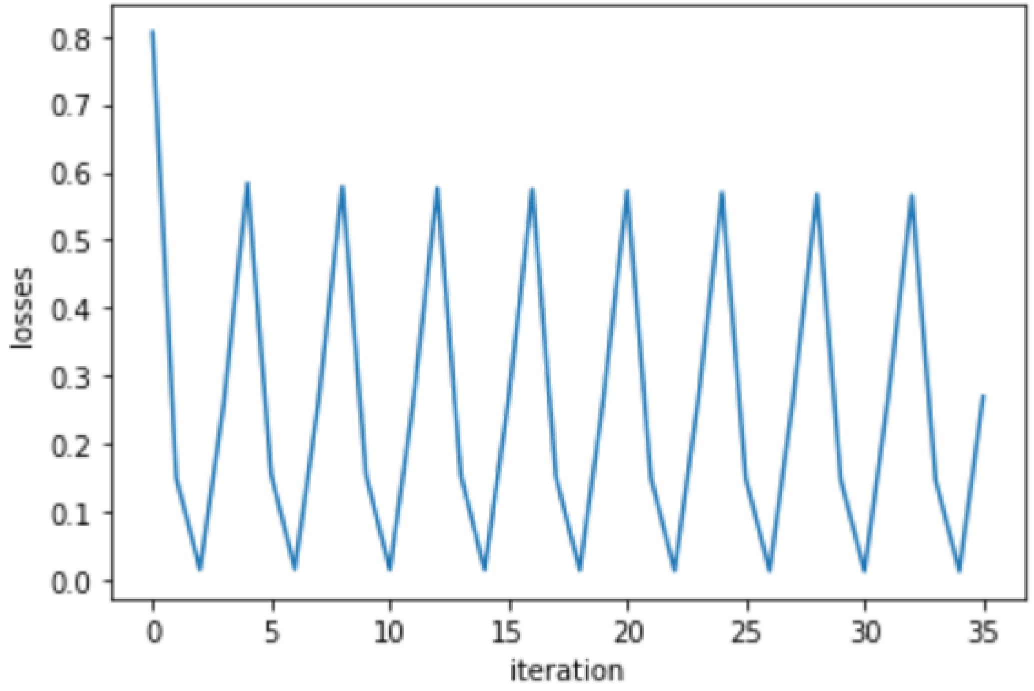
Computational graph

❖ **House price prediction**

❖ **One-sample training: implementation**




```python
import matplotlib.pyplot as plt

plt.plot(losses)
plt.xlabel('iteration')
plt.ylabel('losses')
plt.show()
```



Linear Regression

❖ House price prediction

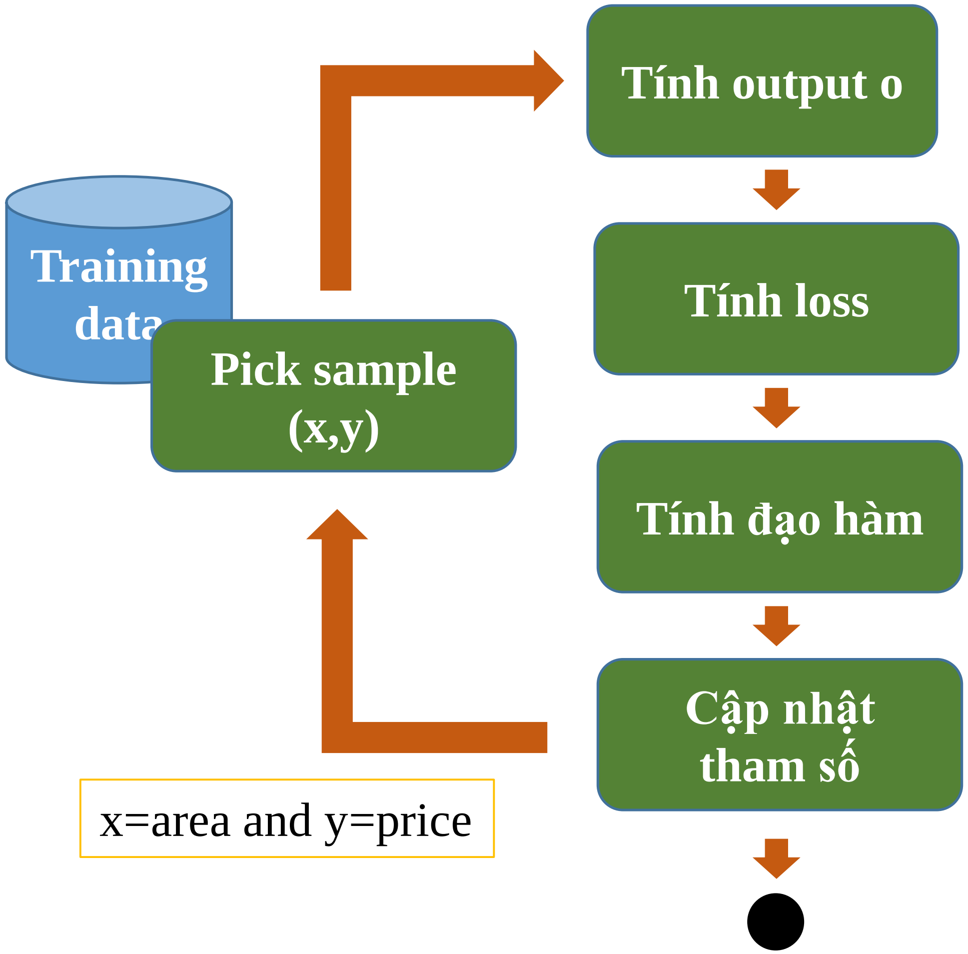
Demo

```
Python 3.7.3 (default, Apr 24 2019, 15:29:51) [MSC v.1915 64 bit (AMD64)] ::
Type "help", "copyright", "credits" or "license" for more information.
>>>
>>>
>>>
>>>
>>>
>>>
>>>
>>> for epoch in range(n_epochs):
...     sum_of_losses = 0
...     gradients = np.zeros((2,1))
...
...     for index in range(4):
...         xi = X_b[index:index+1]
...         yi = y[index:index+1]
```

Computational graph

❖ House price prediction

❖ Implementation: Vectorization




```python
# Implementation - vectorization

# Load data
import numpy as np
from numpy import genfromtxt
import matplotlib.pyplot as plt

data = genfromtxt('data.csv', delimiter=',')
areas  = data[:,0]
prices = data[:,1]
data_size = areas.size

print(type(areas))
print('areas: ', areas)
print('prices: ', prices)
print('data_size: ', data_size)

plt.scatter(areas, prices)
plt.xlabel('Diện tích nhà (x 100$m^2$)')
plt.ylabel('Giá nhà (chục lượng vàng)')
plt.xlim(3,7)
plt.ylim(4,10)
plt.show()
```

Computational graph

❖ House price prediction

❖ Implementation: Vectorization

1) Pick a sample (x, y) from training data

2) Tính output o

$$o = \boldsymbol{\theta}^T \boldsymbol{x}$$

3) Tính loss

$$L = (o - y)^2$$

4) Tính đạo hàm

$$L'_{\boldsymbol{\theta}} = 2\boldsymbol{x}(o - y)$$

5) Cập nhật tham số

$$\boldsymbol{\theta} = \boldsymbol{\theta} - \eta L'_{\boldsymbol{\theta}}$$

η is learning rate

```python
# forward
def predict(x,theta):
    return x.dot(theta)

# compute gradient
def gradient(z,y,x):
    dtheta = 2*x*(z-y)

    return dtheta

# update weights
def update_weight(theta,n,dtheta):
    dtheta_new = theta - n*dtheta

    return dtheta_new
```

Computational graph

❖ House price prediction

❖ Implementation: Vectorization

1) Pick a sample (x, y) from training data

2) Tính output o

$$o = \boldsymbol{\theta}^T \boldsymbol{x}$$

3) Tính loss

$$L = (o - y)^2$$

4) Tính đạo hàm

$$L'_{\boldsymbol{\theta}} = 2\boldsymbol{x}(o - y)$$

5) Cập nhật tham số

$$\boldsymbol{\theta} = \boldsymbol{\theta} - \eta L'_{\boldsymbol{\theta}}$$

η is learning rate

```python
# vector [x, b]
data = np.c_[areas, np.ones((data_size, 1))]
print(data)

# init weight
n = 0.01
theta = np.array([-0.34, 0.04]) #[w, b]
print('theta', theta)
```

Computational graph

❖ House price prediction

❖ Implementation: Vectorization

1) Pick a sample (x, y) from training data

2) Tính output o

$$o = \boldsymbol{\theta}^T \boldsymbol{x}$$

3) Tính loss

$$L = (o - y)^2$$

4) Tính đạo hàm

$$L'_{\boldsymbol{\theta}} = 2\boldsymbol{x}(o - y)$$

5) Cập nhật tham số

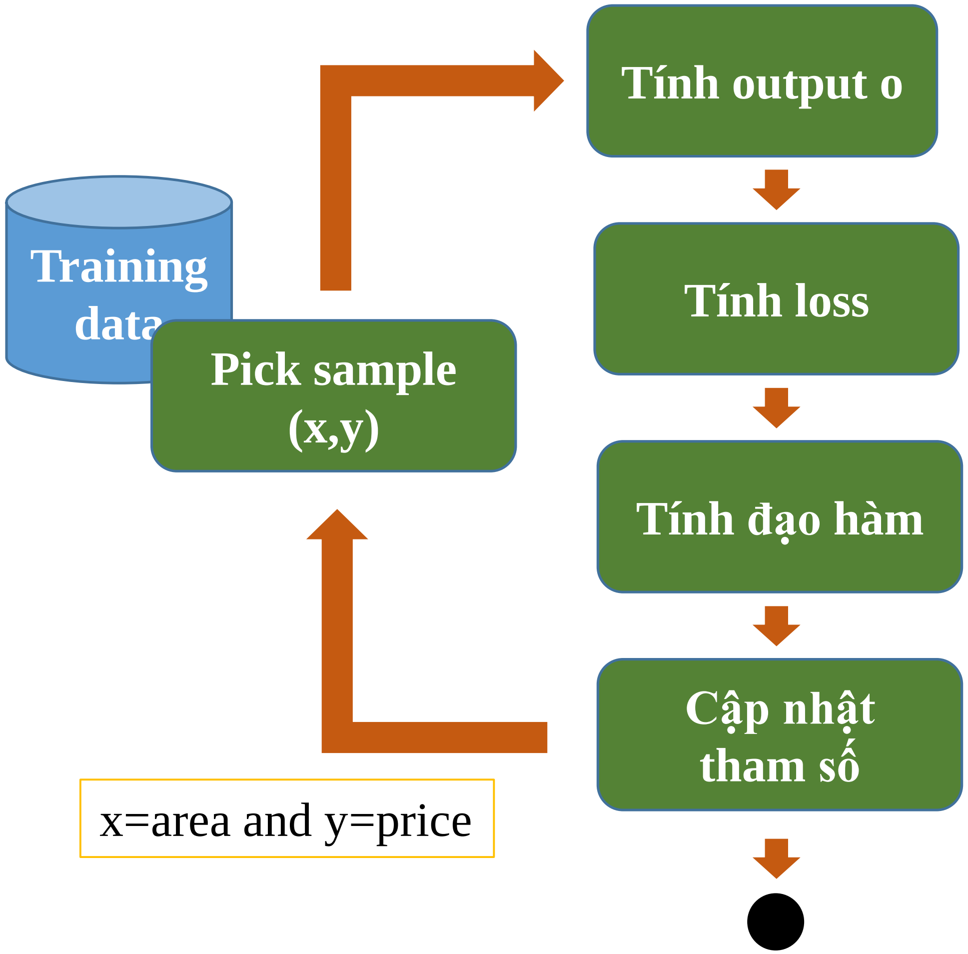
$$\boldsymbol{\theta} = \boldsymbol{\theta} - \eta L'_{\boldsymbol{\theta}}$$

η is learning rate

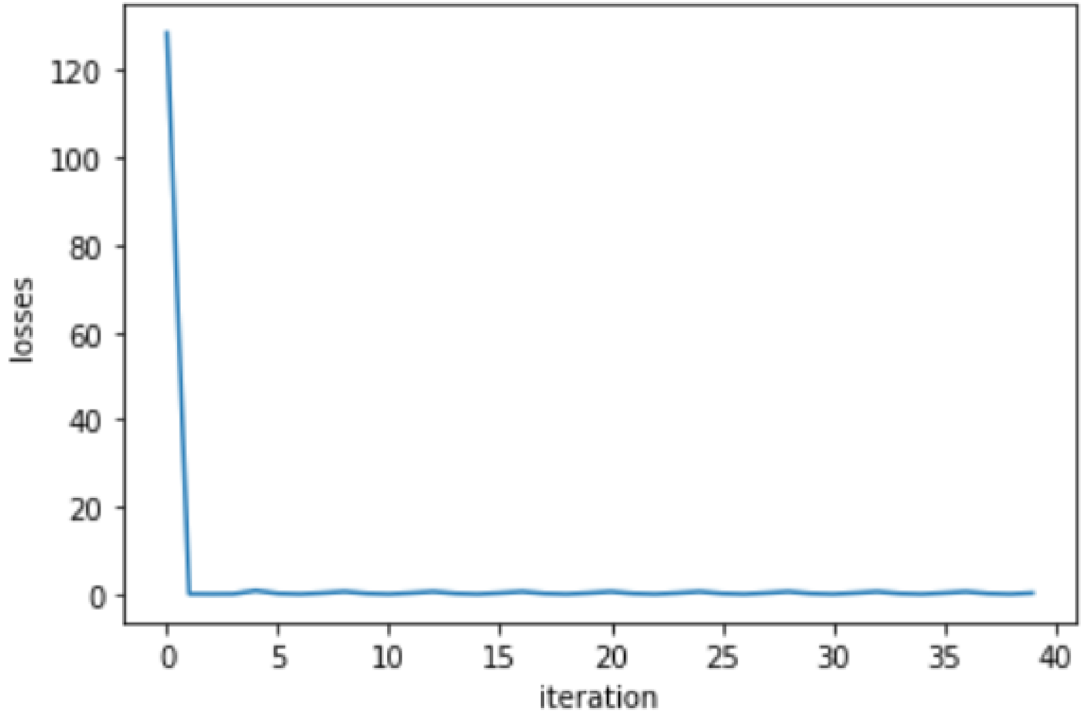
```python
# how long
epoch_max = 10

for epoch in range(epoch_max):
    for i in range(data_size):
        # get a sample
        x = data[i]
        y = prices[i:i+1]
        print('sample: ', x, y)

        # predict z
        z = predict(x, theta)
        print('Input data: ', x)
        print('Theta: ', theta)
        print('z: ', z)

        # compute loss
        loss = (z-y)*(z-y)
        print('Loss: ', loss)

        # compute gradient
        dtheta = gradient(z,y,x)
        print('dtheta: ', dtheta)

        # update weights
        theta = update_weight(theta,n,dtheta)
        print('dtheta_new: ', theta)
        print('\n\n')
```

Computational graph

- House price prediction
  - Implementation: Vectorization




```python
import matplotlib.pyplot as plt

plt.plot(losses)
plt.xlabel('iteration')
plt.ylabel('losses')
plt.show()
```



Linear Regression

❖ House price prediction

Demo

```
Python 3.7.3 (default, Apr 24 2019, 15:29:51) [MSC v.1915 64 bit (AMD64)] ::
Type "help", "copyright", "credits" or "license" for more information.
>>>
>>>
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>>>
>>>
>>>
>>> for epoch in range(n_epochs):
...     sum_of_losses = 0
...     gradients = np.zeros((2,1))
...
...     for index in range(4):
...         xi = X_b[index:index+1]
...         yi = y[index:index+1]
```

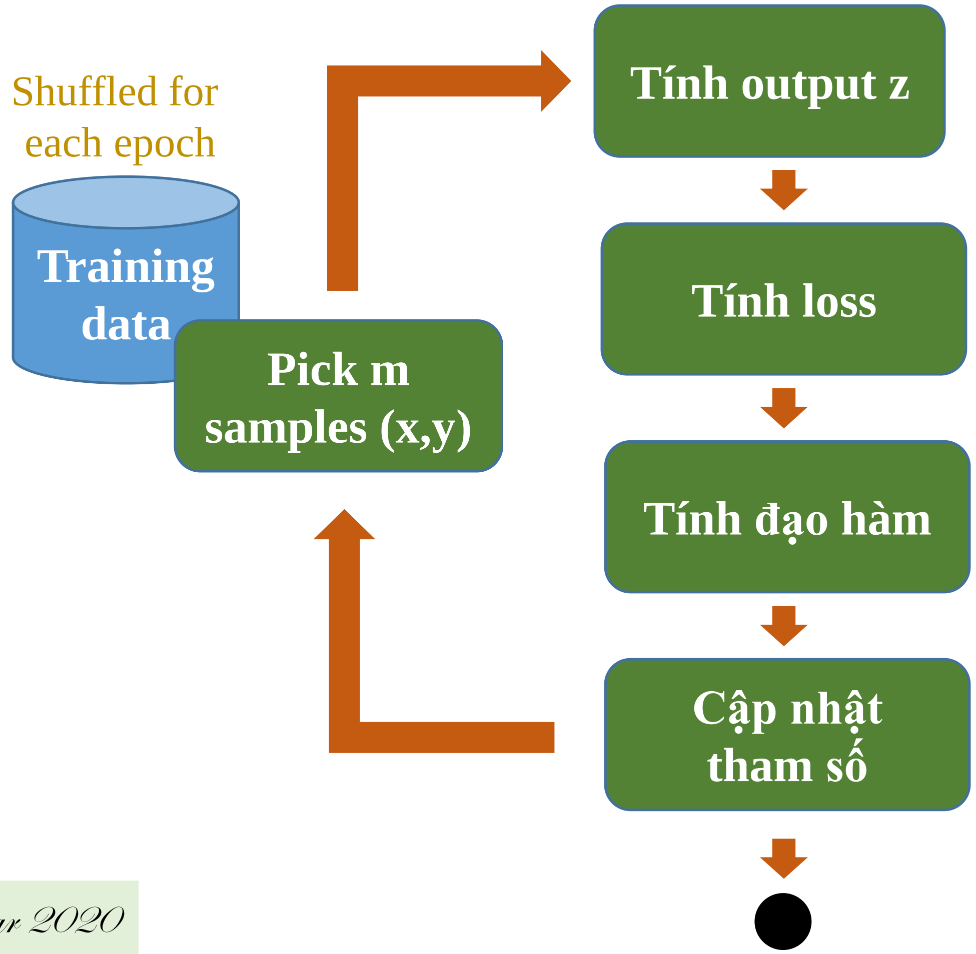
Outline

- Machine Learning
- Derivative/Gradient
- Linear Regression
- Computational Graph
    - 1-sample training
    - m-sample training
- Summary

Computational graph

❖ House price prediction

❖ m-sample training (1<m<N)



1) Pick m samples $(x^{(i)}, y^{(i)})$ from training data

2) Tính output o_i

$$o^{(i)} = wx^{(i)} + b \qquad \text{for } 0 \le i < m$$

3) Tính loss

$$L^{(i)} = (o^{(i)} - y^{(i)})^2 \quad \text{for } 0 \le i < m$$

4) Tính đạo hàm

$$L_w'^{(i)} = 2x(o^{(i)} - y^{(i)})$$
$$L_b'^{(i)} = 2(o^{(i)} - y^{(i)}) \qquad \text{for } 0 \le i < m$$

5) Cập nhật tham số

$$w = w - \eta \frac{\sum_i L_w'^{(i)}}{m}$$

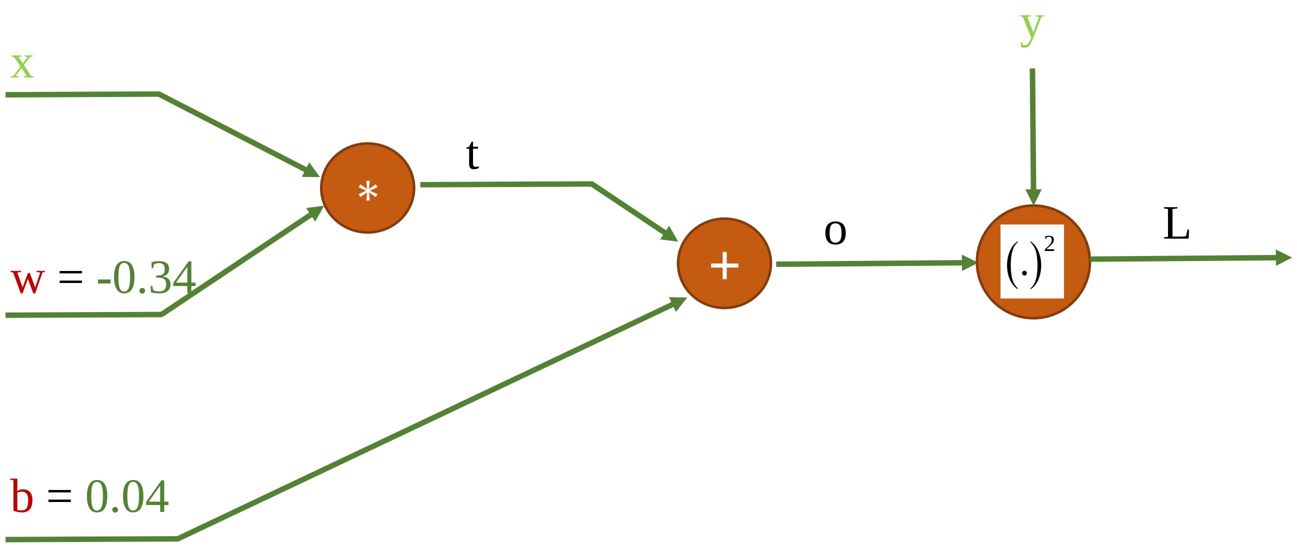
$$b = b - \eta \frac{\sum_i L_b'^{(i)}}{m} \qquad \text{Learning rate } \eta$$

Computational graph

❖ House price prediction

❖ m-sample training (1<m<N)

$t = wx$ $\quad o = t + b$ $\quad L = (o - y)^2$



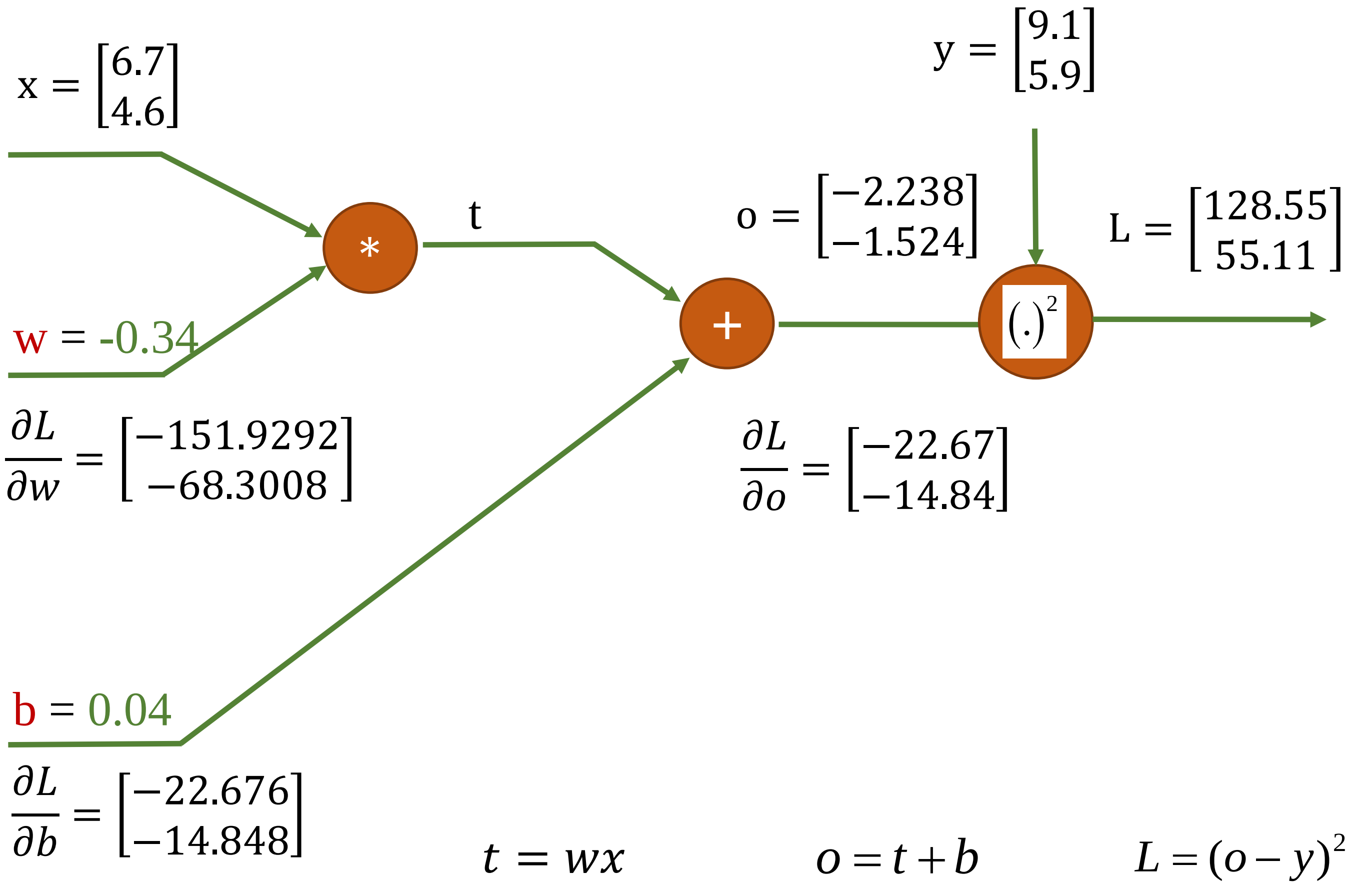
Computational graph

- **House price prediction**
  - **m-sample training (1<m<N)**

m = 2

$$\frac{sum(\frac{\partial L}{\partial w})}{m} = -110.115$$

$$\frac{sum(\frac{\partial L}{\partial b})}{m} = -18.762$$



Computational graph

❖ House price prediction

❖ m-sample training (1<m<N)

Cách cập nhật a và b

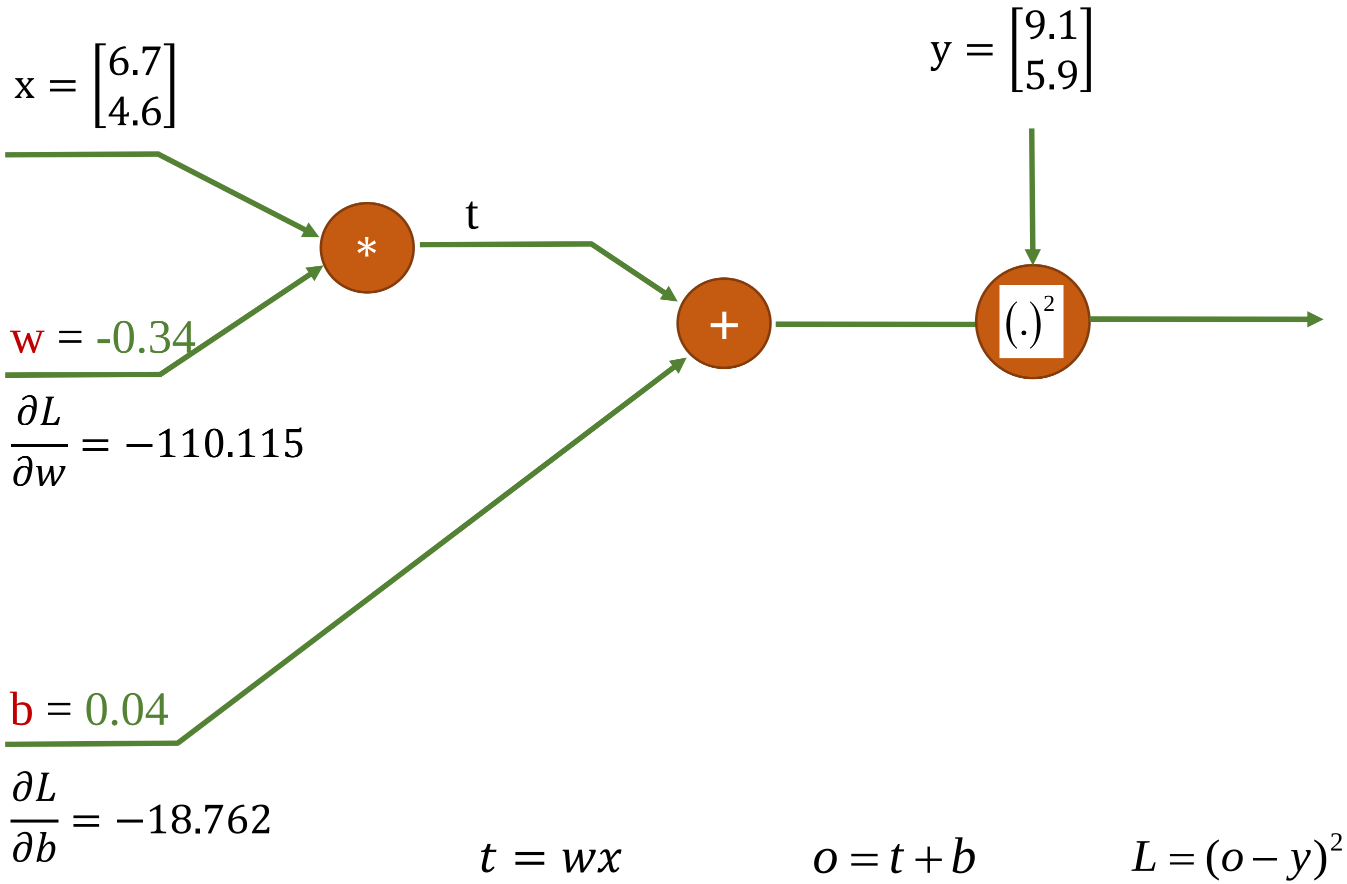
$$w = w - \eta * \frac{\partial L}{\partial w}$$

$$b = b - \eta * \frac{\partial L}{\partial b}$$

Learning rate $\eta = 0.01$

$$w = -0.34 - (0.01 * (-110.115)) = 0.761$$

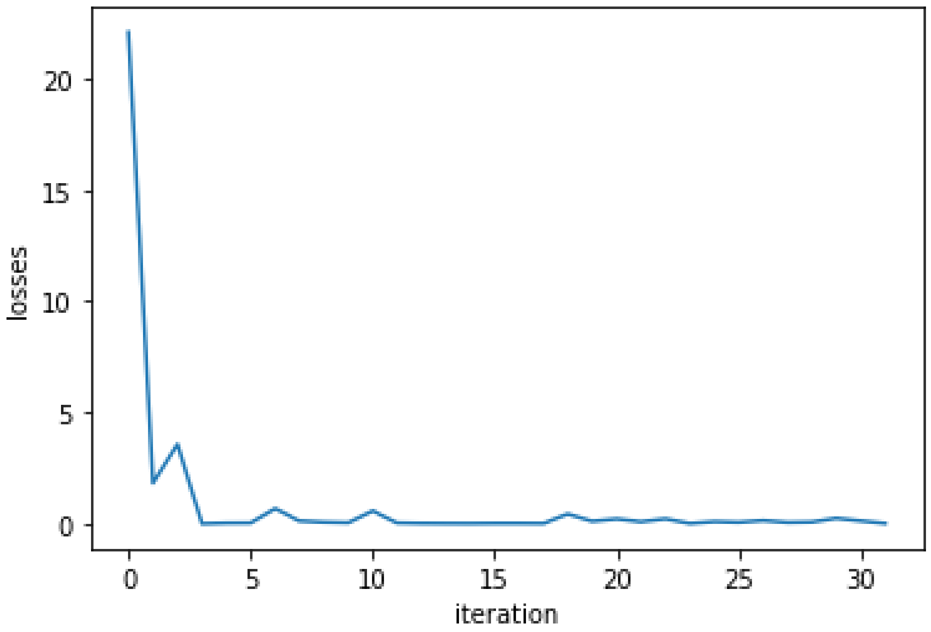
$$b = 0.04 - \left(0.01 * (-18.762)\right) = 0.227$$



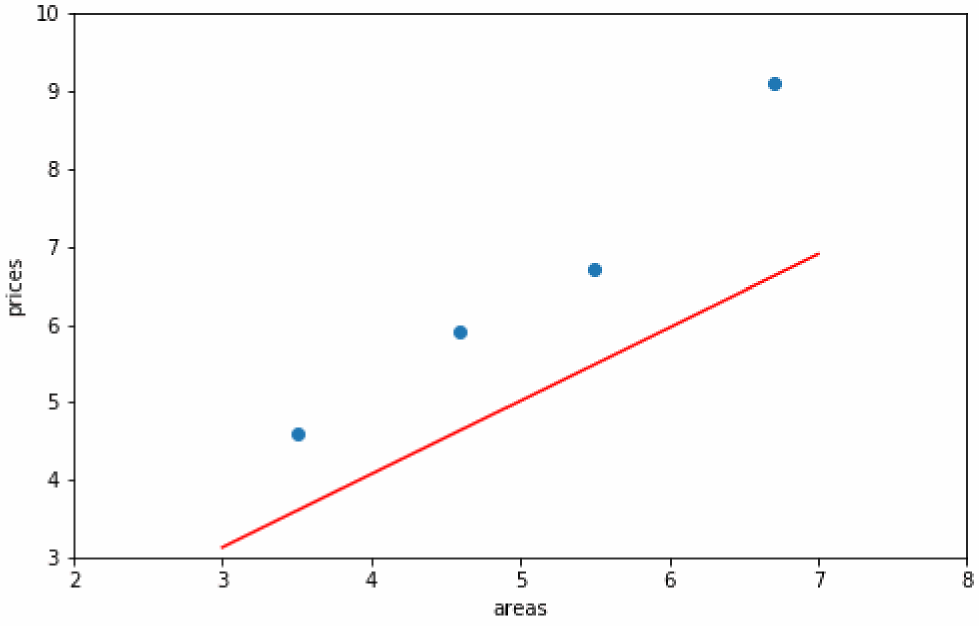
$t = wx$ $\quad o = t + b \quad$ $L = (o - y)^2$

Computational graph

- **House price prediction**
  - **m-sample training (1<m<N)**



Losses for 30 iterations



Model updating for different iterations

Linear Regression (m-samples)

1) Pick m samples $(x^{(i)}, y^{(i)})$ from training data

1.1) Tính output $o^{(i)}$

$$o^{(i)} = wx^{(i)} + b \qquad \text{for } 0 \leq i < m$$

1.2) Tính loss

$$L^{(i)} = (o^{(i)} - y^{(i)})^2 \quad \text{for } 0 \leq i < m$$

1.3) Tính đạo hàm

$$\left.\begin{aligned} L_w'^{(i)} &= 2x(o^{(i)} - y^{(i)}) \\ L_b'^{(i)} &= 2(o^{(i)} - y^{(i)}) \end{aligned}\right\} \text{for } 0 \leq i < m$$

2) Cập nhật tham số

$$w = w - \eta \frac{\sum_i L_w'^{(i)}}{m}$$

$$b = b - \eta \frac{\sum_i L_b'^{(i)}}{m}$$

η is learning rate

Friendly version

1) Pick m samples $(x^{(i)}, y^{(i)})$ from training data

1.1) Tính output $o^{(i)}$

$$o^{(i)} = \boldsymbol{\theta}^T \boldsymbol{x}^{(i)} \qquad \text{for } 0 \leq i < m$$

1.2) Tính loss

$$L^{(i)} = (o^{(i)} - y^{(i)})^2 \qquad \text{for } 0 \leq i < m$$

1.3) Tính đạo hàm

$$L_{\boldsymbol{\theta}}'^{(i)} = 2\boldsymbol{x}(o^{(i)} - y^{(i)}) \qquad \text{for } 0 \leq i < m$$

2) Cập nhật tham số

$$\boldsymbol{\theta} = \boldsymbol{\theta} - \eta \frac{\sum_i L_{\boldsymbol{\theta}}'^{(i)}}{m}$$

η is learning rate

Generalized formula

Linear Regression (m-samples)

1) Pick m samples $(x^{(i)}, y^{(i)})$ from training data

1.1) Tính output $o^{(i)}$

$$o^{(i)} = \boldsymbol{\theta}^T \boldsymbol{x}^{(i)} \qquad \text{for } 0 \le i < m$$

1.2) Tính loss

$$L^{(i)} = (o^{(i)} - y^{(i)})^2 \qquad \text{for } 0 \le i < m$$

1.3) Tính đạo hàm

$$L_{\boldsymbol{\theta}}^{\prime(i)} = 2\boldsymbol{x}(o^{(i)} - y^{(i)}) \qquad \text{for } 0 \le i < m$$

2) Cập nhật tham số

$$\boldsymbol{\theta} = \boldsymbol{\theta} - \eta \frac{\sum_i L_{\boldsymbol{\theta}}^{\prime(i)}}{m}$$

η is learning rate

```python
# load data

import numpy as np
from numpy import genfromtxt
import matplotlib.pyplot as plt

data = genfromtxt('data.csv', delimiter=',')
areas  = data[:,0]
prices = data[:,1]
data_size = areas.size

print(type(areas))
print('areas: ', areas)
print('prices: ', prices)
print('data_size: ', data_size)

plt.scatter(areas, prices)
plt.xlabel('areas')
plt.ylabel('prices')
plt.xlim(3,7)
plt.ylim(4,10)
plt.show()
```


Generalized formula

Linear Regression (m-samples)

1) Pick m samples $(x^{(i)}, y^{(i)})$ from training data

1.1) Tính output $o^{(i)}$

$$o^{(i)} = \boldsymbol{\theta}^T \boldsymbol{x}^{(i)} \qquad \text{for } 0 \le i < m$$

1.2) Tính loss

$$L^{(i)} = (o^{(i)} - y^{(i)})^2 \qquad \text{for } 0 \le i < m$$

1.3) Tính đạo hàm

$$L'^{(i)}_{\boldsymbol{\theta}} = 2\boldsymbol{x}(o^{(i)} - y^{(i)}) \qquad \text{for } 0 \le i < m$$

2) Cập nhật tham số

$$\boldsymbol{\theta} = \boldsymbol{\theta} - \eta \frac{\sum_i L'^{(i)}_{\boldsymbol{\theta}}}{m} \qquad \eta \text{ is learning rate}$$

```python
# vector [x, b]
data = np.c_[areas, np.ones((data_size, 1))]
print(data)

# init weight
eta = 0.01
theta = np.array([-0.34, 0.04]) #[w, b]
print('theta', theta)

# how long
epoch_max = 1

# mini-batch size
m = 2
```


Generalized formula

Linear Regression (m-samples)

1) Pick m samples $(x^{(i)}, y^{(i)})$ from training data

1.1) Tính output $o^{(i)}$

$$o^{(i)} = \boldsymbol{\theta}^T \boldsymbol{x}^{(i)} \quad \text{for } 0 \le i < m$$

1.2) Tính loss

$$L^{(i)} = (o^{(i)} - y^{(i)})^2 \quad \text{for } 0 \le i < m$$

1.3) Tính đạo hàm

$$L'^{(i)}_{\boldsymbol{\theta}} = 2\boldsymbol{x}(o^{(i)} - y^{(i)}) \quad \text{for } 0 \le i < m$$

2) Cập nhật tham số

$$\boldsymbol{\theta} = \boldsymbol{\theta} - \eta \frac{\sum_i L'^{(i)}_{\boldsymbol{\theta}}}{m}$$

η is learning rate

```python
for epoch in range(epoch_max):
    sum_of_losses = 0
    gradients = np.zeros((2,))

    for j in range(0, m, m):
        for index in range(j, j+m):
            xi = data[index]
            yi = prices[index]
            print('\ndata: ', xi, yi)

            # predict z/o
            oi = xi.dot(theta)
            print('z: ', oi)

            # compute loss
            li = (oi - yi)*(oi - yi)
            print('loss: ', index, li)

            # compute gradient
            g_li = 2*(oi - yi)
            print('g_li: ', g_li)
            gradient_i = xi*g_li
            print('gradient_i: ', index, gradient_i)

            gradients = gradients + gradient_i
            sum_of_losses = sum_of_losses + li

        sum_of_losses = sum_of_losses/2
        gradients     = gradients/2
        print('\ngradients: ', gradients)

        theta = theta - eta*gradients
        print('new params: ', theta)
```

Generalized formula

Linear Regression (m-samples)

1) Pick m samples $(x^{(i)}, y^{(i)})$ from training data

1.1) Tính output $o^{(i)}$

$$o^{(i)} = \boldsymbol{\theta}^T \boldsymbol{x}^{(i)} \qquad \text{for } 0 \leq i < m$$

1.2) Tính loss

$$L^{(i)} = (o^{(i)} - y^{(i)})^2 \qquad \text{for } 0 \leq i < m$$

1.3) Tính đạo hàm

$$L'^{(i)}_{\boldsymbol{\theta}} = 2\boldsymbol{x}(o^{(i)} - y^{(i)}) \qquad \text{for } 0 \leq i < m$$

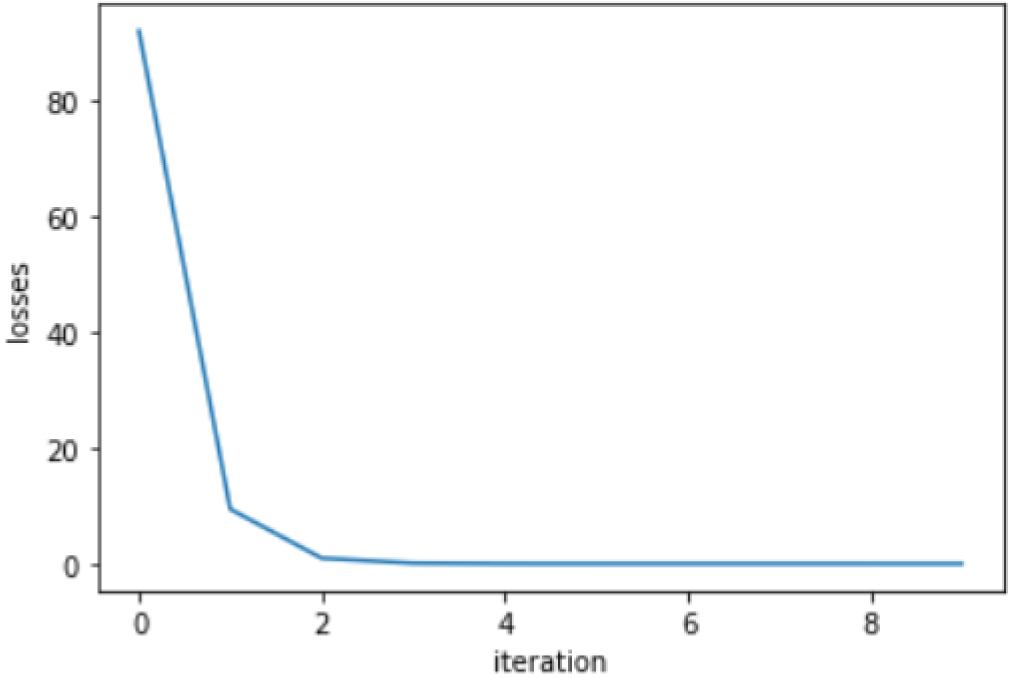
2) Cập nhật tham số

$$\boldsymbol{\theta} = \boldsymbol{\theta} - \eta \frac{\sum_i L'^{(i)}_{\boldsymbol{\theta}}}{m}$$

η is learning rate

```python
import matplotlib.pyplot as plt

plt.plot(losses)
plt.xlabel('iteration')
plt.ylabel('losses')
plt.show()
```



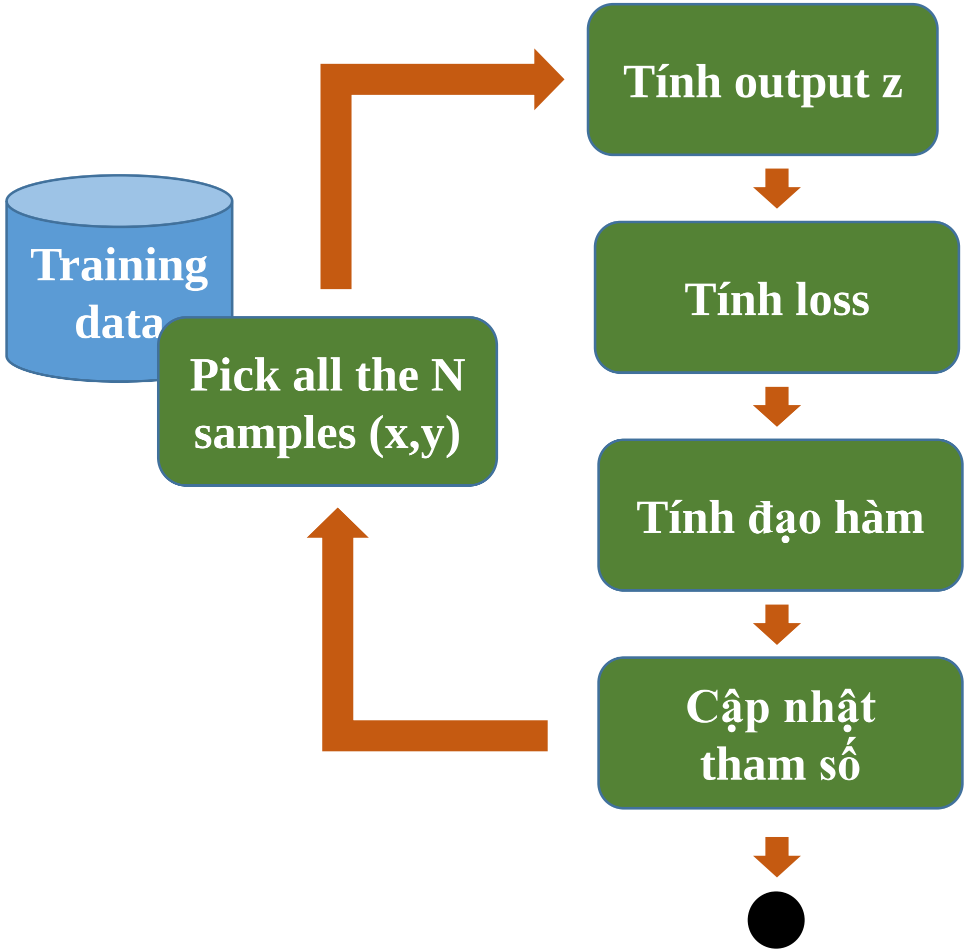
Generalized formula

Outline

- Machine Learning
- Derivative/Gradient
- Linear Regression
- Computational Graph
  - 1-sample training
  - m-sample training
  - N-sample training
- Summary

Computational graph

- **House price prediction**
  - **N-sample training**



1) Pick all the N samples $(x^{(i)}, y^{(i)})$ from training data

2) Tính output o_i

$$o^{(i)} = wx^{(i)} + b \qquad \text{for } 0 \le i < N$$

3) Tính loss

$$L^{(i)} = (o^{(i)} - y^{(i)})^2 \quad \text{for } 0 \le i < N$$

4) Tính đạo hàm

$$L_w'^{(i)} = 2x(o^{(i)} - y^{(i)})$$
$$L_b'^{(i)} = 2(o^{(i)} - y^{(i)}) \quad \text{for } 0 \le i < N$$

5) Cập nhật tham số

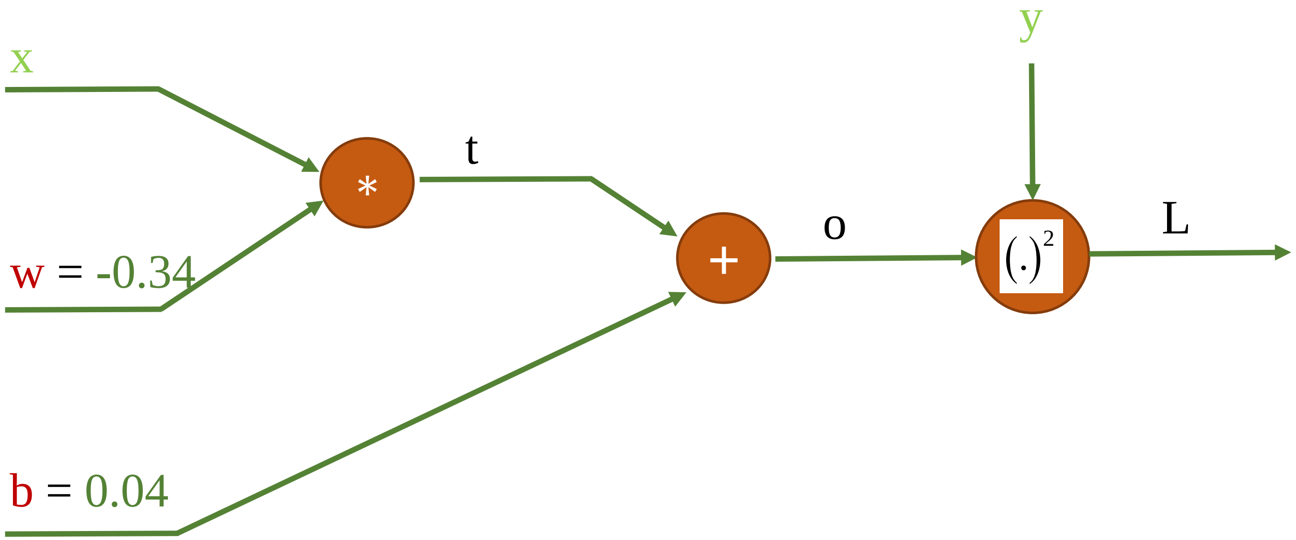
$$w = w - \eta \frac{\sum_i L_w'^{(i)}}{N}$$

$$b = b - \eta \frac{\sum_i L_b'^{(i)}}{N} \qquad \text{Learning rate } \eta$$

Computational graph

- **House price prediction**
  - **N-sample training**

$t = wx$ $\qquad o = t + b$ $\qquad L = (o - y)^2$

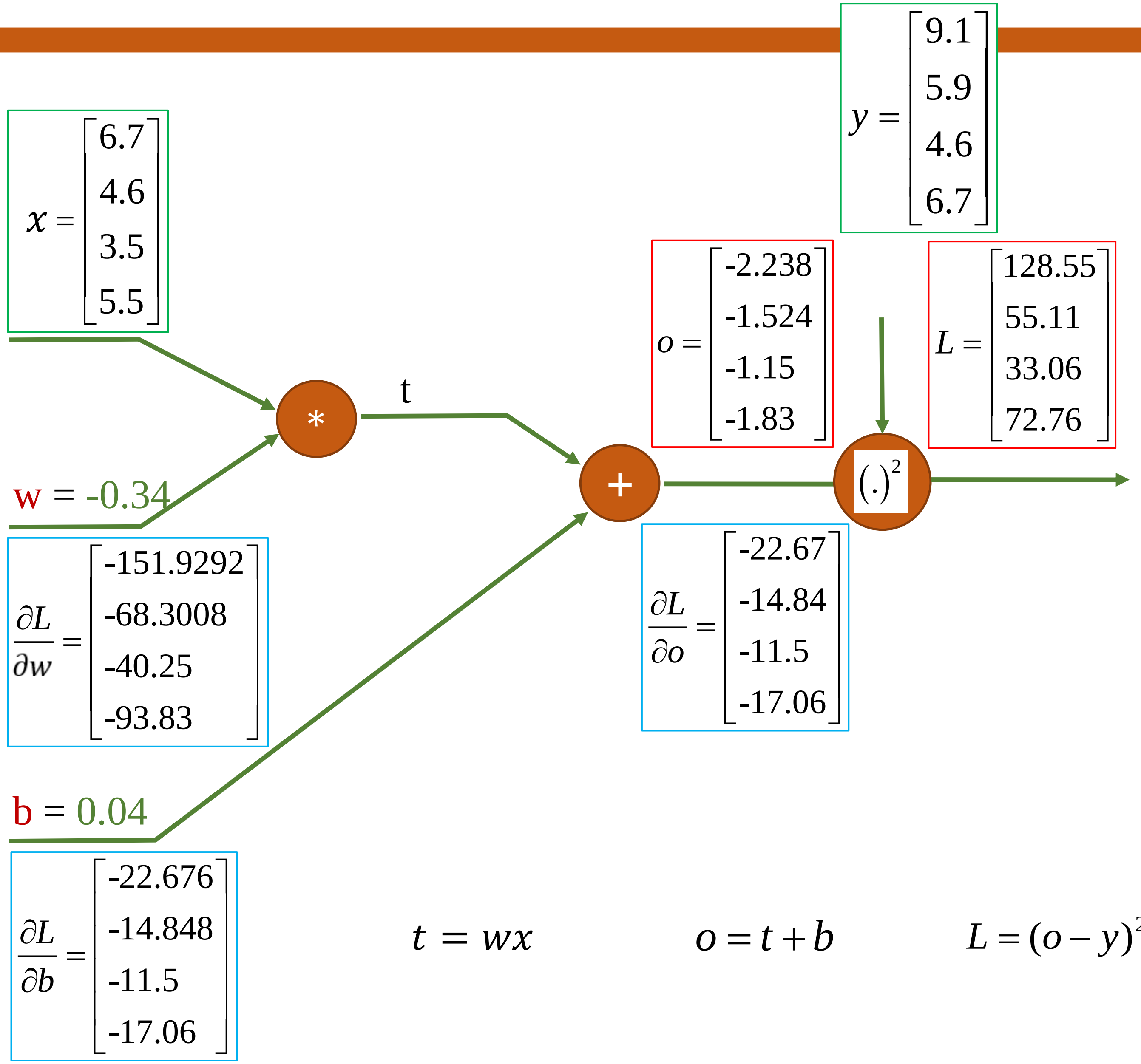


Computational graph

- **House price prediction**
  - **N-sample training**

$$\frac{sum(\frac{\partial L}{\partial w})}{4} = -88.5775$$

$$\frac{sum(\frac{\partial L}{\partial b})}{4} = -16.521$$



Computational graph

- **House price prediction**
  - **N-sample training**

Cách cập nhật a và b

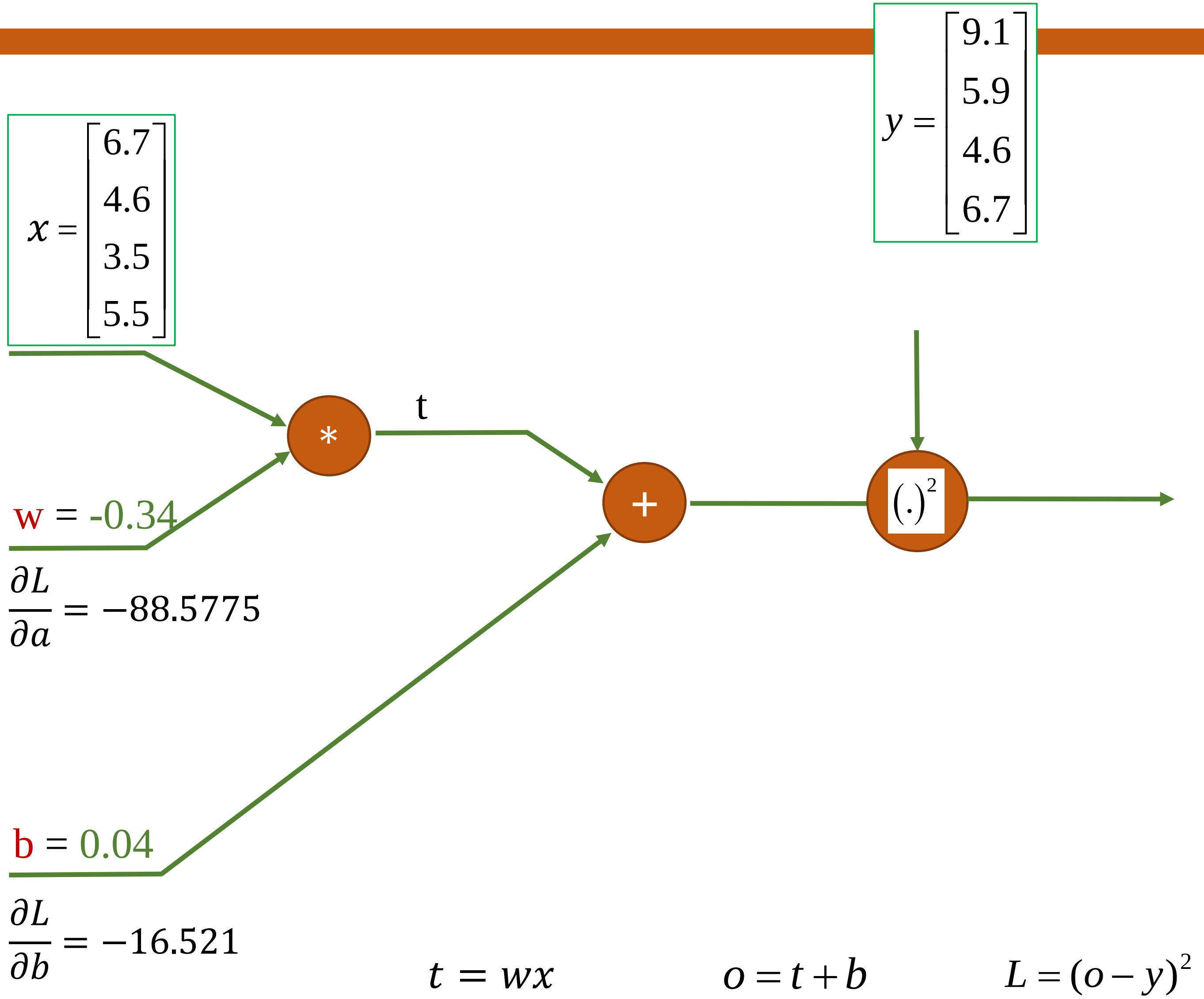
$$w = w - \eta * \frac{\partial L}{\partial w}$$

$$b = b - \eta * \frac{\partial L}{\partial b}$$

Learning rate $\eta = 0.01$

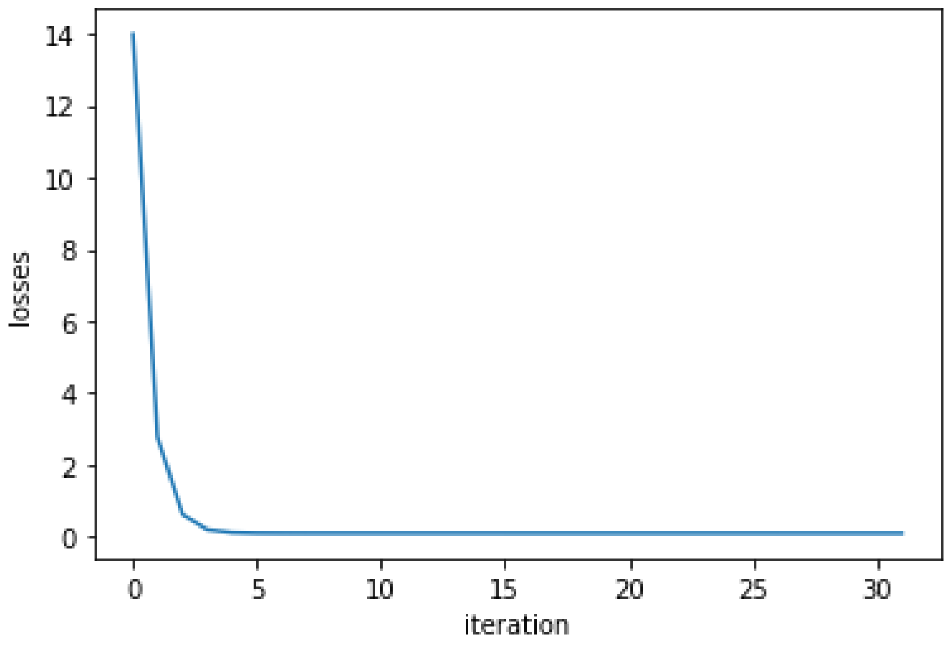
$$\mathrm{w} = -0.34 - (0.01 * (-88.5775)) = 0.54$$

$$\mathrm{b} = 0.04 - \left(0.01 * (-16.521)\right) = 0.205$$

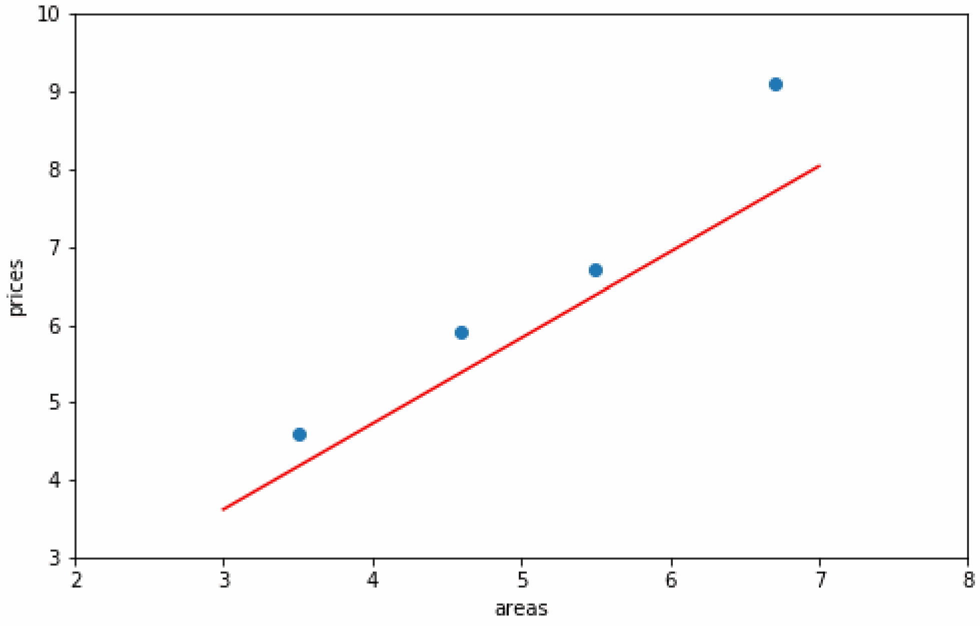


Computational graph

- ❖ House price prediction
  - ❖ N-sample training



Losses for 30 iterations



Model updating for different iterations

Linear Regression (N-samples)

1) Pick all the N samples from training data

2) Tính output $o^{(i)}$

$$o^{(i)} = wx^{(i)} + b \qquad \text{for } 0 \le i < N$$

3) Tính loss

$$L^{(i)} = (o^{(i)} - y^{(i)})^2 \quad \text{for } 0 \le i < N$$

4) Tính đạo hàm

$$L_w'^{(i)} = 2x(o^{(i)} - y^{(i)})$$
$$L_b'^{(i)} = 2(o^{(i)} - y^{(i)}) \quad \text{for } 0 \le i < N$$

5) Cập nhật tham số

$$w = w - \eta \frac{\sum_i L_w'^{(i)}}{N}$$

$$b = b - \eta \frac{\sum_i L_b'^{(i)}}{N}$$

η is learning rate

Friendly version

1) Pick all the N samples from training data

2) Tính output $o^{(i)}$

$$o^{(i)} = \boldsymbol{\theta}^T \boldsymbol{x}^{(i)} \qquad \text{for } 0 \le i < N$$

3) Tính loss

$$L^{(i)} = (o^{(i)} - y^{(i)})^2 \qquad \text{for } 0 \le i < N$$

4) Tính đạo hàm

$$L_{\boldsymbol{\theta}}'^{(i)} = 2\boldsymbol{x}(o^{(i)} - y^{(i)}) \quad \text{for } 0 \le i < N$$

5) Cập nhật tham số

$$\boldsymbol{\theta} = \boldsymbol{\theta} - \eta \frac{\sum_i L_{\boldsymbol{\theta}}'^{(i)}}{N}$$

η is learning rate

Generalized formula

Linear Regression (N-samples)

1) Pick all the N samples from training data

2) Tính output $o^{(i)}$

$$o^{(i)} = \boldsymbol{\theta}^T \boldsymbol{x}^{(i)} \qquad \text{for } 0 \leq i < N$$

3) Tính loss

$$L^{(i)} = (o^{(i)} - y^{(i)})^2 \qquad \text{for } 0 \leq i < N$$

4) Tính đạo hàm

$$L'^{(i)}_{\boldsymbol{\theta}} = 2\boldsymbol{x}(o^{(i)} - y^{(i)}) \qquad \text{for } 0 \leq i < N$$

5) Cập nhật tham số

$$\boldsymbol{\theta} = \boldsymbol{\theta} - \eta \frac{\sum_i L'^{(i)}_{\boldsymbol{\theta}}}{N}$$

η is learning rate

```python
# load data

import numpy as np
from numpy import genfromtxt
import matplotlib.pyplot as plt

data = genfromtxt('data.csv', delimiter=',')
areas  = data[:,0]
prices = data[:,1]
data_size = areas.size

print(type(areas))
print('areas: ', areas)
print('prices: ', prices)
print('data_size: ', data_size)

plt.scatter(areas, prices)
plt.xlabel('areas')
plt.ylabel('prices')
plt.xlim(3,7)
plt.ylim(4,10)
plt.show()
```

Generalized formula

Linear Regression (N-samples)

1) Pick all the N samples from training data

2) Tính output $o^{(i)}$

$$o^{(i)} = \boldsymbol{\theta}^T \boldsymbol{x}^{(i)} \qquad \text{for } 0 \leq i < N$$

3) Tính loss

$$L^{(i)} = (o^{(i)} - y^{(i)})^2 \qquad \text{for } 0 \leq i < N$$

4) Tính đạo hàm

$$L_{\boldsymbol{\theta}}'^{(i)} = 2\boldsymbol{x}(o^{(i)} - y^{(i)}) \qquad \text{for } 0 \leq i < N$$

5) Cập nhật tham số

$$\boldsymbol{\theta} = \boldsymbol{\theta} - \eta \frac{\sum_i L_{\boldsymbol{\theta}}'^{(i)}}{N}$$

η is learning rate

```python
# vector [x, b]
data = np.c_[areas, np.ones((data_size, 1))]
print(data)

n_epochs = 1
eta = 0.01

theta = np.array([[-0.34],[0.04]])
print('theta', theta)
```

Generalized formula

Linear Regression (N-samples)

1) Pick all the N samples from training data

2) Tính output $o^{(i)}$

$$o^{(i)} = \boldsymbol{\theta}^T \boldsymbol{x}^{(i)} \qquad \text{for } 0 \le i < N$$

3) Tính loss

$$L^{(i)} = (o^{(i)} - y^{(i)})^2 \qquad \text{for } 0 \le i < N$$

4) Tính đạo hàm

$$L'^{(i)}_{\boldsymbol{\theta}} = 2\boldsymbol{x}(o^{(i)} - y^{(i)}) \qquad \text{for } 0 \le i < N$$

5) Cập nhật tham số

$$\boldsymbol{\theta} = \boldsymbol{\theta} - \eta \frac{\sum_i L'^{(i)}_{\boldsymbol{\theta}}}{N} \qquad \eta \text{ is learning rate}$$

```python
for epoch in range(n_epochs):
    sum_of_losses = 0
    gradients = np.zeros((2,1))

    for index in range(data_size):
        xi = data[index:index+1]
        yi = prices[index:index+1]
        print('\ndata: ', xi, yi)

        oi = xi.dot(theta)
        li = (oi - yi)*(oi - yi)
        g_li = 2*(oi - yi)

        print('z: ', oi)
        print('loss: ', index, li)
        print('gradient_loss: ', index, g_li)

        cg = xi.T.dot(g_li)
        print('variable gradient: ', index, cg)

        gradients = gradients + cg
        sum_of_losses = sum_of_losses + li

    sum_of_losses = sum_of_losses/data_size
    print('\nsum_of_losses: ', sum_of_losses)

    gradients     = gradients/data_size
    print('\ngradients: ', gradients)

    theta = theta - eta*gradients
    print('new params: ', theta)
```

Generalized formula

Linear Regression (N-samples)

1) Pick all the N samples from training data

2) Tính output $o^{(i)}$

$$o^{(i)} = \boldsymbol{\theta}^T \boldsymbol{x}^{(i)} \qquad \text{for } 0 \leq i < N$$

3) Tính loss

$$L^{(i)} = (o^{(i)} - y^{(i)})^2 \qquad \text{for } 0 \leq i < N$$

4) Tính đạo hàm

$$L_{\boldsymbol{\theta}}^{\prime(i)} = 2\boldsymbol{x}(o^{(i)} - y^{(i)}) \qquad \text{for } 0 \leq i < N$$

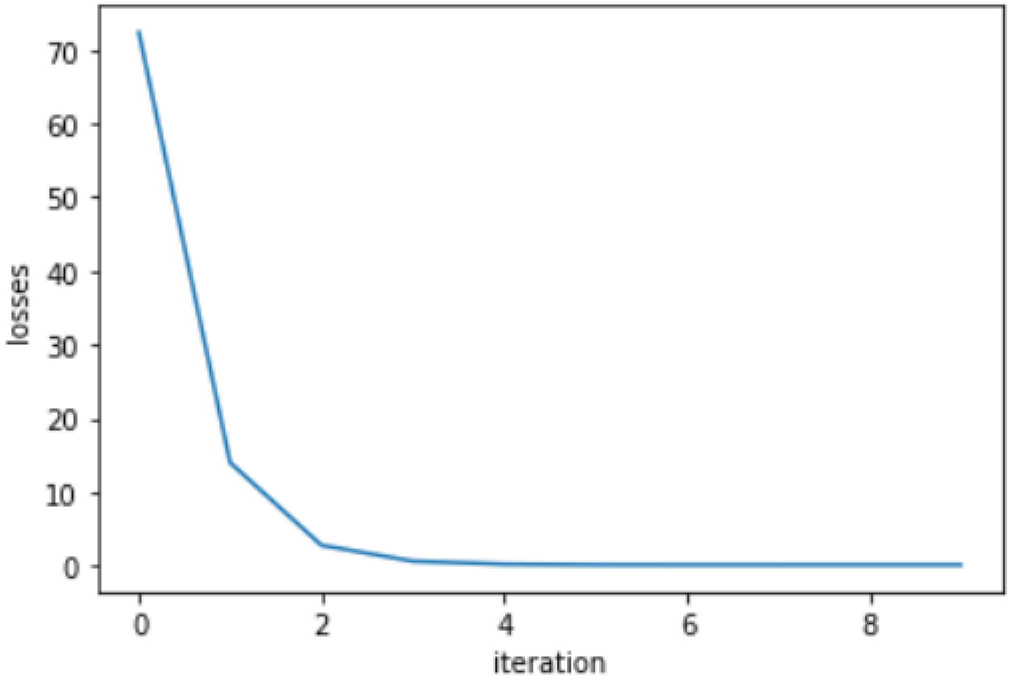
5) Cập nhật tham số

$$\boldsymbol{\theta} = \boldsymbol{\theta} - \eta \frac{\sum_i L_{\boldsymbol{\theta}}^{\prime(i)}}{N}$$

η is learning rate

```python
import matplotlib.pyplot as plt

plt.plot(losses)
plt.xlabel('iteration')
plt.ylabel('losses')
plt.show()
```



Generalized formula

Linear Regression

	Advantages	Disadvantages
1 sample	Simple to understand and implement Faster learning on some problems Noisy update is beneficial sometime	Computationally expensive Noisy gradient signal Convergence problem
m sample	A balance between the robustness of 1-sample and the efficiency of N-sample	
N sample	Computationally efficient More stable error gradient parallel processing	Premature convergence Memory problem Training speed is slower

Outline

- Machine Learning
- Derivative/Gradient
- Linear Regression
- Computational Graph
- Summary

Linear Regression

❖ Generalized formula

House price data

	Feature	Label
	area	price
	6.7	9.1
	4.6	5.9
	3.5	4.6
	5.5	6.7

Model

$$\text{price} = \text{w} * area + b$$

$$\text{y} = wx + b$$

Model (vectorization)

$$\text{y} = \boldsymbol{\theta}^T \boldsymbol{x} \quad \text{where} \quad \boldsymbol{\theta}^T = [b \quad w]^T$$

$$\boldsymbol{x} = [x_0 \quad area]^T$$

$$x_0 = 1$$

Advertising data

Features			Label
TV	Radio	Newspaper	Sales
230.1	37.8	69.2	22.1
44.5	39.3	45.1	10.4
17.2	45.9	69.3	12
151.5	41.3	58.5	16.5
180.8	10.8	58.4	17.9

Model

$$\text{Sale} = w_1 * TV + w_2 * Radio + w_3 * Newspaper + b$$

$$\text{y} = w_1 x_1 + w_2 x_2 + w_3 x_3 + b$$

Model (vectorization)

$$\text{y} = \boldsymbol{\theta}^T \boldsymbol{x} \quad \text{where} \quad \boldsymbol{\theta}^T = [b \quad w_1 \quad w_2 \quad w_3]^T$$

$$\boldsymbol{x} = [x_0 \quad TV \quad Radio \quad Newspaper]^T$$

$$x_0 = 1$$

Linear Regression

❖ Generalized formula

Features | Label

Boston House Price Data

crim	zn	indus	chas	nox	rm	age	dis	rad	tax	ptratio	black	lstat	medv
0.00632	18	2.31	0	0.538	6.575	65.2	4.09	1	296	15.3	396.9	4.98	24
0.02731	0	7.07	0	0.469	6.421	78.9	4.9671	2	242	17.8	396.9	9.14	21.6
0.03237	0	2.18	0	0.458	6.998	45.8	6.0622	3	222	18.7	394.63	2.94	33.4
0.06905	0	2.18	0	0.458	7.147	54.2	6.0622	3	222	18.7	396.9	5.33	36.2
0.08829	12.5	7.87	0	0.524	6.012	66.6	5.5605	5	311	15.2	395.6	12.43	22.9

Model

$$\text{medv} = w_1 * x_1 + \cdots + w_{13} * x_{13} + b$$

Model (vectorization)

$$y = \boldsymbol{\theta}^T \boldsymbol{x} \quad \text{where} \quad \begin{aligned} \boldsymbol{\theta}^T &= [b \quad w_1 \quad \ldots \quad w_{13}]^T \\ \boldsymbol{x} &= [x_0 \quad x_1 \quad \ldots \quad x_{13}]^T \\ x_0 &= 1 \end{aligned}$$

Linear Regression (1-sample)

1) Pick a sample (x, y) from training data

2) Tính output o

$$o = wx + b$$

3) Tính loss

$$L = (o - y)^2$$

4) Tính đạo hàm

$$L'_w = 2x(o - y)$$

$$L'_b = 2(o - y)$$

5) Cập nhật tham số

$$w = w - \eta L'_w$$

$$b = b - \eta L'_b$$

η is learning rate

Friendly version

1) Pick a sample (x, y) from training data

2) Tính output o

$$o = \boldsymbol{\theta}^T \boldsymbol{x}$$

3) Tính loss

$$L = (o - y)^2$$

4) Tính đạo hàm

$$L'_{\boldsymbol{\theta}} = 2\boldsymbol{x}(o - y)$$

5) Cập nhật tham số

$$\boldsymbol{\theta} = \boldsymbol{\theta} - \eta L'_{\boldsymbol{\theta}}$$

η is learning rate

Generalized formula

Linear Regression (m-samples)

1) Pick m samples $(x^{(i)}, y^{(i)})$ from training data

1.1) Tính output $o^{(i)}$

$$o^{(i)} = wx^{(i)} + b \qquad \text{for } 0 \leq i < m$$

1.2) Tính loss

$$L^{(i)} = (o^{(i)} - y^{(i)})^2 \quad \text{for } 0 \leq i < m$$

1.3) Tính đạo hàm

$$\begin{aligned} L'^{(i)}_w &= 2x(o^{(i)} - y^{(i)}) \\ L'^{(i)}_b &= 2(o^{(i)} - y^{(i)}) \end{aligned} \quad \text{for } 0 \leq i < m$$

2) Cập nhật tham số

$$w = w - \eta \frac{\sum_i L'^{(i)}_w}{m}$$

$$b = b - \eta \frac{\sum_i L'^{(i)}_b}{m}$$

η is learning rate

Friendly version

1) Pick m samples $(x^{(i)}, y^{(i)})$ from training data

1.1) Tính output $o^{(i)}$

$$o^{(i)} = \boldsymbol{\theta}^T \boldsymbol{x}^{(i)} \qquad \text{for } 0 \leq i < m$$

1.2) Tính loss

$$L^{(i)} = (o^{(i)} - y^{(i)})^2 \qquad \text{for } 0 \leq i < m$$

1.3) Tính đạo hàm

$$L'^{(i)}_{\boldsymbol{\theta}} = 2\boldsymbol{x}(o^{(i)} - y^{(i)}) \quad \text{for } 0 \leq i < m$$

2) Cập nhật tham số

$$\boldsymbol{\theta} = \boldsymbol{\theta} - \eta \frac{\sum_i L'^{(i)}_{\boldsymbol{\theta}}}{m}$$

η is learning rate

Generalized formula

Linear Regression (N-samples)

1) Pick all the N samples from training data

2) Tính output $o^{(i)}$

$$o^{(i)} = wx^{(i)} + b \qquad \text{for } 0 \leq i < N$$

3) Tính loss

$$L^{(i)} = (o^{(i)} - y^{(i)})^2 \quad \text{for } 0 \leq i < N$$

4) Tính đạo hàm

$$L'^{(i)}_w = 2x(o^{(i)} - y^{(i)})$$
$$L'^{(i)}_b = 2(o^{(i)} - y^{(i)}) \quad \text{for } 0 \leq i < N$$

5) Cập nhật tham số

$$w = w - \eta \frac{\sum_i L'^{(i)}_w}{N}$$
$$b = b - \eta \frac{\sum_i L'^{(i)}_b}{N}$$

η is learning rate

Friendly version

1) Pick all the N samples from training data

2) Tính output $o^{(i)}$

$$o^{(i)} = \boldsymbol{\theta}^T \boldsymbol{x}^{(i)} \qquad \text{for } 0 \leq i < N$$

3) Tính loss

$$L^{(i)} = (o^{(i)} - y^{(i)})^2 \qquad \text{for } 0 \leq i < N$$

4) Tính đạo hàm

$$L'^{(i)}_{\boldsymbol{\theta}} = 2\boldsymbol{x}(o^{(i)} - y^{(i)}) \quad \text{for } 0 \leq i < N$$

5) Cập nhật tham số

$$\boldsymbol{\theta} = \boldsymbol{\theta} - \eta \frac{\sum_i L'^{(i)}_{\boldsymbol{\theta}}}{N}$$

η is learning rate

Generalized formula


Thank you!