

Convolutional Neural Network (Session 3)

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Ph.D. in Computer Science

Classic Feature Extractors

❖ Edge detection



Original



Sobel (Th = 0.05)



Sobel (Th = 0.15)



Canny (Th = 0.05)



Canny (Th = 0.15)



Roberts (Th = 0.05)



Roberts (Th = 0.15)



Fuzzy-BFA



Proposed (Th = 0.18)



Proposed (Th = 0.35)



Proposed (optimal)

Edge Detection via Edge-Strength Estimation
Using Fuzzy Reasoning and Optimal Threshold
Selection Using Particle Swarm Optimization

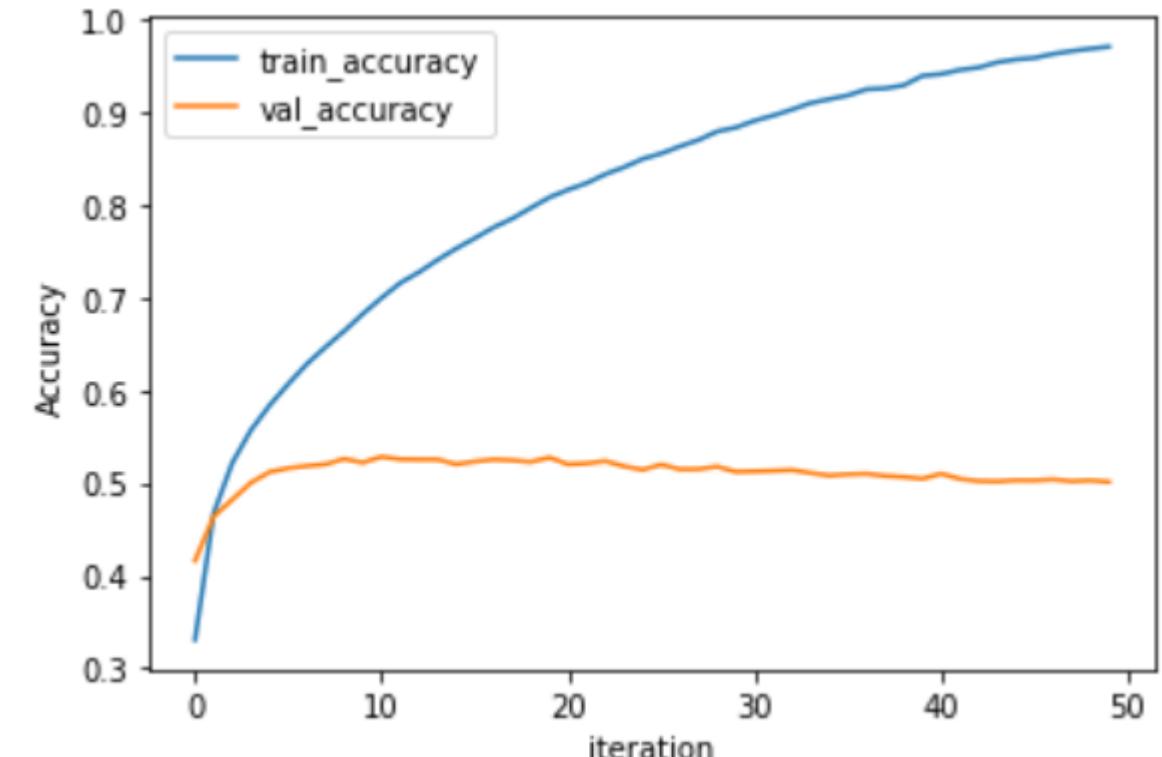
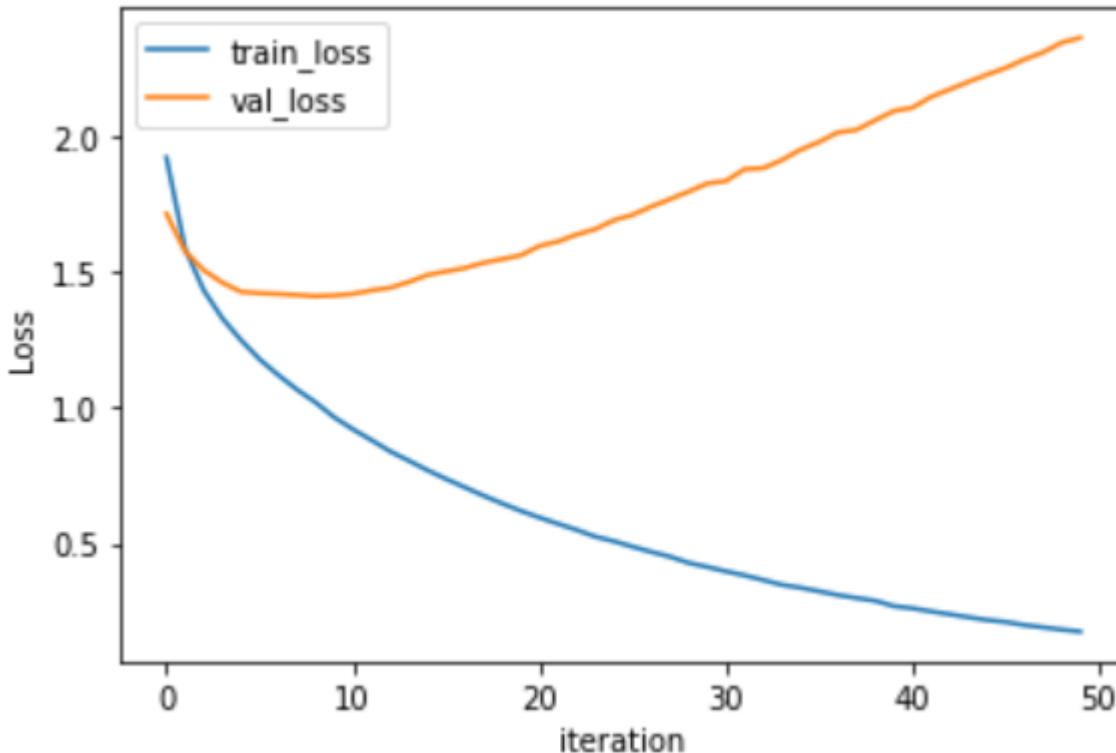
Case Study

❖ Classic feature extractors vs. CNN filters

```
1 import tensorflow as tf
2 from tensorflow import keras
3
4 def construct_model():
5     inp = keras.layers.Input(shape=[32, 32, 3])
6
7     # get edges
8     edges = tf.image.sobel_edges(inp)
9     edges = tf.keras.layers.Reshape((32, 32, 6))(edges)
10
11    dx, dy = tf.image.image_gradients(inp)
12    x = tf.keras.layers.Concatenate(axis=3)([dx, dy, edges])
13
14    x = keras.layers.MaxPooling2D(2)(x)
15    x = keras.layers.Flatten()(x)
16    x = keras.layers.Dense(128, activation='relu')(x)
17    x = keras.layers.Dense(10, activation='softmax')(x)
18
19    return tf.keras.Model(inputs=[inp], outputs=x)
20
21 cifar10 = keras.datasets.cifar10
22 (train_images, train_labels), (test_images, test_labels) = cifar10.load_data()
23
24 train_images = train_images / 255.0
25 test_images = test_images / 255.0
26
27 train_images = tf.reshape(train_images, (50000, 32, 32, 3))
28 test_images = tf.reshape(test_images, (10000, 32, 32, 3))
29
30 # model
31 model = construct_model()
32 model.summary()
33
34 model.compile(optimizer='adam',
35                 loss=tf.keras.losses.SparseCategoricalCrossentropy(),
36                 metrics=['accuracy'])
37 history_data = model.fit(train_images, train_labels, batch_size=1024,
38                           validation_data=(test_images, test_labels), epochs=50)
```

Case Study

❖ Classic feature extractors vs. CNN filters



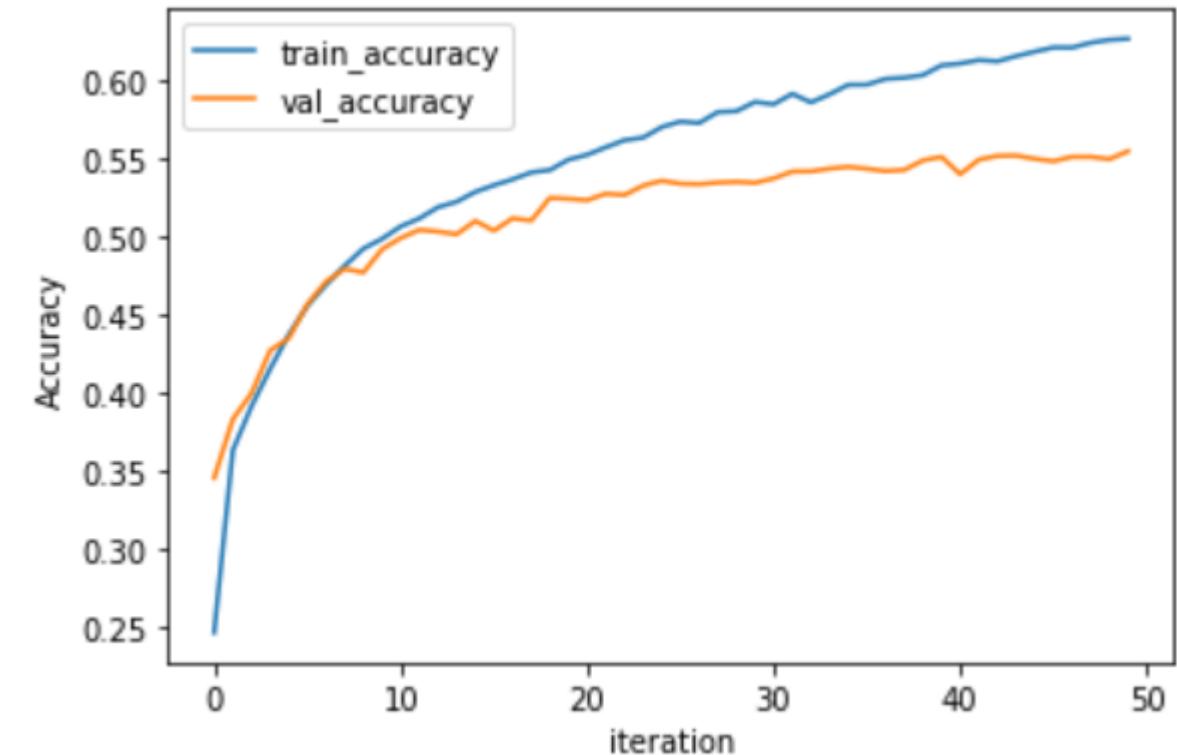
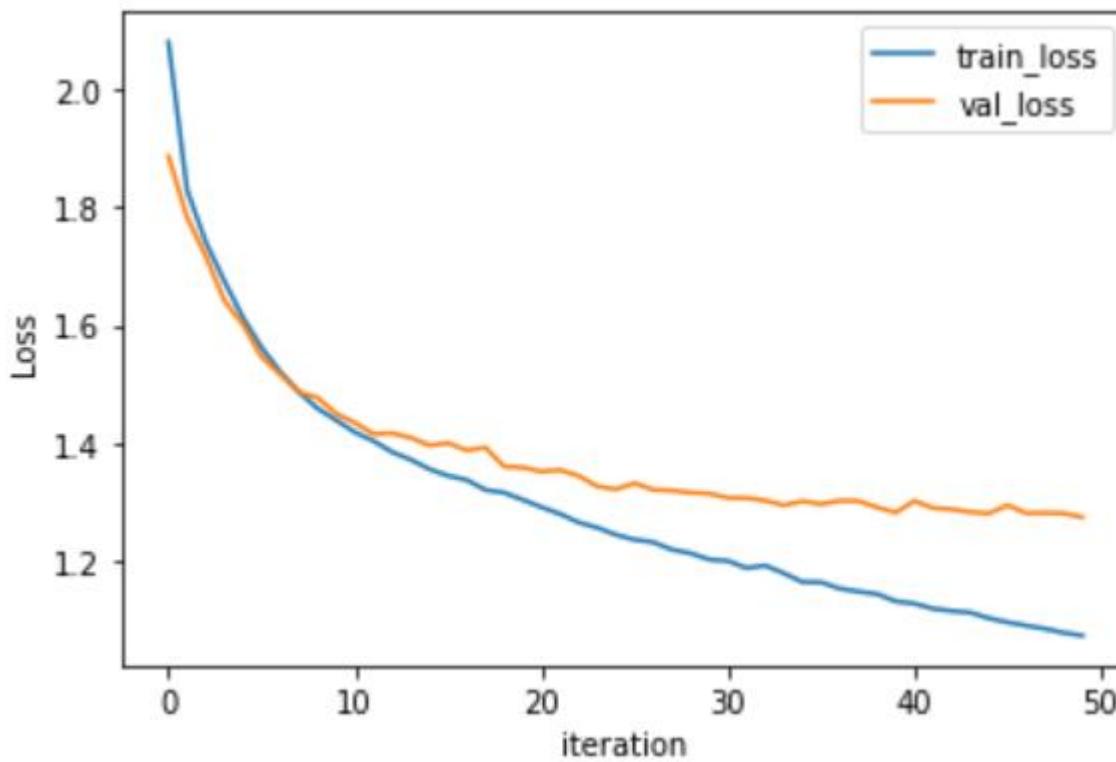
Case Study

❖ Classic feature extractors vs. CNN filters

```
1 import tensorflow as tf
2 from tensorflow import keras
3
4 cifar10 = keras.datasets.cifar10
5 (train_images, train_labels), (test_images, test_labels) = cifar10.load_data()
6
7 train_images = train_images / 255.0
8 test_images = test_images / 255.0
9
10 train_images = tf.reshape(train_images, (50000, 32, 32, 3))
11 test_images = tf.reshape(test_images, (10000, 32, 32, 3))
12
13 # model
14 model = keras.models.Sequential()
15 model.add(tf.keras.Input(shape=(32, 32, 3)))
16 model.add(keras.layers.Conv2D(4, (3, 3), activation='relu'))
17 model.add(keras.layers.MaxPooling2D(2))
18
19 # flatten
20 model.add(keras.layers.Flatten())
21 model.add(keras.layers.Dense(128, activation='relu'))
22 model.add(keras.layers.Dense(10, activation='softmax'))
23
24 model.summary()
25
26 model.compile(optimizer='adam',
27                 loss=tf.keras.losses.SparseCategoricalCrossentropy(),
28                 metrics=['accuracy'])
29 history_data = model.fit(train_images, train_labels, batch_size=1024,
30                           validation_data=(test_images, test_labels), epochs=50)
```

Case Study

❖ Classic feature extractors vs. CNN filters



Down-sample Feature Map

Max pooling: Features are preserved

v_1	v_2	v_3	v_4
v_5	v_6	v_7	v_8
v_9	v_{10}	v_{11}	v_{12}
v_{13}	v_{14}	v_{15}	v_{16}

Data

2x2 max pooling (

v_1	v_2	v_3	v_4
v_5	v_6	v_7	v_8
v_9	v_{10}	v_{11}	v_{12}
v_{13}	v_{14}	v_{15}	v_{16}

) =

m_1	m_2
m_3	m_4

$$m_1 = \max(v_1, v_2, v_5, v_6)$$
$$m_2 = \max(v_3, v_4, v_7, v_8)$$
$$m_3 = \max(v_9, v_{10}, v_{13}, v_{14})$$
$$m_4 = \max(v_{11}, v_{12}, v_{15}, v_{16})$$



Feature map (110x110)

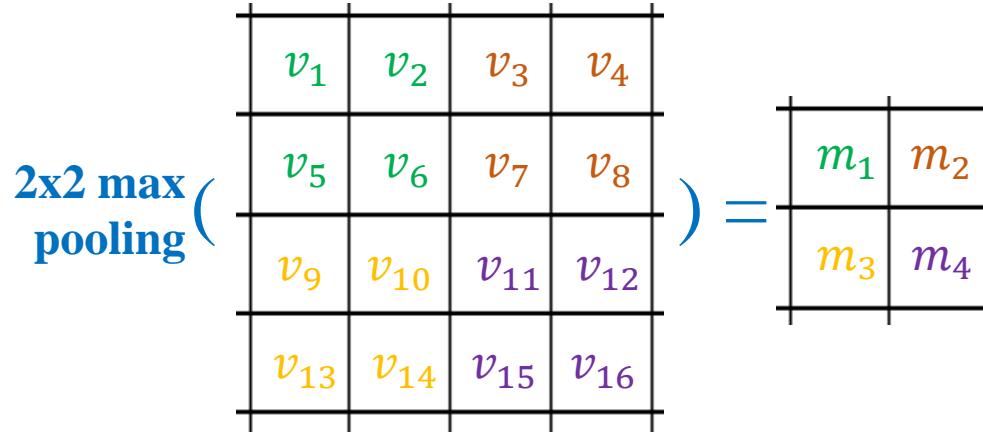
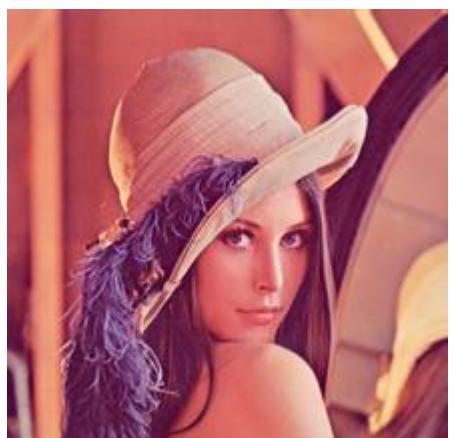
Feature map (55x55)

Down-sample Feature Map

Max pooling: Features are preserved

v_1	v_2	v_3	v_4
v_5	v_6	v_7	v_8
v_9	v_{10}	v_{11}	v_{12}
v_{13}	v_{14}	v_{15}	v_{16}

Data



$$m_1 = \max(v_1, v_2, v_5, v_6)$$
$$m_2 = \max(v_3, v_4, v_7, v_8)$$
$$m_3 = \max(v_9, v_{10}, v_{13}, v_{14})$$
$$m_4 = \max(v_{11}, v_{12}, v_{15}, v_{16})$$

`keras.layers.MaxPooling2D(pool_size=2)`



max pooling
(2x2) →

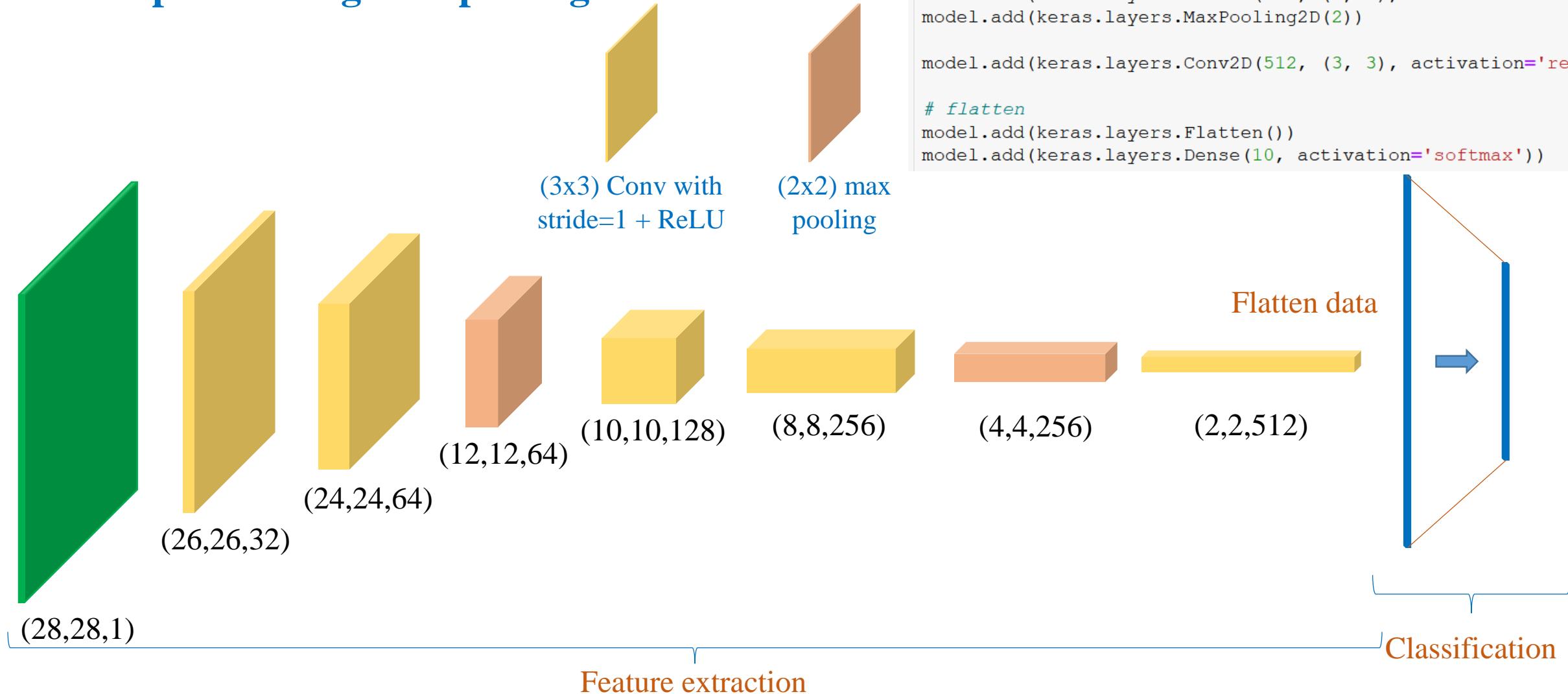


max pooling
(2x2) →



Down-sample Feature Map

Example 1: Using max pooling



Down-sample Feature Map

Average pooling: Features are preserved

v_1	v_2	v_3	v_4
v_5	v_6	v_7	v_8
v_9	v_{10}	v_{11}	v_{12}
v_{13}	v_{14}	v_{15}	v_{16}

Data

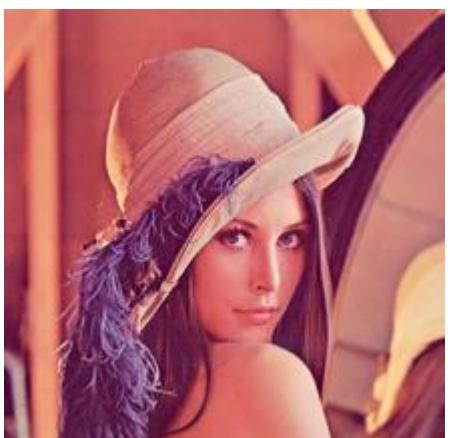
average pooling (

v_1	v_2	v_3	v_4
v_5	v_6	v_7	v_8
v_9	v_{10}	v_{11}	v_{12}
v_{13}	v_{14}	v_{15}	v_{16}

) =

m_1	m_2
m_3	m_4

$$m_1 = \text{mean}(v_1, v_2, v_5, v_6)$$
$$m_2 = \text{mean}(v_3, v_4, v_7, v_8)$$
$$m_3 = \text{mean}(v_9, v_{10}, v_{13}, v_{14})$$
$$m_4 = \text{mean}(v_{11}, v_{12}, v_{15}, v_{16})$$



Feature map (220x220)

Average
Pooling (2x2) \rightarrow



Feature map
(110x110)

Average
Pooling (2x2) \rightarrow



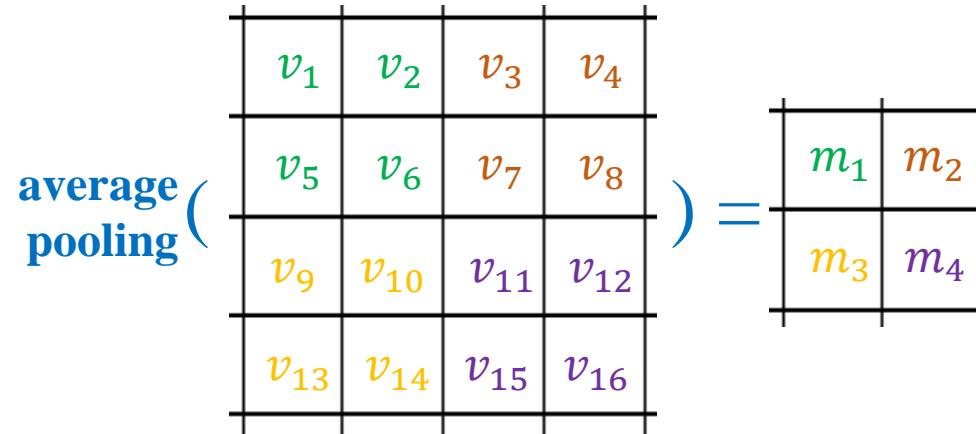
Feature map
(55x55)

Down-sample Feature Map

Average pooling: Features are preserved

v_1	v_2	v_3	v_4
v_5	v_6	v_7	v_8
v_9	v_{10}	v_{11}	v_{12}
v_{13}	v_{14}	v_{15}	v_{16}

Data



$$m_1 = \text{mean}(v_1, v_2, v_5, v_6)$$
$$m_2 = \text{mean}(v_3, v_4, v_7, v_8)$$
$$m_3 = \text{mean}(v_9, v_{10}, v_{13}, v_{14})$$
$$m_4 = \text{mean}(v_{11}, v_{12}, v_{15}, v_{16})$$

keras.layers.AveragePooling2D (pool_size=2)



Feature map (220x220)

Average
Pooling (2x2) \rightarrow



Feature map
(110x110)

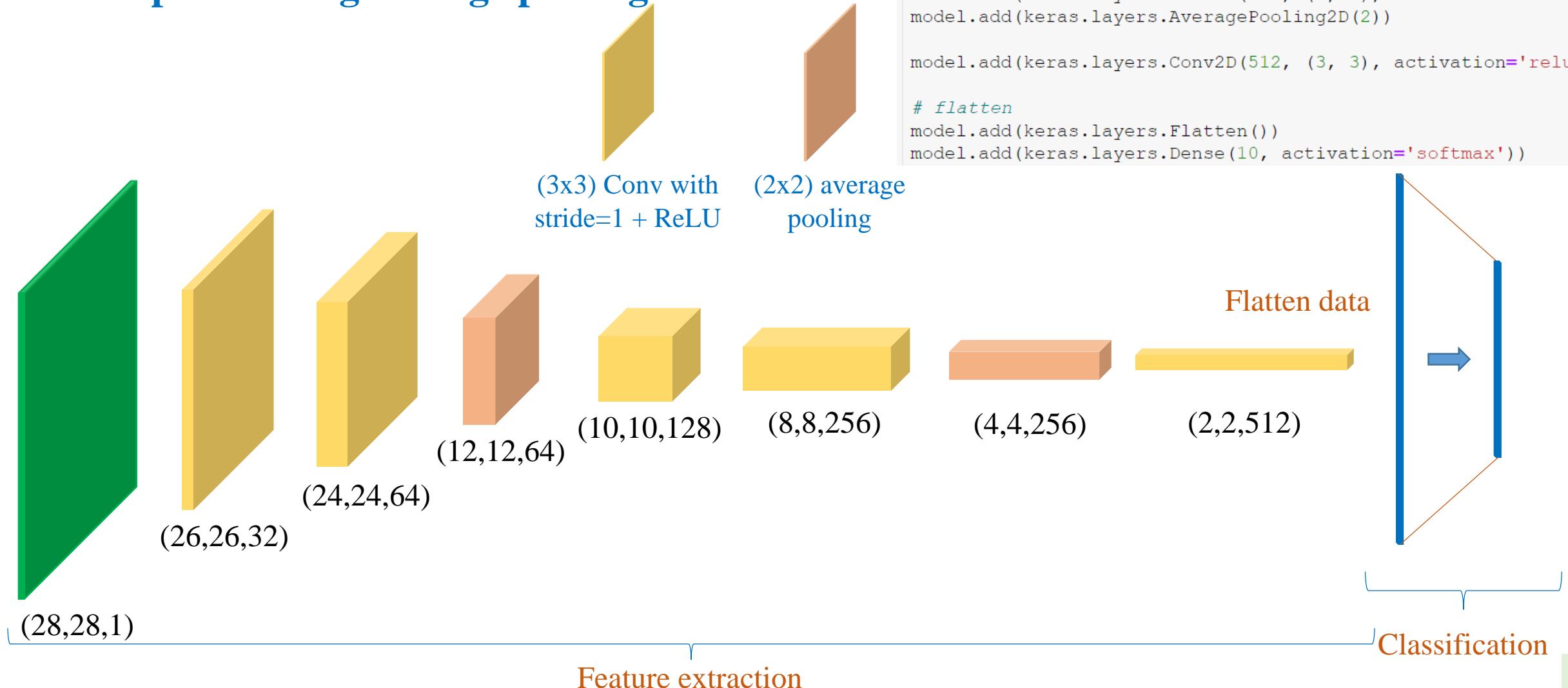
Average
Pooling (2x2) \rightarrow



Feature map
(55x55)

Down-sample Feature Map

Example 2: Using average pooling



Down-sample Feature Map

Max pooling vs. Average pooling



Feature map (220x220)



Feature map (220x220)

max pooling
(2x2)



max pooling
(2x2)



Feature map
(55x55)

Average
Pooling (2x2)



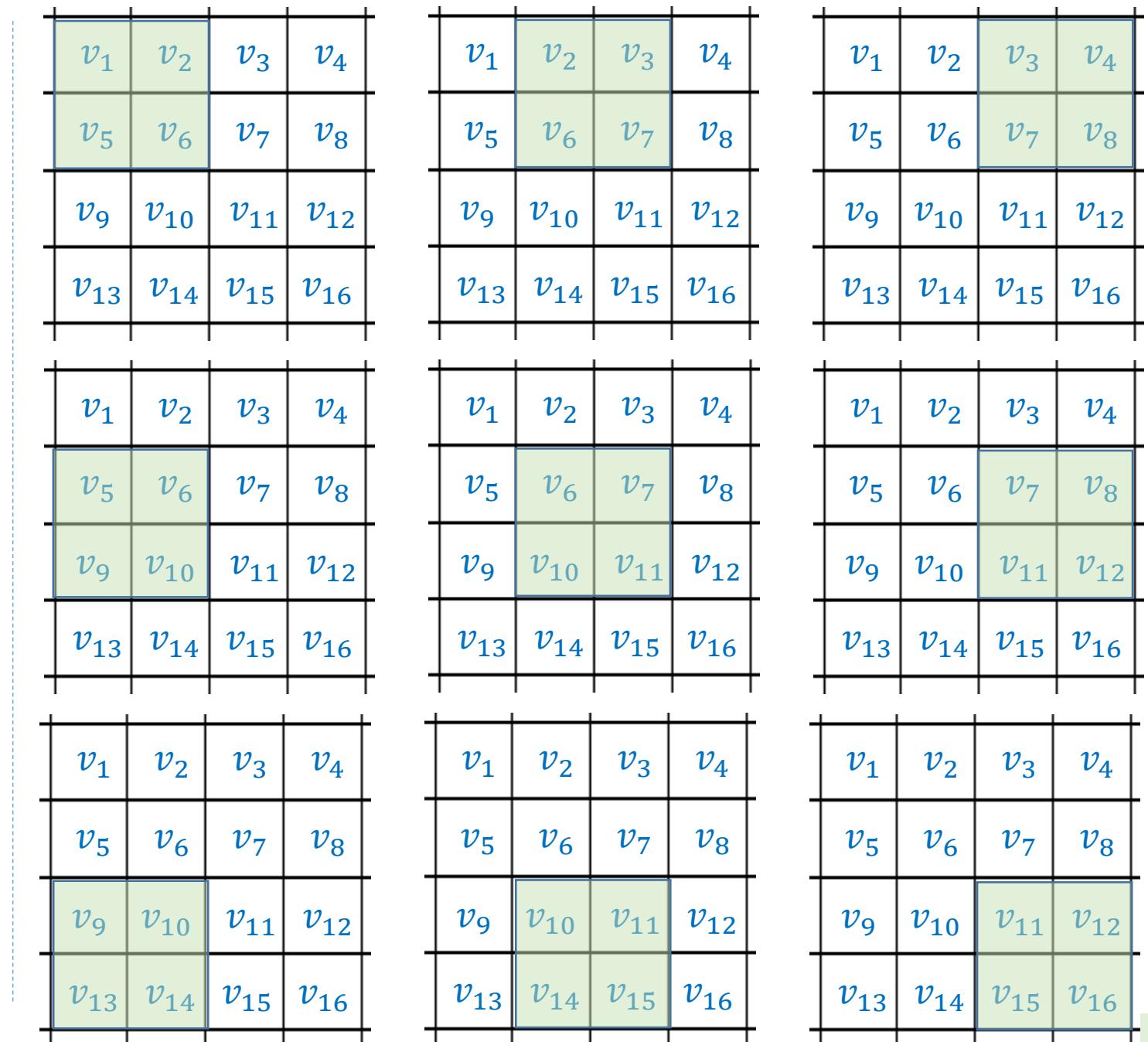
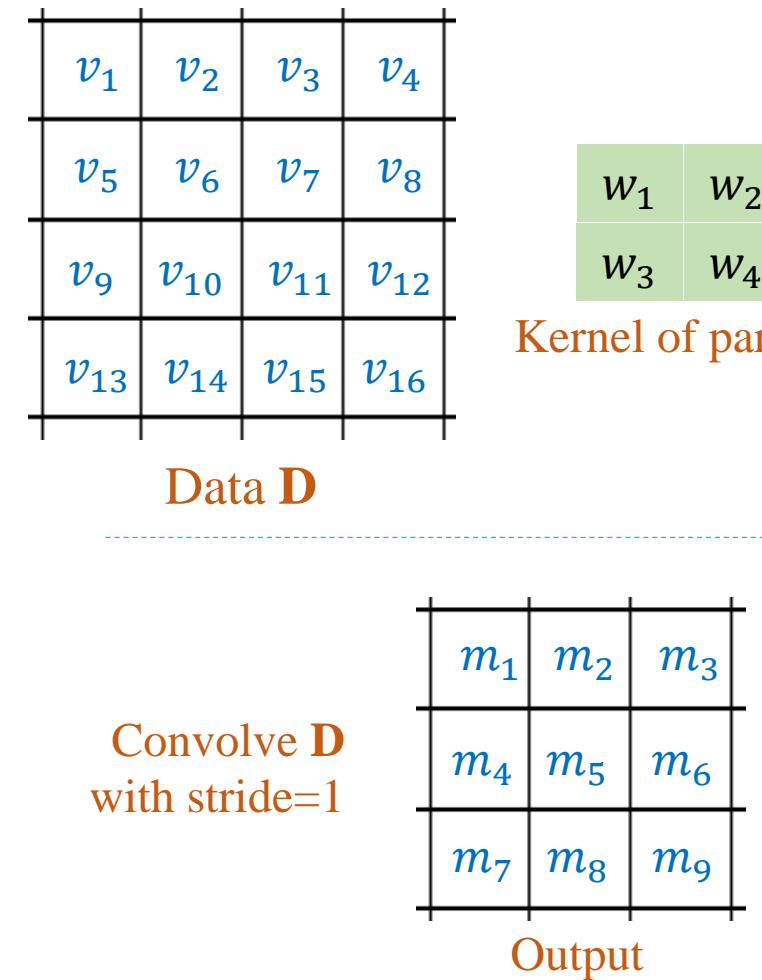
Average
Pooling (2x2)



Feature map
(55x55)

Down-sample Feature Map

❖ Convolve with stride



Down-sample Feature Map

❖ Convolve with stride

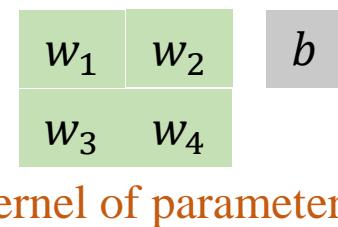
v_1	v_2	v_3	v_4
v_5	v_6	v_7	v_8
v_9	v_{10}	v_{11}	v_{12}
v_{13}	v_{14}	v_{15}	v_{16}

Data D

Convolve D
with stride=2

m_1	m_2
m_3	m_4

Output



v_1	v_2	v_3	v_4
v_5	v_6	v_7	v_8
v_9	v_{10}	v_{11}	v_{12}
v_{13}	v_{14}	v_{15}	v_{16}

v_1	v_2	v_3	v_4
v_5	v_6	v_7	v_8
v_9	v_{10}	v_{11}	v_{12}
v_{13}	v_{14}	v_{15}	v_{16}

v_1	v_2	v_3	v_4
v_5	v_6	v_7	v_8
v_9	v_{10}	v_{11}	v_{12}
v_{13}	v_{14}	v_{15}	v_{16}

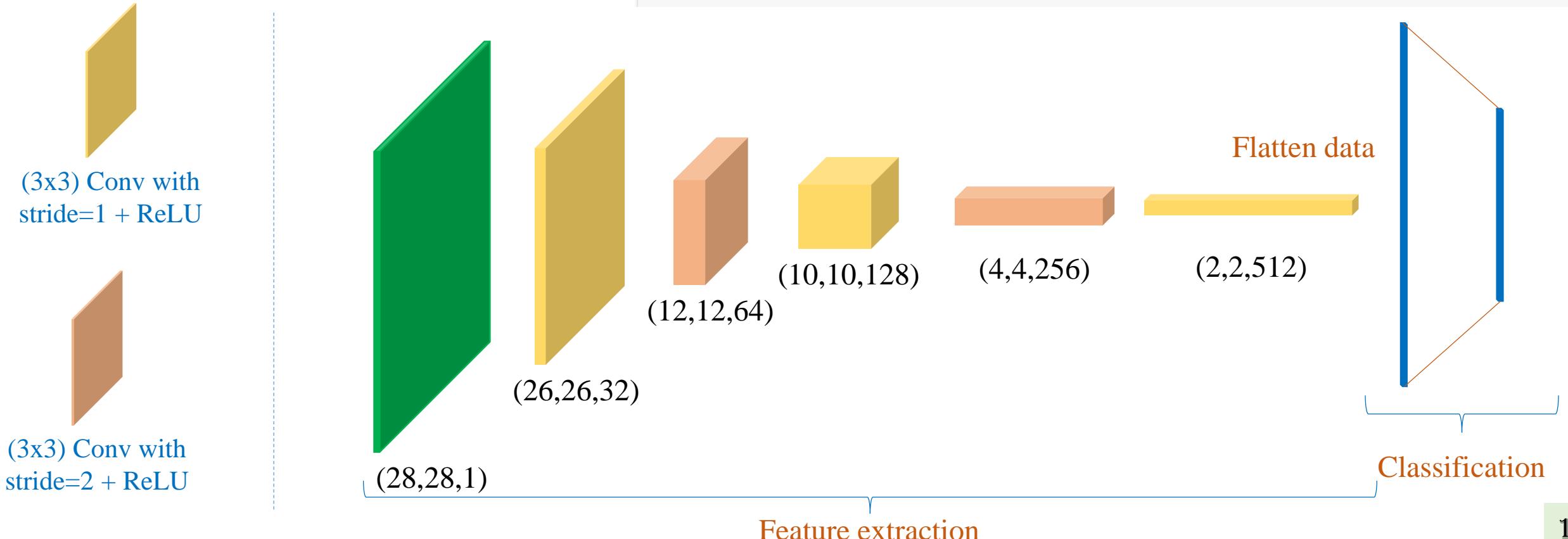
v_1	v_2	v_3	v_4
v_5	v_6	v_7	v_8
v_9	v_{10}	v_{11}	v_{12}
v_{13}	v_{14}	v_{15}	v_{16}

`keras.layers.Conv2D(32, (3, 3), strides=1, activation='relu')`

`keras.layers.Conv2D(32, (3, 3), strides=2, activation='relu')`

Down-sample Feature Map

Example 2: Using Conv with strides



Padding

Goal: Keep resolution

of feature map

v_1	v_2	v_3	v_4
v_5	v_6	v_7	v_8
v_9	v_{10}	v_{11}	v_{12}
v_{13}	v_{14}	v_{15}	v_{16}

Data D
(4x4)

w_1	w_2	w_3	b
w_4	w_5	w_6	
w_7	w_8	w_9	

Kernel of parameters

Without using padding
or padding=0

Convolve
with stride=1 (D) =

m_1	m_2
m_4	m_5

Output
(2x2)

Padding = 1

v	v	v	v	v	v
v	v_1	v_2	v_3	v_4	v
v	v_5	v_6	v_7	v_8	v
v	v_9	v_{10}	v_{11}	v_{12}	v
v	v_{13}	v_{14}	v_{15}	v_{16}	v
v	v	v	v	v	v

Data D_p

Convolve
with stride=1 (D_p) =

m_1	m_2	m_3	m_4
m_5	m_6	m_7	m_8
m_9	m_{10}	m_{11}	m_{12}
m_{13}	m_{14}	m_{15}	m_{16}

Output
(4x4)

Padding

Example

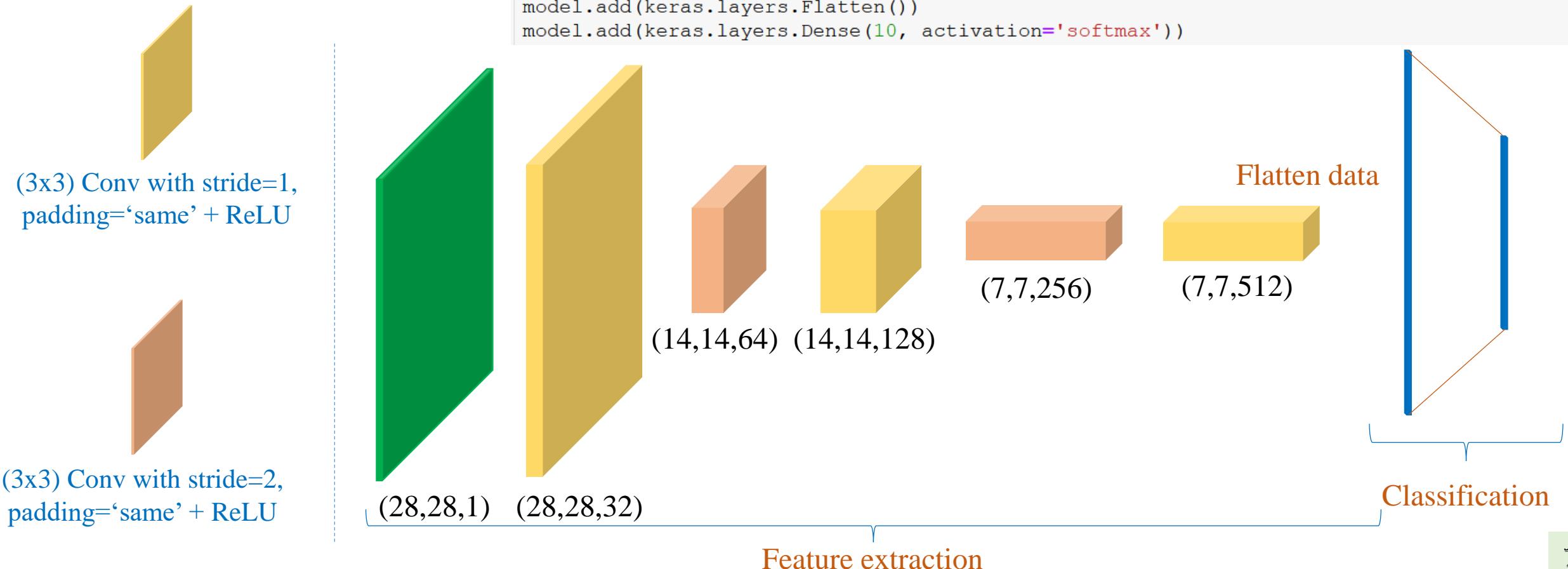
```
# model
model = keras.models.Sequential()
model.add(tf.keras.Input(shape=(28, 28, 1)))

model.add(keras.layers.Conv2D(32, (3, 3), strides=1, padding='same', activation='relu'))
model.add(keras.layers.Conv2D(64, (3, 3), strides=2, padding='same', activation='relu'))

model.add(keras.layers.Conv2D(128, (3, 3), strides=1, padding='same', activation='relu'))
model.add(keras.layers.Conv2D(256, (3, 3), strides=2, padding='same', activation='relu'))

model.add(keras.layers.Conv2D(512, (3, 3), strides=1, padding='same', activation='relu'))

# flatten
model.add(keras.layers.Flatten())
model.add(keras.layers.Dense(10, activation='softmax'))
```



Outline

- Common CNN Architecture
- 1x1 Convolution
- Examples
- Backpropagation

Size of Feature Maps

Summary. To summarize, the Conv Layer:

- Accepts a volume of size $W_1 \times H_1 \times D_1$
- Requires four hyperparameters:
 - Number of filters K ,
 - their spatial extent F ,
 - the stride S ,
 - the amount of zero padding P .
- Produces a volume of size $W_2 \times H_2 \times D_2$ where:
 - $W_2 = (W_1 - F + 2P)/S + 1$
 - $H_2 = (H_1 - F + 2P)/S + 1$ (i.e. width and height are computed equally by symmetry)
 - $D_2 = K$
- With parameter sharing, it introduces $F \cdot F \cdot D_1$ weights per filter, for a total of $(F \cdot F \cdot D_1) \cdot K$ weights and K biases.
- In the output volume, the d -th depth slice (of size $W_2 \times H_2$) is the result of performing a valid convolution of the d -th filter over the input volume with a stride of S , and then offset by d -th bias.

Image Classification

Cifar-10 dataset

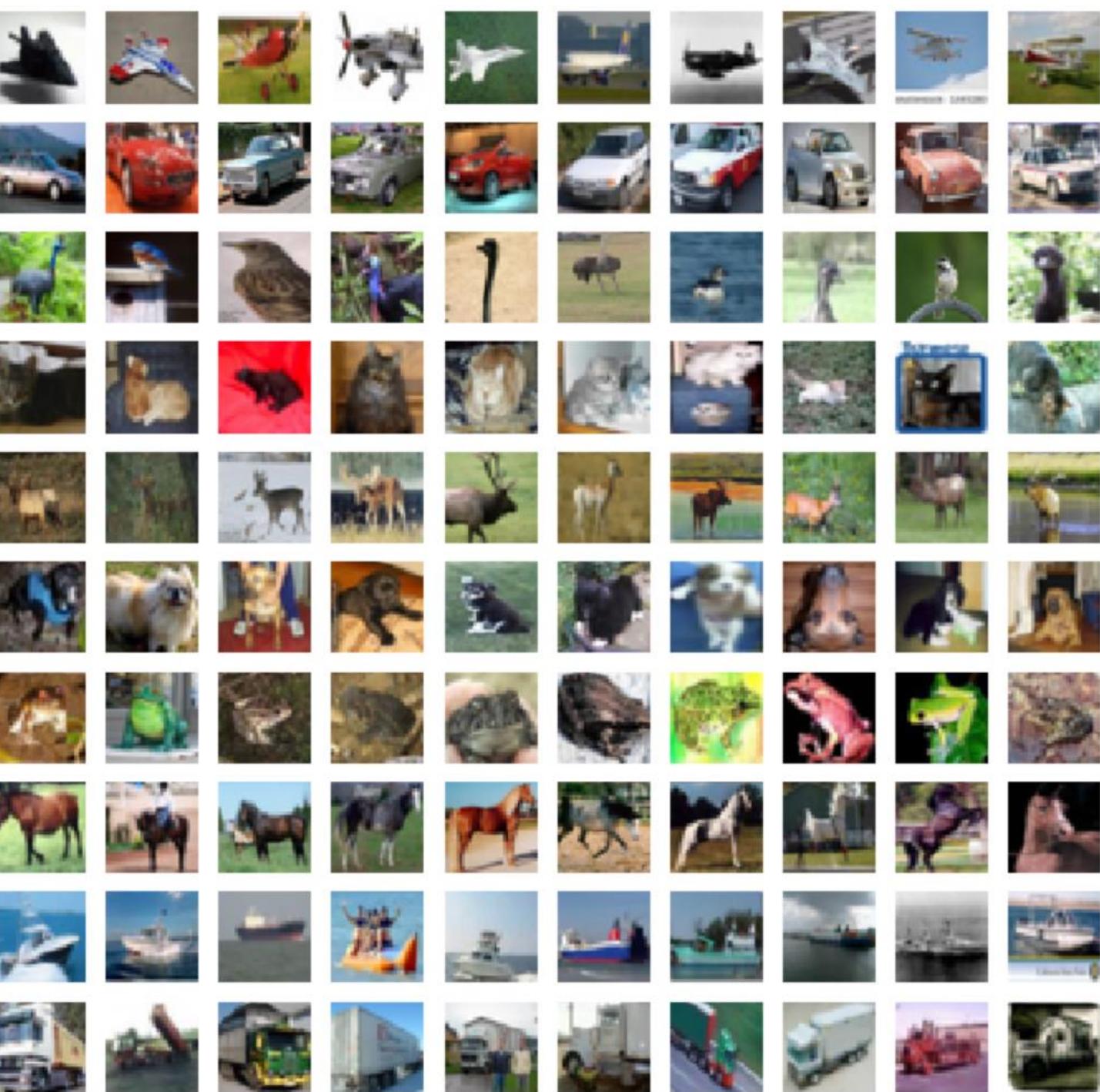
Color images

Resolution=32x32

Training set: 50000 samples

Testing set: 10000 samples

airplane



Cifar-10 Image Classification

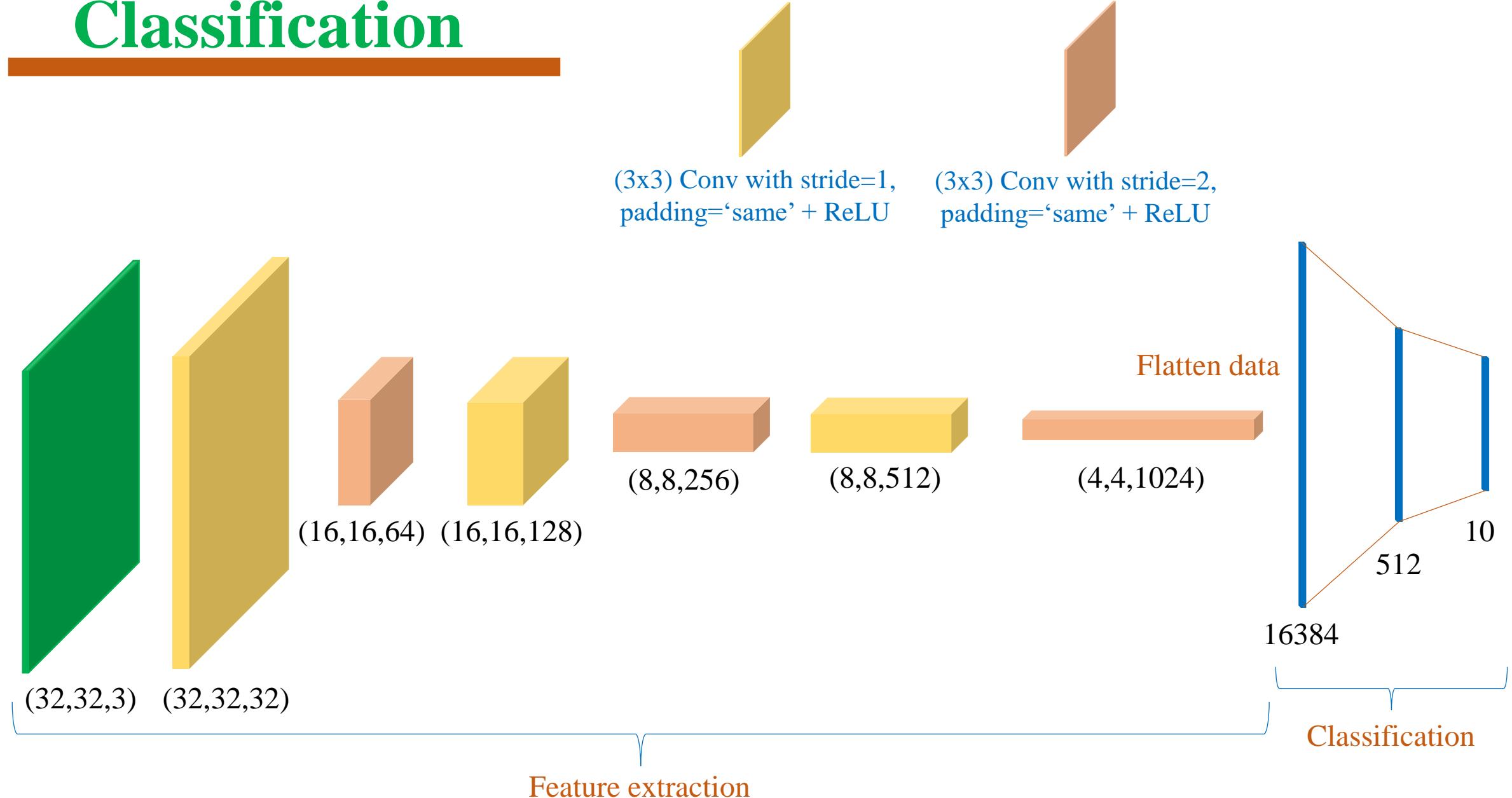


Image Classification

Cifar-10

```
1 import tensorflow as tf
2
3 # data preparation
4 cifar10 = tf.keras.datasets.cifar10
5 (x_train, y_train), (x_test, y_test) = cifar10.load_data()
6
7 # normalize
8 x_train, x_test = x_train / 255.0, x_test / 255.0
```

```
# model
model = keras.models.Sequential()
model.add(tf.keras.Input(shape=(height, width, 3)))

model.add(keras.layers.Conv2D(32, (3, 3), strides=1, padding='same', activation = 'relu'))
model.add(keras.layers.Conv2D(64, (3, 3), strides=2, padding='same', activation='relu'))

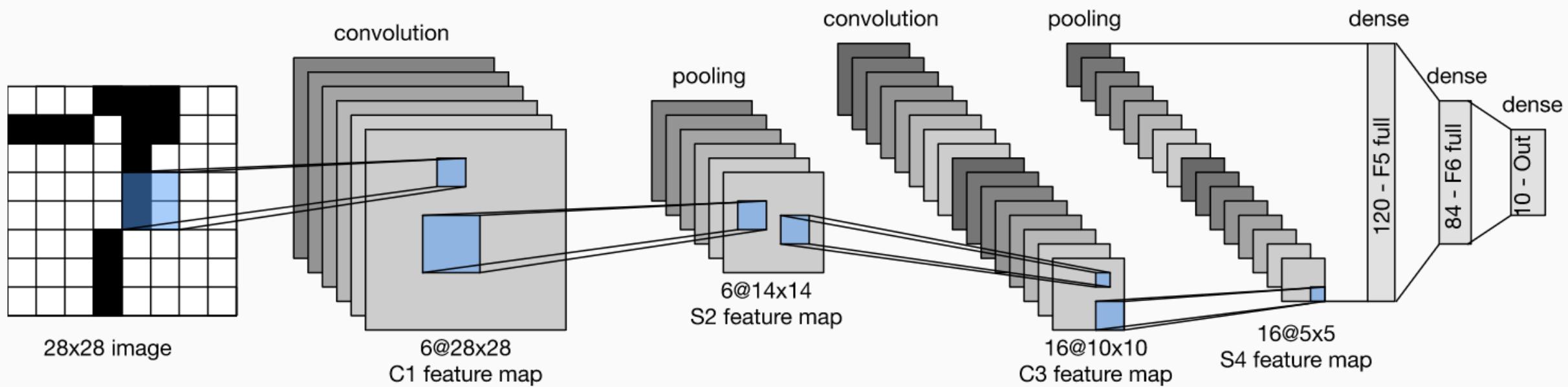
model.add(keras.layers.Conv2D(128, (3, 3), strides=1, padding='same', activation='relu'))
model.add(keras.layers.Conv2D(256, (3, 3), strides=2, padding='same', activation='relu'))

model.add(keras.layers.Conv2D(512, (3, 3), strides=1, padding='same', activation='relu'))
model.add(keras.layers.Conv2D(1024, (3, 3), strides=2, padding='same', activation='relu'))

# flatten
model.add(keras.layers.Flatten())
model.add(keras.layers.Dense(512, activation='relu'))
model.add(keras.layers.Dense(10, activation='softmax'))
model.summary()
```

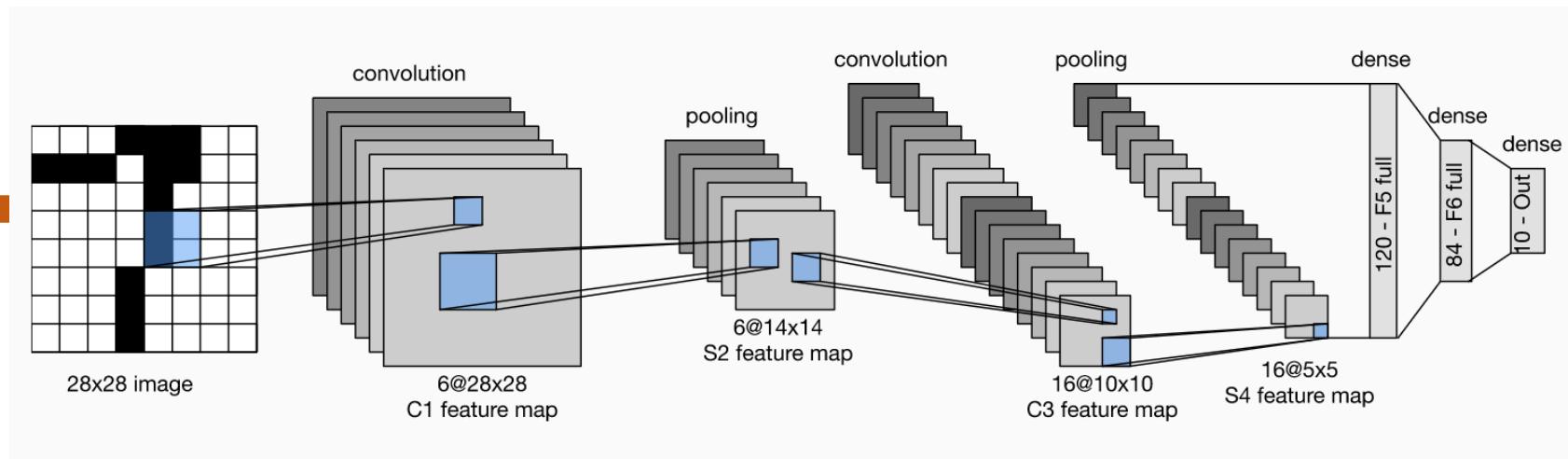
LeNet Architecture

❖ Model Construction



LeNet Architecture

❖ Model Construction



```
# model architecture
model = tf.keras.Sequential()
# input shape (28,28,1)
model.add(tf.keras.Input(shape=(28, 28, 1)))

# convolution 1 and max pooling 1
model.add(tf.keras.layers.Conv2D(6, (5,5), padding='same', activation='relu'))
model.add(tf.keras.layers.MaxPooling2D(pool_size=2))

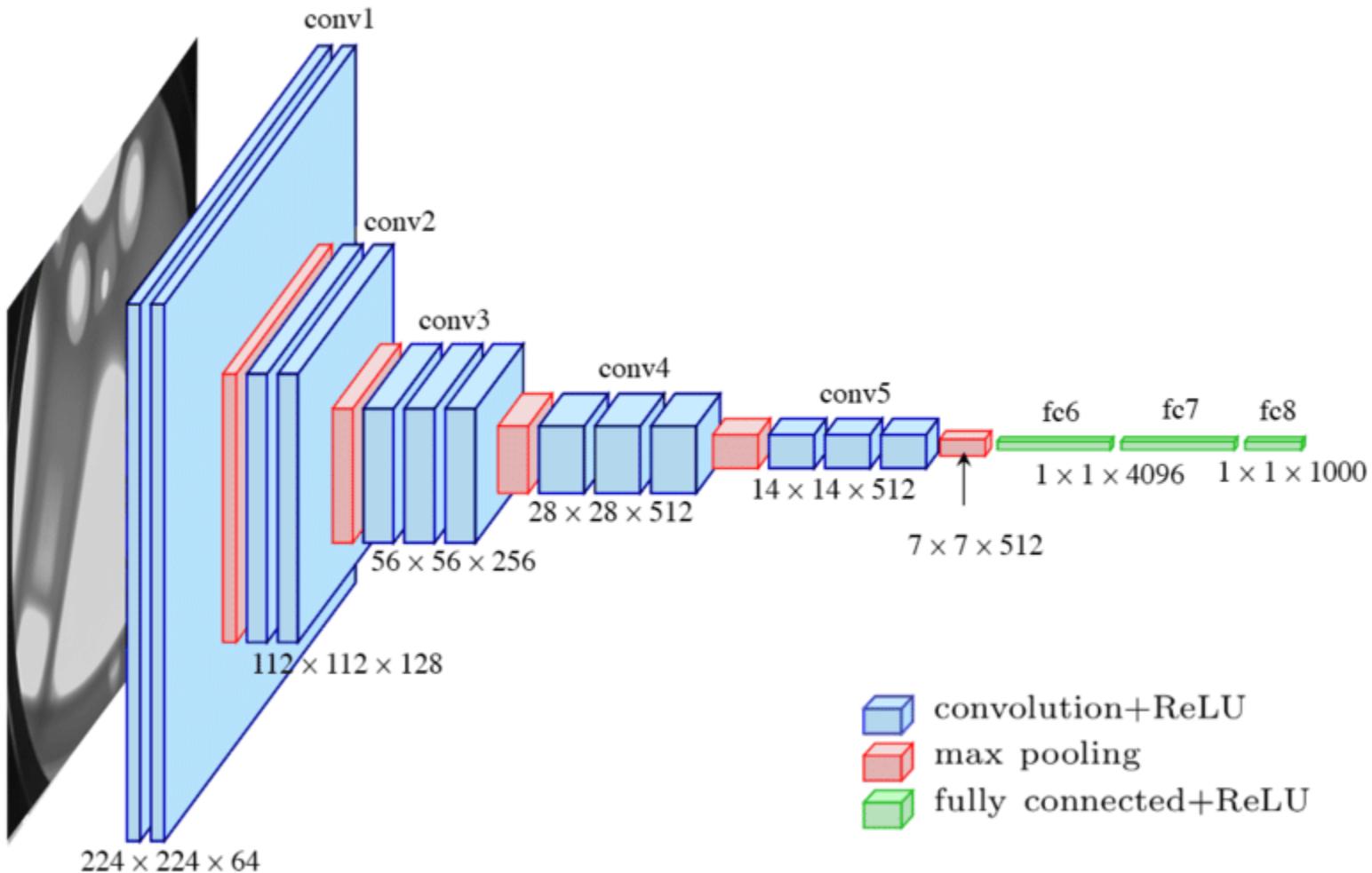
# convolution 2 and max pooling 2
model.add(tf.keras.layers.Conv2D(filters=16, kernel_size=5, activation='relu'))
model.add(tf.keras.layers.MaxPooling2D(pool_size=2))

# Flatten
model.add(tf.keras.layers.Flatten())

# fully connected
model.add(tf.keras.layers.Dense(120, activation='relu'))
model.add(tf.keras.layers.Dense(84, activation='relu'))
model.add(tf.keras.layers.Dense(10, activation='softmax'))
```

VGG16 Architecture

❖ Model Construction



VGG16 Architecture

❖ Model Construction

```
def VGG_16():
    model = Sequential()
    model.add(Convolution2D(64, 3, padding='same', activation='relu',
                           input_shape=(224,224, 1)))
    model.add(Convolution2D(64, 3, padding='same', activation='relu'))
    model.add(MaxPooling2D((2,2), strides=(2,2)))

    model.add(Convolution2D(128, 3, padding='same', activation='relu'))
    model.add(Convolution2D(128, 3, padding='same', activation='relu'))
    model.add(MaxPooling2D((2,2), strides=(2,2)))

    model.add(Convolution2D(256, 3, padding='same', activation='relu'))
    model.add(Convolution2D(256, 3, padding='same', activation='relu'))
    model.add(Convolution2D(256, 3, padding='same', activation='relu'))
    model.add(MaxPooling2D((2,2), strides=(2,2)))

    model.add(Convolution2D(512, 3, padding='same', activation='relu'))
    model.add(Convolution2D(512, 3, padding='same', activation='relu'))
    model.add(Convolution2D(512, 3, padding='same', activation='relu'))
    model.add(MaxPooling2D((2,2), strides=(2,2)))

    model.add(Convolution2D(512, 3, padding='same', activation='relu'))
    model.add(Convolution2D(512, 3, padding='same', activation='relu'))
    model.add(Convolution2D(512, 3, padding='same', activation='relu'))
    model.add(MaxPooling2D((2,2), strides=(2,2)))

    model.add(Flatten())
    model.add(Dense(4096, activation='relu'))
    model.add(Dense(4096, activation='relu'))
    model.add(Dense(1000, activation='softmax'))

return model
```

Outline

- From MLP to CNN
- Feature Map Down-sampling
- Padding
- 1x1 Convolution
- Image classification: Cifar-10 data
- Backpropagation

1x1 Convolution

- ❖ Why 1x1 Convolution
- ❖ Flexible input size



Yann LeCun

April 7, 2015 ·

...

In Convolutional Nets, there is no such thing as "fully-connected layers".
There are only convolution layers with 1x1 convolution kernels and a full connection table.

It's a too-rarely-understood fact that ConvNets don't need to have a fixed-size input. You can train them on inputs that happen to produce a single output vector (with no spatial extent), and then apply them to larger images. Instead of a single output vector, you then get a spatial map of output vectors. Each vector sees input windows at different locations on the input.

In that scenario, the "fully connected layers" really act as 1x1 convolutions.

Yann LeCun

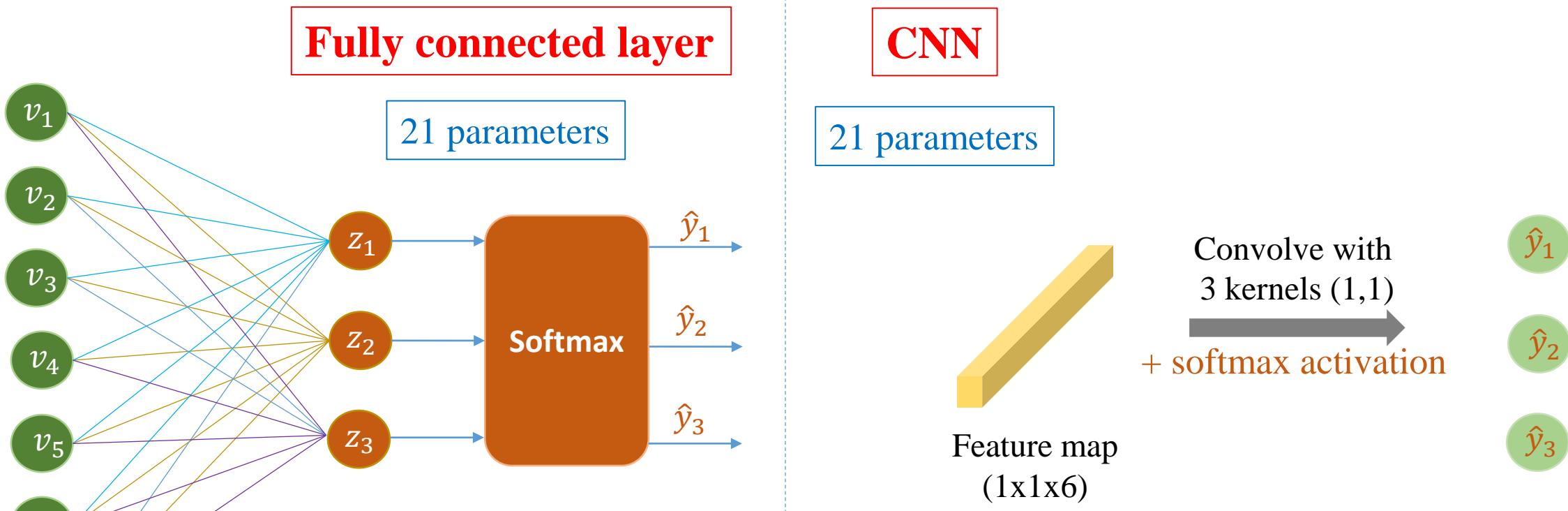


Yann LeCun in 2018

Born	July 8, 1960 (age 60)
	Soisy-sous-Montmorency, France
Alma mater	ESIEE Paris (MSc) Pierre and Marie Curie University (PhD)
Known for	Deep learning
Awards	Turing Award (2018) AAAI Fellow (2019) Legion of Honour (2020)
	Scientific career
Institutions	Bell Labs (1988-1996) New York University Facebook
Thesis	<i>Modèles connexionnistes de l'apprentissage (connectionist learning models)</i> (1987a)
Doctoral advisor	Maurice Milgram
Website	yann.lecun.com

1x1 Convolution

❖ Comparison



1x1 Convolution

```
import numpy as np
import tensorflow as tf
import tensorflow.keras as keras

# data preparation
(x_train, y_train), (x_test, y_test) = tf.keras.datasets.cifar10.load_data()

# normalize
x_train, x_test = x_train / 255.0, x_test / 255.0

# model
model = keras.models.Sequential()
model.add(tf.keras.Input(shape=(32, 32, 3)))

model.add(keras.layers.Conv2D(32, (3, 3), strides=1, activation='relu'))
model.add(keras.layers.Conv2D(64, (3, 3), strides=2, activation='relu'))

model.add(keras.layers.Conv2D(128, (3, 3), strides=1, activation='relu'))
model.add(keras.layers.Conv2D(256, (3, 3), strides=2, activation='relu'))
model.add(keras.layers.Conv2D(512, (5, 5), strides=1, activation='relu'))

# flatten
model.add(keras.layers.Flatten())
model.add(keras.layers.Dense(128, activation='relu'))
model.add(keras.layers.Dense(10, activation='softmax'))
model.summary()

# training
model.compile(optimizer='adam', loss='sparse_categorical_crossentropy',
              metrics=['accuracy'])
history = model.fit(x_train, y_train, batch_size=1024,
                     validation_data=(x_test, y_test), epochs=20, verbose=1)
```

conv2d_10 (Conv2D)	(None, 14, 14, 64)	18496
conv2d_11 (Conv2D)	(None, 12, 12, 128)	73856
conv2d_12 (Conv2D)	(None, 5, 5, 256)	295168
conv2d_13 (Conv2D)	(None, 1, 1, 512)	3277312
flatten_2 (Flatten)	(None, 512)	0
dense_4 (Dense)	(None, 128)	65664
dense_5 (Dense)	(None, 10)	1290
=====		
Total params: 3,732,682		
Trainable params: 3,732,682		
Non-trainable params: 0		

```

import numpy as np
import tensorflow as tf
import tensorflow.keras as keras

# data preparation
(x_train, y_train), (x_test, y_test) = tf.keras.datasets.cifar10.load_data()

# normalize
x_train, x_test = x_train / 255.0, x_test / 255.0

# model
model = keras.models.Sequential()
model.add(tf.keras.Input(shape=(32, 32, 3)))

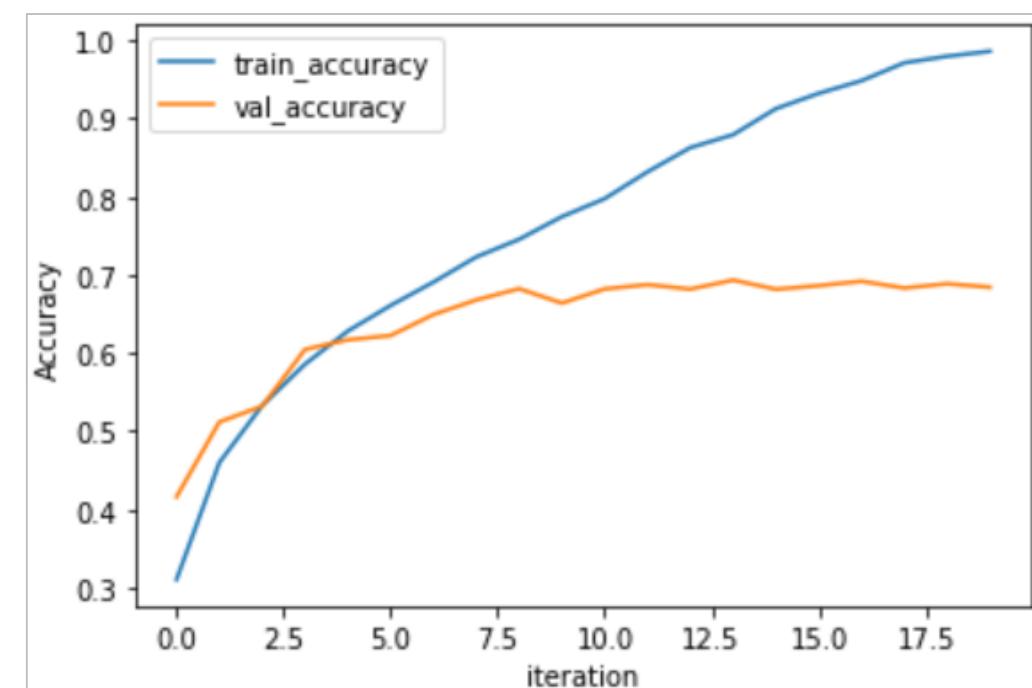
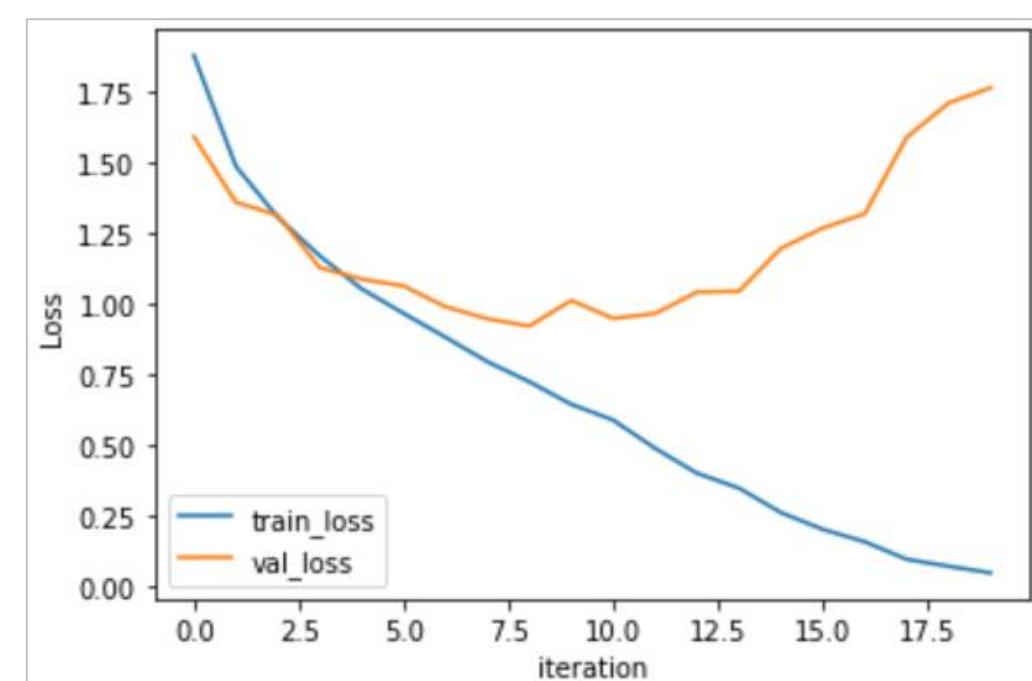
model.add(keras.layers.Conv2D(32, (3, 3), strides=1, activation='relu'))
model.add(keras.layers.Conv2D(64, (3, 3), strides=2, activation='relu'))

model.add(keras.layers.Conv2D(128, (3, 3), strides=1, activation='relu'))
model.add(keras.layers.Conv2D(256, (3, 3), strides=2, activation='relu'))
model.add(keras.layers.Conv2D(512, (5, 5), strides=1, activation='relu'))

# flatten
model.add(keras.layers.Flatten())
model.add(keras.layers.Dense(128, activation='relu'))
model.add(keras.layers.Dense(10, activation='softmax'))
model.summary()

# training
model.compile(optimizer='adam', loss='sparse_categorical_crossentropy',
              metrics=['accuracy'])
history = model.fit(x_train, y_train, batch_size=1024,
                     validation_data=(x_test, y_test), epochs=20, verbose=1)

```



```

import numpy as np
import tensorflow as tf
import tensorflow.keras as keras

# data preparation
(x_train, y_train), (x_test, y_test) = tf.keras.datasets.cifar10.load_data()

# normalize
x_train, x_test = x_train / 255.0, x_test / 255.0

# model
model = keras.models.Sequential()
model.add(tf.keras.Input(shape=(32, 32, 3)))

model.add(keras.layers.Conv2D(32, (3, 3), strides=1, activation='relu'))
model.add(keras.layers.Conv2D(64, (3, 3), strides=2, activation='relu'))

model.add(keras.layers.Conv2D(128, (3, 3), strides=1, activation='relu'))
model.add(keras.layers.Conv2D(256, (3, 3), strides=2, activation='relu'))
model.add(keras.layers.Conv2D(512, (5, 5), strides=1, activation='relu'))

model.add(keras.layers.Conv2D(128, (1, 1), strides=1, activation='relu'))
model.add(keras.layers.Conv2D(10, (1, 1), activation='softmax'))
model.add(keras.layers.Reshape((10,)))
model.summary()

# training
model.compile(optimizer='adam', loss='sparse_categorical_crossentropy',
              metrics=['accuracy'])
history = model.fit(x_train, y_train, batch_size=1024,
                     validation_data=(x_test, y_test), epochs=20, verbose=1)

```

1x1 Convolution

conv2d_22 (Conv2D)	(None, 14, 14, 64)	18496
conv2d_23 (Conv2D)	(None, 12, 12, 128)	73856
conv2d_24 (Conv2D)	(None, 5, 5, 256)	295168
conv2d_25 (Conv2D)	(None, 1, 1, 512)	3277312
conv2d_26 (Conv2D)	(None, 1, 1, 128)	65664
conv2d_27 (Conv2D)	(None, 1, 1, 10)	1290
reshape (Reshape)	(None, 10)	0
=====		
Total params: 3,732,682		
Trainable params: 3,732,682		
Non-trainable params: 0		

```

import numpy as np
import tensorflow as tf
import tensorflow.keras as keras

# data preparation
(x_train, y_train), (x_test, y_test) = tf.keras.datasets.cifar10.load_data()

# normalize
x_train, x_test = x_train / 255.0, x_test / 255.0

# model
model = keras.models.Sequential()
model.add(tf.keras.Input(shape=(32, 32, 3)))

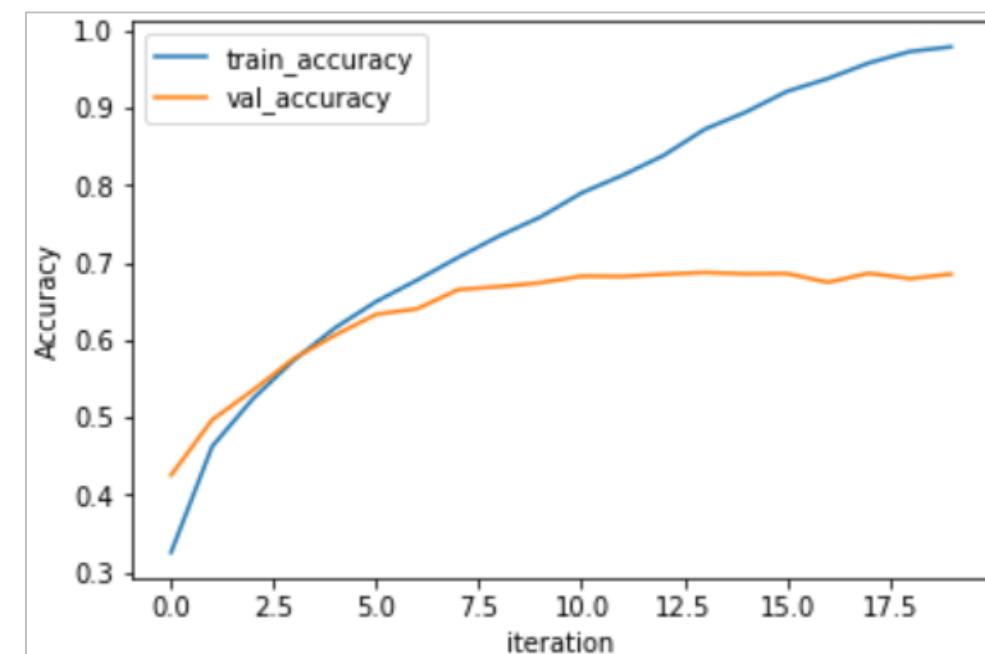
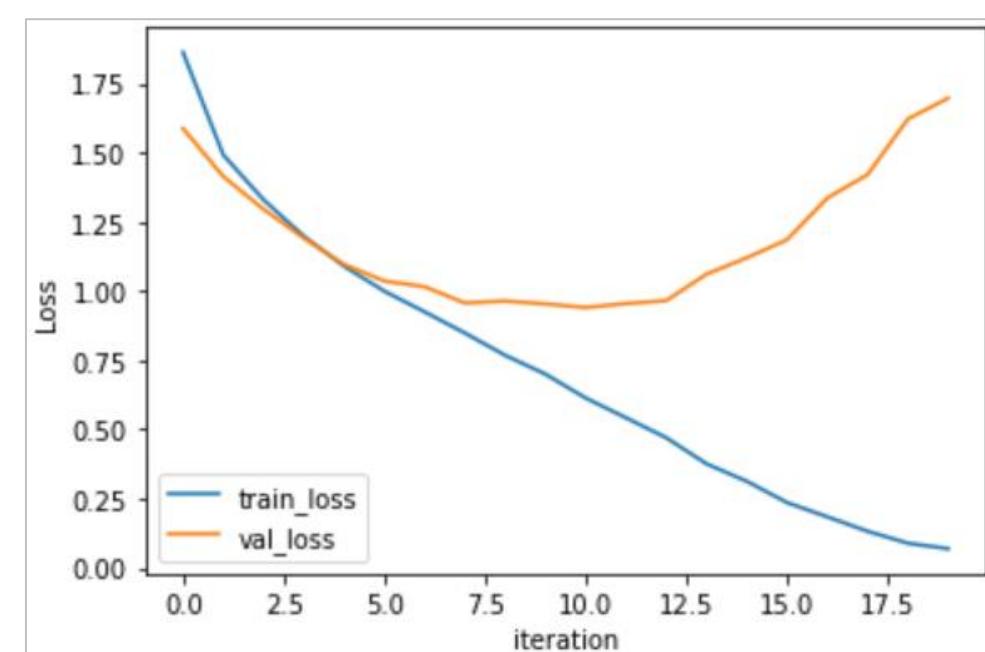
model.add(keras.layers.Conv2D(32, (3, 3), strides=1, activation='relu'))
model.add(keras.layers.Conv2D(64, (3, 3), strides=2, activation='relu'))

model.add(keras.layers.Conv2D(128, (3, 3), strides=1, activation='relu'))
model.add(keras.layers.Conv2D(256, (3, 3), strides=2, activation='relu'))
model.add(keras.layers.Conv2D(512, (5, 5), strides=1, activation='relu'))

model.add(keras.layers.Conv2D(128, (1, 1), strides=1, activation='relu'))
model.add(keras.layers.Conv2D(10, (1, 1), activation='softmax'))
model.add(keras.layers.Reshape((10,)))
model.summary()

# training
model.compile(optimizer='adam', loss='sparse_categorical_crossentropy',
              metrics=['accuracy'])
history = model.fit(x_train, y_train, batch_size=1024,
                     validation_data=(x_test, y_test), epochs=20, verbose=1)

```



1x1 Convolution

- ❖ Convert from FCL to 1x1 Conv
- ❖ Another example

1x1 Convolution

```
import numpy as np
import tensorflow as tf
import tensorflow.keras as keras
import tensorflow.keras.layers as layers

# data preparation
(x_train, y_train), (x_test, y_test) = tf.keras.datasets.cifar10.load_data()

# normalize
x_train, x_test = x_train / 255.0, x_test / 255.0

# model
model = keras.models.Sequential()
model.add(tf.keras.Input(shape=(32, 32, 3)))

model.add(layers.Conv2D(32, 3, strides=1, padding='same', activation='relu'))
model.add(layers.Conv2D(64, 3, strides=2, padding='same', activation='relu'))

model.add(layers.Conv2D(128, 3, strides=1, padding='same', activation='relu'))
model.add(layers.Conv2D(256, 3, strides=2, padding='same', activation='relu'))

# flatten
model.add(keras.layers.Flatten())
model.add(keras.layers.Dense(512, activation='relu'))
model.add(keras.layers.Dense(10, activation='softmax'))
model.summary()

# training
model.compile(optimizer='adam', loss='sparse_categorical_crossentropy',
              metrics=['accuracy'])
history = model.fit(x_train, y_train, batch_size=1024,
                     validation_data=(x_test, y_test), epochs=20, verbose=1)
```

Layer (type)	Output Shape	Param #
=====		
conv2d (Conv2D)	(None, 32, 32, 32)	896
conv2d_1 (Conv2D)	(None, 16, 16, 64)	18496
conv2d_2 (Conv2D)	(None, 16, 16, 128)	73856
conv2d_3 (Conv2D)	(None, 8, 8, 256)	295168
flatten (Flatten)	(None, 16384)	0
dense (Dense)	(None, 512)	8389120
dense_1 (Dense)	(None, 10)	5130
=====		
Total params:	8,782,666	
Trainable params:	8,782,666	
Non-trainable params:	0	

```

import numpy as np
import tensorflow as tf
import tensorflow.keras as keras
import tensorflow.keras.layers as layers

# data preparation
(x_train, y_train), (x_test, y_test) = tf.keras.datasets.cifar10.load_data()

# normalize
x_train, x_test = x_train / 255.0, x_test / 255.0

# model
model = keras.models.Sequential()
model.add(tf.keras.Input(shape=(32, 32, 3)))

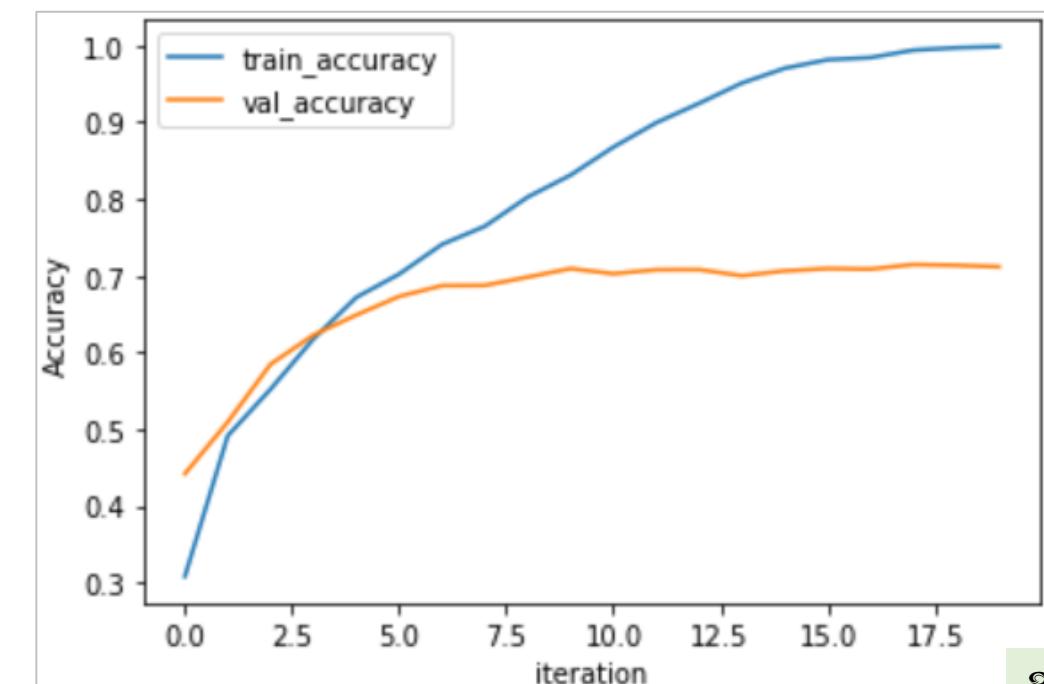
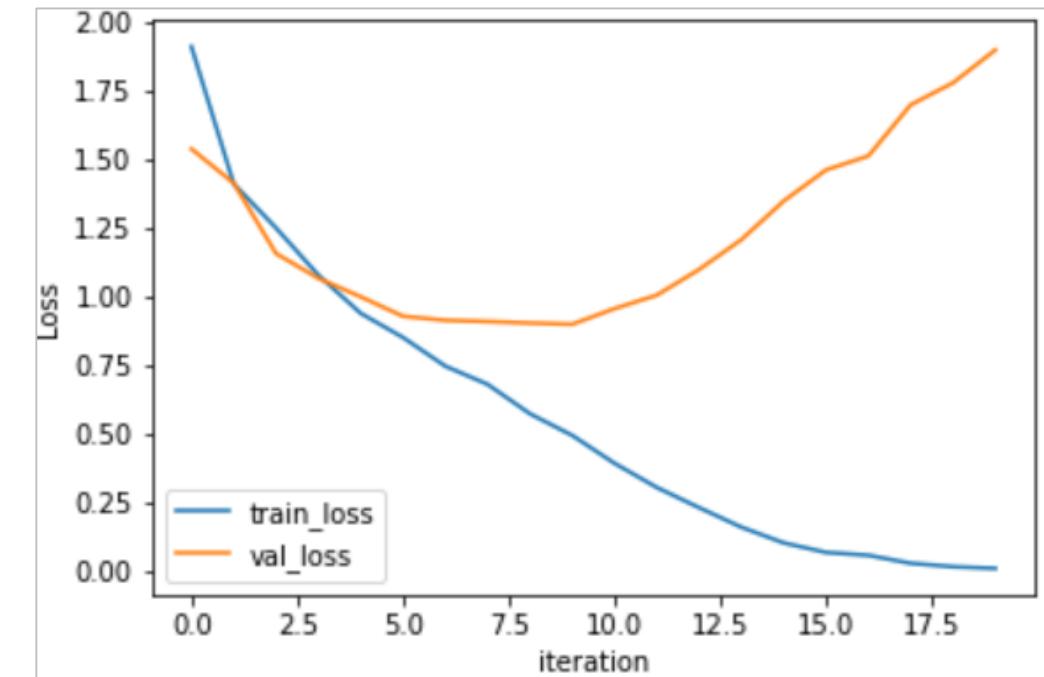
model.add(layers.Conv2D(32, 3, strides=1, padding='same', activation='relu'))
model.add(layers.Conv2D(64, 3, strides=2, padding='same', activation='relu'))

model.add(layers.Conv2D(128, 3, strides=1, padding='same', activation='relu'))
model.add(layers.Conv2D(256, 3, strides=2, padding='same', activation='relu'))

# flatten
model.add(keras.layers.Flatten())
model.add(keras.layers.Dense(512, activation='relu'))
model.add(keras.layers.Dense(10, activation='softmax'))
model.summary()

# training
model.compile(optimizer='adam', loss='sparse_categorical_crossentropy',
              metrics=['accuracy'])
history = model.fit(x_train, y_train, batch_size=1024,
                     validation_data=(x_test, y_test), epochs=20, verbose=1)

```



```

import numpy as np
import tensorflow as tf
import tensorflow.keras as keras
import tensorflow.keras.layers as layers

# data preparation
(x_train, y_train), (x_test, y_test) = tf.keras.datasets.cifar10.load_data()

# normalize
x_train, x_test = x_train / 255.0, x_test / 255.0

# model
model = keras.models.Sequential()
model.add(tf.keras.Input(shape=(32, 32, 3)))

model.add(layers.Conv2D(32, 3, strides=1, padding='same', activation='relu'))
model.add(layers.Conv2D(64, 3, strides=2, padding='same', activation='relu'))

model.add(layers.Conv2D(128, 3, strides=1, padding='same', activation='relu'))
model.add(layers.Conv2D(256, 3, strides=2, padding='same', activation='relu'))

# flatten
model.add(layers.Conv2D(512, (8, 8), activation='relu'))
model.add(layers.Conv2D(10, (1,1), activation='softmax'))
model.add(layers.Reshape((10,)))
model.summary()

# training
model.compile(optimizer='adam', loss='sparse_categorical_crossentropy',
              metrics=['accuracy'])
history = model.fit(x_train, y_train, batch_size=1024,
                     validation_data=(x_test, y_test), epochs=20, verbose=1)

```

1x1 Convolution

Layer (type)	Output Shape	Param #
<hr/>		
conv2d_34 (Conv2D)	(None, 32, 32, 32)	896
conv2d_35 (Conv2D)	(None, 16, 16, 64)	18496
conv2d_36 (Conv2D)	(None, 16, 16, 128)	73856
conv2d_37 (Conv2D)	(None, 8, 8, 256)	295168
conv2d_38 (Conv2D)	(None, 1, 1, 512)	8389120
conv2d_39 (Conv2D)	(None, 1, 1, 10)	5130
reshape_3 (Reshape)	(None, 10)	0
<hr/>		
Total params: 8,782,666		
Trainable params: 8,782,666		
Non-trainable params: 0		

```

import numpy as np
import tensorflow as tf
import tensorflow.keras as keras
import tensorflow.keras.layers as layers

# data preparation
(x_train, y_train), (x_test, y_test) = tf.keras.datasets.cifar10.load_data()

# normalize
x_train, x_test = x_train / 255.0, x_test / 255.0

# model
model = keras.models.Sequential()
model.add(tf.keras.Input(shape=(32, 32, 3)))

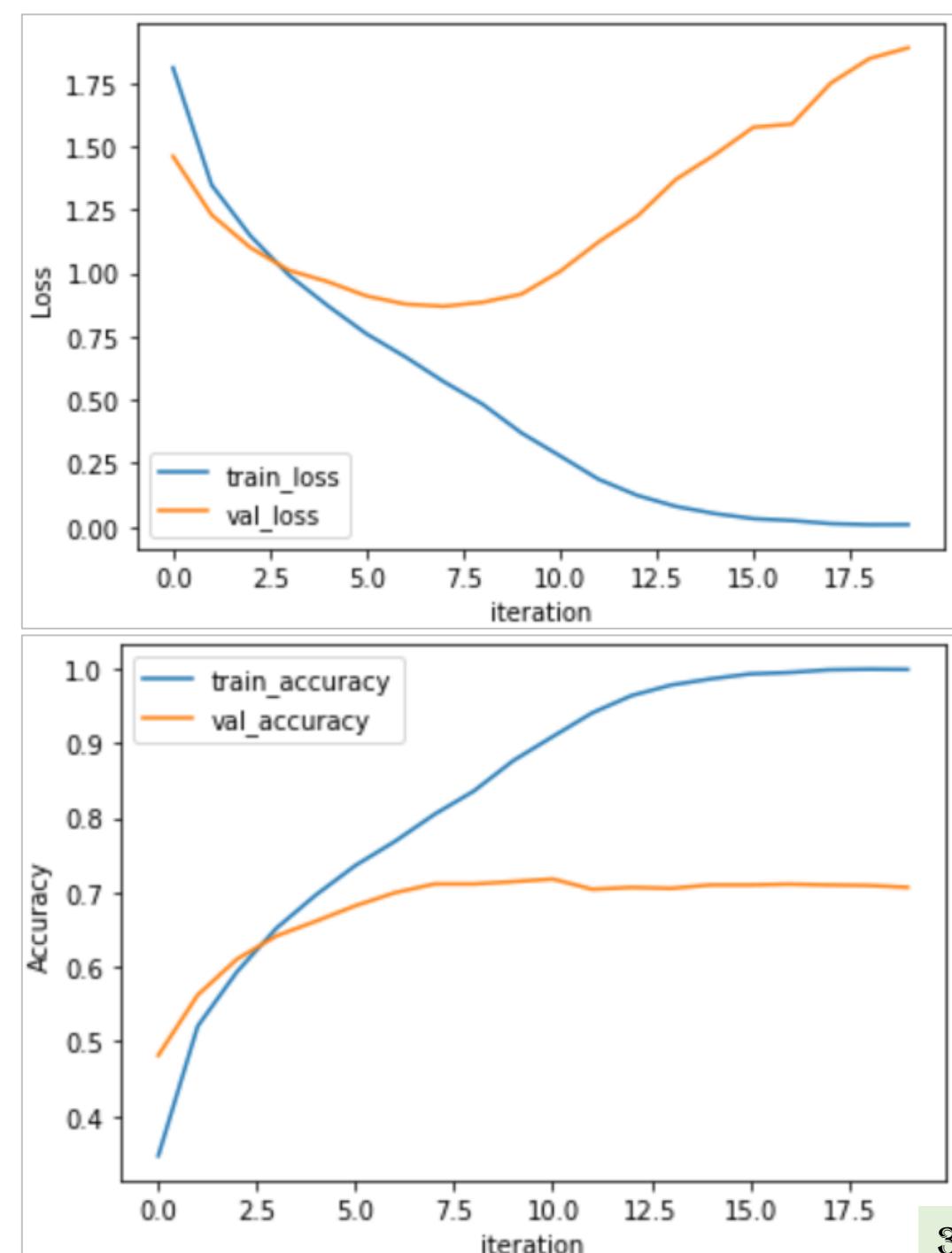
model.add(layers.Conv2D(32, 3, strides=1, padding='same', activation='relu'))
model.add(layers.Conv2D(64, 3, strides=2, padding='same', activation='relu'))

model.add(layers.Conv2D(128, 3, strides=1, padding='same', activation='relu'))
model.add(layers.Conv2D(256, 3, strides=2, padding='same', activation='relu'))

# flatten
model.add(layers.Conv2D(512, (8, 8), activation='relu'))
model.add(layers.Conv2D(10, (1,1), activation='softmax'))
model.add(layers.Reshape((10,)))
model.summary()

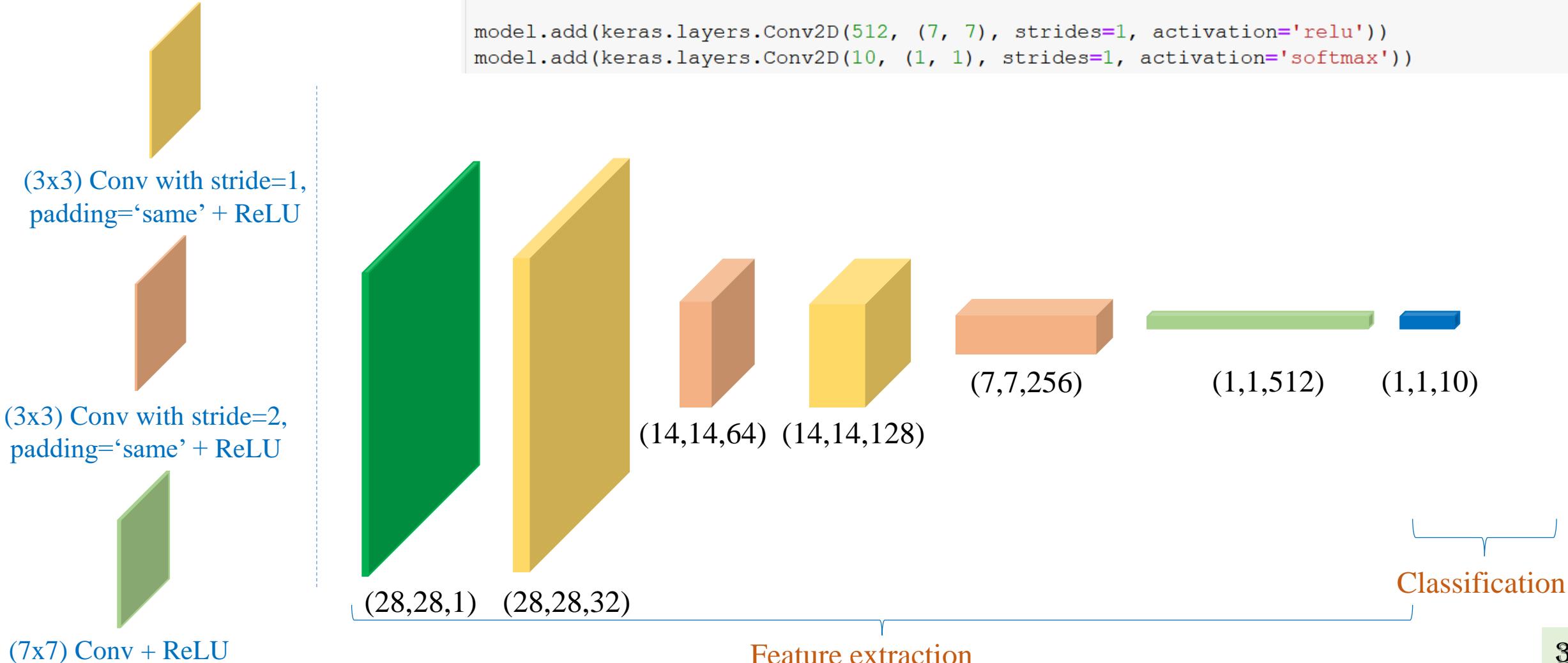
# training
model.compile(optimizer='adam', loss='sparse_categorical_crossentropy',
              metrics=['accuracy'])
history = model.fit(x_train, y_train, batch_size=1024,
                     validation_data=(x_test, y_test), epochs=20, verbose=1)

```



1x1 Convolution

Replace FC by 1x1 Conv



1x1 Convolution

Dynamic input sizes

```
# model
model = keras.models.Sequential()
model.add(tf.keras.Input(shape=(None, None, 1)))

model.add(keras.layers.Conv2D(32, (3, 3), strides=1, padding='same', activation='relu'))
model.add(keras.layers.Conv2D(64, (3, 3), strides=2, padding='same', activation='relu'))

model.add(keras.layers.Conv2D(128, (3, 3), strides=1, padding='same', activation='relu'))
model.add(keras.layers.Conv2D(256, (3, 3), strides=2, padding='same', activation='relu'))

model.add(keras.layers.Conv2D(512, (7, 7), strides=1, activation='relu'))
model.add(keras.layers.Conv2D(10, (1, 1), strides=1, activation='softmax'))
```

```
1 shape = (32,28,28,1)
2 data = tf.random.normal(shape)
3 print(data.shape)
4
5 output = model.predict(data)
6 print(output.shape)
```

```
(32, 28, 28, 1)
(32, 1, 1, 10)
```

```
1 shape = (32,64,64,1)
2 data = tf.random.normal(shape)
3 print(data.shape)
4
5 output = model.predict(data)
6 print(output.shape)
```

```
(32, 64, 64, 1)
(32, 10, 10, 10)
```

Shape=(batch size, height, width, channel)

1x1 Convolution

Dynamic
input sizes

```
# model
model = keras.models.Sequential()
model.add(tf.keras.Input(shape=(None, None, 1)))

model.add(keras.layers.Conv2D(32, (3, 3), activation='relu'))
model.add(keras.layers.Conv2D(64, (3, 3), activation='relu'))
model.add(keras.layers.MaxPooling2D(2))

model.add(keras.layers.Conv2D(128, (3, 3), activation='relu'))
model.add(keras.layers.Conv2D(256, (3, 3), activation='relu'))
model.add(keras.layers.MaxPooling2D(2))

model.add(keras.layers.Conv2D(512, (3, 3), activation='relu'))

# flatten
model.add(keras.layers.Flatten())
model.add(keras.layers.Dense(10, activation='softmax'))
```

```
C:\installed\lib\site-packages\tensorflow_core\python\keras\layers\core.py in build(self, input_shape)
1013     input_shape = tensor_shape.TensorShape(input_shape)
1014     if tensor_shape.dimension_value(input_shape[-1]) is None:
-> 1015         raise ValueError('The last dimension of the inputs to `Dense` '
1016                         'should be defined. Found `None`.')
1017     last_dim = tensor_shape.dimension_value(input_shape[-1])
```

ValueError: The last dimension of the inputs to `Dense` should be defined. Found `None`.

Outline

- From MLP to CNN
- Feature Map Down-sampling
- Padding
- 1x1 Convolution
- Image classification: Cifar-10 data
- Backpropagation

Sigmoid Function

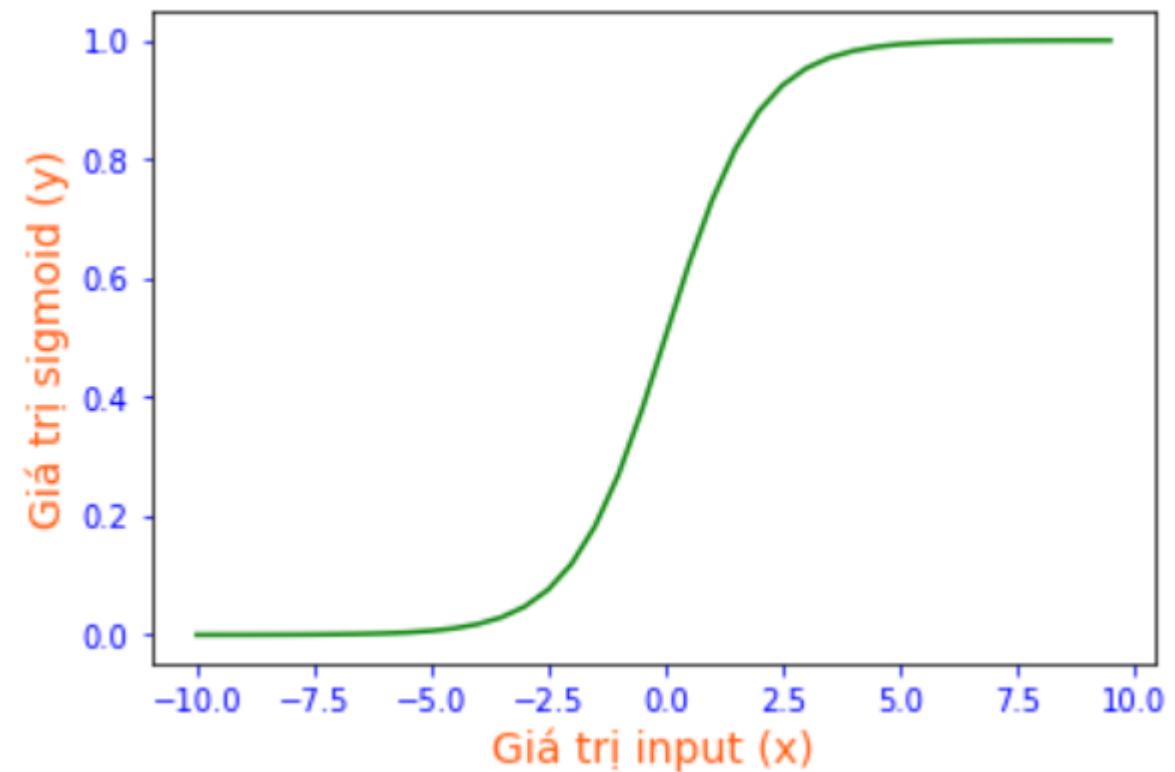
❖ Sigmoid function

$$\text{sigmoid}(x) = \frac{1}{1 + e^{-x}}$$

data = [1 | 5 | -4 | 3 | -2]

data_a = sigmoid(**data**)

data_a = [0.731 | 0.993 | 0.017 | 0.95 | 0.119]



Sigmoid Function

Sigmoid function

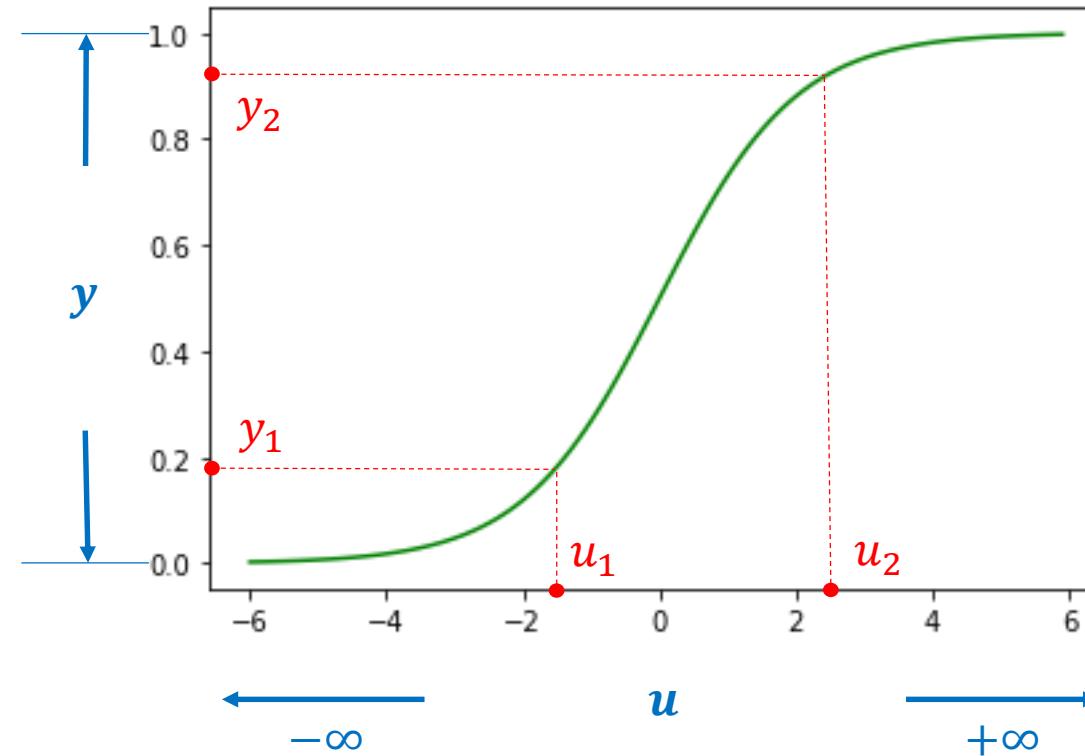
$$y = \sigma(u) = \frac{1}{1 + e^{-u}}$$

$$u \in (-\infty, +\infty)$$

$$y \in (0, 1)$$

Property

$$\forall u_1, u_2 \in [a, b] \text{ và } u_1 \leq u_2 \rightarrow \sigma(u_1) \leq \sigma(u_2)$$



Sigmoid Function

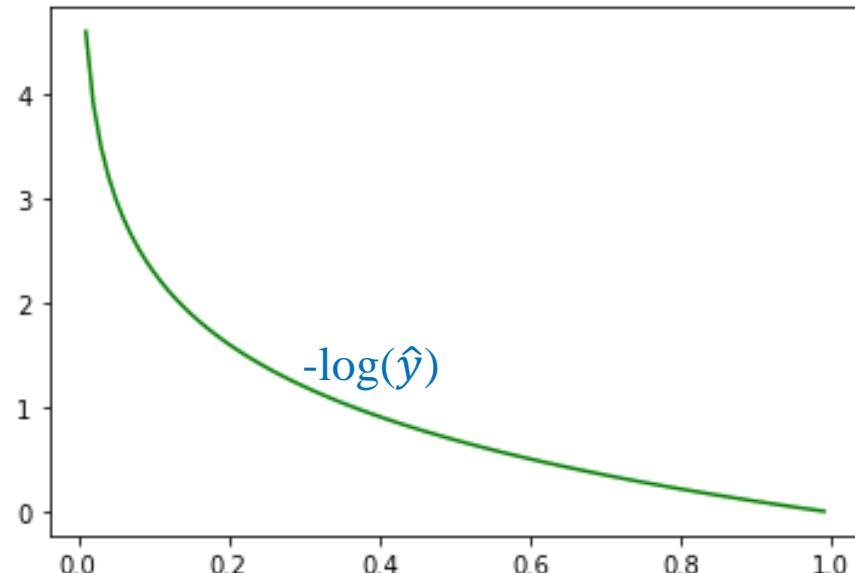
❖ Construct loss

$$z = \theta^T x$$

$$\hat{y} = \sigma(z) = \frac{1}{1 + e^{-z}}$$

$$L = \frac{1}{N} [-y^T \log(\hat{y}) - (1 - y^T) \log(1 - \hat{y})]$$

Model and Loss



Derivative

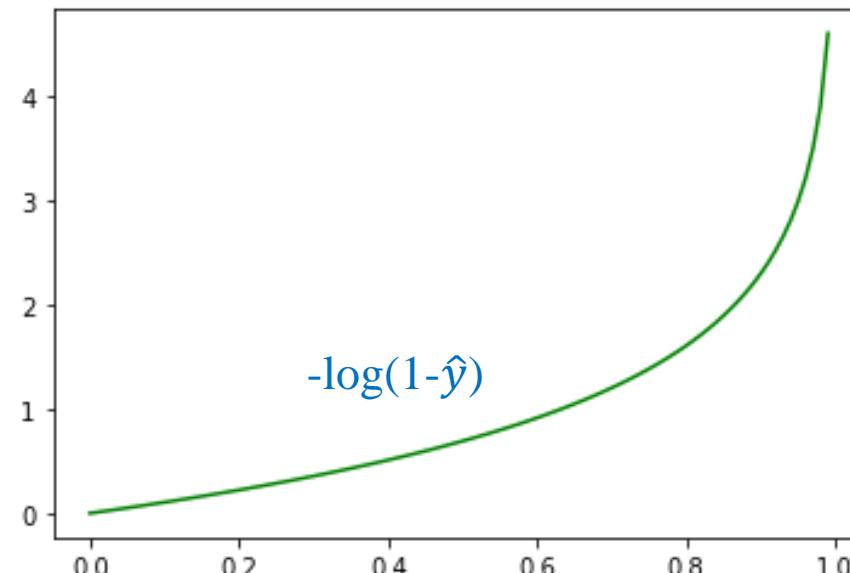
$$\frac{\partial L}{\partial \theta} = \frac{\partial L}{\partial \hat{y}} \frac{\partial \hat{y}}{\partial z} \frac{\partial z}{\partial \theta}$$

$$\frac{\partial L}{\partial \hat{y}} = \frac{1}{N} \left(-\frac{y}{\hat{y}} + \frac{1-y}{1-\hat{y}} \right) = \frac{1}{N} \frac{\hat{y} - y}{\hat{y}(1-\hat{y})}$$

$$\frac{\partial \hat{y}}{\partial z} = \hat{y}(1-\hat{y})$$

$$\frac{\partial z}{\partial \theta} = x$$

$$\frac{\partial L}{\partial \theta} = \frac{1}{N} x^T (\hat{y} - y)$$



Sigmoid Function

```
1 # sigmoid
2
3 import tensorflow as tf
4 import tensorflow.keras as keras
5 import numpy as np
6
7 sigmoid_activation = keras.activations.sigmoid
8
9 data_X = np.array([[-3, 9, 2],
10                  [5, -8, 4],
11                  [-7, 3, 5]])
12 data_X = data_X.reshape(1, 3, 3, 1)
13 data_X = tf.Variable(data_X, dtype=tf.float32)
14
15 data_y = np.ones((1, 3, 3, 1))
16 data_y = tf.Variable(data_y, dtype=tf.float32)
17
18 print('\n\n data_X: \n', data_X[0,:,:,:0])
19 print('\n\n data_y: \n', data_y[0,:,:,:0])
20
21 with tf.GradientTape(persistent=True) as tape:
22     data_after = sigmoid_activation(data_X)
23     print('\n\n data_after: \n', data_after[0,:,:,:0])
24
25     loss = tf.reduce_mean((data_after-data_y)**2)
26     print('\n\n loss: \n', loss)
27
28 dloss = tape.gradient(loss, data_after)
29 print('\n\n dloss: \n', dloss[0,:,:,:0])
30
31 ddata_X = tape.gradient(loss, data_X)
32 print('\n\n ddata_X: \n', ddata_X[0,:,:,:0])
```

```
data_X:
tf.Tensor(
[[[-3.  9.  2.]
 [ 5. -8.  4.]
 [-7.  3.  5.]]], shape=(3, 3), dtype=float32)

data_y:
tf.Tensor(
[[[1.  1.  1.]
 [1.  1.  1.]
 [1.  1.  1.]]], shape=(3, 3), dtype=float32)

data_after_mp:
tf.Tensor(
[[[4.7425866e-02 9.9987662e-01 8.8079709e-01]
 [9.9330711e-01 3.3542514e-04 9.8201376e-01]
 [9.1102719e-04 9.5257413e-01 9.9330717e-01]], shape=(3, 3), dtype=float32)

loss:
tf.Tensor(0.32464194, shape=(), dtype=float32)

dloss:
tf.Tensor(
[[[-2.1168314e-01 -2.7418137e-05 -2.6489535e-02]
 [-1.4873081e-03 -2.2214767e-01 -3.9969417e-03]
 [-2.2201978e-01 -1.0539082e-02 -1.4872948e-03]], shape=(3, 3), dtype=float32)

ddata_X:
tf.Tensor(
[[[-9.5631359e-03 -3.3824765e-09 -2.7812310e-03]
 [-9.8877599e-06 -7.4488918e-05 -7.0596914e-05]
 [-2.0208179e-04 -4.7612048e-04 -9.8875844e-06]], shape=(3, 3), dtype=float32)
```

Tanh Function

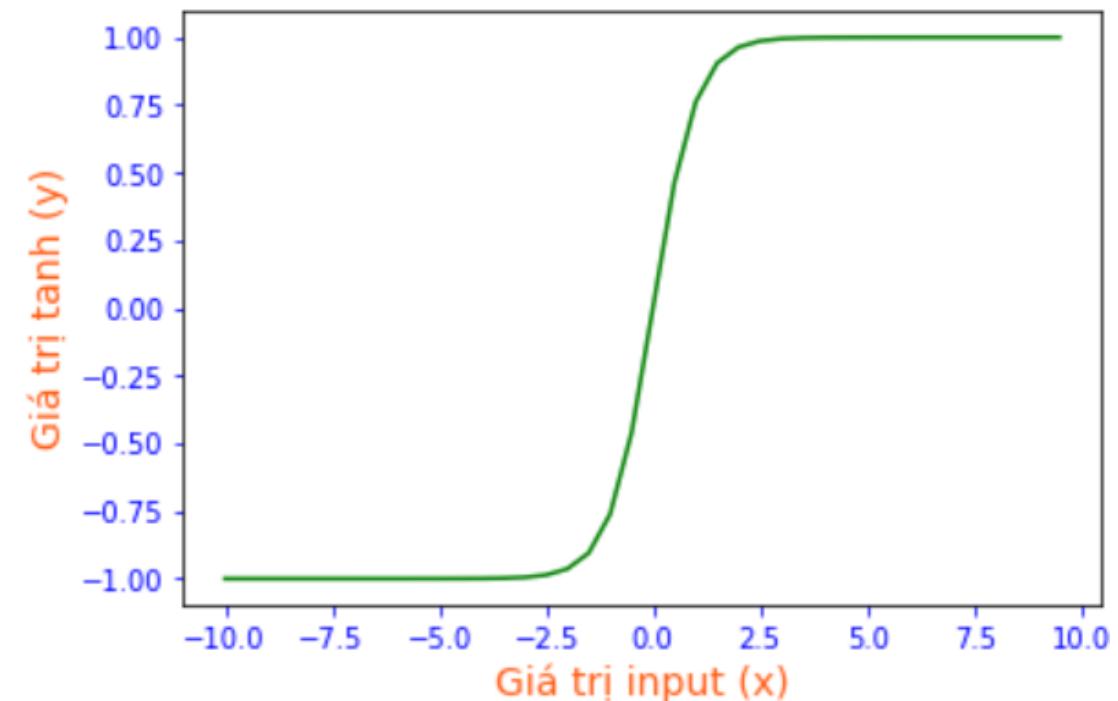
❖ Tanh function

$$\tanh(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}} = \frac{2}{1 + e^{-2x}} - 1$$

data = [1, 5, -4, 3, -2]

data_a = tanh(data)

data_a = [0.761, 0.999, -0.999, 0.995, -0.964]



Tanh Function

❖ Construct loss

$$z = \theta^T x$$

$$\hat{y} = \tanh(z) = \frac{e^z - e^{-z}}{e^z + e^{-z}}$$

$$L = \frac{1}{N} [-y^T \log(\hat{y}) - (1 - y^T) \log(1 - \hat{y})]$$

Model and Loss

$$\frac{\partial L}{\partial \theta} = \frac{\partial L}{\partial \hat{y}} \frac{\partial \hat{y}}{\partial z} \frac{\partial z}{\partial \theta}$$

$$\frac{\partial L}{\partial \hat{y}} = \frac{1}{N} \left(-\frac{y}{\hat{y}} + \frac{1-y}{1-\hat{y}} \right) = \frac{1}{N} \frac{\hat{y} - y}{\hat{y}(1-\hat{y})}$$

$$\frac{\partial \hat{y}}{\partial z} = 1 - \hat{y}^2$$

$$\frac{\partial z}{\partial \theta} = x$$

Derivative

$$\frac{\partial L}{\partial \theta} = \frac{1}{N} x^T \frac{(\hat{y} - y)(1 + \hat{y})}{\hat{y}}$$

Tanh Function

```
1 # tanh
2
3 import tensorflow as tf
4 import tensorflow.keras as keras
5 import numpy as np
6
7 tanh_activation = keras.activations.tanh
8
9 data_X = np.array([[-1, 0.6, 0.3],
10                   [5, -8, 4],
11                   [-7, 3, 5]])
12 data_X = data_X.reshape(1, 3, 3, 1)
13 data_X = tf.Variable(data_X, dtype=tf.float32)
14
15 data_y = np.ones((1, 3, 3, 1))
16 data_y = tf.Variable(data_y, dtype=tf.float32)
17
18 print('\n\n data_X: \n', data_X[0,:,:,:])
19 print('\n\n data_y: \n', data_y[0,:,:,:])
20
21 with tf.GradientTape(persistent=True) as tape:
22     data_after = tanh_activation(data_X)
23     print('\n\n data_after: \n', data_after[0,:,:,:])
24
25     loss = tf.reduce_mean((data_after-data_y)**2)
26     print('\n\n loss: \n', loss)
27
28 dloss = tape.gradient(loss, data_after)
29 print('\n\n dloss: \n', dloss[0,:,:,:])
30
31 ddata_X = tape.gradient(loss, data_X)
32 print('\n\n ddata_X: \n', ddata_X[0,:,:,:])
```

```
data_X:
tf.Tensor(
[-1.  0.6  0.3]
[ 5. -8.   4. ]
[-7.  3.   5. ]], shape=(3, 3), dtype=float32)

data_y:
tf.Tensor(
[1. 1. 1.]
[1. 1. 1.]
[1. 1. 1.]], shape=(3, 3), dtype=float32)

data_after:
tf.Tensor(
[-0.7615942  0.53704965  0.2913126 ]
[ 0.99990916 -0.99999976  0.9993292 ]
[-0.99999833  0.9950547   0.99990916]], shape=(3, 3), dtype=float32)

loss:
tf.Tensor(1.3133101, shape=(), dtype=float32)

dloss:
tf.Tensor(
[-3.91465366e-01 -1.02877855e-01 -1.57486096e-01]
[-2.01861058e-05 -4.44444388e-01 -1.49064595e-04]
[-4.44444090e-01 -1.09895074e-03 -2.01861058e-05]], shape=(3, 3), dtype=float32)

ddata_X:
tf.Tensor(
[-1.6440541e-01 -7.3205583e-02 -1.4412135e-01]
[-3.6673100e-09 -2.1192760e-07 -1.9991120e-07]
[-1.4834922e-06 -1.0842378e-05 -3.6673100e-09]], shape=(3, 3), dtype=float32)
```

ReLU Function

$$\text{ReLU}(x) = \begin{cases} 0 & \text{if } x < 0 \\ x & \text{if } x \geq 0 \end{cases}$$

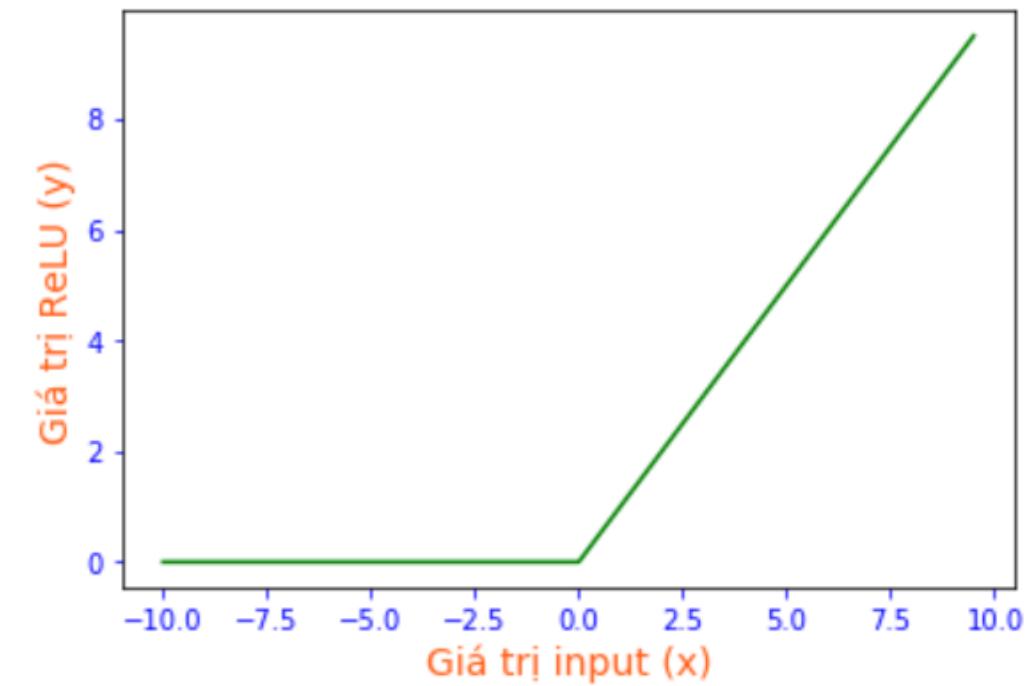
data =

1	5	-4	3	-2
---	---	----	---	----

data_a = ReLU(data)

data_a =

1	5	0	3	0
---	---	---	---	---



ReLU Function

```
1 # relu
2
3 import tensorflow as tf
4 import tensorflow.keras as keras
5 import numpy as np
6
7 relu_activation = keras.activations.relu
8
9 data_X = np.array([[1, 5, 3, -7],
10                   [7, -3, 9, 2],
11                   [3, 5, -8, 4],
12                   [8, -7, 3, 5]])
13 data_X = data_X.reshape(1, 4, 4, 1)
14 data_X = tf.Variable(data_X, dtype=tf.float32)
15
16 data_y = np.ones((1, 4, 4, 1))
17 data_y = tf.Variable(data_y, dtype=tf.float32)
18
19 print('\n\n data_X: \n', data_X[0,:,:,:])
20 print('\n\n data_y: \n', data_y[0,:,:,:])
21
22 with tf.GradientTape(persistent=True) as tape:
23     data_after = relu_activation(data_X)
24     print('\n\n data_after: \n', data_after[0,:,:,:])
25
26     loss = tf.reduce_mean((data_after-data_y)**2)
27     print('\n\n loss: \n', loss)
28
29 dloss = tape.gradient(loss, data_after)
30 print('\n\n dloss: \n', dloss[0,:,:,:])
31
32 ddata_X = tape.gradient(loss, data_X)
33 print('\n\n ddata_X: \n', ddata_X[0,:,:,:])
```

```
data_X:
tf.Tensor(
[[ 1.  5.  3. -7.]
 [ 7. -3.  9.  2.]
 [ 3.  5. -8.  4.]
 [ 8. -7.  3.  5.]], shape=(4, 4), dtype=float32)

data_y:
tf.Tensor(
[[1. 1. 1. 1.]
 [1. 1. 1. 1.]
 [1. 1. 1. 1.]
 [1. 1. 1. 1.]], shape=(4, 4), dtype=float32)

data_after:
tf.Tensor(
[[1. 5. 3. 0.]
 [7. 0. 9. 2.]
 [3. 5. 0. 4.]
 [8. 0. 3. 5.]], shape=(4, 4), dtype=float32)

loss:
tf.Tensor(13.9375, shape=(), dtype=float32)

dloss:
tf.Tensor(
[[ 0.      0.5     0.25   -0.125]
 [ 0.75   -0.125   1.      0.125]
 [ 0.25     0.5    -0.125   0.375]
 [ 0.875  -0.125   0.25    0.5   ]], shape=(4, 4), dtype=float32)

ddata_X:
tf.Tensor(
[[ 0.      0.5     0.25   -0.    ]
 [ 0.75   -0.      1.      0.125]
 [ 0.25     0.5    -0.      0.375]
 [ 0.875  -0.      0.25    0.5   ]], shape=(4, 4), dtype=float32)
```

PRelu Function

❖ PReLU function

$$\text{PRelu}(x) = \begin{cases} \alpha x & \text{if } x < 0 \\ x & \text{if } x \geq 0 \end{cases}$$

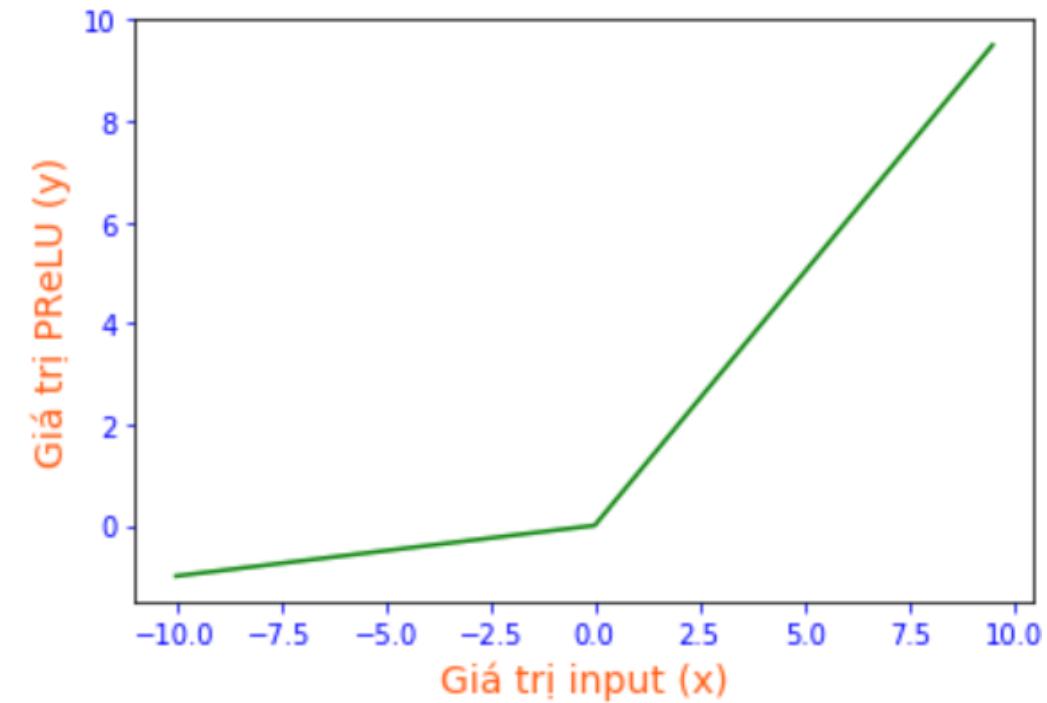
data =

1	5	-4	3	-2
---	---	----	---	----

data_a = PRELU(data)

data_a =

1	5	-0.4	3	-0.2
---	---	------	---	------



ELU Function

❖ ELU function

$$\text{ELU}(x) = \begin{cases} \alpha(e^x - 1) & \text{if } x < 0 \\ x & \text{if } x \geq 0 \end{cases}$$

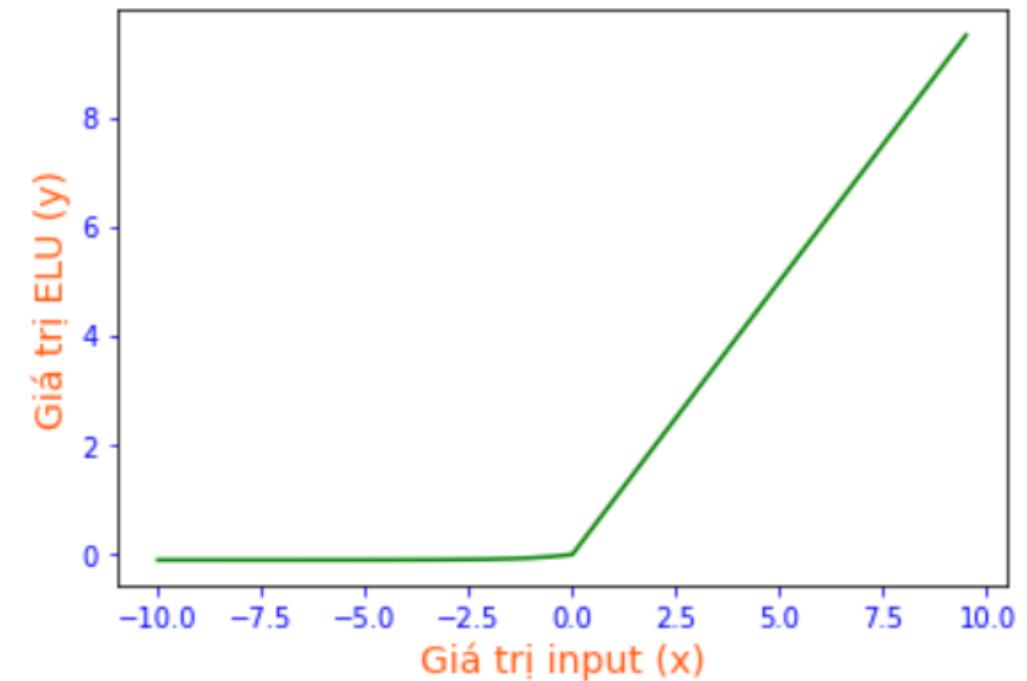
data =

1	5	-4	3	-2
---	---	----	---	----

data_a = **ELU(data)**

data_a =

1	5	-0.098	3	-0.086
---	---	--------	---	--------



Softplus Function

❖ Softplus function

$$\text{softplus}(x) = \log(1 + e^x)$$

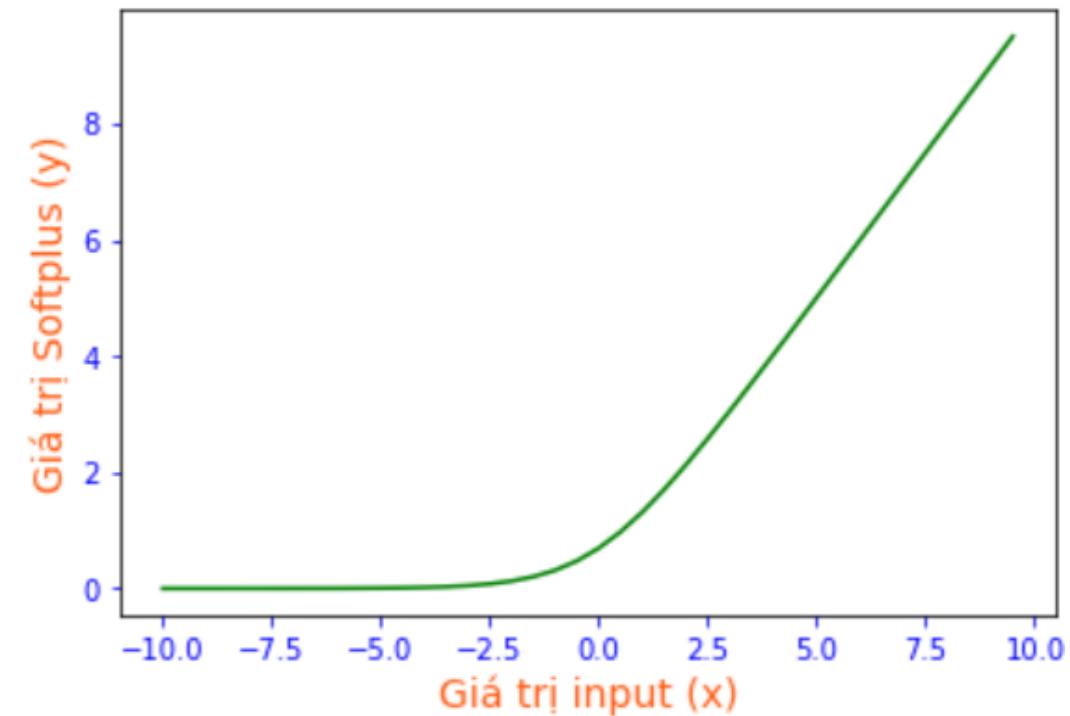
data =

1	5	-4	3	-2
---	---	----	---	----

data_a = **softplus(data)**

data_a =

1.313	5.006	0.018	3.048	0.126
-------	-------	-------	-------	-------



Softmax function

Chuyển các giá trị của một vector thành các giá trị xác suất

Formula

$$f(x_i) = \frac{e^{x_i}}{\sum_j e^{x_j}}$$

$$0 \leq f(x_i) \leq 1$$

$$\sum_i f(x_i) = 1$$

Input

$$x_1 = 1.0$$

$$x_2 = 2.0$$

$$x_3 = 3.0$$

Softmax

Probability

$$f(x_1) = 0.09$$

$$f(x_2) = 0.24$$

$$f(x_3) = 0.67$$

```
1 import numpy as np
```

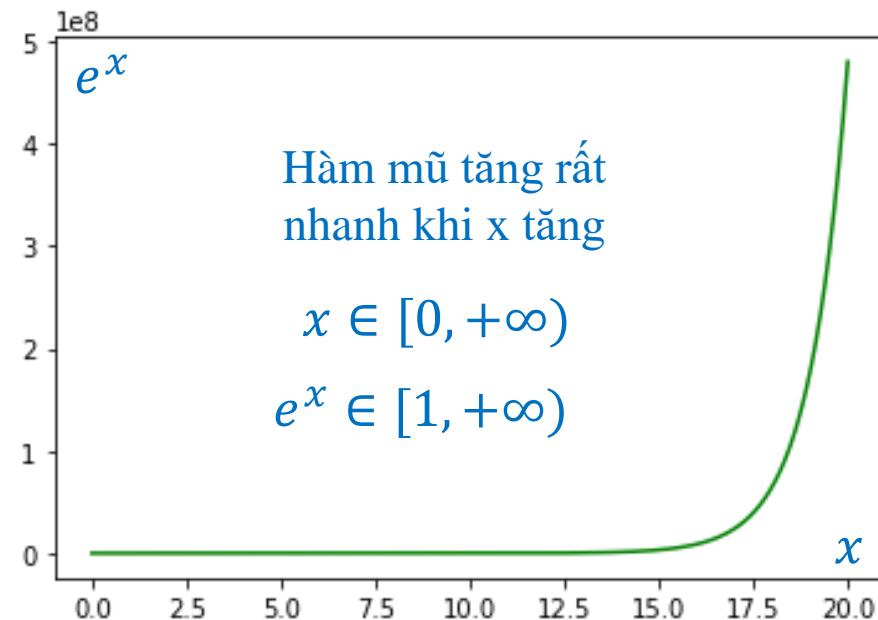
```
2
3 def softmax(X):
4     exps = np.exp(X)
5     return exps / np.sum(exps)
```

```
1 X = np.array([1.0, 2.0, 3.0])
2 f = softmax(X)
3 print(f)
```

```
[0.09003057 0.24472847 0.66524096]
```

```
1 X = np.array([1000.0, 1001.0, 1002.0])
2 f = softmax(X)
3 print(f)
```

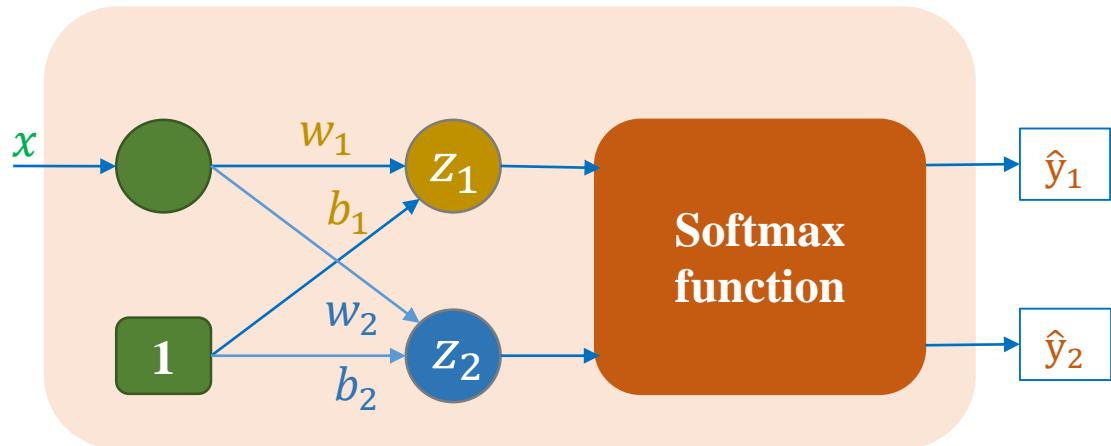
```
[nan nan nan]
```



Giá trị nan vì e^x vượt giới hạn lưu trữ của biến

Softmax function

Model



$$L(\theta) = - \sum_{i=1}^2 \delta(i, y) \log \hat{y}_i$$

$$\hat{y}_1 = \frac{e^{z_1}}{\sum_{j=1}^2 e^{z_j}}$$
$$\hat{y}_2 = \frac{e^{z_2}}{\sum_{j=1}^2 e^{z_j}}$$

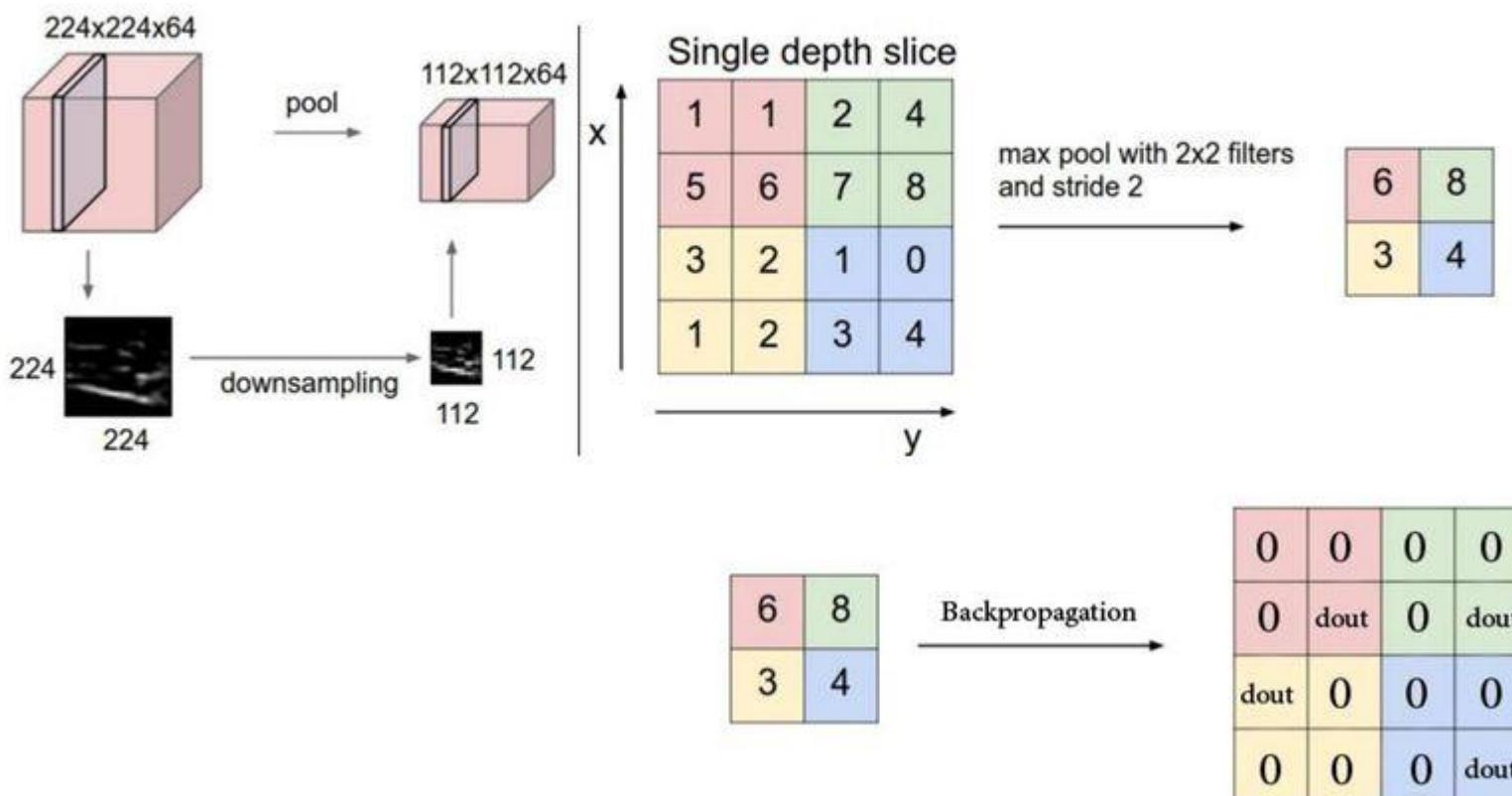
Derivative

$$\frac{\partial \hat{y}_i}{\partial z_j} = \hat{y}_i (\delta(i, j) - \hat{y}_j)$$

$$\frac{\partial L}{\partial z_i} = \hat{y}_i - \delta(i, y)$$

Max Pooling

Forward and Backward Max Pooling



Max Pooling

```
1 # max_pooling
2
3 import tensorflow as tf
4 import tensorflow.keras as keras
5 import numpy as np
6
7 max_pooling = keras.layers.MaxPooling2D(pool_size=2, strides=2)
8
9 data_X = np.array([[1, 5, 3, 7],
10                   [7, 3, 9, 2],
11                   [3, 5, 8, 4],
12                   [8, 7, 3, 5]])
13 data_X = data_X.reshape(1, 4, 4, 1)
14 data_X = tf.Variable(data_X, dtype=tf.float32)
15
16 data_y = np.ones((1, 2, 2, 1))
17 data_y = tf.Variable(data_y, dtype=tf.float32)
18
19 print('\n\n data_X: \n', data_X[0,:,:,:])
20 print('\n\n data_y: \n', data_y[0,:,:,:])
21
22 with tf.GradientTape(persistent=True) as tape:
23     data_after = max_pooling(data_X)
24     print('\n\n data_after: \n', data_after[0,:,:,:])
25
26     loss = tf.reduce_mean((data_after-data_y)**2)
27     print('\n\n loss: \n', loss)
28
29 dloss = tape.gradient(loss, data_after)
30 print('\n\n dloss: \n', dloss[0,:,:,:])
31
32 ddata_X = tape.gradient(loss, data_X)
33 print('\n\n ddata_X: \n', ddata_X[0,:,:,:])
```

```
data_X:
tf.Tensor(
[[[1. 5. 3. 7.]
[7. 3. 9. 2.]
[3. 5. 8. 4.]
[8. 7. 3. 5.]]], shape=(4, 4), dtype=float64)

data_y:
tf.Tensor(
[[[1. 1.]
[1. 1.]]], shape=(2, 2), dtype=float64)

data_after:
tf.Tensor(
[[[7. 9.]
[8. 8.]]], shape=(2, 2), dtype=float64)

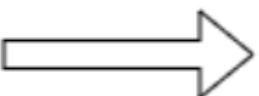
loss:
tf.Tensor(49.5, shape=(), dtype=float64)

dloss:
tf.Tensor(
[[[3. 4.]
[3.5 3.5]]], shape=(2, 2), dtype=float64)

ddata_X:
tf.Tensor(
[[[0. 0. 0. 0. ]
[3. 0. 4. 0. ]
[0. 0. 3.5 0. ]
[3.5 0. 0. 0. ]]], shape=(4, 4), dtype=float64)
```

Average Pooling

4	3	1	5
1	3	4	8
4	5	4	3
6	5	9	4



2.8	4.5
5.3	5.0

Formula

Average Pooling

```

1 # AveragePooling2D
2
3 import tensorflow as tf
4 import tensorflow.keras as keras
5 import numpy as np
6
7 avg_pooling = keras.layers.AveragePooling2D(pool_size=2, strides=2)
8
9 data_X = np.array([[1, 5, 3, 7],
10                   [7, 3, 9, 2],
11                   [3, 5, 8, 4],
12                   [8, 7, 3, 5]])
13 data_X = data_X.reshape(1, 4, 4, 1)
14 data_X = tf.Variable(data_X, dtype=tf.float32)
15
16 data_y = np.ones((1, 2, 2, 1))
17 data_y = tf.Variable(data_y, dtype=tf.float32)
18
19 print('\n\n data_X: \n', data_X[0,:,:,:])
20 print('\n\n data_y: \n', data_y[0,:,:,:])
21
22 with tf.GradientTape(persistent=True) as tape:
23     data_after = avg_pooling(data_X)
24     print('\n\n data_after_ap: \n', data_after[0,:,:,:])
25
26     loss = tf.reduce_mean((data_after-data_y)**2)
27     print('\n\n loss: \n', loss)
28
29 dloss = tape.gradient(loss, data_after)
30 print('\n\n dloss: \n', dloss[0,:,:,:])
31
32 ddata_X = tape.gradient(loss, data_X)
33 print('\n\n ddata_X: \n', ddata_X[0,:,:,:])

```

```

data_X:
tf.Tensor(
[[1. 5. 3. 7.]
 [7. 3. 9. 2.]
 [3. 5. 8. 4.]
 [8. 7. 3. 5.]], shape=(4, 4), dtype=float32)

data_y:
tf.Tensor(
[[1. 1.]
 [1. 1.]], shape=(2, 2), dtype=float32)

data_after_ap:
tf.Tensor(
[[4. 5.25]
 [5.75 5. ]], shape=(2, 2), dtype=float32)

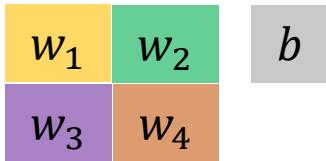
loss:
tf.Tensor(16.40625, shape=(), dtype=float32)

dloss:
tf.Tensor(
[[1.5 2.125]
 [2.375 2. ]], shape=(2, 2), dtype=float32)

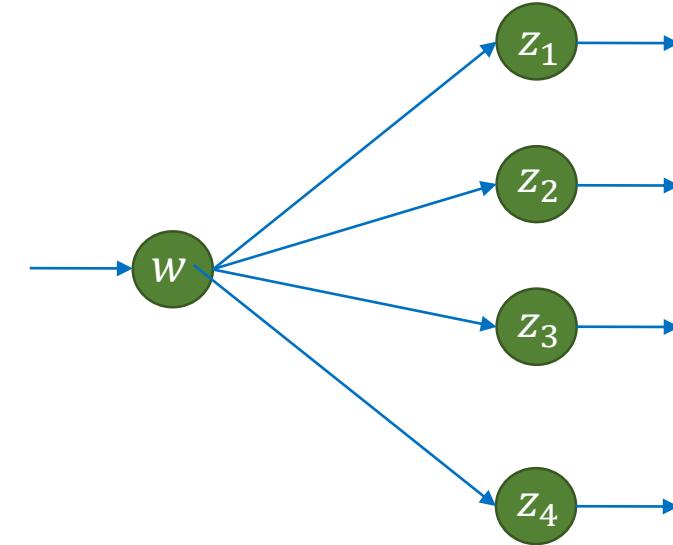
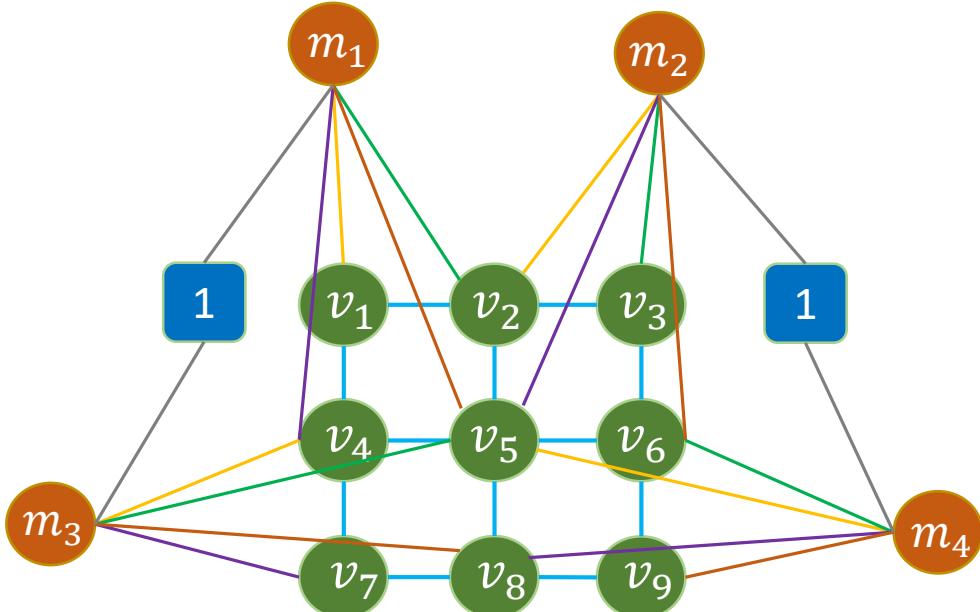
ddata_X:
tf.Tensor(
[[0.375 0.375 0.53125 0.53125]
 [0.375 0.375 0.53125 0.53125]
 [0.59375 0.59375 0.5 0.5 ]
 [0.59375 0.59375 0.5 0.5 ]], shape=(4, 4), dtype=float32)

```

Convolution



Kernel of parameters

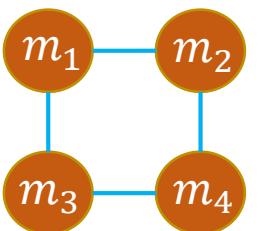
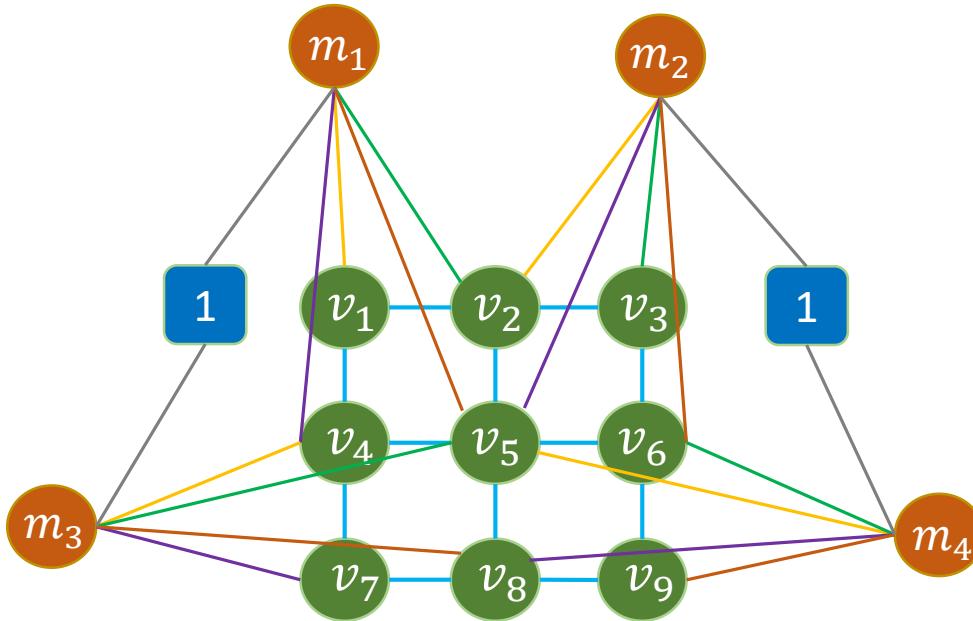


$$\frac{\partial L}{\partial w} = \sum_{i=1}^4 \frac{\partial L}{\partial z_i} \frac{\partial z_i}{\partial w}$$

Convolution

$$\begin{matrix} w_1 & w_2 \\ w_3 & w_4 \end{matrix} \quad b$$

Kernel of parameters



Feature Map

$$m_1 = v_1w_1 + v_2w_2 + v_4w_3 + v_5w_4 + b$$

$$m_2 = v_2w_1 + v_3w_2 + v_5w_3 + v_6w_4 + b$$

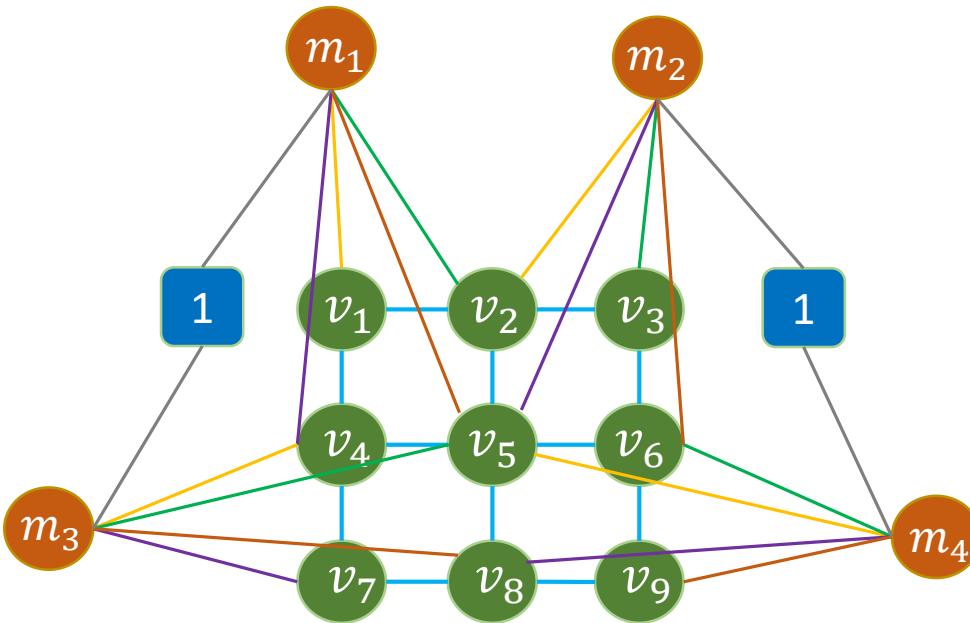
$$m_3 = v_4w_1 + v_5w_2 + v_7w_3 + v_8w_4 + b$$

$$m_4 = v_5w_1 + v_6w_2 + v_8w_3 + v_9w_4 + b$$

Convolution

$$\begin{matrix} w_1 & w_2 \\ w_3 & w_4 \end{matrix} \quad b$$

Kernel of parameters



Feature Map

$$\frac{\partial L}{\partial w_1} = \sum_i \frac{\partial L}{\partial m_i} \frac{\partial m_i}{\partial w_1}$$

$$m_1 = v_1w_1 + v_2w_2 + v_4w_3 + v_5w_4 + b$$

$$m_2 = v_2w_1 + v_3w_2 + v_5w_3 + v_6w_4 + b$$

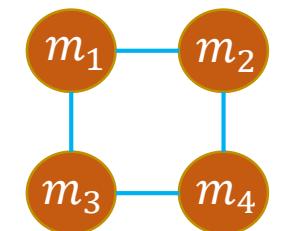
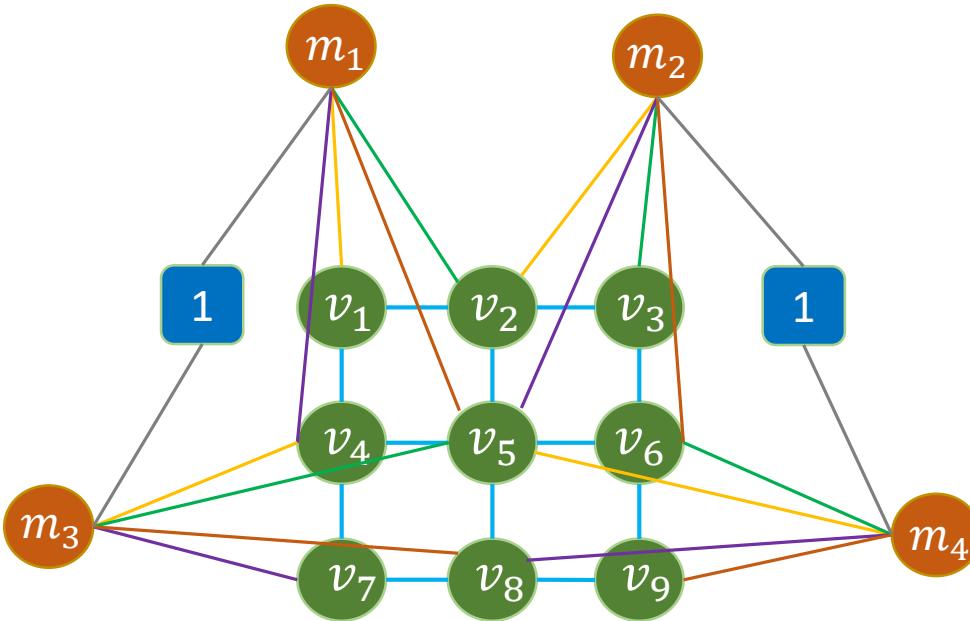
$$m_3 = v_4w_1 + v_5w_2 + v_7w_3 + v_8w_4 + b$$

$$m_4 = v_5w_1 + v_6w_2 + v_8w_3 + v_9w_4 + b$$

Convolution

$$\begin{matrix} w_1 & w_2 \\ w_3 & w_4 \end{matrix} \quad b$$

Kernel of parameters



Feature Map

$$\frac{\partial L}{\partial w_1} = \sum_i v_i \frac{\partial L}{\partial m_i}$$

$$m_1 = v_1 w_1 + v_2 w_2 + v_4 w_3 + v_5 w_4 + b$$

$$m_2 = v_2 w_1 + v_3 w_2 + v_5 w_3 + v_6 w_4 + b$$

$$m_3 = v_4 w_1 + v_5 w_2 + v_7 w_3 + v_8 w_4 + b$$

$$m_4 = v_5 w_1 + v_6 w_2 + v_8 w_3 + v_9 w_4 + b$$

Convolution

```

1 import tensorflow as tf
2 import tensorflow.keras as keras
3 import numpy as np
4
5 conv2D = keras.layers.Conv2D(1, (2, 2), activation='relu')
6
7 data_X = np.array([[1, 1, 2],
8                   [1, 2, 1],
9                   [2, 1, 1]])
10 data_X = data_X.reshape(1, 3, 3, 1)
11 data_X = tf.Variable(data_X, dtype=tf.float32)
12
13 data_y = np.ones((1, 2, 2, 1))
14 data_y = tf.Variable(data_y, dtype=tf.float32)
15
16 print('\n\n data_X: \n', data_X[0,:,:,:])
17 print('\n\n data_y: \n', data_y[0,:,:,:])
18
19 with tf.GradientTape(persistent=True) as tape:
20     data_after_conv = conv2D(data_X)
21     print('\n\n data_after_conv: \n', data_after_conv[0,:,:,:])
22
23     # Check the values of the current conv weight
24     print('\n\n conv_weight: \n', conv2D.trainable_weights[0][:,:,:,:])
25     print('\n conv_bias: \n', conv2D.trainable_weights[1][0])
26
27     loss = tf.reduce_mean((data_after_conv-data_y)**2)
28     print('\n\n loss: \n', loss)
29
30 dloss = tape.gradient(loss, data_after_conv)
31 print('\n\n dloss: \n', dloss[0,:,:,:])
32
33 dconv_weight = tape.gradient(loss, conv2D.trainable_weights)
34 print('\n\n dconv_weight: \n', dconv_weight[0][:,:,:,:])
35 print('\n dconv_bias: \n', dconv_weight[1][0])

```

```

data_X:
tf.Tensor(
[[1. 1. 2.]
 [1. 2. 1.]
 [2. 1. 1.]], shape=(3, 3), dtype=float32)

data_y:
tf.Tensor(
[[1. 1.]
 [1. 1.]], shape=(2, 2), dtype=float32)

data_after_conv:
tf.Tensor(
[[0.9292677 1.2405912]
 [1.2405912 1.6272811]], shape=(2, 2), dtype=float32)

conv_weight:
tf.Tensor(
[[ 0.6779961 -0.3282364 ]
 [ 0.61954254 -0.02001727]], shape=(2, 2), dtype=float32)

conv_bias:
tf.Tensor(0.0, shape=(), dtype=float32)

loss:
tf.Tensor(0.12856321, shape=(), dtype=float32)

dloss:
tf.Tensor(
[[-0.03536615  0.12029558]
 [ 0.12029558  0.31364053]], shape=(2, 2), dtype=float32)

dconv_weight:
tf.Tensor(
[[0.83250606 0.75945675]
 [0.75945675 0.4834994 ]], shape=(2, 2), dtype=float32)

dconv_bias:
tf.Tensor(0.5188656, shape=(), dtype=float32)

```

Reading and Exercises

❖ Exercises

- 1) Use LeNet for the fashion-MNIST and Cifar-10 data sets
- 2) What are the advantages of using 1x1 Conv instead of FC

❖ Reading

<https://cs231n.github.io/convolutional-networks/>

<https://towardsdatascience.com/a-comprehensive-guide-to-convolutional-neural-networks-the-eli5-way-3bd2b1164a53>

<https://stanford.edu/~shervine/teaching/cs-230/cheatsheet-convolutional-neural-networks>

Thank you!