

Functional programming

Introduction

- High level of abstraction
- Based on lambda calculus
- Language of choice: Haskell
- Functional programming -> expressions > statements.

Example:

```
--Add the first ten numbers together  
sum[1..10]
```

- Install Hugs, Haskell interpreter (ghci is another interpreter)

First steps

- Start hugs in terminal

Examples:

```
> head[1,2,3,4] --take first element  
1  
> tail[1,2,3,4] --remove first element  
[2,3,4]  
> [1,2,3,4] !! 2 --element #2  
3  
> take 3 [1,2,3,4] --generalization of head  
[1,2,3]  
> drop 3 [1,2,3,4,5] --generalization of tail  
[4,5]  
> [1,2,3]++[4,5] --append  
[1,2,3,4,5]
```

- First element in list has index 0
- List different to array -> indexing bad idea, not in constant time but in linear

Function application: function application is denoted by space. Higher priority

```
f a b + c*d --f(a,b) + c d from math
f a + b --f(a) + b from math
```

- Haskell file (script) -> **.hs**
- Define function in script, then open Hugs with script as argument so that functions are available. If script is changed use **:reload**. Also possible to load using **:load script**
- *Infix operator*: **xfy --> f x y**
- *Naming*:
- function and parameter name must begin with lowercase
- can use quotes (*prime*)
- type has to start with uppercase
- convention -> **s** at the end means list, **ss** list of lists
- Indentation like Python, implicit grouping
- Useful commands -> **:load script**, **:reload**, **:edit script**, **:type expression, :?**
- Comments: one line **--comment**, nested:

```
{-
very long
comment goes
here
-}
```

Types and classes

- **Type**: name for a collection of related values. Example **Bool**
- Applying a function to a wrong type makes a *type error*
- **e :: t -> e** has type **t**
- *Type inference*: compiler calculates type of expression prior to execution. Haskell programs are *type safe*, type error never happens in run time
- **:type <exp>** to calculate type of expression

Table 1: Different types in Haskell

Type	Explanation
Bool	Logical value: True or False
Char	Single character, enclosed in single quotes: 'a'
String	String of characters, double quotes: "abc"
Int	Fixed precision integer
Integer	Arbitrary precision integer, doesn't overflow
Float	Single precision floating point number

- **List:** sequence of values with same type. Can be infinite. Examples:

```
[False, True, False] :: [Bool] -- list of elements type Bool
['a', 'b'] :: [Char] -- list of elements type char
```

- **Tuple:** sequence of values of different type. Number of elements is called *arity*. Finite number of elements because type of all have to be calculated. Example:

```
(False, 'a') :: (Bool, Char) -- length appears in type
```

- **Function:** mapping from values of a type to values of another type. Examples:

```
not :: Bool -> Bool
isDigit :: Char -> Bool
```

```
function :: t1 -> t2 -- from domain to range in general
```

- *Curried function:* functions that return arguments one at a time (functions can return functions): $a \rightarrow (a \rightarrow a)$ equivalent to $a \rightarrow a \rightarrow a$, arrow associates to the right. Any function that returns more than one values can be curried. Useful for *partially applying functions*. **Most** functions applied in curried form, if tuples are not explicitly declared.
- *Polymorphic function:* functions not defined for a particular type. Example:

```
length :: [a] -> Int
```

- Price for polymorphism: type variables start with lowercase and types with uppercase
- *Overloaded function:* functions with same name but different types. In Haskell overloading means that there is a restriction in the type class. Example:

```
sum :: Num [a] => [a] -> Int -- only numeric values allowed
```

Defining functions

Conditional expressions

Example:

```
abs :: Int -> Int
abs n = if n >= 0 then n else -n
```

- Can be nested
- Conditional expressions **must** have an **else** branch

Guarded equation:

- Sequence of logical expressions
- Alternative to conditional (Haskell people prefer this)

```
abs n | n >= 0    = n -- /= such that
      | otherwise = -n
```

- Can be used to make definitions involving multiple conditions

Pattern matching

```
(&&) :: Bool -> Bool -> Bool
True && True = True
_     && _    = False --anything but True&&True
```

More efficient way using wildcard + lazy evaluation:

```
(&&) :: Bool -> Bool -> Bool
True && b = b -- True && something --> something
False && _ = False -- always False
```

- Order is important
- Patterns may not repeat variables: all the variables inside the pattern have to be different
- *Lists in pattern matching*: use *cons* definition $(:)$ ¹. Only matches not empty list. These pattern must be parenthesized because function application has higher priority.

```
head :: [a] -> a
head (x : _) = x
```

¹Lists are constructed one element at a time from the empty list using *cons operator*
[1,2,3] = 1:(2:(3:[]))= 1:2:3:[]

Lambda expressions

Functions can be constructed without naming using *lambda expressions*:

```
\x -> x + x --\x = \lambda x (from lambda calculus)
```

- Useful for currying:

```
add x y = x + y
add = \x -> (\y -> x+y)
```

- For returning functions as results
- For avoiding naming functions only used once:

```
odds n = map f [0..n-1]
  where
    f x = x*2 + 1
```

```
odds n = map (\x -> x*2 + 1) [0..n-1] --pass lambda as parameter to map
```

Sections

Operator written between two arguments can be used in curried way using parenthesis:

```
--Examples:
(1+) --sucessor
(1/) --reciprocate
(*2) --double
(/2) --half
```

For avoiding naming.

List comprehensions

- Code that manipulates collections
- Favorite collection for mathematicians: **sets**. Problems with sets:
- No duplication
- Deal with equality

- Haskell has tricks to deal with sets as lists
- Set comprehensions in Math: $\{x^2 | x \in \{1, \dots, 5\}\}$
- List comprehension in Haskell: `[x^2 | x <- [1..5]]`
- `x <- [1..5]` is called *generator*
- Comprehensions can have multiple generators (similar to nested loop, x is the outer loop and y is the inner):

```
> [(x,y) | x <- [1,2,3], y <- [4,5]]
> [(1,4),(1,5),(2,4),(2,5),(3,4),(3,5)]
> [(x,y) | y <- [4,5], x <- [1,2,3]]
> [(1,4),(2,4),(3,4),(1,5),(2,5),(3,5)]
```

- Generators can depend on each other, as in loops:

```
> [(x,y) | x <- [1,2,3], y <- [x..3]]
> [(1,1),(1,2),(1,3),(2,2),(2,3),(3,3)]
```

- Very concise code
- Filters (*guards*): `[x | x <- [1..10], even x]`

The zip function

- Combines two list to a list of pairs
- Useful when programming with list comprehensions.

```
zip :: [a] -> [b] -> [(a,b)]
```

String comprehension

- Strings = character list `[Char]`
- Everything that can be done in lists will work with strings

Recursive functions

- *Tail call elimination*

```
factorial 1 = 0
factorial n = n * factorial (n-1)
```

- Some functions are simpler to define using recursions
- *Induction* can be used to prove properties of recursive functions
- Recursion can be used also in lists:

```
product :: [Int] -> Int
product [] = 1
product (n:ns) = n * product ns
```

Note: `:` appends element to list, `++` concatenates lists

- *Quicksort*: algorithm for sorting integers. Two rules:
- The empty list is already sorted
- Bolzano in the rest

```
qsort :: [Int] -> Int
qsort [] = []
qsort (x:xs)=
  qsort smaller ++ [x] ++ qsort larger
  where
    smaller = [a | a <- xs, a <= x]
    larger  = [b | b <- xs, b > x]
```