Test 2

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1 Damped Harmonic Oscillator

The damped harmonic oscillator is a classical mechanical system that experiences damping, which is a dissipative force that opposes the motion of the oscillator. The equation of motion for a damped harmonic oscillator is typically given by:

$$m\frac{d^2x}{dt^2} + c\frac{dx}{dt} + kx = 0 \tag{1}$$

where:

m is the mass of the oscillator,
c is the damping coefficient,
k is the spring constant,
x is the displacement from the equilibrium position.

1.1 Obtaining the Solution for the Damped Harmonic Oscillator

The differential equation for the damped harmonic oscillator is given by:

$$m\frac{d^2x}{dt^2} + c\frac{dx}{dt} + kx = 0$$

Assume a solution of the form $x(t) = e^{rt}$, where r is a complex number.
Substitute this solution into the differential equation:

$$mr^2e^{rt} + cre^{rt} + ke^{rt} = 0$$

Factor out e^{rt}:

$$e^{rt}(mr^2 + cr + k) = 0$$

For a nontrivial solution (where $e^{rt} \neq 0$), the expression in the parentheses must be equal to zero:

$$mr^2 + cr + k = 0$$

This is a quadratic equation in r. Solve for r:

$$r = \frac{-c \pm \sqrt{c^2 - 4mk}}{2m}$$

The solutions to this quadratic equation yield two values of r. Depending on the nature of the roots (real, complex, or repeated), we get different forms for the solution. If the roots are complex, the solution takes the form:

$$x(t) = Ae^{-\frac{c}{2m}t}\cos(\omega_d t + \phi) \tag{2}$$

where:

$$A \text{ is the amplitude,}$$
$$\omega_d = \sqrt{\omega_0^2 - \left(\frac{c}{2m}\right)^2} \text{ (damped angular frequency),}$$
$$\omega_0 = \sqrt{\frac{k}{m}} \text{ (natural angular frequency),}$$
$$\phi \text{ is the phase constant.}$$

1.2 Example solution

The solution to this differential equation (1) depends on the type of damping. One common form of solution (2) describes the exponentially decaying oscillatory motion of the damped harmonic oscillator (Figure 1).

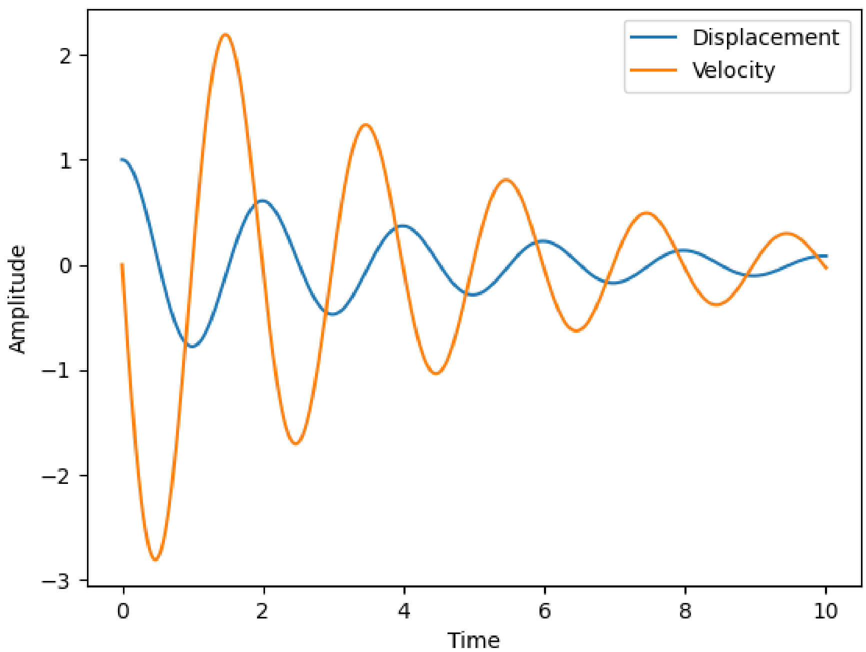

Figure 1: Damped Harmonic Oscillator Solution