Lectures Report 2 - The "Production" of Power: A CAS Analysis of Tullock's (1980) "Efficient Rent Seeking"

Ondřej Marvan, 17 November 2025

AI X CAS Workflow

My analysis followed the 6-step workflow, with the full script in the attached Jupyter Notebook.

1. AI Reconnaissance:

- AI Suggests: The core of the paper is a model called the **Contest Success Function** (CSF), which acts as the "production function". The inputs are lobbying
- expenses (X_A, X_B), and the output is the probability of winning (p_A).

2. CAS Specification:

- **CAS Verifies:** I specified the model in SymPy.
    - **Production Function (p_A):** $p_A\ (X_A, X_B) = \frac{X^r A}{X^r A + X^r B}$
    - **Utility Function (U_A):** $U_A = (p_A * V) - X_A$
    - Where V is the prize value and $_r$ is the "decisiveness" of the contest.

3. Property Tests:

- **AI Suggests:** For the model to be realistic, it must be well-behaved. Spending more should help, but with diminishing returns.
- **CAS Verifies:**
  - **Monotonicity:** $\partial p_A / \partial X_A > 0$. **Passed.** (More spending always helps).
  - **Concavity:** $\partial^2 p_A / \partial^2 X_A < 0$. **Passed.** (Confirmed by testing for $r=1$, which shows diminishing returns).
  - **Returns to Scale:** The most important test. My notebook shows the function is **homogeneous of degree 0** ($p_A(t * X_A, t * X_B) = p_A$). This is a huge insight: **if both players double their spending, their odds of winning *do not change***. It mathematically proves Tullock's "social waste" argument.

4. Internal Validity:

- **AI Suggests:** Tullock claims there is a predictable "Nash Equilibrium" level of wasteful spending.
- **CAS Verifies: Passed.** I used SymPy to solve the first-order condition ($\partial U_A / \partial X_A = 0$) for a symmetric equilibrium. The CAS symbolically derived the exact equilibrium formulas:
  - **Equilibrium Spending per Player (*x**):** $x^* = \frac{V*r}{4}$.
  - *Total Social Waste ($x_A + x_B^*$):* Total Waste = $\frac{V*r}{2}$
  - The model is perfectly internally consistent.

5. External Validity:

- **AI Suggests:** The model must align with reality. The "Tullock Paradox" is the empirical observation that real-world lobbying spending (Total Waste) is often *much lower* than the prize value (V).
- **CAS Verifies:** Our formula, Total Waste = $\frac{V*r}{2}$, explains this paradox perfectly.

  The model's external validity depends entirely on r:
  - If $r = 2$, Total Waste = V. (100% of the prize is wasted).
  - If $r = 1$, Total Waste = $\frac{V}{2}$. (50% is wasted).
  - If $r = 0.5$, Total Waste = $\frac{V}{4}$. (Only 25% is wasted).
- **Conclusion:** The model is only empirical if $r < 2$. The "Tullock Paradox" is simply a case where real-world political contests are "indecisive" (i.e., $r < 1$), meaning spending has very high diminishing returns.

6. Long Story Short:

This AI X CAS approach is effective. It allowed me to prove the intuitions of a foundational economic paper and use its own math to define its real-world limitations.