Hierarchical Active Fault Diagnosis for Stochastic Large Scale Systems with Coupled Faults

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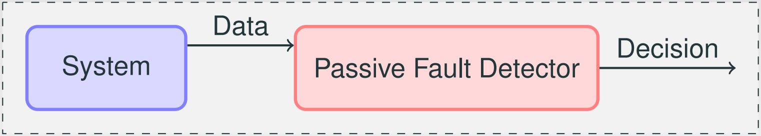
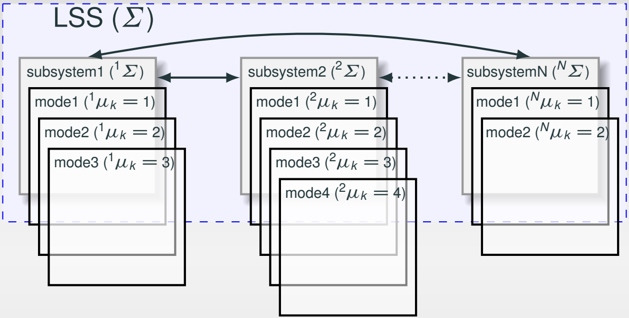
Active vs. Passive Fault Diagnosis



Large Scale System, Multiple Model Framework – Illustration



$$\Sigma : \underbrace{\begin{bmatrix} {}^1\mathbf{x}_{k+1} \\ \vdots \\ {}^N\mathbf{x}_{k+1} \end{bmatrix}}_{\mathbf{x}_{k+1}} = \underbrace{\begin{bmatrix} {}^1\mathbf{f}(\mathbf{x}_k, {}^1\mu_k, {}^1\mathbf{u}_k) \\ \vdots \\ {}^N\mathbf{f}(\mathbf{x}_k, {}^N\mu_k, {}^N\mathbf{u}_k) \end{bmatrix}}_{\mathbf{f}(\mathbf{x}_k, \boldsymbol{\mu}_k, \mathbf{u}_k)} + \mathbf{F}(\boldsymbol{\mu}_k) \underbrace{\begin{bmatrix} {}^1\mathbf{w}_k \\ \vdots \\ {}^N\mathbf{w}_k \end{bmatrix}}_{\mathbf{w}_k},$$

$$\underbrace{\begin{bmatrix} {}^1\mathbf{y}_k \\ \vdots \\ {}^N\mathbf{y}_k \end{bmatrix}} = \underbrace{\begin{bmatrix} {}^1\mathbf{h}({}^1\mathbf{x}_k, {}^1\mu_k) \\ \vdots \\ {}^N\mathbf{h}({}^N\mathbf{x}_k, {}^N\mu_k) \end{bmatrix}} + \mathbf{H}(\boldsymbol{\mu}_k) \underbrace{\begin{bmatrix} {}^1\mathbf{v}_k \\ \vdots \\ {}^N\mathbf{v}_k \end{bmatrix}},$$

Active Fault Diagnosis – Problem Formulation

- LSS Σ consists of N weakly coupled subsystems ${}^{n}\Sigma$:

$$
\begin{aligned}
{}^{n}\Sigma : \quad {}^{n}\mathbf{x}_{k+1} &= {}^{n}\mathbf{f}\left(\mathbf{x}_k, {}^{n}\mu_k, {}^{n}\mathbf{u}_k\right) + {}^{n}\mathbf{F}({}^{n}\mu_k)\,{}^{n}\mathbf{w}_k \\
{}^{n}\mathbf{y}_k &= {}^{n}\mathbf{h}\left({}^{n}\mathbf{x}_k, {}^{n}\mu_k\right) + {}^{n}\mathbf{H}({}^{n}\mu_k)\,{}^{n}\mathbf{v}_k
\end{aligned}
$$

- Assumptions:
  1. Noises ${}^{n}\mathbf{w}_k$ and ${}^{n}\mathbf{v}_k$ white and mutually independent
  2. Initial mode indices ${}^{n}\mu_0$ and initial states ${}^{n}\mathbf{x}_0$ are independent
  3. Mode indices ${}^{n}\mu_k$ are modelled as Markov processes
     ${}^{n}\mu_k = 1$ – fault-free behavior, ${}^{n}\mu_k \in \{2, \ldots, {}^{n}M\}$ – faulty behavior

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Dependent Faults

- Model indices assumptions

$$\Pr(\boldsymbol{\mu}_{k+1}|\boldsymbol{\mu}_k) = \begin{cases} \prod_{n=1}^{N} \Pr({}^n\mu_{k+1}|\,{}^n\mu_k) & \text{in Straka, Punčochář (2019)} \\ \prod_{n=1}^{N} \Pr({}^n\mu_{k+1}|\boldsymbol{\mu}_k) & \text{in Straka, Punčochář (2020)} \end{cases}$$

- Insufficient in some cases as the faults in different subsystems tend to appear simultaneously or even system-wide faults may emerge.
- This behavior can be formally described as dependent faults.
- Goal of the paper: design AFD for dependent faults

Active Fault Diagnosis – Problem Formulation

- AFD system:

$$\Delta : \quad \begin{bmatrix} \mathbf{d}_k \\ \mathbf{u}_k \end{bmatrix} = \begin{bmatrix} \boldsymbol{\sigma}_k\left(\mathbf{I}_0^k\right) \\ \boldsymbol{\gamma}_k\left(\mathbf{I}_0^k\right) \end{bmatrix},$$

  where $\mathbf{I}_0^k \triangleq [\mathbf{y}_0^k, \mathbf{u}_0^{k-1}]$ is the information available at k

- AFD goal: generate $\mathbf{d}_k$ and $\mathbf{u}_k$ to minimize discounted criterion

$$J(\boldsymbol{\sigma}_0^\infty, \boldsymbol{\gamma}_0^\infty) = \lim_{F \to \infty} \mathrm{E}\left\{ \sum_{k=0}^{F} \eta^k L^{\mathrm{d}}(\boldsymbol{\mu}_k, \mathbf{d}_k) \right\}$$

- Example of cost function

$$L^{\mathrm{d}}(\boldsymbol{\mu}_k, \mathbf{d}_k) = \sum_{n=1}^{N} {}^nL^{\mathrm{d}}\left({}^n\mu_k, {}^nd_k\right),$$

  where ${}^nL^{\mathrm{d}}$ penalizes discrepancy between ${}^n\mu_k$ and ${}^nd_k$ (missed detections, false alarms, incorrect fault identifications)

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General Approach to AFD design

AFD algorithm stages:

1. off-line: design of the excitation input and decision generators
2. on-line: state estimation and utilization of the generators

Solution: adopt dynamic programming and find Bellman function

Dim. of $\mathbf{I}_0^k$ increases with $k \to \infty \implies$ problem reformulation:

- unknown system state $\mathbf{s}_k = [\mathbf{x}_k^\mathsf{T}, \mu_k]^\mathsf{T}$
- recast the **imperfect state information model** (unknown $\mathbf{s}_k$) to a **perfect state information model** with a new (known) information state ξ_k
- ξ_k consists of sufficient statistics related to $p(\mathbf{s}_k|\mathbf{I}_0^k)$ obtained by an estimator

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- $\boldsymbol{\xi}_k$ consists of sufficient statistics related to $p(\mathbf{s}_k|\mathbf{I}_0^k)$ obtained by an estimator

Design of AFD Systems – Problem Reformulation – Illustration

Imperfect state information problem

System model

$$\mathbf{x}_{k+1} = f(\mathbf{x}_k, \mu_k, \mathbf{u}_k) + \mathbf{F}_{\mu_k}\mathbf{w}_k,$$
$$\mathbf{y}_k = \mathbf{h}(\mathbf{x}_k, \mu_k) + \mathbf{H}_{\mu_k}\mathbf{v}_k,$$
$$P(\mu_{k+1} = j | \mu_k = i)$$

Estimate $\boldsymbol{\xi}_k$

Criterion

$$J = \lim_{F\to\infty} \mathrm{E}\left\{\sum_{k=0}^{F} \eta^k L^d(\mu_k, d_k)\right\}$$

AFD

$$\begin{bmatrix} d_k \\ \mathbf{u}_k \end{bmatrix} = \boldsymbol{\rho}_k(\mathbf{I}_0^k)$$

Problem reformulation

Perfect state information problem

System model and estimator

$$\boldsymbol{\xi}_{k+1} = \boldsymbol{\phi}(\boldsymbol{\xi}_k, \mathbf{u}_k, \mathbf{y}_{k+1}),$$

Criterion

$$J = \lim_{F\to\infty} \mathrm{E}\left\{\sum_{k=0}^{F} \eta^k L^d(\boldsymbol{\xi}_k, d_k)\right\}$$

AFD

$$\begin{bmatrix} d_k \\ \mathbf{u}_k \end{bmatrix} = \bar{\boldsymbol{\rho}}_k(\boldsymbol{\xi}_k)$$

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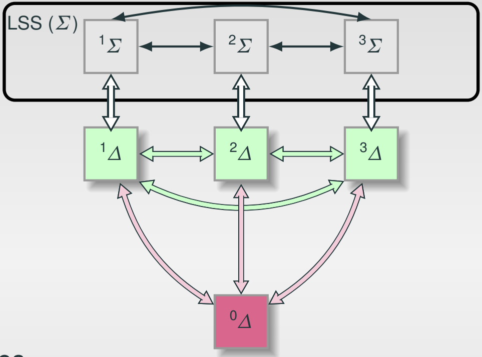
Hierarchical AFD Architecture

- off-line design will employ the computationally efficient decentralized architecture
- on-line estimation will work in a hierarchical architecture that has the ability to take into account the faults dependency and the coupling through $\mathbf{x}_k$.

Hierarchical AFD Architecture



- local nodes
  - estimate the continuous state of the subsystems
  - select the optimal excitations
  - send local likelihoods to the central node.
- central node
  - generates the decisions
  - submits the respective model probabilities to the local nodes

Numerical Illustration – System Specification

Two weakly coupled multiple-model linear subsystems ${}^{i}\Sigma,\ i = 1, 2$

$$
\begin{aligned}
{}^{i}\Sigma: \quad {}^{i}\mathbf{x}_{k+1} &= {}^{i}\mathbf{A}_{({}^{i}\mu_k)}\mathbf{x}_k + {}^{i}B_{({}^{i}\mu_k)}\,{}^{i}u_k + {}^{i}G_{({}^{i}\mu_k)}\,{}^{i}w_k, \\
{}^{i}z_k &= {}^{i}C_{({}^{i}\mu_k)}\,{}^{i}\mathbf{x}_k + {}^{i}H_{({}^{i}\mu_k)}\,{}^{i}v_k
\end{aligned}
$$

where both subsystems have two modes with the following matrices

$$
{}^{1}\Sigma: \quad \begin{cases} {}^{1}\mathbf{A}_{(1)} = [0.98\ 0.01], {}^{1}B_{(1)} = 0.01, {}^{1}G_{(1)} = \sqrt{0.003}, {}^{1}C_{(1)} = 1, {}^{1}H_{(1)} = 0.01, \\ {}^{1}\mathbf{A}_{(2)} = [0.92\ 0.03], {}^{1}B_{(2)} = 0.08, {}^{1}G_{(2)} = \sqrt{0.003}, {}^{1}C_{(2)} = 1, {}^{1}H_{(2)} = 0.01, \end{cases}
$$

$$
{}^{2}\Sigma: \quad \begin{cases} {}^{2}\mathbf{A}_{(1)} = [0.02\ 0.93], {}^{2}B_{(1)} = 0.07, {}^{2}G_{(1)} = \sqrt{0.002}, {}^{2}C_{(1)} = 1, {}^{2}H_{(1)} = 0.01, \\ {}^{2}\mathbf{A}_{(2)} = [0.01\ 0.93], {}^{2}B_{(2)} = 0.09, {}^{2}G_{(2)} = \sqrt{0.002}\,{}^{2}C_{(2)} = 1, {}^{2}H_{(2)} = 0.01. \end{cases}
$$

Numerical Illustration – Transition probabilities

	$({}^1\mu_k, {}^2\mu_k)$			
$({}^1\mu_{k+1}, {}^2\mu_{k+1})$	(1,1)	(1,2)	(2,1)	(2,2)
(1,1)	0.95	0.04	0.04	0.01
(1,2)	0.02	0.80	0.01	0.02
(2,1)	0.02	0.01	0.80	0.02
(2,2)	0.01	0.15	0.15	0.95

Numerical illustration – Performance Evaluation

- 10^5 Monte Carlo simulations
- Finite time horizon: $F = 400$
- PFD – passive fault diagnosis with $u_k = \text{sign}(\sin(k))$
- Results:

	$\hat{J}$	P_{MD}	P_{FA}	$T_{\text{on-line}}$
decentralized AFD	2.876	12.94%	10.08%	0.150 s
distributed AFD	1.852	2.06%	2.56%	0.421 s
hierarchical AFD	1.691	1.97%	2.30%	0.487 s

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Conclusion

- The paper focused on the active fault diagnosis of a large scale stochastic system with coupled faults.
- The system was represented using multiple models, which describe the fault-free and faulty behavior.
- The off-line design was carried out in a decentralized architecture to achieve computational and storage tractability.
- The on-line estimation used a hierarchical architecture.
- The numerical example demonstrated improved detection quality of the proposed hierarchical architecture.

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