

# **Gungnir's Standard Code Library**

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# Chapter 1

## 计算几何

### 1.1 二维

#### 1.1.1 基础

```
1 typedef double DB;
2 const DB eps = 1e-8;
3
4 int sign(DB x) {
5     return x < -eps ? -1 : ( x > eps ? 1 : 0 );
6 }
7 DB msqrt(DB x) {
8     return sign(x) > 0 ? sqrt(x) : 0;
9 }
10
11 struct Point {
12     DB x, y;
13     Point(): x(0), y(0) {}
14     Point(DB x, DB y): x(x), y(y) {}
15     Point operator+(const Point &rhs) const {
16         return Point(x + rhs.x, y + rhs.y);
17     }
18     Point operator-(const Point &rhs) const {
19         return Point(x - rhs.x, y - rhs.y);
20     }
21     Point operator*(DB k) const {
22         return Point(x * k, y * k);
23     }
24     Point operator/(DB k) const {
25         assert(sign(k));
26         return Point(x / k, y / k);
27     }
28     Point rotate(DB ang) const { // 逆时针旋转 ang 弧度
29         return Point(cos(ang) * x - sin(ang) * y,
30                     cos(ang) * y + sin(ang) * x);
31     }
32     Point turn90() const { // 逆时针旋转 90 度
33         return Point(-y, x);
34     }
35 }
```

```

35     Point unit() const {
36         return *this / len();
37     }
38 };
39 DB dot(const Point& a, const Point& b) {
40     return a.x * b.x + a.y * b.y;
41 }
42 DB det(const Point& a, const Point& b) {
43     return a.x * b.y - a.y * b.x;
44 }
45 #define cross(p1,p2,p3) ((p2.x-p1.x)*(p3.y-p1.y)-(p3.x-p1.x)*(p2.y-p1.y))
46 #define crossOp(p1,p2,p3) sign(cross(p1,p2,p3))
47 bool isLL(const Line& l1, const Line& l2, Point& p) { // 直线与直线交点
48     DB s1 = det(l2.b - l2.a, l1.a - l2.a),
49         s2 = -det(l2.b - l2.a, l1.b - l2.a);
50     if (!sign(s1 + s2)) return false;
51     p = (l1.a * s2 + l1.b * s1) / (s1 + s2);
52     return true;
53 }
54 bool onSeg(const Line& l, const Point& p) { // 点在线段上
55     return sign(det(p - l.a, l.b - l.a)) == 0 && sign(dot(p - l.a, p - l.b)) <= 0;
56 }
57 Point projection(const Line & l, const Point& p) {
58     return l.a + (l.b - l.a) * (dot(p - l.a, l.b - l.a) / (l.b - l.a).len2());
59 }
60 DB disToLine(const Line& l, const Point& p) { // 点到 * 直线 * 距离
61     return fabs(det(p - l.a, l.b - l.a) / (l.b - l.a).len());
62 }
63 DB disToSeg(const Line& l, const Point& p) { // 点到线段距离
64     return sign(dot(p - l.a, l.b - l.a)) * sign(dot(p - l.b, l.a - l.b)) == 1 ? disToLine(l, p) :
65     ↪ std::min((p - l.a).len(), (p - l.b).len());
66 }
67 // 圆与直线交点
68 bool isCL(Circle a, Line l, Point& p1, Point& p2) {
69     DB x = dot(l.a - a.o, l.b - l.a),
70         y = (l.b - l.a).len2(),
71         d = x * x - y * ((l.a - a.o).len2() - a.r * a.r);
72     if (sign(d) < 0) return false;
73     Point p = l.a - ((l.b - l.a) * (x / y)), delta = (l.b - l.a) * (msqrt(d) / y);
74     p1 = p + delta; p2 = p - delta;
75     return true;
76 }
77 // 圆与圆的交面积
78 DB areaCC(const Circle& c1, const Circle& c2) {
79     DB d = (c1.o - c2.o).len();
80     if (sign(d - (c1.r + c2.r)) >= 0) return 0;
81     if (sign(d - std::abs(c1.r - c2.r)) <= 0) {
82         DB r = std::min(c1.r, c2.r);
83         return r * r * PI;
84     }
85     DB x = (d * d + c1.r * c1.r - c2.r * c2.r) / (2 * d),
86         t1 = acos(x / c1.r), t2 = acos((d - x) / c2.r);
87     return c1.r * c1.r * t1 + c2.r * c2.r * t2 - d * c1.r * sin(t1);
88 }

```

```

88 // 圆与圆交点
89 bool isCC(Circle a, Circle b, P& p1, P& p2) {
90     DB s1 = (a.o - b.o).len();
91     if (sign(s1 - a.r - b.r) > 0 || sign(s1 - std::abs(a.r - b.r)) < 0) return false;
92     DB s2 = (a.r * a.r - b.r * b.r) / s1;
93     DB aa = (s1 + s2) * 0.5, bb = (s1 - s2) * 0.5;
94     P o = (b.o - a.o) * (aa / (aa + bb)) + a.o;
95     P delta = (b.o - a.o).unit().turn90() * msqrt(a.r * a.r - aa * aa);
96     p1 = o + delta, p2 = o - delta;
97     return true;
98 }
99 // 求点到圆的切点, 按关于点的顺时针方向返回两个点
100 bool tanCP(const Circle &c, const Point &p0, Point &p1, Point &p2) {
101     double x = (p0 - c.o).len2(), d = x - c.r * c.r;
102     if (d < eps) return false; // 点在圆上认为没有切点
103     Point p = (p0 - c.o) * (c.r * c.r / x);
104     Point delta = ((p0 - c.o) * (-c.r * sqrt(d) / x)).turn90();
105     p1 = c.o + p + delta;
106     p2 = c.o + p - delta;
107     return true;
108 }
109 // 求圆到圆的内共切线, 按关于 c1.o 的顺时针方向返回两条线
110 std::vector<Line> intanCC(const Circle &c1, const Circle &c2) {
111     std::vector<Line> ret;
112     Point p = (c1.o * c2.r + c2.o * c1.r) / (c1.r + c2.r);
113     Point p1, p2, q1, q2;
114     if (tanCP(c1, p, p1, p2) && tanCP(c2, p, q1, q2)) { // 两圆相切认为没有切线
115         ret.push_back(Line(p1, q1));
116         ret.push_back(Line(p2, q2));
117     }
118     return ret;
119 }
120 // 点在多边形内
121 bool inPolygon(const Point& p, const std::vector<Point>& poly) {
122     int n = poly.size();
123     int counter = 0;
124     for (int i = 0; i < n; ++i) {
125         P a = poly[i], b = poly[(i + 1) % n];
126         if (onSeg(Line(a, b), p)) return false; // 边界上不算
127         int x = sign(det(p - a, b - a));
128         int y = sign(a.y - p.y);
129         int z = sign(b.y - p.y);
130         if (x > 0 && y <= 0 && z > 0) ++ counter;
131         if (x < 0 && z <= 0 && y > 0) -- counter;
132     }
133     return counter != 0;
134 }
135 // 用半平面 (q1,q2) 的逆时针方向去切凸多边形
136 std::vector<Point> convexCut(const std::vector<Point>&ps, Point q1, Point q2) {
137     std::vector<Point> qs; int n = ps.size();
138     for (int i = 0; i < n; ++i) {
139         Point p1 = ps[i], p2 = ps[(i + 1) % n];
140         int d1 = crossOp(q1,q2,p1), d2 = crossOp(q1,q2,p2);
141         if (d1 >= 0) qs.push_back(p1);

```

```

142         if (d1 * d2 < 0) qs.push_back(isSS(p1, p2, q1, q2));
143     }
144     return qs;
145 }
146 // 求凸包
147 std::vector<Point> convexHull(std::vector<Point> ps) {
148     int n = ps.size(); if (n <= 1) return ps;
149     std::sort(ps.begin(), ps.end());
150     std::vector<Point> qs;
151     for (int i = 0; i < n; qs.push_back(ps[i ++]))
152         while (qs.size() > 1 && sign(det(qs[qs.size() - 2], qs.back(), ps[i])) <= 0)
153             qs.pop_back();
154     for (int i = n - 2, t = qs.size(); i >= 0; qs.push_back(ps[i --]))
155         while ((int)qs.size() > t && sign(det(qs[qs.size() - 2], qs.back(), ps[i])) <= 0)
156             qs.pop_back();
157     return qs;

```

### 1.1.2 凸包

```

1 // 凸包中的点按逆时针方向
2 struct Convex {
3     int n;
4     std::vector<Point> a, upper, lower;
5     void make_shell(const std::vector<Point>& p,
6                     std::vector<Point>& shell) { // p needs to be sorted.
7         clear(shell); int n = p.size();
8         for (int i = 0, j = 0; i < n; i++, j++) {
9             for (; j >= 2 && sign(det(shell[j-1] - shell[j-2],
10                                     p[i] - shell[j-2])) <= 0; --j) shell.pop_back();
11             shell.push_back(p[i]);
12         }
13     }
14     void make_convex() {
15         std::sort(a.begin(), a.end());
16         make_shell(a, lower);
17         std::reverse(a.begin(), a.end());
18         make_shell(a, upper);
19         a = lower; a.pop_back();
20         a.insert(a.end(), upper.begin(), upper.end());
21         if ((int)a.size() >= 2) a.pop_back();
22         n = a.size();
23     }
24     void init(const std::vector<Point>& _a) {
25         clear(a); a = _a; n = a.size();
26         make_convex();
27     }
28     void read(int _n) { // Won't make convex.
29         clear(a); n = _n; a.resize(n);
30         for (int i = 0; i < n; i++)
31             a[i].read();
32     }
33     std::pair<DB, int> get_tangent(
34         const std::vector<Point>& convex, const Point& vec) {

```



```

35     int l = 0, r = (int)convex.size() - 2;
36     assert(r >= 0);
37     for (; l + 1 < r; ) {
38         int mid = (l + r) / 2;
39         if (sign(det(convex[mid + 1] - convex[mid], vec)) > 0)
40             r = mid;
41         else l = mid;
42     }
43     return std::max(std::make_pair(det(vec, convex[r]), r),
44                    std::make_pair(det(vec, convex[0]), 0));
45 }
46 int binary_search(Point u, Point v, int l, int r) {
47     int s1 = sign(det(v - u, a[l % n] - u));
48     for (; l + 1 < r; ) {
49         int mid = (l + r) / 2;
50         int smid = sign(det(v - u, a[mid % n] - u));
51         if (smid == s1) l = mid;
52         else r = mid;
53     }
54     return l % n;
55 }
56 // 求凸包上和向量 vec 叉积最大的点, 返回编号, 共线的多个切点返回任意一个
57 int get_tangent(Point vec) {
58     std::pair<DB, int> ret = get_tangent(upper, vec);
59     ret.second = (ret.second + (int)lower.size() - 1) % n;
60     ret = std::max(ret, get_tangent(lower, vec));
61     return ret.second;
62 }
63 // 求凸包和直线 u, v 的交点, 如果不相交返回 false, 如果有则是和 (i, next(i)) 的交点, 交在点上不确
64 // 定返回前后两条边其中之一
65 bool get_intersection(Point u, Point v, int &i0, int &i1) {
66     int p0 = get_tangent(u - v), p1 = get_tangent(v - u);
67     if (sign(det(v - u, a[p0] - u)) * sign(det(v - u, a[p1] - u)) <= 0) {
68         if (p0 > p1) std::swap(p0, p1);
69         i0 = binary_search(u, v, p0, p1);
70         i1 = binary_search(u, v, p1, p0 + n);
71         return true;
72     }
73     else return false;
74 }
75 };

```

## 1.2 三维

### 1.2.1 基础

```

1 // 三维绕轴旋转, 大拇指指向 axis 向量方向, 四指弯曲方向转 w 弧度
2 Point rotate(const Point& s, const Point& axis, DB w) {
3     DB x = axis.x, y = axis.y, z = axis.z;
4     DB s1 = x * x + y * y + z * z, ss1 = msqrt(s1),
5         cosw = cos(w), sinw = sin(w);
6     DB a[4][4];
7     memset(a, 0, sizeof a);

```

```
8   a[3][3] = 1;
9   a[0][0] = ((y * y + z * z) * cosw + x * x) / s1;
10  a[0][1] = x * y * (1 - cosw) / s1 + z * sinw / ss1;
11  a[0][2] = x * z * (1 - cosw) / s1 - y * sinw / ss1;
12  a[1][0] = x * y * (1 - cosw) / s1 - z * sinw / ss1;
13  a[1][1] = ((x * x + z * z) * cosw + y * y) / s1;
14  a[1][2] = y * z * (1 - cosw) / s1 + x * sinw / ss1;
15  a[2][0] = x * z * (1 - cosw) / s1 + y * sinw / ss1;
16  a[2][1] = y * z * (1 - cosw) / s1 - x * sinw / ss1;
17  a[2][2] = ((x * x + y * y) * cos(w) + z * z) / s1;
18  DB ans[4] = {0, 0, 0, 0}, c[4] = {s.x, s.y, s.z, 1};
19  for (int i = 0; i < 4; ++ i)
20      for (int j = 0; j < 4; ++ j)
21          ans[i] += a[j][i] * c[j];
22  return Point(ans[0], ans[1], ans[2]);
23 }
```

## Chapter 2

# 数论

### 2.1 $O(m^2 \log n)$ 求线性递推数列第 $n$ 项

Given  $a_0, a_1, \dots, a_{m-1}$

$$a_n = c_0 \times a_{n-m} + \dots + c_{m-1} \times a_{n-1}$$

Solve for  $a_n = v_0 \times a_0 + v_1 \times a_1 + \dots + v_{m-1} \times a_{m-1}$

```
1 void linear_recurrence(long long n, int m, int a[], int c[], int p) {
2     long long v[M] = {1 % p}, u[M << 1], msk = !n;
3     for(long long i(n); i > 1; i >= 1) {
4         msk <= 1;
5     }
6     for(long long x(0); msk; msk >= 1, x <= 1) {
7         fill_n(u, m << 1, 0);
8         int b(!(n & msk));
9         x |= b;
10        if(x < m) {
11            u[x] = 1 % p;
12        } else {
13            for(int i(0); i < m; i++) {
14                for(int j(0), t(i + b); j < m; j++, t++) {
15                    u[t] = (u[t] + v[i] * v[j]) % p;
16                }
17            }
18            for(int i((m << 1) - 1); i >= m; i--) {
19                for(int j(0), t(i - m); j < m; j++, t++) {
20                    u[t] = (u[t] + c[j] * u[i]) % p;
21                }
22            }
23        }
24        copy(u, u + m, v);
25    }
26    //a[n] = v[0] * a[0] + v[1] * a[1] + ... + v[m - 1] * a[m - 1].
27    for(int i(m); i < 2 * m; i++) {
28        a[i] = 0;
29        for(int j(0); j < m; j++) {
30            a[i] = (a[i] + (long long)c[j] * a[i + j - m]) % p;
31        }
32    }
```

```

33     for(int j(0); j < m; j++) {
34         b[j] = 0;
35         for(int i(0); i < m; i++) {
36             b[j] = (b[j] + v[i] * a[i + j]) % p;
37         }
38     }
39     for(int j(0); j < m; j++) {
40         a[j] = b[j];
41     }
42 }

```

## 2.2 求逆元

```

1 void ex_gcd(long long a, long long b, long long &x, long long &y) {
2     if (b == 0) {
3         x = 1;
4         y = 0;
5         return;
6     }
7     long long xx, yy;
8     ex_gcd(b, a % b, xx, yy);
9     y = xx - a / b * yy;
10    x = yy;
11 }
12
13 long long inv(long long x, long long MODN) {
14     long long inv_x, y;
15     ex_gcd(x, MODN, inv_x, y);
16     return (inv_x % MODN + MODN) % MODN;
17 }

```

## 2.3 中国剩余定理

```

1 // 返回 (ans, M), 其中 ans 是模 M 意义下的解
2 std::pair<long long, long long> CRT(const std::vector<long long>& m, const std::vector<long long>& a) {
3     long long M = 1, ans = 0;
4     int n = m.size();
5     for (int i = 0; i < n; i++) M *= m[i];
6     for (int i = 0; i < n; i++) {
7         ans = (ans + (M / m[i]) * a[i] % M * inv(M / m[i], m[i])) % M; // 可能需要大整数相乘取模
8     }
9     return std::make_pair(ans, M);
10 }

```

## Chapter 3

# 代数

### 3.1 快速傅里叶变换

```
1 // n 必须是 2 的次幂
2 void fft(Complex a[], int n, int f) {
3     for (int i = 0; i < n; ++i)
4         if (R[i] < i) swap(a[i], a[R[i]]);
5     for (int i = 1, h = 0; i < n; i <= 1, h++) {
6         Complex wn = Complex(cos(pi / i), f * sin(pi / i));
7         Complex w = Complex(1, 0);
8         for (int k = 0; k < i; ++k, w = w * wn) tmp[k] = w;
9         for (int p = i < 1, j = 0; j < n; j += p) {
10             for (int k = 0; k < i; ++k) {
11                 Complex x = a[j + k], y = a[j + k + i] * tmp[k];
12                 a[j + k] = x + y; a[j + k + i] = x - y;
13             }
14         }
15     }
16 }
```



## Chapter 4

# 字符串

### 4.1 后缀数组

```
1 const int MAXN = MAXL * 2 + 1;
2 int a[MAXN], x[MAXN], y[MAXN], c[MAXN], sa[MAXN], rank[MAXN], height[MAXN];
3 void calc_sa(int n) {
4     int m = alphabet, k = 1;
5     memset(c, 0, sizeof(*c) * (m + 1));
6     for (int i = 1; i <= n; ++i) c[x[i] = a[i]]++;
7     for (int i = 1; i <= m; ++i) c[i] += c[i - 1];
8     for (int i = n; i; --i) sa[c[x[i]]--] = i;
9     for (; k <= n; k <= 1) {
10         int tot = k;
11         for (int i = n - k + 1; i <= n; ++i) y[i - n + k] = i;
12         for (int i = 1; i <= n; ++i)
13             if (sa[i] > k) y[++tot] = sa[i] - k;
14         memset(c, 0, sizeof(*c) * (m + 1));
15         for (int i = 1; i <= n; ++i) c[x[i]]++;
16         for (int i = 1; i <= m; ++i) c[i] += c[i - 1];
17         for (int i = n; i; --i) sa[c[x[y[i]]]--] = y[i];
18         for (int i = 1; i <= n; ++i) y[i] = x[i];
19         tot = 1; x[sa[1]] = 1;
20         for (int i = 2; i <= n; ++i) {
21             if (max(sa[i], sa[i - 1]) + k > n || y[sa[i]] != y[sa[i - 1]] || y[sa[i] + k] != y[sa[i - 1]
22 ↪ + k]) ++tot;
23             x[sa[i]] = tot;
24         }
25         if (tot == n) break; else m = tot;
26     }
27 void calc_height(int n) {
28     for (int i = 1; i <= n; ++i) rank[sa[i]] = i;
29     for (int i = 1; i <= n; ++i) {
30         height[rank[i]] = max(0, height[rank[i - 1]] - 1);
31         if (rank[i] == 1) continue;
32         int j = sa[rank[i] - 1];
33         while (max(i, j) + height[rank[i]] <= n && a[i + height[rank[i]]] == a[j + height[rank[i]]])
34 ↪ ++height[rank[i]];
35     }
```

```
35 }
```

## 4.2 后缀自动机

```
1 static const int MAXL = MAXN * 2; // MAXN is original length
2 static const int alphabet = 26; // sometimes need changing
3 int l, last, cnt, trans[MAXL][alphabet], par[MAXL], sum[MAXL], seq[MAXL], mxl[MAXL], size[MAXL]; // mxl
   ↳ is maxlength, size is the size of right
4 char str[MAXL];
5 inline void init() {
6     l = strlen(str + 1); cnt = last = 1;
7     for (int i = 0; i <= l * 2; ++i) memset(trans[i], 0, sizeof(trans[i]));
8     memset(par, 0, sizeof(*par) * (l * 2 + 1));
9     memset(mxl, 0, sizeof(*mxl) * (l * 2 + 1));
10    memset(size, 0, sizeof(*size) * (l * 2 + 1));
11 }
12 inline void extend(int pos, int c) {
13     int p = last, np = last = ++cnt;
14     mxl[np] = mxl[p] + 1; size[np] = 1;
15     for (; p && !trans[p][c]; p = par[p]) trans[p][c] = np;
16     if (!p) par[np] = 1;
17     else {
18         int q = trans[p][c];
19         if (mxl[p] + 1 == mxl[q]) par[np] = q;
20         else {
21             int nq = ++cnt;
22             mxl[nq] = mxl[p] + 1;
23             memcpy(trans[nq], trans[q], sizeof(trans[nq]));
24             par[nq] = par[q];
25             par[np] = par[q] = nq;
26             for (; trans[p][c] == q; p = par[p]) trans[p][c] = nq;
27         }
28     }
29 }
30 inline void buildsam() {
31     for (int i = 1; i <= l; ++i) extend(i, str[i] - 'a');
32     memset(sum, 0, sizeof(*sum) * (l * 2 + 1));
33     for (int i = 1; i <= cnt; ++i) sum[mxl[i]]++;
34     for (int i = 1; i <= l; ++i) sum[i] += sum[i - 1];
35     for (int i = cnt; i; --i) seq[sum[mxl[i]] - 1] = i;
36     for (int i = cnt; i; --i) size[par[seq[i]]] += size[seq[i]];
37 }
```

## 4.3 回文自动机

```
1 int nT, nStr, last, c[MAXT][26], fail[MAXT], r[MAXN], l[MAXN], s[MAXN];
2 int allocate(int len) {
3     l[nT] = len;
4     r[nT] = 0;
5     fail[nT] = 0;
6     memset(c[nT], 0, sizeof(c[nT]));
```



```
7     return nT++;
8 }
9 void init() {
10     nT = nStr = 0;
11     int newE = allocate(0);
12     int new0 = allocate(-1);
13     last = newE;
14     fail[newE] = new0;
15     fail[new0] = newE;
16     s[0] = -1;
17 }
18 void add(int x) {
19     s[++nStr] = x;
20     int now = last;
21     while (s[nStr - l[now] - 1] != s[nStr]) now = fail[now];
22     if (!c[now][x]) {
23         int newnode = allocate(l[now] + 2, &newfail = fail[newnode]);
24         newfail = fail[now];
25         while (s[nStr - l[newfail] - 1] != s[nStr]) newfail = fail[newfail];
26         newfail = c[newfail][x];
27         c[now][x] = newnode;
28     }
29     last = c[now][x];
30     r[last]++;
31 }
32 void count() {
33     for (int i = nT - 1; i >= 0; i--) {
34         r[fail[i]] += r[i];
35     }
36 }
```



## Chapter 5

# 图论

### 5.1 基础

```
1 struct Graph { // Remember to call .init()!
2     int e, nxt[M], v[M], adj[N], n;
3     bool base;
4     __inline void init(bool _base, int _n = 0) {
5         assert(n < N);
6         n = _n; base = _base;
7         e = 0; memset(adj + base, -1, sizeof(*adj) * n);
8     }
9     __inline int new_node() {
10         adj[n + base] = -1;
11         assert(n + base + 1 < N);
12         return n++ + base;
13     }
14     __inline void ins(int u0, int v0) { // directional
15         assert(u0 < n + base && v0 < n + base);
16         v[e] = v0; nxt[e] = adj[u0]; adj[u0] = e++;
17         assert(e < M);
18     }
19     __inline void bi_ins(int u0, int v0) { // bi-directional
20         ins(u0, v0); ins(v0, u0);
21     }
22 };
```

### 5.2 KM

```
1 struct KM {
2     // Truly  $O(n^3)$ 
3     // 邻接矩阵，不能连的边设为 -INF，求最小权匹配时边权取负，但不能连的还是 -INF，使用时先对 1 -> n
4     // 调用 hungary()，再 get_ans() 求值
5     int w[N][N];
6     int lx[N], ly[N], match[N], way[N], slack[N];
7     bool used[N];
8     void init() {
9         for (int i = 1; i <= n; i++) {
10             match[i] = 0;
```

```

10     lx[i] = 0;
11     ly[i] = 0;
12     way[i] = 0;
13 }
14 }
15 void hungary(int x) {
16     match[0] = x;
17     int j0 = 0;
18     for (int j = 0; j <= n; j++) {
19         slack[j] = INF;
20         used[j] = false;
21     }
22
23     do {
24         used[j0] = true;
25         int i0 = match[j0], delta = INF, j1 = 0;
26         for (int j = 1; j <= n; j++) {
27             if (used[j] == false) {
28                 int cur = -w[i0][j] - lx[i0] - ly[j];
29                 if (cur < slack[j]) {
30                     slack[j] = cur;
31                     way[j] = j0;
32                 }
33                 if (slack[j] < delta) {
34                     delta = slack[j];
35                     j1 = j;
36                 }
37             }
38         }
39         for (int j = 0; j <= n; j++) {
40             if (used[j]) {
41                 lx[match[j]] += delta;
42                 ly[j] -= delta;
43             }
44             else slack[j] -= delta;
45         }
46         j0 = j1;
47     } while (match[j0] != 0);
48
49     do {
50         int j1 = way[j0];
51         match[j0] = match[j1];
52         j0 = j1;
53     } while (j0);
54 }
55
56 int get_ans() {
57     int sum = 0;
58     for(int i = 1; i <= n; i++) {
59         if (w[match[i]][i] == -INF) ; // 无解
60         if (match[i] > 0) sum += w[match[i]][i];
61     }
62     return sum;
63 }

```

```
64 } km;
```

## 5.3 点双连通分量

bcc.forest is a set of connected tree whose vertices are chequered with cut-vertex and BCC.

```
1  const bool BCC_VERTEX = 0, BCC_EDGE = 1;
2  struct BCC { // N = N0 + M0. Remember to call init(&raw_graph).
3      Graph *g, forest; // g is raw graph ptr.
4      int dfn[N], DFN, low[N];
5      int stack[N], top;
6      int expand_to[N]; // Where edge i is expanded to in expanded graph.
7      // Vertex i expanded to i.
8      int compress_to[N]; // Where vertex i is compressed to.
9      bool vertex_type[N], cut[N], compress_cut[N], branch[M];
10     //std::vector<int> BCC_component[N]; // Cut vertex belongs to none.
11     __inline void init(Graph *raw_graph) {
12         g = raw_graph;
13     }
14     void DFS(int u, int pe) {
15         dfn[u] = low[u] = ++DFN; cut[u] = false;
16         if (!g->adj[u]) {
17             cut[u] = 1;
18             compress_to[u] = forest.new_node();
19             compress_cut[compress_to[u]] = 1;
20         }
21         for (int e = g->adj[u]; ~e; e = g->nxt[e]) {
22             int v = g->v[e];
23             if ((e ^ pe) > 1 && dfn[v] > 0 && dfn[v] < dfn[u]) {
24                 stack[top++] = e;
25                 low[u] = std::min(low[u], dfn[v]);
26             }
27             else if (!dfn[v]) {
28                 stack[top++] = e; branch[e] = 1;
29                 DFS(v, e);
30                 low[u] = std::min(low[v], low[u]);
31                 if (low[v] >= dfn[u]) {
32                     if (!cut[u]) {
33                         cut[u] = 1;
34                         compress_to[u] = forest.new_node();
35                         compress_cut[compress_to[u]] = 1;
36                     }
37                     int cc = forest.new_node();
38                     forest.bi_ins(compress_to[u], cc);
39                     compress_cut[cc] = 0;
40                     //BCC_component[cc].clear();
41                     do {
42                         int cur_e = stack[--top];
43                         compress_to[expand_to[cur_e]] = cc;
44                         compress_to[expand_to[cur_e^1]] = cc;
45                         if (branch[cur_e]) {
46                             int v = g->v[cur_e];
47                             if (cut[v])
```

```

48         forest.bi_ins(cc, compress_to[v]);
49     else {
50         //BCC_component[cc].push_back(v);
51         compress_to[v] = cc;
52     }
53 }
54 } while (stack[top] != e);
55 }
56 }
57 }
58 }
59 void solve() {
60     forest.init(g->base);
61     int n = g->n;
62     for (int i = 0; i < g->e; i++) {
63         expand_to[i] = g->new_node();
64     }
65     memset(branch, 0, sizeof(*branch) * g->e);
66     memset(dfn + g->base, 0, sizeof(*dfn) * n); DFN = 0;
67     for (int i = 0; i < n; i++)
68         if (!dfn[i + g->base]) {
69             top = 0;
70             DFS(i + g->base, -1);
71         }
72 }
73 } bcc;
74
75 bcc.init(&raw_graph);
76 bcc.solve();
77 // Do something with bcc.forest ...

```

## 5.4 边双连通分量

```

1 struct BCC {
2     Graph *g, forest;
3     int dfn[N], low[N], stack[N], tot[N], belong[N], vis[N], top, dfs_clock;
4     // tot[] is the size of each BCC, belong[] is the BCC that each node belongs to
5     pair<int, int> ori[M]; // bridge in raw_graph(raw node)
6     bool is_bridge[M];
7     __inline void init(Graph *raw_graph) {
8         g = raw_graph;
9         memset(is_bridge, false, sizeof(*is_bridge) * g -> e);
10        memset(vis + g -> base, 0, sizeof(*vis) * g -> n);
11    }
12    void tarjan(int u, int from) {
13        dfn[u] = low[u] = ++dfs_clock; vis[u] = 1; stack[++top] = u;
14        for (int p = g -> adj[u]; ~p; p = g -> nxt[p]) {
15            if ((p ^ 1) == from) continue;
16            int v = g -> v[p];
17            if (vis[v]) {
18                if (vis[v] == 1) low[u] = min(low[u], dfn[v]);
19            } else {
20                tarjan(v, p);

```

```
21         low[u] = min(low[u], low[v]);
22         if (low[v] > dfn[u]) is_bridge[p / 2] = true;
23     }
24 }
25 if (dfn[u] != low[u]) return;
26 tot[forest.new_node()] = 0;
27 do {
28     belong[stack[top]] = forest.n;
29     vis[stack[top]] = 2;
30     tot[forest.n]++;
31     --top;
32 } while (stack[top + 1] != u);
33 }
34 void solve() {
35     forest.init(g -> base);
36     int n = g -> n;
37     for (int i = 0; i < n; ++i)
38         if (!vis[i + g -> base]) {
39             top = dfs_clock = 0;
40             tarjan(i + g -> base, -1);
41         }
42     for (int i = 0; i < g -> e / 2; ++i)
43         if (is_bridge[i]) {
44             int e = forest.e;
45             forest.bi_ins(belong[g -> v[i * 2]], belong[g -> v[i * 2 + 1]], g -> w[i * 2]);
46             ori[e] = make_pair(g -> v[i * 2 + 1], g -> v[i * 2]);
47             ori[e + 1] = make_pair(g -> v[i * 2], g -> v[i * 2 + 1]);
48         }
49 }
50 } bcc;
```





## Chapter 6

# 技巧

### 6.1 真正的释放 STL 容器内存空间

```
1 template <typename T>
2 __inline void clear(T& container) {
3     container.clear(); // 或者删除了一堆元素
4     T(container).swap(container);
5 }
```

### 6.2 无敌的大整数相乘取模

Time complexity  $O(1)$ .

```
1 // 需要保证 x 和 y 非负
2 long long mult(long long x, long long y, long long MODN) {
3     long long t = (x * y - (long long)((long double)x / MODN * y + 1e-3) * MODN) % MODN;
4     return t < 0 ? t + MODN : t;
5 }
```

### 6.3 无敌的读入优化

```
1 // getchar() 读入优化 << 关同步 cin << 此优化
2 // 用 isdigit() 会小幅变慢
3 // 返回 false 表示读到文件尾
4 namespace Reader {
5     const int L = (1 << 15) + 5;
6     char buffer[L], *S, *T;
7     __inline bool getchar(char &ch) {
8         if (S == T) {
9             T = (S = buffer) + fread(buffer, 1, L, stdin);
10            if (S == T) {
11                ch = EOF;
12                return false;
13            }
14        }
15        ch = *S++;
16    }
```

```
16     return true;
17 }
18 __inline bool getint(int &x) {
19     char ch; bool neg = 0;
20     for (; getchar(ch) && (ch < '0' || ch > '9'); ) neg ^= ch == '-';
21     if (ch == EOF) return false;
22     x = ch - '0';
23     for (; getchar(ch), ch >= '0' && ch <= '9'; )
24         x = x * 10 + ch - '0';
25     if (neg) x = -x;
26     return true;
27 }
28 }
```

## 6.4 控制 cout 输出实数精度

```
1 std::cout << std::fixed << std::setprecision(5);
```

# Chapter 7

## 提示

### 7.1 线性规划转对偶

maximize  $\mathbf{c}^T \mathbf{x}$   
subject to  $\mathbf{Ax} \leq \mathbf{b}, \mathbf{x} \geq 0$

$\iff$

minimize  $\mathbf{y}^T \mathbf{b}$   
subject to  $\mathbf{y}^T \mathbf{A} \geq \mathbf{c}^T, \mathbf{y} \geq 0$

### 7.2 32-bit/64-bit 随机素数

32-bit	64-bit
73550053	1249292846855685773
148898719	1701750434419805569
189560747	3605499878424114901
459874703	5648316673387803781
1202316001	6125342570814357977
1431183547	6215155308775851301
1438011109	6294606778040623451
1538762023	6347330550446020547
1557944263	7429632924303725207
1981315913	8524720079480389849

### 7.3 NTT 素数及其原根

Prime	Primitive root
1053818881	7
1051721729	6
1045430273	3
1012924417	5
1007681537	3