# Gungnir'l Standard Code Library\*

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Dated: November 20, 2016

 $<sup>{\</sup>rm *https://github.com/footoredo/Gungnir}$ 

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### Chapter 1

## 计算几何

### 1.1 二维

### 1.1.1 基础

```
typedef double DB;
  const DB eps = 1e-8;
  int sign(DB x) {
      return x < -eps ? -1 : (x > eps ? 1 : 0);
5
  }
6
  DB msqrt(DB x) {
      return sign(x) > 0 ? sqrt(x) : 0;
8
  }
9
10
  struct Point {
11
      DB x, y;
12
      Point(): x(0), y(0) {}
13
      Point(DB x, DB y): x(x), y(y) {}
14
15
      Point operator+(const Point &rhs) const {
          return Point(x + rhs.x, y + rhs.y);
16
17
      Point operator-(const Point &rhs) const {
18
          return Point(x - rhs.x, y - rhs.y);
19
20
      Point operator*(DB k) const {
21
          return Point(x * k, y * k);
22
23
      Point operator/(DB k) const {
24
          assert(sign(k));
25
          return Point(x / k, y / k);
26
27
      Point rotate(DB ang) const { // 逆时针旋转 ang 弧度
28
          return Point(cos(ang) * x - sin(ang) * y,
29
                   cos(ang) * y + sin(ang) * x);
30
      }
31
```

CHAPTER 1. 计算几何

```
Point turn90() const { // 逆时针旋转 90 度
32
          return Point(-y, x);
33
34
      Point unit() const {
35
          return *this / len();
36
37
  };
38
  DB dot(const Point& a, const Point& b) {
39
      return a.x * b.x + a.y * b.y;
40
41
  DB det(const Point& a, const Point& b) {
42
      return a.x * b.y - a.y * b.x;
43
44
  #define cross(p1,p2,p3) ((p2.x-p1.x)*(p3.y-p1.y)-(p3.x-p1.x)*(p2.y-p1.y))
45
  #define crossOp(p1,p2,p3) sign(cross(p1,p2,p3))
46
  bool isLL(const Line& l1, const Line& l2, Point& p) { // 直线与直线交点
47
      DB s1 = det(l2.b - l2.a, l1.a - l2.a),
48
         s2 = -det(l2.b - l2.a, l1.b - l2.a);
49
      if (!sign(s1 + s2)) return false;
50
      p = (l1.a * s2 + l1.b * s1) / (s1 + s2);
51
      return true;
52
53
  bool onSeg(const Line& l, const Point& p) { // 点在线段上
54
      return sign(det(p - l.a, l.b - l.a)) == \emptyset && sign(dot(p - l.a, p - l.b)) <= \emptyset;
55
56
  Point projection(const Line & l, const Point& p) {
57
      return l.a + (l.b - l.a) * (dot(p - l.a, l.b - l.a) / (l.b - l.a).len2());
58
59
  DB disToLine(const Line& l, const Point& p) { // 点到 * 直线 * 距离
60
      return fabs(det(p - l.a, l.b - l.a) / (l.b - l.a).len());
61
 |}
62
63 DB disToSeg(const Line& l, const Point& p) { // 点到线段距离
      return sign(dot(p - l.a, l.b - l.a)) * sign(dot(p - l.b, l.a - l.b)) == 1 ?

    disToLine(l, p) : std::min((p - l.a).len(), (p - l.b).len());
  }
65
  // 圆与直线交点
66
  bool isCL(Circle a, Line l, Point& p1, Point& p2) {
67
      DB x = dot(l.a - a.o, l.b - l.a),
68
         y = (l.b - l.a).len2(),
69
         d = x * x - y * ((l.a - a.o).len2() - a.r * a.r);
70
      if (sign(d) < 0) return false;
71
      Point p = l.a - ((l.b - l.a) * (x / y)), delta = (l.b - l.a) * (msqrt(d) / y);
      p1 = p + delta; p2 = p - delta;
73
      return true;
74
75
  //圆与圆的交面积
76
 DB areaCC(const Circle& c1, const Circle& c2) {
78
      DB d = (c1.o - c2.o).len();
79
      if (sign(d - (c1.r + c2.r)) >= 0) return 0;
```

1.1. **二维** 7

```
if (sign(d - std::abs(c1.r - c2.r)) \le 0) {
80
           DB r = std::min(c1.r, c2.r);
           return r * r * PI;
82
83
       DB x = (d * d + c1.r * c1.r - c2.r * c2.r) / (2 * d),
84
           t1 = acos(x / c1.r), t2 = acos((d - x) / c2.r);
85
       return c1.r * c1.r * t1 + c2.r * c2.r * t2 - d * c1.r * sin(t1);
86
  }
87
  // 圆与圆交点
88
  bool isCC(Circle a, Circle b, P& p1, P& p2) {
       DB s1 = (a.o - b.o).len();
90
       if (sign(s1 - a.r - b.r) > 0 \mid | sign(s1 - std::abs(a.r - b.r)) < 0) return
91
    → false:
       DB s2 = (a.r * a.r - b.r * b.r) / s1;
92
       DB aa = (s1 + s2) * 0.5, bb = (s1 - s2) * 0.5;
93
       P \circ = (b.o - a.o) * (aa + bb)) + a.o;
94
95
       P 	ext{ delta} = (b.o - a.o).unit().turn90() * msqrt(a.r * a.r - aa * aa);
       p1 = o + delta, p2 = o - delta;
96
       return true;
97
  }
98
   // 求点到圆的切点,按关于点的顺时针方向返回两个点
99
   bool tanCP(const Circle &c, const Point &p0, Point &p1, Point &p2) {
100
       double x = (p0 - c.o).len2(), d = x - c.r * c.r;
101
       if (d < eps) return false; // 点在圆上认为没有切点
102
       Point p = (p0 - c.o) * (c.r * c.r / x);
103
       Point delta = ((p0 - c.o) * (-c.r * sqrt(d) / x)).turn90();
104
105
       p1 = c.o + p + delta;
       p2 = c.o + p - delta;
106
       return true;
107
108
  // 求圆到圆的内共切线,按关于 c1.o 的顺时针方向返回两条线
109
  std::vector<Line> intanCC(const Circle &c1, const Circle &c2) {
       std::vector<Line> ret;
111
       Point p = (c1.0 * c2.r + c2.0 * c1.r) / (c1.r + c2.r);
112
       Point p1, p2, q1, q2;
113
       if (tanCP(c1, p, p1, p2) && tanCP(c2, p, q1, q2)) { // 两圆相切认为没有切线
114
           ret.push_back(Line(p1, q1));
115
           ret.push_back(Line(p2, q2));
116
       }
117
       return ret;
118
  }
119
  // 点在多边形内
120
  bool inPolygon(const Point& p, const std::vector<Point>& poly) {
121
       int n = poly.size();
122
       int counter = 0;
123
       for (int i = 0; i < n; ++ i) {
124
           P = poly[i], b = poly[(i + 1) % n];
125
126
           if (onSeg(Line(a, b), p)) return false; // 边界上不算
127
           int x = sign(det(p - a, b - a));
```

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```
int y = sign(a.y - p.y);
128
           int z = sign(b.y - p.y);
           if (x > 0 \& \& y \le 0 \& \& z > 0) ++ counter;
130
           if (x < 0 \&\& z \le 0 \&\& y > 0) -- counter;
131
132
133
       return counter != 0;
   }
134
   // 用半平面(q1,q2)的逆时针方向去切凸多边形
135
   std::vector<Point> convexCut(const std::vector<Point>&ps, Point q1, Point q2) {
       std::vector<Point> qs; int n = ps.size();
137
       for (int i = 0; i < n; ++i) {
138
           Point p1 = ps[i], p2 = ps[(i + 1) % n];
139
           int d1 = cross0p(q1,q2,p1), d2 = cross0p(q1,q2,p2);
140
           if (d1 \ge 0) qs.push_back(p1);
141
           if (d1 * d2 < 0) qs.push_back(isSS(p1, p2, q1, q2));
142
143
       return qs;
144
   }
145
   // 求凸包
146
   std::vector<Point> convexHull(std::vector<Point> ps) {
147
       int n = ps.size(); if (n <= 1) return ps;</pre>
148
       std::sort(ps.begin(), ps.end());
149
       std::vector<Point> qs;
150
       for (int i = 0; i < n; qs.push_back(ps[i ++]))
151
           while (qs.size() > 1 \&\& sign(det(qs[qs.size() - 2], qs.back(), ps[i])) <= 0)
152
                qs.pop_back();
153
       for (int i = n - 2, t = qs.size(); i \ge 0; qs.push_back(ps[i --]))
154
           while ((int)qs.size() > t \& sign(det(qs[qs.size() - 2], qs.back(), ps[i]))
155
     <= ∅)</p>
                qs.pop_back();
156
       return qs;
157
```

#### 1.1.2 凸包

```
// 凸包中的点按逆时针方向
  struct Convex {
2
      int n;
3
      std::vector<Point> a, upper, lower;
4
      void make_shell(const std::vector<Point>& p,
5
               std::vector<Point>& shell) { // p needs to be sorted.
6
          clear(shell); int n = p.size();
7
           for (int i = 0, j = 0; i < n; i++, j++) {
8
               for (; j \ge 2 \& sign(det(shell[j-1] - shell[j-2],
g
                                p[i] - shell[j-2])) <= 0; --j) shell.pop_back();</pre>
10
               shell.push_back(p[i]);
11
          }
12
      }
13
      void make_convex() {
14
          std::sort(a.begin(), a.end());
15
```

1.1. 二维

```
make_shell(a, lower);
16
          std::reverse(a.begin(), a.end());
17
          make shell(a, upper);
18
          a = lower; a.pop_back();
19
          a.insert(a.end(), upper.begin(), upper.end());
20
          if ((int)a.size() >= 2) a.pop_back();
21
          n = a.size();
22
23
      void init(const std::vector<Point>& _a) {
24
          clear(a); a = _a; n = a.size();
25
          make_convex();
26
      void read(int _n) { // Won't make convex.
28
          clear(a); n = _n; a.resize(n);
29
          for (int i = 0; i < n; i++)
30
              a[i].read();
31
32
      std::pair<DB, int> get_tangent(
33
              const std::vector<Point>& convex, const Point& vec) {
34
          int l = 0, r = (int)convex.size() - 2;
35
          assert(r >= 0);
36
          for (; l + 1 < r; ) {
37
              int mid = (l + r) / 2;
38
              if (sign(det(convex[mid + 1] - convex[mid], vec)) > 0)
39
                  r = mid;
40
              else l = mid;
41
          }
42
          return std::max(std::make_pair(det(vec, convex[r]), r),
43
                  std::make_pair(det(vec, convex[0]), 0));
45
      int binary_search(Point u, Point v, int l, int r) {
46
          int s1 = sign(det(v - u, a[l % n] - u));
47
          for (; l + 1 < r; ) {
48
              int mid = (l + r) / 2;
49
              int smid = sign(det(v - u, a[mid % n] - u));
50
              if (smid == s1) l = mid;
51
52
              else r = mid;
          }
53
          return l % n;
54
55
      // 求凸包上和向量 vec 叉积最大的点,返回编号,共线的多个切点返回任意一个
56
      int get tangent(Point vec) {
57
          std::pair<DB, int> ret = get_tangent(upper, vec);
58
          ret.second = (ret.second + (int)lower.size() - 1) % n;
59
          ret = std::max(ret, get_tangent(lower, vec));
60
          return ret.second;
61
62
      // 求凸包和直线 u, v 的交点,如果不相交返回 false,如果有则是和(i, next(i))的交点,交在
63

→ 点上不确定返回前后两条边其中之一
```

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```
bool get_intersection(Point u, Point v, int &i0, int &i1) {
64
           int p0 = get_tangent(u - v), p1 = get_tangent(v - u);
           if (sign(det(v - u, a[p0] - u)) * sign(det(v - u, a[p1] - u)) <= 0) {
66
               if (p0 > p1) std::swap(p0, p1);
67
               i0 = binary_search(u, v, p0, p1);
68
69
               i1 = binary_search(u, v, p1, p0 + n);
70
               return true;
71
           else return false;
72
      }
73
74 | };
```

#### 1.2 三维

#### 1.2.1 基础

```
|// 三维绕轴旋转,大拇指指向 axis 向量方向,四指弯曲方向转 w 弧度
  Point rotate(const Point& s, const Point& axis, DB w) {
2
      DB x = axis.x, y = axis.y, z = axis.z;
3
      DB s1 = x * x + y * y + z * z, ss1 = msqrt(s1),
4
         cosw = cos(w), sinw = sin(w);
5
6
      DB a[4][4];
      memset(a, 0, sizeof a);
7
      a[3][3] = 1;
8
      a[0][0] = ((y * y + z * z) * cosw + x * x) / s1;
9
      a[0][1] = x * y * (1 - cosw) / s1 + z * sinw / ss1;
10
      a[0][2] = x * z * (1 - cosw) / s1 - y * sinw / ss1;
11
      a[1][0] = x * y * (1 - cosw) / s1 - z * sinw / ss1;
12
      a[1][1] = ((x * x + z * z) * cosw + y * y) / s1;
13
      a[1][2] = y * z * (1 - cosw) / s1 + x * sinw / ss1;
14
      a[2][0] = x * z * (1 - cosw) / s1 + y * sinw / ss1;
15
      a[2][1] = y * z * (1 - cosw) / s1 - x * sinw / ss1;
16
      a[2][2] = ((x * x + y * y) * cos(w) + z * z) / s1;
17
      DB ans [4] = \{0, 0, 0, 0\}, c[4] = \{s.x, s.y, s.z, 1\};
18
      for (int i = 0; i < 4; ++ i)
19
          for (int j = 0; j < 4; ++ j)
20
21
              ans[i] += a[j][i] * c[j];
      return Point(ans[0], ans[1], ans[2]);
22
23 | }
```

#### 1.2.2 凸包

1.2. 三维

```
7 | }
8
    inline DB mix(const P& a, const P& b, const P& c) {
9
      return dot(cross(a, b), c);
10
11
12
13
    _inline DB volume(const P& a, const P& b, const P& c, const P& d) {
      return mix(b - a, c - a, d - a);
14
15 }
16
  struct Face {
17
      int a, b, c;
18
      __inline Face() {}
19
      __inline Face(int _a, int _b, int _c):
20
           a(_a), b(_b), c(_c) {}
21
      __inline DB area() const {
22
23
           return 0.5 * cross(p[b] - p[a], p[c] - p[a]).len();
24
      __inline P normal() const {
25
           return cross(p[b] - p[a], p[c] - p[a]).unit();
26
27
      __inline DB dis(const P& p0) const {
28
          return dot(normal(), p0 - p[a]);
29
      }
30
  };
31
32
  std::vector<Face> face, tmp; // Should be O(n).
33
  int mark[N][N], Time, n;
35
   _inline void add(int v) {
36
      ++ Time;
37
      clear(tmp);
38
      for (int i = 0; i < (int) face.size(); ++ i) {
39
           int a = face[i].a, b = face[i].b, c = face[i].c;
40
           if (sign(volume(p[v], p[a], p[b], p[c])) > 0) {
41
               mark[a][b] = mark[b][a] = mark[a][c] =
42
                   mark[c][a] = mark[b][c] = mark[c][b] = Time;
43
           }
44
          else {
45
               tmp.push_back(face[i]);
46
           }
47
48
      clear(face); face = tmp;
49
      for (int i = 0; i < (int)tmp.size(); ++ i) {</pre>
50
           int a = face[i].a, b = face[i].b, c = face[i].c;
51
           if (mark[a][b] == Time) face.emplace_back(v, b, a);
52
           if (mark[b][c] == Time) face.emplace_back(v, c, b);
53
           if (mark[c][a] == Time) face.emplace_back(v, a, c);
           assert(face.size() < 500u);
55
```

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```
}
56
  }
57
58
  void reorder() {
59
        for (int i = 2; i < n; ++ i) {
60
            P tmp = cross(p[i] - p[0], p[i] - p[1]);
if (sign(tmp.len())) {
61
62
                 std::swap(p[i], p[2]);
63
                  for (int j = 3; j < n; ++ j)
64
                       if (sign(volume(p[0], p[1], p[2], p[j]))) {
65
                            std::swap(p[j], p[3]);
66
                            return;
67
                       }
68
            }
69
        }
70
  }
71
72
  void build_convex() {
73
        reorder();
74
        clear(face);
75
       face.emplace_back(0, 1, 2);
face.emplace_back(0, 2, 1);
76
77
        for (int i = 3; i < n; ++ i)
78
             add(i);
79
80 }
```

### Chapter 2

## 数论

### 2.1 $O(m^2 \log n)$ 求线性递推数列第 n 项

```
Given a_0, a_1, \dots, a_{m-1}

a_n = c_0 \times a_{n-m} + \dots + c_{m-1} \times a_{n-1}

Solve for a_n = v_0 \times a_0 + v_1 \times a_1 + \dots + v_{m-1} \times a_{m-1}
```

```
void linear_recurrence(long long n, int m, int a[], int c[], int p) {
2
      long long v[M] = \{1 \% p\}, u[M << 1], msk = !!n;
      for(long long i(n); i > 1; i >>= 1) {
3
           msk <<= 1;
4
5
      for(long long x(0); msk; msk >>= 1, x <<= 1) {
6
           fill_n(u, m << 1, 0);
7
           int b(!!(n & msk));
8
           x \mid = b;
9
           if(x < m) {
10
               u[x] = 1 % p;
11
12
           }else {
               for(int i(0); i < m; i++) {
13
                   for(int j(0), t(i + b); j < m; j++, t++) {
14
                        u[t] = (u[t] + v[i] * v[j]) % p;
15
                   }
16
17
               for(int i((m << 1) - 1); i >= m; i--) {
18
                   for(int j(0), t(i - m); j < m; j++, t++) {
19
                        u[t] = (u[t] + c[j] * u[i]) % p;
20
                   }
21
               }
22
           }
23
           copy(u, u + m, v);
24
25
      //a[n] = v[0] * a[0] + v[1] * a[1] + ... + v[m - 1] * a[m - 1].
26
      for(int i(m); i < 2 * m; i++) {
27
           a[i] = 0;
28
           for(int j(0); j < m; j++) {
29
```

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```
a[i] = (a[i] + (long long)c[j] * a[i + j - m]) % p;
30
           }
31
32
       for(int j(0); j < m; j++) {
33
           b[j] = 0;
34
35
           for(int i(0); i < m; i++) {
               b[j] = (b[j] + v[i] * a[i + j]) % p;
36
37
38
       for(int j(0); j < m; j++) {
39
           a[j] = b[j];
40
       }
41
  }
42
```

### 2.2 求逆元

```
void ex_gcd(long long a, long long b, long long &x, long long &y) {
1
      if (b == 0) {
2
           x = 1;
3
4
           y = 0;
           return;
5
6
      long long xx, yy;
7
      ex_gcd(b, a % b, xx, yy);
8
9
      y = xx - a / b * yy;
      x = yy;
10
11 | }
12
  long long inv(long long x, long long MODN) {
13
      long long inv_x, y;
14
      ex_gcd(x, MODN, inv_x, y);
15
      return (inv x % MODN + MODN) % MODN;
16
17 | }
```

### 2.3 中国剩余定理

```
1 // 返回 (ans, M), 其中 ans 是模 M 意义下的解
 std::pair<long long, long long> CRT(const std::vector<long long>& m, const

    std::vector<long long>& a) {
     long long M = 1, ans = 0;
3
     int n = m.size();
4
     for (int i = 0; i < n; i++) M *= m[i];
5
     for (int i = 0; i < n; i++) {
6
         ans = (ans + (M / m[i]) * a[i] % M * inv(M / m[i], m[i])) % M; // 可能需要大
7
   →整数相乘取模
     }
8
```

2.4. 素性测试 15

```
return std::make_pair(ans, M);
}
```

### 2.4 素性测试

```
int strong_pseudo_primetest(long long n,int base) {
1
2
      long long n2=n-1, res;
      int s=0;
3
      while(n2\%2==0) n2>>=1,s++;
4
      res=powmod(base,n2,n);
5
      if((res==1)||(res==n-1)) return 1;
6
7
      s--;
      while(s >= 0) {
8
           res=mulmod(res,res,n);
9
           if(res==n-1) return 1;
10
11
      }
12
      return 0; // n is not a strong pseudo prime
13
  }
14
  int isprime(long long n) {
15
      static LL testNum[]={2,3,5,7,11,13,17,19,23,29,31,37};
16
      static LL lim[]={4,0,1373653LL,25326001LL,25000000000LL,2152302898747LL,
17
    → 3474749660383LL,341550071728321LL,0,0,0,0);
      if(n<2||n==3215031751LL) return 0;
18
      for(int i=0;i<12;++i){
19
           if(n<lim[i]) return 1;</pre>
20
           if(strong_pseudo_primetest(n,testNum[i])==0) return 0;
21
22
      return 1;
23
 |}
24
```

#### 2.5 质因数分解

```
int ansn; LL ans[1000];
  LL func(LL x,LL n) { return(mod_mul(x,x,n)+1)%n; }
  LL Pollard(LL n){
3
      LL i,x,y,p;
4
      if(Rabin_Miller(n)) return n;
5
      if(!(n&1)) return 2;
6
      for(i=1;i<20;i++){
7
          x=i; y=func(x,n); p=gcd(y-x,n);
8
          while(p==1) {x=func(x,n); y=func(func(y,n),n); p=gcd((y-x+n)%n,n)%n;}
9
          if(p==0||p==n) continue;
10
           return p;
11
      }
12
13 }
14 void factor(LL n){
```

CHAPTER 2. 数论

```
LL x;
x=Pollard(n);
if(x==n){ ans[ansn++]=x; return; }
factor(x), factor(n/x);
}
```

### 2.6 线下整点

### Chapter 3

## 代数

### 3.1 快速傅里叶变换

```
1 // n 必须是 2 的次幂
  void fft(Complex a[], int n, int f) {
       for (int i = 0; i < n; ++i)
           if (R[i] < i) swap(a[i], a[R[i]]);</pre>
       for (int i = 1, h = 0; i < n; i <<= 1, h++) {
5
           Complex wn = Complex(cos(pi / i), f * sin(pi / i));
6
           Complex w = Complex(1, 0);
for (int k = 0; k < i; ++k, w = w * wn) tmp[k] = w;
7
8
           for (int p = i \ll 1, j = 0; j < n; j += p) {
9
                for (int k = 0; k < i; ++k) {
10
                    Complex x = a[j + k], y = a[j + k + i] * tmp[k];
11
                    a[j + k] = x + y; a[j + k + i] = x - y;
12
               }
13
           }
       }
15
16 | }
```

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### Chapter 4

## 字符串

### 4.1 后缀数组

```
_{1} const int MAXN = MAXL * 2 + 1;
2 int a[MAXN], x[MAXN], y[MAXN], c[MAXN], sa[MAXN], rank[MAXN], height[MAXN];
  void calc_sa(int n) {
3
      int m = alphabet, k = 1;
4
      memset(c, 0, sizeof(*c) * (m + 1));
5
      for (int i = 1; i \le n; ++i) c[x[i] = a[i]]++;
6
      for (int i = 1; i \le m; ++i) c[i] += c[i - 1];
7
      for (int i = n; i; --i) sa[c[x[i]]--] = i;
8
9
      for (; k <= n; k <<= 1) {
           int tot = k;
10
           for (int i = n - k + 1; i \le n; ++i) y[i - n + k] = i;
11
           for (int i = 1; i \le n; ++i)
12
               if (sa[i] > k) y[++tot] = sa[i] - k;
13
           memset(c, 0, sizeof(*c) * (m + 1));
14
           for (int i = 1; i \le n; ++i) c[x[i]]++;
15
           for (int i = 1; i \le m; ++i) c[i] += c[i - 1];
16
           for (int i = n; i; --i) sa[c[x[y[i]]]--] = y[i];
17
           for (int i = 1; i \le n; ++i) y[i] = x[i];
18
           tot = 1; x[sa[1]] = 1;
19
           for (int i = 2; i \le n; ++i) {
20
               if (max(sa[i], sa[i-1]) + k > n || y[sa[i]] != y[sa[i-1]] || y[sa[i]
21
    \rightarrow + k] != y[sa[i - 1] + k]) ++tot;
               x[sa[i]] = tot;
23
           if (tot == n) break; else m = tot;
24
25
  }
26
  void calc_height(int n) {
27
      for (int i = 1; i <= n; ++i) rank[sa[i]] = i;</pre>
28
      for (int i = 1; i \le n; ++i) {
29
           height[rank[i]] = max(0, height[rank[i - 1]] - 1);
30
           if (rank[i] == 1) continue;
31
           int j = sa[rank[i] - 1];
32
```

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```
while (max(i, j) + height[rank[i]] <= n && a[i + height[rank[i]]] == a[j +
    height[rank[i]]]) ++height[rank[i]];
}

34
35
}</pre>
```

### 4.2 后缀自动机

```
1 static const int MAXL = MAXN * 2; // MAXN is original length
static const int alphabet = 26; // sometimes need changing
 int l, last, cnt, trans[MAXL][alphabet], par[MAXL], sum[MAXL], seq[MAXL], mxl[MAXL],
    → size[MAXL]; // mxl is maxlength, size is the size of right
  char str[MAXL];
  inline void init() {
5
      l = strlen(str + 1); cnt = last = 1;
6
      for (int i = 0; i \le l * 2; ++i) memset(trans[i], 0, sizeof(trans[i]));
7
      memset(par, 0, sizeof(*par) * (l * 2 + 1));
8
      memset(mxl, 0, sizeof(*mxl) * (l * 2 + 1));
9
      memset(size, 0, sizeof(*size) * (l * 2 + 1));
10
  }
11
12
  inline void extend(int pos, int c) {
      int p = last, np = last = ++cnt;
13
      mxl[np] = mxl[p] + 1; size[np] = 1;
14
      for (; p && !trans[p][c]; p = par[p]) trans[p][c] = np;
15
      if (!p) par[np] = 1;
16
      else {
17
           int q = trans[p][c];
18
          if (mxl[p] + 1 == mxl[q]) par[np] = q;
19
          else {
20
               int nq = ++cnt;
21
               mxl[nq] = mxl[p] + 1;
22
               memcpy(trans[nq], trans[q], sizeof(trans[nq]));
23
               par[nq] = par[q];
24
               par[np] = par[q] = nq;
25
               for (; trans[p][c] == q; p = par[p]) trans[p][c] = nq;
26
          }
27
      }
28
  }
29
  inline void buildsam() {
30
      for (int i = 1; i <= l; ++i) extend(i, str[i] - 'a');
31
      memset(sum, 0, sizeof(*sum) * (l * 2 + 1));
32
      for (int i = 1; i <= cnt; ++i) sum[mxl[i]]++;</pre>
33
      for (int i = 1; i \le l; ++i) sum[i] += sum[i - 1];
34
      for (int i = cnt; i; --i) seq[sum[mxl[i]]--] = i;
35
      for (int i = cnt; i; --i) size[par[seq[i]]] += size[seq[i]];
36
37 | }
```

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### 4.3 EX 后缀自动机

```
inline void add_node(int x, int &last) {
      int lastnode = last;
2
      if (c[lastnode][x]) {
3
          int nownode = c[lastnode][x];
          if (l[nownode] == l[lastnode] + 1) last = nownode;
5
6
               int auxnode = ++cnt; l[auxnode] = l[lastnode] + 1;
7
               for (int i = 0; i < alphabet; ++i) c[auxnode][i] = c[nownode][i];
8
               par[auxnode] = par[nownode]; par[nownode] = auxnode;
9
               for (; lastnode && c[lastnode][x] == nownode; lastnode = par[lastnode])
10
    → {
                   c[lastnode][x] = auxnode;
11
               }
12
               last = auxnode;
13
          }
14
      } else {
15
          int newnode = ++cnt; l[newnode] = l[lastnode] + 1;
16
          for (; lastnode && !c[lastnode][x]; lastnode = par[lastnode]) c[lastnode][x]
17
          if (!lastnode) par[newnode] = 1;
18
          else {
19
               int nownode = c[lastnode][x];
20
               if (l[lastnode] + 1 == l[nownode]) par[newnode] = nownode;
21
               else {
22
                   int auxnode = ++cnt; l[auxnode] = l[lastnode] + 1;
23
                   for (int i = 0; i < alphabet; ++i) c[auxnode][i] = c[nownode][i];
24
                   par[auxnode] = par[nownode]; par[nownode] = par[newnode] = auxnode;
25
                   for (; lastnode && c[lastnode][x] == nownode; lastnode =
26
    → par[lastnode]) {
                       c[lastnode][x] = auxnode;
27
                   }
28
               }
29
30
          last = newnode;
31
      }
32
 }
33
```

### 4.4 回文自动机

```
int nT, nStr, last, c[MAXT][26], fail[MAXT], r[MAXN], l[MAXN], s[MAXN];
int allocate(int len) {
    l[nT] = len;
    r[nT] = 0;
    fail[nT] = 0;
    memset(c[nT], 0, sizeof(c[nT]));
    return nT++;
```

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```
8 | }
9 void init() {
      nT = nStr = 0;
10
      int newE = allocate(0);
11
      int new0 = allocate(-1);
12
13
      last = newE;
14
      fail[newE] = new0;
      fail[new0] = newE;
15
      s[0] = -1;
16
17 }
  void add(int x) {
18
      s[++nStr] = x;
19
      int now = last;
20
      while (s[nStr - l[now] - 1] != s[nStr]) now = fail[now];
21
      if (!c[now][x]) {
22
           int newnode = allocate(l[now] + 2), &newfail = fail[newnode];
23
           newfail = fail[now];
24
           while (s[nStr - l[newfail] - 1] != s[nStr]) newfail = fail[newfail];
25
           newfail = c[newfail][x];
26
           c[now][x] = newnode;
27
28
      last = c[now][x];
29
      r[last]++;
30
31 }
  void count() {
      for (int i = nT - 1; i \ge 0; i--) {
33
           r[fail[i]] += r[i];
34
      }
35
36 }
```

### Chapter 5

## 数据结构

### 5.1 KD-Tree

```
1 long long norm(const long long &x) {
             For manhattan distance
2
      //
      return std::abs(x);
3
      //
             For euclid distance
4
      return x * x;
5
  }
6
7
  struct Point {
8
9
      int x, y, id;
10
      const int& operator [] (int index) const {
11
           if (index == 0) {
12
13
               return x;
           } else {
14
               return y;
15
           }
16
      }
17
18
      friend long long dist(const Point &a, const Point &b) {
19
           long long result = 0;
20
           for (int i = 0; i < 2; ++i) {
21
               result += norm(a[i] - b[i]);
22
23
24
           return result;
25
  } point[N];
26
27
  struct Rectangle {
28
      int min[2], max[2];
29
30
      Rectangle() {
31
           min[0] = min[1] = INT_MAX;
                                        // sometimes int is not enough
32
           max[0] = max[1] = INT_MIN;
33
```

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```
}
34
35
       void add(const Point &p) {
36
           for (int i = 0; i < 2; ++i) {
37
               min[i] = std::min(min[i], p[i]);
38
39
               max[i] = std::max(max[i], p[i]);
           }
40
       }
41
42
       long long dist(const Point &p) {
43
           long long result = 0;
44
           for (int i = 0; i < 2; ++i) {
45
                      For minimum distance
                result += norm(std::min(std::max(p[i], min[i]), max[i]) - p[i]);
47
                      For maximum distance
48
                result += std::max(norm(max[i] - p[i]), norm(min[i] - p[i]));
49
50
           return result;
51
       }
52
  };
53
  struct Node {
55
       Point seperator;
56
       Rectangle rectangle;
57
       int child[2];
58
59
       void reset(const Point &p) {
60
           seperator = p;
61
           rectangle = Rectangle();
62
           rectangle.add(p);
63
           child[0] = child[1] = 0;
64
65
  } tree[N << 1];</pre>
66
67
  int size, pivot;
68
69
  bool compare(const Point &a, const Point &b) {
70
       if (a[pivot] != b[pivot]) {
71
           return a[pivot] < b[pivot];</pre>
72
73
       return a.id < b.id;</pre>
74
  }
75
76
  // 左閉右開: build(1, n + 1)
77
  int build(int l, int r, int type = 1) {
78
       pivot = type;
79
       if (l >= r) {
80
81
           return 0;
82
       }
```

5.1. KD-TREE 25

```
int x = ++size;
83
       int mid = l + r \gg 1;
       std::nth_element(point + l, point + mid, point + r, compare);
85
       tree[x].reset(point[mid]);
86
       for (int i = l; i < r; ++i) {
87
88
           tree[x].rectangle.add(point[i]);
89
       tree[x].child[0] = build(l, mid, type ^ 1);
90
       tree[x].child[1] = build(mid + 1, r, type ^1);
91
       return x;
92
   }
93
   int insert(int x, const Point &p, int type = 1) {
95
       pivot = type;
96
       if (x == 0) {
97
           tree[++size].reset(p);
98
99
           return size;
100
       tree[x].rectangle.add(p);
101
       if (compare(p, tree[x].seperator)) {
102
           tree[x].child[0] = insert(tree[x].child[0], p, type ^ 1);
103
104
           tree[x].child[1] = insert(tree[x].child[1], p, type ^ 1);
105
106
       return x;
107
   }
108
109
   // For minimum distance
110
   // For maximum: 下面递归 query 时 0, 1 换顺序;< and >;min and max
111
   void query(int x, const Point &p, std::pair<long long, int> &answer, int type = 1) {
112
       pivot = type;
113
       if (x == 0 || tree[x].rectangle.dist(p) > answer.first) {
114
           return;
115
       }
116
       answer = std::min(answer,
117
                 std::make_pair(dist(tree[x].seperator, p), tree[x].seperator.id));
118
       if (compare(p, tree[x].seperator)) {
119
           query(tree[x].child[0], p, answer, type ^ 1);
120
           query(tree[x].child[1], p, answer, type ^ 1);
121
122
           query(tree[x].child[1], p, answer, type ^ 1);
123
           query(tree[x].child[0], p, answer, type ^ 1);
124
       }
125
126
127
   std::priority_queue<std::pair<long long, int> > answer;
128
129
   void query(int x, const Point &p, int k, int type = 1) {
131
       pivot = type;
```

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```
if (x == 0 \mid | (int)answer.size() == k \&\& tree[x].rectangle.dist(p) >
132
     → answer.top().first) {
           return;
133
       }
134
       answer.push(std::make_pair(dist(tree[x].seperator, p), tree[x].seperator.id));
135
136
       if ((int)answer.size() > k) {
           answer.pop();
137
       }
138
       if (compare(p, tree[x].seperator)) {
139
           query(tree[x].child[0], p, k, type ^ 1);
140
           query(tree[x].child[1], p, k, type ^ 1);
141
       } else {
142
           query(tree[x].child[1], p, k, type ^ 1);
143
           query(tree[x].child[0], p, k, type ^ 1);
144
       }
145
146 }
```

### 5.2 Treap

```
struct Node{
2
       int mn, key, size, tag;
       bool rev;
3
       Node* ch[2];
4
       Node(int mn, int key, int size): mn(mn), key(key), size(size), rev(0), tag(0){}
5
       void downtag();
6
       Node* update(){
7
            mn = min(ch[0] \rightarrow mn, min(key, ch[1] \rightarrow mn));
8
            size = ch[0] -> size + 1 + ch[1] -> size;
9
            return this;
10
       }
11
12 };
13 typedef pair<Node*, Node*> Pair;
14 Node *null, *root;
  void Node::downtag(){
15
       if(rev){
16
            for(int i = 0; i < 2; i++)
17
                 if(ch[i] != null){
18
                     ch[i] -> rev ^= 1;
19
                     swap(ch[i] \rightarrow ch[0], ch[i] \rightarrow ch[1]);
20
21
            rev = 0;
22
       }
23
       if(tag){
24
            for(int i = 0; i < 2; i++)
25
                 if(ch[i] != null){
26
                     ch[i] \rightarrow key += tag;
27
                     ch[i] -> mn += tag;
28
                     ch[i] \rightarrow tag += tag;
29
```

5.2. TREAP 27

```
30
            tag = 0;
32
  }
33
  int r(){
34
35
       static int s = 3023192386;
       return (s += (s << 3) + 1) & (\sim0u >> 1);
36
  }
37
  bool random(int x, int y){
38
       return r() % (x + y) < x;
39
  }
40
  Node* merge(Node *p, Node *q){
41
       if(p == null) return q;
42
       if(q == null) return p;
43
       p -> downtag();
44
       q -> downtag();
45
46
       if(random(p -> size, q -> size)){
            p -> ch[1] = merge(p -> ch[1], q);
47
            return p -> update();
48
       }else{
49
            q \rightarrow ch[0] = merge(p, q \rightarrow ch[0]);
50
            return q -> update();
51
52
  }
53
  Pair split(Node *x, int n){
       if(x == null) return make_pair(null, null);
55
       x -> downtag();
56
       if(n \ll x \rightarrow ch[0] \rightarrow size){
57
            Pair ret = split(x \rightarrow ch[0], n);
58
            x \rightarrow ch[0] = ret.second;
59
            return make_pair(ret.first, x -> update());
60
61
       Pair ret = split(x \rightarrow ch[1], n - x \rightarrow ch[0] \rightarrow size - 1);
62
       x \rightarrow ch[1] = ret.first;
63
       return make_pair(x -> update(), ret.second);
64
  }
65
  pair<Node*, Pair> get_segment(int l, int r){
66
       Pair ret = split(root, l - 1);
67
       return make_pair(ret.first, split(ret.second, r - l + 1));
68
  }
69
  int main(){
70
       null = new Node(INF, INF, 0);
71
       null \rightarrow ch[0] = null \rightarrow ch[1] = null;
72
       root = null;
73
74 }
```

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### 5.3 Link/cut Tree

```
inline void reverse(int x) {
      tr[x].rev ^= 1; swap(tr[x].c[0], tr[x].c[1]);
2
  }
3
  inline void rotate(int x, int k) {
5
      int y = tr[x].fa, z = tr[y].fa;
6
      tr[x].fa = z; tr[z].c[tr[z].c[1] == y] = x;
7
      tr[tr[x].c[k ^ 1]].fa = y; tr[y].c[k] = tr[x].c[k ^ 1];
8
      tr[x].c[k ^ 1] = y; tr[y].fa = x;
9
  }
10
11
  inline void splay(int x, int w) {
12
      int z = x; pushdown(x);
13
      while (tr[x].fa != w) {
14
           int y = tr[x].fa; z = tr[y].fa;
15
           if (z == w) {
16
               pushdown(z = y); pushdown(x);
17
               rotate(x, tr[y].c[1] == x);
18
               update(y); update(x);
19
           } else {
20
               pushdown(z); pushdown(y); pushdown(x);
21
               int t1 = tr[y].c[1] == x, t2 = tr[z].c[1] == y;
22
               if (t1 == t2) rotate(y, t2), rotate(x, t1);
23
               else rotate(x, t1), rotate(x, t2);
24
               update(z); update(y); update(x);
25
           }
26
      }
27
      update(x);
      if (x != z) par[x] = par[z], par[z] = 0;
29
  }
30
31
  inline void access(int x) {
32
      for (int y = 0; x; y = x, x = par[x]) {
33
           splay(x, 0);
34
           if (tr[x].c[1]) par[tr[x].c[1]] = x, tr[tr[x].c[1]].fa = 0;
35
           tr[x].c[1] = y; par[y] = 0; tr[y].fa = x; update(x);
36
      }
37
38
  }
39
  inline void makeroot(int x) {
40
      access(x); splay(x, 0); reverse(x);
41
  }
42
43
  inline void link(int x, int y) {
44
      makeroot(x); par[x] = y;
45
46 | }
47
```

5.3. LINK/CUT TREE

```
inline void cut(int x, int y) {
    access(x); splay(y, 0);
    if (par[y] != x) swap(x, y), access(x), splay(y, 0);
    par[y] = 0;
}
inline void split(int x, int y) { // x will be the root of the tree
    makeroot(y); access(x); splay(x, 0);
}
```

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### Chapter 6

## 图论

### 6.1 基础

```
struct Graph { // Remember to call .init()!
1
2
      int e, nxt[M], v[M], adj[N], n;
      bool base;
3
      __inline void init(bool _base, int _n = 0) {
           assert(n < N);
5
           n = _n; base = _base;
6
           e = 0; memset(adj + base, -1, sizeof(*adj) * n);
7
8
      __inline int new_node() {
9
           adj[n + base] = -1;
10
           assert(n + base + 1 < N);
11
           return n++ + base;
12
13
      __inline void ins(int u0, int v0) { // directional
14
           assert(u0 < n + base \&\& v0 < n + base);
15
           v[e] = v0; nxt[e] = adj[u0]; adj[u0] = e++;
16
           assert(e < M);
17
      }
18
      __inline void bi_ins(int u0, int v0) { // bi-directional
19
           ins(u0, v0); ins(v0, u0);
20
      }
21
22 | };
```

### 6.2 KM

```
struct KM {
    // Truly 0(n^3)
    // 邻接矩阵,不能连的边设为 -INF,求最小权匹配时边权取负,但不能连的还是 -INF,使用时先对 1
    →-> n 调用 hungary() ,再 get_ans() 求值
    int w[N] [N];
    int lx[N], ly[N], match[N], way[N], slack[N];
    bool used[N];
```

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```
void init() {
7
           for (int i = 1; i <= n; i++) {
8
                match[i] = 0;
9
                lx[i] = 0;
10
                ly[i] = 0;
11
12
                way[i] = 0;
13
14
       void hungary(int x) {
15
           match[0] = x;
16
           int j0 = 0;
17
           for (int j = 0; j <= n; j++) {
18
                slack[j] = INF;
19
                used[j] = false;
20
           }
21
22
           do {
23
                used[j0] = true;
24
                int i0 = match[j0], delta = INF, j1 = 0;
25
                for (int j = 1; j \le n; j++) {
26
                     if (used[j] == false) {
27
                         int cur = -w[i0][j] - lx[i0] - ly[j];
28
                         if (cur < slack[j]) {</pre>
29
                              slack[j] = cur;
30
                              way[j] = j0;
31
                         }
32
                         if (slack[j] < delta) {</pre>
33
                              delta = slack[j];
34
                              j1 = j;
35
                         }
36
                     }
37
38
                for (int j = 0; j \le n; j++) {
39
                     if (used[j]) {
40
                         lx[match[j]] += delta;
41
                         ly[j] -= delta;
42
                     }
43
                     else slack[j] -= delta;
44
                }
45
                j0 = j1;
46
           } while (match[j0] != 0);
47
48
           do {
49
                int j1 = way[j0];
50
                match[j0] = match[j1];
51
                j0 = j1;
52
           } while (j0);
53
       }
54
55
```

6.3. 点双连通分量 33

```
int get_ans() {
    int sum = 0;
    for(int i = 1; i <= n; i++) {
        if (w[match[i]][i] == -INF); // 无解
        if (match[i] > 0) sum += w[match[i]][i];
    }
    return sum;
}
km;
```

### 6.3 点双连通分量

bcc.forest is a set of connected tree whose vertices are chequered with cut-vertex and BCC.

```
const bool BCC_VERTEX = 0, BCC_EDGE = 1;
  struct BCC { // N = N0 + M0. Remember to call init(&raw_graph).
2
       Graph *g, forest; // g is raw graph ptr.
3
       int dfn[N], DFN, low[N];
       int stack[N], top;
5
       int expand_to[N];
                                // Where edge i is expanded to in expaned graph.
6
       // Vertex i expaned to i.
       int compress_to[N]; // Where vertex i is compressed to.
bool vertex_type[N], cut[N], compress_cut[N], branch[M];
8
9
       //std::vector<int> BCC_component[N]; // Cut vertex belongs to none.
10
       __inline void init(Graph *raw_graph) {
11
           q = raw graph;
12
13
       void DFS(int u, int pe) {
           dfn[u] = low[u] = ++DFN; cut[u] = false;
15
           if (!\sim q->adi[u]) {
16
                cut[u] = 1;
17
                compress_to[u] = forest.new_node();
18
                compress_cut[compress_to[u]] = 1;
19
20
           for (int e = g->adj[u]; \sim e; e = g->nxt[e]) {
21
                int v = g -> v[e];
                if ((e ^pe) > 1 \& dfn[v] > 0 \& dfn[v] < dfn[u]) {
23
                    stack[top++] = e;
24
                    low[u] = std::min(low[u], dfn[v]);
25
26
                else if (!dfn[v]) {
27
                    stack[top++] = e; branch[e] = 1;
28
                    DFS(v, e);
29
                    low[u] = std::min(low[v], low[u]);
30
                    if (low[v] >= dfn[u]) {
31
                         if (!cut[u]) {
32
                             cut[u] = 1;
33
                             compress_to[u] = forest.new_node();
                             compress cut[compress to[u]] = 1;
35
```

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```
}
36
                        int cc = forest.new_node();
                        forest.bi_ins(compress_to[u], cc);
38
                        compress_cut[cc] = 0;
39
                        //BCC_component[cc].clear();
40
41
                        do {
                             int cur_e = stack[--top];
42
                             compress_to[expand_to[cur_e]] = cc;
43
                             compress_to[expand_to[cur_e^1]] = cc;
44
                             if (branch[cur_e]) {
45
                                 int v = g -> v[cur_e];
46
                                 if (cut[v])
                                      forest.bi_ins(cc, compress_to[v]);
49
                                      //BCC_component[cc].push_back(v);
50
                                      compress_to[v] = cc;
51
52
53
                        } while (stack[top] != e);
54
                    }
55
               }
56
           }
57
58
      void solve() {
59
           forest.init(g->base);
60
           int n = g->n;
61
           for (int i = 0; i < g -> e; i ++) {
62
               expand_to[i] = g->new_node();
63
           memset(branch, 0, sizeof(*branch) * g->e);
65
           memset(dfn + g->base, 0, sizeof(*dfn) * n); DFN = 0;
66
           for (int i = 0; i < n; i++)
67
               if (!dfn[i + g->base]) {
68
                    top = 0;
69
                    DFS(i + g -> base, -1);
70
               }
71
72
  } bcc;
73
74
75 bcc.init(&raw_graph);
76 bcc.solve();
77 // Do something with bcc.forest ...
```

### 6.4 边双连通分量

```
struct BCC {
    Graph *g, forest;
    int dfn[N], low[N], stack[N], tot[N], belong[N], vis[N], top, dfs_clock;
```

6.4. 边双连通分量 35

```
// tot[] is the size of each BCC, belong[] is the BCC that each node belongs to
4
       pair<int, int > ori[M]; // bridge in raw_graph(raw node)
5
       bool is_bridge[M];
6
       __inline void init(Graph *raw_graph) {
7
8
           g = raw_graph;
           memset(is_bridge, false, sizeof(*is_bridge) * g -> e);
9
           memset(vis + g \rightarrow base, 0, sizeof(*vis) * g \rightarrow n);
10
11
       void tarjan(int u, int from) {
12
           dfn[u] = low[u] = ++dfs\_clock; vis[u] = 1; stack[++top] = u;
13
           for (int p = g \rightarrow adj[u]; \sim p; p = g \rightarrow nxt[p]) {
14
                if ((p ^ 1) == from) continue;
15
                int v = g \rightarrow v[p];
16
                if (vis[v]) {
17
                     if (vis[v] == 1) low[u] = min(low[u], dfn[v]);
18
19
                } else {
20
                     tarjan(v, p);
                     low[u] = min(low[u], low[v]);
21
                     if (low[v] > dfn[u]) is_bridge[p / 2] = true;
22
                }
23
           }
           if (dfn[u] != low[u]) return;
25
           tot[forest.new_node()] = 0;
26
           do {
27
                belong[stack[top]] = forest.n;
28
                vis[stack[top]] = 2;
29
                tot[forest.n]++;
30
                --top;
31
           } while (stack[top + 1] != u);
32
33
       void solve() {
34
           forest.init(g -> base);
35
            int n = g -> n;
36
           for (int i = 0; i < n; ++i)
                if (!vis[i + g \rightarrow base]) {
38
                     top = dfs_clock = 0;
39
                     tarjan(i + g \rightarrow base, -1);
40
41
           for (int i = 0; i < g -> e / 2; ++i)
42
                if (is_bridge[i]) {
43
                     int e = forest.e;
44
                     forest.bi_ins(belong[g \rightarrow v[i * 2]], belong[g \rightarrow v[i * 2 + 1]], g \rightarrow
45
    \hookrightarrow W[i * 2]);
                     ori[e] = make_pair(g -> v[i * 2 + 1], g -> v[i * 2]);
46
                     ori[e + 1] = make_pair(g -> v[i * 2], g -> v[i * 2 + 1]);
47
                }
48
49
50 | } bcc;
```

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### 6.5 最小树形图

```
1 const int MAXN, INF;// INF >= sum( W_ij )
_{2} int from [MAXN + 10] [MAXN * 2 + 10], n, m, edge [MAXN + 10] [MAXN * 2 + 10];
\frac{1}{2} int sel[MAXN * 2 + 10], fa[MAXN * 2 + 10], vis[MAXN * 2 + 10];
4 | int getfa(int x){if(x == fa[x]) return x; return fa[x] = getfa(fa[x]);}
  void liuzhu(){ // 1-base: root is 1, answer = (sel[i], i) for i in [2..n]
5
      fa[1] = 1;
6
      for(int i = 2; i \le n; ++i){
7
           sel[i] = 1; fa[i] = i;
8
           for(int j = 1; j <= n; ++j) if(fa[j] != i)
9
               if(from[j][i] = i, edge[sel[i]][i] > edge[j][i]) sel[i] = j;
10
      }
11
      int limit = n;
12
      while(1){
13
           int prelimit = limit; memset(vis, 0, sizeof(vis)); vis[1] = 1;
           for(int i = 2; i \le prelimit; ++i) if(fa[i] == i && !vis[i]){
15
               int j = i; while(!vis[j]) vis[j] = i, j = getfa(sel[j]);
16
               if(j == 1 || vis[j] != i) continue; vector<int> C; int k = j;
17
               do C.push_back(k), k = getfa(sel[k]); while(k != j);
18
               ++limit:
19
               for(int i = 1; i \le n; ++i){
20
                   edge[i][limit] = INF, from[i][limit] = limit;
21
22
               fa[limit] = vis[limit] = limit;
23
               for(int i = 0; i < int(C.size()); ++i){</pre>
24
                    int x = C[i], fa[x] = limit;
25
                   for(int j = 1; j <= n; ++j)
26
                        if(edge[j][x] != INF \&\& edge[j][limit] > edge[j][x] -
27
    \rightarrow edge[sel[x]][x]){
                            edge[j][limit] = edge[j][x] - edge[sel[x]][x];
28
                            from[j][limit] = x;
29
                        }
30
               }
31
               for(int j=1;j<=n;++j) if(getfa(j)==limit) edge[j][limit] = INF;</pre>
32
               sel[limit] = 1;
33
               for(int j = 1; j <= n; ++j)
34
                   if(edge[sel[limit]][limit] > edge[j][limit]) sel[limit] = j;
35
36
           if(prelimit == limit) break;
37
38
      for(int i = limit; i > 1; --i) sel[from[sel[i]][i]] = sel[i];
39
40 | }
```

### 6.6 带花树

```
vector<int> link[maxn];
int n,match[maxn],Queue[maxn],head,tail;
```

6.6. 带花树 37

```
int pred[maxn],base[maxn],start,finish,newbase;
 bool InQueue[maxn], InBlossom[maxn];
5 void push(int u){ Queue[tail++]=u;InQueue[u]=true; }
6 int pop(){ return Queue[head++]; }
  int FindCommonAncestor(int u,int v){
8
      bool InPath[maxn];
9
      for(int i=0;i<n;i++) InPath[i]=0;</pre>
      while(true){ u=base[u];InPath[u]=true;if(u==start) break;u=pred[match[u]]; }
10
      while(true){ v=base[v];if(InPath[v]) break;v=pred[match[v]]; }
11
      return v;
12
  }
13
  void ResetTrace(int u){
14
      int v;
15
      while(base[u]!=newbase){
16
           v=match[u];
17
           InBlossom[base[u]]=InBlossom[base[v]]=true;
18
           u=pred[v];
19
           if(base[u]!=newbase) pred[u]=v;
20
      }
21
  }
22
  void BlossomContract(int u,int v){
23
      newbase=FindCommonAncestor(u,v);
24
      for (int i=0;i<n;i++)</pre>
25
      InBlossom[i]=0;
26
      ResetTrace(u);ResetTrace(v);
27
      if(base[u]!=newbase) pred[u]=v;
28
      if(base[v]!=newbase) pred[v]=u;
29
      for(int i=0;i<n;++i)</pre>
30
      if(InBlossom[base[i]]){
31
           base[i]=newbase;
32
           if(!InQueue[i]) push(i);
33
      }
34
  }
35
  bool FindAugmentingPath(int u){
36
      bool found=false;
37
      for(int i=0;i<n;++i) pred[i]=-1,base[i]=i;</pre>
38
      for (int i=0;i<n;i++) InQueue[i]=0;</pre>
39
      start=u;finish=-1; head=tail=0; push(start);
40
      while(head<tail){</pre>
41
           int u=pop();
42
           for(int i=link[u].size()-1;i>=0;i--){
43
               int v=link[u][i]:
44
               if(base[u]!=base[v]&&match[u]!=v)
                    if(v==start||(match[v]>=0&&pred[match[v]]>=0))
                        BlossomContract(u,v);
47
                    else if(pred[v]==-1){
48
                        pred[v]=u;
49
                        if(match[v]>=0) push(match[v]);
50
                        else{ finish=v; return true; }
51
```

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```
}
52
           }
54
       return found;
55
  }
56
57
  void AugmentPath(){
58
       int u=finish,v,w;
       while(u>=0){ v=pred[u]; w=match[v]; match[v]=u; match[u]=v; u=w; }
59
  }
60
  void FindMaxMatching(){
61
       for(int i=0;i<n;++i) match[i]=-1;</pre>
62
       for(int i=0;i<n;++i) if(match[i]==-1) if(FindAugmentingPath(i)) AugmentPath();</pre>
63
  }
```

#### 6.7 Dominator Tree

```
vector<int> prec[N], succ[N];
  vector<int> ord;
  int stamp, vis[N];
  int num[N];
5
  int fa[N];
  void dfs(int u) {
6
      vis[u] = stamp;
7
      num[u] = ord.size();
8
      ord.push back(u);
9
      for (int i = 0; i < (int)succ[u].size(); ++i) {
10
           int v = succ[u][i];
11
           if (vis[v] != stamp) {
12
               fa[v] = u;
13
               dfs(v);
14
           }
15
      }
16
  }
17
  int fs[N], mins[N], dom[N], sem[N];
18
  int find(int u) {
19
      if (u != fs[u]) {
20
           int v = fs[u];
21
           fs[u] = find(fs[u]);
           if (\min s[v] != -1 \&\& num[sem[mins[v]]] < num[sem[mins[u]]]) {
23
               mins[u] = mins[v];
24
           }
25
      }
26
      return fs[u];
27
  }
28
  void merge(int u, int v) { fs[u] = v; }
29
30 | vector<int> buf[N];
31 int buf2[N];
32 | void mark(int source) {
```

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```
ord.clear();
33
       ++stamp;
34
       dfs(source);
35
       for (int i = 0; i < (int)ord.size(); ++i) {
36
           int u = ord[i];
37
38
           fs[u] = u, mins[u] = -1, buf2[u] = -1;
39
       for (int i = (int) ord.size() - 1; i > 0; --i) {
40
           int u = ord[i], p = fa[u];
41
           sem[u] = p;
42
           for (int j = 0; j < (int)prec[u].size(); ++j) {
43
                int v = prec[u][j];
                if (use[v] != stamp) continue;
                if (num[v] > num[u]) {
46
                    find(v); v = sem[mins[v]];
47
                }
48
                if (num[v] < num[sem[u]]) {</pre>
49
                    sem[u] = v;
50
                }
51
           }
52
           buf[sem[u]].push_back(u);
           mins[u] = u;
54
           merge(u, p);
55
           while (buf[p].size()) {
56
                int v = buf[p].back();
57
                buf[p].pop_back();
58
                find(v);
59
                if (sem[v] == sem[mins[v]]) {
60
                    dom[v] = sem[v];
61
                } else {
62
                    buf2[v] = mins[v];
63
64
           }
65
       }
66
       dom[ord[0]] = ord[0];
67
       for (int i = 0; i < (int)ord.size(); ++i) {</pre>
68
69
           int u = ord[i];
70
           if (~buf2[u]) {
                dom[u] = dom[buf2[u]];
71
           }
72
       }
73
74 }
```

#### 6.8 无向图最小割

```
int cost[maxn][maxn],seq[maxn],len[maxn],n,m,pop,ans;
bool used[maxn];
void Init(){
```

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```
int i,j,a,b,c;
4
       for(i=0;i<n;i++) for(j=0;j<n;j++) cost[i][j]=0;</pre>
5
       for(i=0;i<m;i++){
6
           scanf("%d %d %d",&a,&b,&c); cost[a][b]+=c; cost[b][a]+=c;
7
8
9
       pop=n; for(i=0;i<n;i++) seq[i]=i;
  }
10
  void Work(){
11
       ans=inf; int i,j,k,l,mm,sum,pk;
12
       while(pop > 1){
13
           for(i=1;i<pop;i++) used[seq[i]]=0; used[seq[0]]=1;</pre>
14
           for(i=1;i<pop;i++) len[seq[i]]=cost[seq[0]][seq[i]];</pre>
15
           pk=0; mm=-inf; k=-1;
16
           for(i=1;i<pop;i++) if(len[seq[i]] > mm){ mm=len[seq[i]]; k=i; }
17
           for(i=1;i<pop;i++){
18
               used[seq[l=k]]=1;
19
20
                if(i==pop-2) pk=k;
               if(i==pop-1) break;
21
               mm=-inf;
22
               for(j=1;j<pop;j++) if(!used[seq[j]])</pre>
23
                    if((len[seq[j]]+=cost[seq[l]][seq[j]]) > mm)
                        mm=len[seq[j]], k=j;
25
           }
26
           sum=0;
27
           for(i=0;i<pop;i++) if(i != k) sum+=cost[seq[k]][seq[i]];</pre>
28
           ans=min(ans,sum);
29
           for(i=0;i<pop;i++)</pre>
30
               cost[seq[k]][seq[i]]=cost[seq[i]][seq[k]]+=cost[seq[pk]][seq[i]];
31
           seq[pk]=seq[--pop];
32
33
       printf("%d\n",ans);
34
35 | }
```

## Chapter 7

# 其他

#### 7.1 Dancing Links

```
struct Node {
2
       Node *1, *r, *u, *d, *col;
       int size, line_no;
3
       Node() {
            size = 0; line_no = -1;
5
6
            l = r = u = d = col = NULL;
7
  } *root;
8
9
  void cover(Node *c) {
10
       c->l->r = c->r; c->r->l = c->l;
11
       for (Node *u = c->d; u != c; u = u->d)
12
            for (Node *v = u->r; v != u; v = v->r) {
13
                v->d->u = v->u;
14
                v\rightarrow u\rightarrow d = v\rightarrow d;
15
                -- v->col->size;
16
            }
17
  }
18
19
  void uncover(Node *c) {
20
       for (Node *u = c->u; u != c; u = u->u) {
21
            for (Node *v = u->l; v != u; v = v->l) {
22
                ++ v->col->size;
23
                v\rightarrow u\rightarrow d = v;
24
                v->d->u = v;
25
            }
26
27
       c->l->r = c; c->r->l = c;
28
29
30
31 std::vector<int> answer;
32 bool search(int k) {
       if (root->r == root) return true;
```

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```
Node *r = NULL;
34
       for (Node *u = root->r; u != root; u = u->r)
35
           if (r == NULL || u->size < r->size)
36
                r = u;
37
       if (r == NULL || r->size == 0) return false;
38
39
       else {
           cover(r);
40
           bool succ = false;
41
           for (Node *u = r -> d; u != r \&\& !succ; u = u -> d) {
42
                answer.push_back(u->line_no);
43
                for (Node *v = u \rightarrow r; v != u; v = v \rightarrow r) // Cover row
44
                    cover(v->col);
45
                succ \mid = search(k + 1);
                for (Node *v = u -> 1; v != u; v = v -> 1)
47
                    uncover(v->col);
48
                if (!succ) answer.pop_back();
49
50
           }
           uncover(r);
51
           return succ;
52
       }
53
  }
54
55
  bool entry[CR][CC];
57 Node *who[CR][CC];
 int cr, cc;
59
  void construct() {
60
       root = new Node();
61
       Node *last = root;
62
       for (int i = 0; i < cc; ++ i) {
63
           Node *u = new Node();
64
           last->r = u; u->l = last;
65
           Node *v = u; u->line_no = i;
66
           last = u;
67
           for (int j = 0; j < cr; ++ j)
68
                if (entry[j][i]) {
69
70
                    ++ u->size;
                    Node *cur = new Node();
71
                    who[j][i] = cur;
72
                    cur->line_no = j;
73
                    cur->col = u;
74
                    cur->u = v; v->d = cur;
75
                    v = cur;
76
77
           v->d = u; u->u = v;
78
79
       last->r = root; root->l = last;
80
       for (int j = 0; j < cr; ++ j) {
81
82
           Node *last = NULL;
```

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```
for (int i = cc - 1; i \ge 0; -- i)
83
                if (entry[j][i]) {
                     last = who[j][i];
85
                     break;
86
                }
87
            for (int i = 0; i < cc; ++ i)
88
89
                if (entry[j][i]) {
                     last->r = who[j][i];
90
                     who[j][i]->l = last;
91
                     last = who[j][i];
                }
93
       }
94
   }
95
96
   void destruct() {
97
       for (Node *u = root->r; u != root; ) {
98
            for (Node *v = u -> d; v != u; ) {
99
100
                Node *nxt = v->d;
                delete(v);
101
                v = nxt;
102
            }
103
            Node *nxt = u->r;
104
            delete(u); u = nxt;
105
106
       delete root;
107
108 }
```

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### Chapter 8

# 技巧

#### 8.1 真正的释放 STL 容器内存空间

```
template <typename T>
__inline void clear(T& container) {
    container.clear(); // 或者删除了一堆元素
    T(container).swap(container);
}
```

#### 8.2 无敌的大整数相乘取模

Time complexity O(1).

#### 8.3 无敌的读入优化

```
|// getchar() 读入优化 << 关同步        cin << 此优化
 // 用 isdigit() 会小幅变慢
 |// 返回 false 表示读到文件尾
  namespace Reader {
      const int L = (1 << 15) + 5;
5
      char buffer[L], *S, *T;
6
      __inline bool getchar(char &ch) {
          if (S == T) {
8
              T = (S = buffer) + fread(buffer, 1, L, stdin);
9
              if (S == T) {
10
                  ch = EOF;
11
                  return false;
12
```

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```
}
13
           }
14
           ch = *S++;
15
           return true;
16
17
      __inline bool getint(int &x) {
18
19
           char ch; bool neg = 0;
           for (; getchar(ch) && (ch < '0' || ch > '9'); ) neg ^= ch == '-';
20
           if (ch == EOF) return false;
21
           x = ch - '0';
22
           for (; getchar(ch), ch >= '0' && ch <= '9'; )
23
               x = x * 10 + ch - '0';
24
           if (neg) x = -x;
25
           return true;
26
      }
27
28 | }
```

# Chapter 9

# 提示

### 9.1 控制 cout 输出实数精度

```
std::cout << std::fixed << std::setprecision(5);
```

#### 9.2 vimrc

```
set nu
set sw=4
set sts=4
set ts=4
syntax on
set cindent
```

### 9.3 让 make 支持 c ++ 11

In .bashrc or whatever:

```
export CXXFLAGS='-std=c++11 -Wall'
```

#### 9.4 线性规划转对偶

$$\begin{array}{l} \text{maximize } \mathbf{c}^T \mathbf{x} \\ \text{subject to } \mathbf{A} \mathbf{x} \leq \mathbf{b}, \mathbf{x} \geq 0 \\ \end{array} \Longrightarrow \begin{array}{l} \text{minimize } \mathbf{y}^T \mathbf{b} \\ \text{subject to } \mathbf{y}^T \mathbf{A} \geq \mathbf{c}^T, \mathbf{y} \geq 0 \end{array}$$

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# 9.5 32-bit/64-bit 随机素数

32-bit	64-bit
73550053	1249292846855685773
148898719	1701750434419805569
189560747	3605499878424114901
459874703	5648316673387803781
1202316001	6125342570814357977
1431183547	6215155308775851301
1438011109	6294606778040623451
1538762023	6347330550446020547
1557944263	7429632924303725207
1981315913	8524720079480389849

### 9.6 NTT 素数及其原根

Prime	Primitive	root
1053818881	7	
1051721729	6	
1045430273	3	
1012924417	5	
1007681537	3	