## Gungnir's Standard Code Library

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# 计算几何

#### 1.1 二维

#### 1.1.1 基础

```
typedef double DB;
  const DB eps = 1e-8;
  int sign(DB x) {
      return x < -eps ? -1 : (x > eps ? 1 : 0);
5
  }
6
7
  DB msqrt(DB x) {
8
      return sign(x) > 0 ? sqrt(x) : 0;
9 }
10
  struct Point {
11
      DB x, y;
12
13
      Point(): x(0), y(0) {}
14
      Point(DB x, DB y): x(x), y(y) {}
15
      Point operator+(const Point &rhs) const {
          return Point(x + rhs.x, y + rhs.y);
16
17
      Point operator-(const Point &rhs) const {
18
           return Point(x - rhs.x, y - rhs.y);
19
20
21
      Point operator*(DB k) const {
22
          return Point(x * k, y * k);
23
      Point operator/(DB k) const {
24
          assert(sign(k));
25
           return Point(x / k, y / k);
26
27
      Point rotate(DB ang) const { // 逆时针旋转 ang 弧度
28
           return Point(cos(ang) * x - sin(ang) * y,
29
                   cos(ang) * y + sin(ang) * x);
30
31
      Point turn90() const { // 逆时针旋转 90 度
32
          return Point(-y, x);
33
34
```

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```
Point unit() const {
35
           return *this / len();
36
37
38
  };
  DB dot(const Point& a, const Point& b) {
39
40
      return a.x * b.x + a.y * b.y;
  | }
41
  DB det(const Point& a, const Point& b) {
42
       return a.x * b.y - a.y * b.x;
43
44 | }
45 | \text{#define cross}(p1,p2,p3) ((p2.x-p1.x)*(p3.y-p1.y)-(p3.x-p1.x)*(p2.y-p1.y)) |
46 | #define crossOp(p1,p2,p3) sign(cross(p1,p2,p3))
47 bool isLL(const Line& l1, const Line& l2, Point& p) { // 直线与直线交点
48
      DB s1 = det(l2.b - l2.a, l1.a - l2.a),
          s2 = -det(l2.b - l2.a, l1.b - l2.a);
49
      if (!sign(s1 + s2)) return false;
50
      p = (l1.a * s2 + l1.b * s1) / (s1 + s2);
51
52
      return true;
53
  bool onSeg(const Line& l, const Point& p) { // 点在线段上
55
      return sign(det(p - l.a, l.b - l.a)) == 0 \& sign(dot(p - l.a, p - l.b)) <= 0;
56
57 Point projection(const Line & l, const Point& p) {
      return l.a + (l.b - l.a) * (dot(p - l.a, l.b - l.a) / (l.b - l.a).len2());
58
59 }
60|DB disToLine(const Line& l, const Point& p) { // 点到 * 直线 * 距离
      return fabs(det(p - l.a, l.b - l.a) / (l.b - l.a).len());
61
62 }
  |DB disToSeg(const Line& l, const Point& p) {    // 点到线段距离
63
      return sign(dot(p - l.a, l.b - l.a)) * sign(dot(p - l.b, l.a - l.b)) == 1 ? disToLine(l, p) :
     \rightarrow std::min((p - l.a).len(), (p - l.b).len());
  }
65
  // 圆与直线交点
66
  bool isCL(Circle a, Line l, Point& p1, Point& p2) {
67
      DB x = dot(l.a - a.o, l.b - l.a),
68
69
          y = (l.b - l.a).len2(),
          d = x * x - y * ((l.a - a.o).len2() - a.r * a.r);
70
71
       if (sign(d) < 0) return false;</pre>
      Point p = l.a - ((l.b - l.a) * (x / y)), delta = (l.b - l.a) * (msqrt(d) / y);
72
      p1 = p + delta; p2 = p - delta;
73
      return true;
74
75 }
76 //圆与圆的交面积
77 DB areaCC(const Circle& c1, const Circle& c2) {
      DB d = (c1.0 - c2.0).len();
78
79
       if (sign(d - (c1.r + c2.r)) >= 0) return 0;
      if (sign(d - std::abs(c1.r - c2.r)) \le 0) {
80
          DB r = std::min(c1.r, c2.r);
81
82
           return r * r * PI;
83
      DB x = (d * d + c1.r * c1.r - c2.r * c2.r) / (2 * d),
84
          t1 = acos(x / c1.r), t2 = acos((d - x) / c2.r);
85
       return c1.r * c1.r * t1 + c2.r * c2.r * t2 - d * c1.r * sin(t1);
86
87 }
```

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```
88 // 圆与圆交点
   bool isCC(Circle a, Circle b, P& p1, P& p2) {
       DB s1 = (a.o - b.o).len();
90
       if (sign(s1 - a.r - b.r) > 0 \mid | sign(s1 - std::abs(a.r - b.r)) < 0) return false;
91
       DB s2 = (a.r * a.r - b.r * b.r) / s1;
92
       DB aa = (s1 + s2) * 0.5, bb = (s1 - s2) * 0.5;
93
       P o = (b.o - a.o) * (aa + bb)) + a.o;
94
       P delta = (b.o - a.o).unit().turn90() * msqrt(a.r * a.r - aa * aa);
95
       p1 = o + delta, p2 = o - delta;
96
       return true;
97
   }
98
   // 求点到圆的切点,按关于点的顺时针方向返回两个点
99
   bool tanCP(const Circle &c, const Point &p0, Point &p1, Point &p2) {
       double x = (p0 - c.o).len2(), d = x - c.r * c.r;
101
       if (d < eps) return false; // 点在圆上认为没有切点
102
103
       Point p = (p0 - c.o) * (c.r * c.r / x);
       Point delta = ((p0 - c.o) * (-c.r * sqrt(d) / x)).turn90();
104
       p1 = c.o + p + delta;
105
       p2 = c.o + p - delta;
106
107
       return true;
108
   }
   // 求圆到圆的内共切线,按关于 c1.o 的顺时针方向返回两条线
109
   std::vector<Line> intanCC(const Circle &c1, const Circle &c2) {
       std::vector<Line> ret;
111
       Point p = (c1.0 * c2.r + c2.0 * c1.r) / (c1.r + c2.r);
112
       Point p1, p2, q1, q2;
113
       if (tanCP(c1, p, p1, p2) && tanCP(c2, p, q1, q2)) { // 两圆相切认为没有切线
114
           ret.push_back(Line(p1, q1));
115
           ret.push back(Line(p2, q2));
116
       }
117
118
       return ret;
119 }
120 // 点在多边形内
   bool inPolygon(const Point& p, const std::vector<Point>& poly) {
121
       int n = poly.size();
122
       int counter = 0;
123
       for (int i = 0; i < n; ++ i) {
124
           P = poly[i], b = poly[(i + 1) % n];
125
           if (onSeg(Line(a, b), p)) return false; // 边界上不算
126
127
           int x = sign(det(p - a, b - a));
           int y = sign(a.y - p.y);
128
           int z = sign(b.y - p.y);
129
           if (x > 0 \& y \le 0 \& z > 0) ++ counter;
130
           if (x < 0 \&\& z \le 0 \&\& y > 0) -- counter;
131
       }
132
       return counter != 0;
133
134 | }
135 // 用半平面 (q1,q2) 的逆时针方向去切凸多边形
   std::vector<Point> convexCut(const std::vector<Point>&ps, Point q1, Point q2) {
136
       std::vector<Point> qs; int n = ps.size();
137
138
       for (int i = 0; i < n; ++i) {
           Point p1 = ps[i], p2 = ps[(i + 1) % n];
139
140
           int d1 = cross0p(q1,q2,p1), d2 = cross0p(q1,q2,p2);
           if (d1 >= 0) qs.push_back(p1);
141
```

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```
if (d1 * d2 < 0) qs.push_back(isSS(p1, p2, q1, q2));
142
143
144
        return qs;
145
   // 求凸包
146
    std::vector<Point> convexHull(std::vector<Point> ps) {
147
        int n = ps.size(); if (n <= 1) return ps;</pre>
148
        std::sort(ps.begin(), ps.end());
149
        std::vector<Point> qs;
150
        for (int i = 0; i < n; qs.push_back(ps[i ++]))</pre>
151
152
            while (qs.size() > 1 \&\& sign(det(qs[qs.size() - 2], qs.back(), ps[i])) <= 0)
153
                qs.pop back();
        for (int i = n - 2, t = qs.size(); i \ge 0; qs.push_back(ps[i --]))
            while ((int)qs.size() > t \& sign(det(qs[qs.size() - 2], qs.back(), ps[i])) <= 0)
155
                qs.pop_back();
156
        return qs;
157
```

#### 1.1.2 凸包

```
// 凸包中的点按逆时针方向
2
  struct Convex {
3
       int n:
       std::vector<Point> a, upper, lower;
4
5
       void make_shell(const std::vector<Point>& p,
6
               std::vector<Point>& shell) { // p needs to be sorted.
7
           clear(shell); int n = p.size();
8
           for (int i = 0, j = 0; i < n; i++, j++) {
9
               for (; j \ge 2 \&\& sign(det(shell[j-1] - shell[j-2],
                                p[i] - shell[j-2])) \le 0; --j) shell.pop_back();
10
               shell.push_back(p[i]);
11
           }
12
       }
13
       void make_convex() {
14
           std::sort(a.begin(), a.end());
15
           make_shell(a, lower);
16
17
           std::reverse(a.begin(), a.end());
18
           make_shell(a, upper);
           a = lower; a.pop_back();
19
           a.insert(a.end(), upper.begin(), upper.end());
20
           if ((int)a.size() >= 2) a.pop_back();
21
22
           n = a.size();
       }
23
24
       void init(const std::vector<Point>& _a) {
           clear(a); a = _a; n = a.size();
25
           make_convex();
26
       void read(int _n) { // Won't make convex.
28
           clear(a); n = _n; a.resize(n);
29
           for (int i = 0; i < n; i++)
30
               a[i].read();
31
32
       std::pair<DB, int> get_tangent(
33
               const std::vector<Point>& convex, const Point& vec) {
34
```

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```
int l = 0, r = (int)convex.size() - 2;
35
          assert(r >= 0);
36
          for (; l + 1 < r; ) {
37
              int mid = (l + r) / 2;
38
              if (sign(det(convex[mid + 1] - convex[mid], vec)) > 0)
39
                  r = mid;
40
              else l = mid;
41
          }
42
          return std::max(std::make pair(det(vec, convex[r]), r),
43
                  std::make pair(det(vec, convex[0]), 0));
44
45
      int binary_search(Point u, Point v, int l, int r) {
46
47
          int s1 = sign(det(v - u, a[l % n] - u));
          for (; l + 1 < r; ) {
48
              int mid = (l + r) / 2;
49
              int smid = sign(det(v - u, a[mid % n] - u));
50
51
              if (smid == s1) l = mid;
52
              else r = mid;
          }
53
          return 1 % n;
54
      }
55
      // 求凸包上和向量 vec 叉积最大的点,返回编号,共线的多个切点返回任意一个
56
      int get tangent(Point vec) {
57
          std::pair<DB, int> ret = get_tangent(upper, vec);
58
          ret.second = (ret.second + (int)lower.size() - 1) % n;
59
60
          ret = std::max(ret, get_tangent(lower, vec));
          return ret.second;
61
62
      // 求凸包和直线 u, v 的交点, 如果不相交返回 false, 如果有则是和 (i, next(i)) 的交点, 交在点上不确
63
     → 定返回前后两条边其中之一
      bool get_intersection(Point u, Point v, int &i0, int &i1) {
64
65
          int p0 = get_tangent(u - v), p1 = get_tangent(v - u);
          if (sign(det(v - u, a[p0] - u)) * sign(det(v - u, a[p1] - u)) <= 0) {
66
              if (p0 > p1) std::swap(p0, p1);
67
              i0 = binary_search(u, v, p0, p1);
68
              i1 = binary_search(u, v, p1, p0 + n);
69
70
              return true;
71
          else return false;
72
      }
73
74 };
```

#### 1.2 三维

#### 1.2.1 基础

```
// 三维绕轴旋转,大拇指指向 axis 向量方向,四指弯曲方向转 w 弧度
Point rotate(const Point& s, const Point& axis, DB w) {
    DB x = axis.x, y = axis.y, z = axis.z;
    DB s1 = x * x + y * y + z * z, ss1 = msqrt(s1),
        cosw = cos(w), sinw = sin(w);
    DB a[4][4];
    memset(a, 0, sizeof a);
```

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```
8
       a[3][3] = 1;
       a[0][0] = ((y * y + z * z) * cosw + x * x) / s1;
9
       a[0][1] = x * y * (1 - cosw) / s1 + z * sinw / ss1;
10
       a[0][2] = x * z * (1 - cosw) / s1 - y * sinw / ss1;

a[1][0] = x * y * (1 - cosw) / s1 - z * sinw / ss1;
11
12
       a[1][1] = ((x * x + z * z) * cosw + y * y) / s1;
13
       a[1][2] = y * z * (1 - cosw) / s1 + x * sinw / ss1;
14
       a[2][0] = x * z * (1 - cosw) / s1 + y * sinw / ss1;
15
       a[2][1] = y * z * (1 - cosw) / s1 - x * sinw / ss1;
16
17
       a[2][2] = ((x * x + y * y) * cos(w) + z * z) / s1;
       DB ans[4] = \{0, 0, 0, 0\}, c[4] = \{s.x, s.y, s.z, 1\};
18
       for (int i = 0; i < 4; ++ i)
19
           for (int j = 0; j < 4; ++ j)
20
                ans[i] += a[j][i] * c[j];
21
22
       return Point(ans[0], ans[1], ans[2]);
23 }
```

## 数论

### 2.1 $O(m^2 \log n)$ 求线性递推数列第 n 项

```
Given a_0, a_1, \ldots, a_{m-1}

a_n = c_0 \times a_{n-m} + \cdots + c_{m-1} \times a_{n-1}

Solve for a_n = v_0 \times a_0 + v_1 \times a_1 + \cdots + v_{m-1} \times a_{m-1}
```

```
void linear_recurrence(long long n, int m, int a[], int c[], int p) {
1
2
       long long v[M] = \{1 \% p\}, u[M << 1], msk = !!n;
3
       for(long long i(n); i > 1; i >>= 1) {
           msk <<= 1;
4
5
6
       for(long long x(0); msk; msk >>= 1, x <<= 1) {
           fill_n(u, m << 1, 0);
7
           int b(!!(n & msk));
8
           x \mid = b;
10
           if(x < m) {
               u[x] = 1 % p;
11
           }else {
12
               for(int i(0); i < m; i++) {</pre>
13
                    for(int j(0), t(i + b); j < m; j++, t++) {
14
15
                        u[t] = (u[t] + v[i] * v[j]) % p;
16
               }
17
               for(int i((m << 1) - 1); i >= m; i--) {
18
                   for(int j(0), t(i - m); j < m; j++, t++) {
19
                        u[t] = (u[t] + c[j] * u[i]) % p;
20
21
22
               }
           }
23
           copy(u, u + m, v);
24
25
       //a[n] = v[0] * a[0] + v[1] * a[1] + ... + v[m - 1] * a[m - 1].
26
27
       for(int i(m); i < 2 * m; i++) {
28
           a[i] = 0;
           for(int j(0); j < m; j++) {
29
               a[i] = (a[i] + (long long)c[j] * a[i + j - m]) % p;
30
           }
31
       }
32
```

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```
for(int j(0); j < m; j++) {
33
           b[j] = 0;
34
            for(int i(0); i < m; i++) {</pre>
35
                b[j] = (b[j] + v[i] * a[i + j]) % p;
36
37
       }
38
       for(int j(0); j < m; j++) {
39
           a[j] = b[j];
40
41
  |}
42
```

### 2.2 求逆元

```
void ex_gcd(long long a, long long b, long long &x, long long &y) {
1
2
       if (b == 0) {
3
           x = 1;
4
           y = 0;
           return;
5
6
       long long xx, yy;
7
       ex_gcd(b, a % b, xx, yy);
8
       y = xx - a / b * yy;
9
10
       x = yy;
11 }
12
  long long inv(long long x, long long MODN) {
13
       long long inv_x, y;
14
       ex_gcd(x, MODN, inv_x, y);
15
       return (inv_x % MODN + MODN) % MODN;
16
17
  }
```

### 2.3 中国剩余定理

```
// 返回 (ans, M), 其中 ans 是模 M 意义下的解
  std::pair<long long, long long> CRT(const std::vector<long long>& m, const std::vector<long long>& a) {
2
      long long M = 1, ans = 0;
3
      int n = m.size();
4
      for (int i = 0; i < n; i++) M *= m[i];
5
      for (int i = 0; i < n; i++) {
6
7
          ans = (ans + (M / m[i]) * a[i] % M * inv(M / m[i], m[i])) % M; // 可能需要大整数相乘取模
8
      return std::make_pair(ans, M);
9
10 }
```

# 代数

### 3.1 快速傅里叶变换

```
// n 必须是 2 的次幂
2
  void fft(Complex a[], int n, int f) {
      for (int i = 0; i < n; ++i)
3
           if (R[i] < i) swap(a[i], a[R[i]]);</pre>
      for (int i = 1, h = 0; i < n; i <<= 1, h++) {
5
           Complex wn = Complex(cos(pi / i), f * sin(pi / i));
6
           Complex w = Complex(1, 0);
7
8
           for (int k = 0; k < i; ++k, w = w * wn) tmp[k] = w;
9
           for (int p = i \ll 1, j = 0; j < n; j += p) {
10
               for (int k = 0; k < i; ++k) {
                   Complex x = a[j + k], y = a[j + k + i] * tmp[k];
11
                   a[j + k] = x + y; a[j + k + i] = x - y;
12
13
           }
14
      }
15
16 }
```

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# 字符串

#### 4.1 后缀数组

```
1 const int MAXN = MAXL * 2 + 1;
2 \mid \text{int a[MAXN]}, x[MAXN], y[MAXN], c[MAXN], sa[MAXN], rank[MAXN], height[MAXN];
3
  void calc_sa(int n) {
       int m = alphabet, k = 1;
4
       memset(c, 0, sizeof(*c) * (m + 1));
5
       for (int i = 1; i \le n; ++i) c[x[i] = a[i]]++;
6
       for (int i = 1; i \le m; ++i) c[i] += c[i - 1];
8
       for (int i = n; i; --i) sa[c[x[i]]--] = i;
9
       for (; k <= n; k <<= 1) {
10
           int tot = k;
11
           for (int i = n - k + 1; i \le n; ++i) y[i - n + k] = i;
           for (int i = 1; i <= n; ++i)
12
               if (sa[i] > k) y[++tot] = sa[i] - k;
13
14
           memset(c, 0, sizeof(*c) * (m + 1));
           for (int i = 1; i \le n; ++i) c[x[i]]++;
15
           for (int i = 1; i \le m; ++i) c[i] += c[i - 1];
16
           for (int i = n; i; --i) sa[c[x[y[i]]]--] = y[i];
17
18
           for (int i = 1; i \le n; ++i) y[i] = x[i];
19
           tot = 1; x[sa[1]] = 1;
20
           for (int i = 2; i <= n; ++i) {
                \text{if } (\max(sa[i], \, sa[i-1]) \, + \, k \, > \, n \, \mid \mid \, y[sa[i]] \, \mid = \, y[sa[i-1]] \, \mid \mid \, y[sa[i] \, + \, k] \, \mid = \, y[sa[i-1] 
21
     \hookrightarrow + k]) ++tot;
22
               x[sa[i]] = tot;
23
           if (tot == n) break; else m = tot;
24
25
26 }
  void calc_height(int n) {
27
       for (int i = 1; i <= n; ++i) rank[sa[i]] = i;
28
29
       for (int i = 1; i <= n; ++i) {
           height[rank[i]] = max(0, height[rank[i - 1]] - 1);
30
31
           if (rank[i] == 1) continue;
           int j = sa[rank[i] - 1];
32
           33

→ ++height[rank[i]];

      }
34
```

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35 }

#### 4.2 后缀自动机

```
static const int MAXL = MAXN * 2; // MAXN is original length
2 static const int alphabet = 26; // sometimes need changing
int l, last, cnt, trans[MAXL][alphabet], par[MAXL], sum[MAXL], seq[MAXL], mxl[MAXL], size[MAXL]; // mxl
     char str[MAXL];
  inline void init() {
5
      l = strlen(str + 1); cnt = last = 1;
6
7
      for (int i = 0; i \le l * 2; ++i) memset(trans[i], 0, sizeof(trans[i]));
      memset(par, 0, sizeof(*par) * (l * 2 + 1));
8
9
      memset(mxl, 0, sizeof(*mxl) * (l * 2 + 1));
      memset(size, 0, sizeof(*size) * (l * 2 + 1));
10
11 | }
  inline void extend(int pos, int c) {
12
      int p = last, np = last = ++cnt;
13
      mxl[np] = mxl[p] + 1; size[np] = 1;
14
      for (; p && !trans[p][c]; p = par[p]) trans[p][c] = np;
15
16
      if (!p) par[np] = 1;
17
      else {
          int q = trans[p][c];
18
          if (mxl[p] + 1 == mxl[q]) par[np] = q;
19
          else {
20
               int nq = ++cnt;
21
               mxl[nq] = mxl[p] + 1;
22
               memcpy(trans[nq], trans[q], sizeof(trans[nq]));
23
               par[nq] = par[q];
24
25
               par[np] = par[q] = nq;
26
               for (; trans[p][c] == q; p = par[p]) trans[p][c] = nq;
27
          }
      }
28
29
  }
  inline void buildsam() {
30
      for (int i = 1; i <= l; ++i) extend(i, str[i] - 'a');
31
32
      memset(sum, 0, sizeof(*sum) * (l * 2 + 1));
33
      for (int i = 1; i <= cnt; ++i) sum[mxl[i]]++;
      for (int i = 1; i \le l; ++i) sum[i] += sum[i - 1];
34
      for (int i = cnt; i; --i) seq[sum[mxl[i]]--] = i;
35
      for (int i = cnt; i; --i) size[par[seq[i]]] += size[seq[i]];
36
37 | }
```

### 4.3 回文自动机

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```
7
       return nT++;
8 }
9 void init() {
       nT = nStr = 0;
10
11
       int newE = allocate(0);
       int new0 = allocate(-1);
12
       last = newE;
13
       fail[newE] = new0;
14
       fail[new0] = newE;
15
       s[0] = -1;
16
17 }
18 void add(int x) {
      s[++nStr] = x;
19
       int now = last;
20
       while (s[nStr - l[now] - 1] != s[nStr]) now = fail[now];
21
22
       if (!c[now][x]) {
           int newnode = allocate(l[now] + 2), &newfail = fail[newnode];
23
           newfail = fail[now];
24
           while (s[nStr - l[newfail] - 1] != s[nStr]) newfail = fail[newfail];
25
           newfail = c[newfail][x];
26
           c[now][x] = newnode;
27
28
29
       last = c[now][x];
30
       r[last]++;
31 }
32 void count() {
       for (int i = nT - 1; i >= 0; i--) {
33
           r[fail[i]] += r[i];
34
35
36 | }
```

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## 图论

### 5.1 基础

```
struct Graph { // Remember to call .init()!
       int e, nxt[M], v[M], adj[N], n;
2
3
       bool base;
       __inline void init(bool _base, int _n = 0) {
4
           assert(n < N);</pre>
5
6
           n = _n; base = _base;
           e = 0; memset(adj + base, -1, sizeof(*adj) * n);
8
       __inline int new_node() {
9
           adj[n + base] = -1;
10
           assert(n + base + 1 < N);
11
           return n++ + base;
12
13
       __inline void ins(int u0, int v0) { // directional
14
           assert(u0 < n + base && v0 < n + base);
15
           v[e] = v0; nxt[e] = adj[u0]; adj[u0] = e++;
16
17
           assert(e < M);</pre>
18
       __inline void bi_ins(int u0, int v0) { // bi-directional
19
           ins(u0, v0); ins(v0, u0);
20
       }
21
22 };
```

#### 5.2 KM

```
struct KM {
2
     // Truly 0(n^3)
     // 邻接矩阵,不能连的边设为 -INF, 求最小权匹配时边权取负,但不能连的还是 -INF, 使用时先对 1 -> n
3
    → 调用 hungary() ,再 get_ans() 求值
     int w[N][N];
     int lx[N], ly[N], match[N], way[N], slack[N];
5
     bool used[N];
6
     void init() {
7
         for (int i = 1; i \le n; i++) {
8
            match[i] = 0;
```

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```
lx[i] = 0;
10
11
                ly[i] = 0;
                way[i] = 0;
12
           }
13
14
       void hungary(int x) {
15
           match[0] = x;
16
           int j0 = 0;
17
           for (int j = 0; j <= n; j++) {
18
                slack[j] = INF;
19
20
                used[j] = false;
           }
21
22
           do {
23
                used[j0] = true;
24
25
                int i0 = match[j0], delta = INF, j1 = 0;
26
                for (int j = 1; j \le n; j++) {
                    if (used[j] == false) {
27
                         int cur = -w[i0][j] - lx[i0] - ly[j];
28
                         if (cur < slack[j]) {</pre>
29
                             slack[j] = cur;
30
                             way[j] = j0;
31
32
                        }
33
                         if (slack[j] < delta) {</pre>
34
                             delta = slack[j];
                             j1 = j;
35
                        }
36
                    }
37
38
                }
                for (int j = 0; j \le n; j++) {
39
                    if (used[j]) {
40
                        lx[match[j]] += delta;
41
                         ly[j] -= delta;
42
43
44
                    else slack[j] -= delta;
45
                }
                j0 = j1;
46
           } while (match[j0] != 0);
47
48
           do {
49
50
                int j1 = way[j0];
51
                match[j0] = match[j1];
52
                j0 = j1;
           } while (j0);
53
       }
54
55
       int get_ans() {
56
57
           int sum = 0;
58
           for(int i = 1; i <= n; i++) {
                if (w[match[i]][i] == -INF); // 无解
59
                if (match[i] > 0) sum += w[match[i]][i];
60
61
62
           return sum;
63
       }
```

5.3. 点双连通分量 21

64|} km;

#### 5.3 点双连通分量

bcc.forest is a set of connected tree whose vertices are chequered with cut-vertex and BCC.

```
const bool BCC_VERTEX = 0, BCC_EDGE = 1;
  struct BCC { // N = N0 + M0. Remember to call init(&raw_graph).
2
       Graph *g, forest; // g is raw graph ptr.
3
4
       int dfn[N], DFN, low[N];
5
       int stack[N], top;
6
       int expand_to[N];
                               // Where edge i is expanded to in expaned graph.
       // Vertex i expaned to i.
       int compress_to[N]; // Where vertex i is compressed to.
8
       bool vertex_type[N], cut[N], compress_cut[N], branch[M];
9
       //std::vector<int> BCC_component[N]; // Cut vertex belongs to none.
10
       __inline void init(Graph *raw_graph) {
11
           g = raw_graph;
12
13
       }
       void DFS(int u, int pe) {
14
           dfn[u] = low[u] = ++DFN; cut[u] = false;
15
           if (!\sim g->adj[u]) {
16
17
               cut[u] = 1;
               compress_to[u] = forest.new_node();
18
19
               compress_cut[compress_to[u]] = 1;
20
           for (int e = g->adj[u]; \sim e; e = g->nxt[e]) {
21
22
               int v = g -> v[e];
               if ((e^p) > 1 \& dfn[v] > 0 \& dfn[v] < dfn[u]) {
23
                   stack[top++] = e;
24
25
                    low[u] = std::min(low[u], dfn[v]);
               }
26
               else if (!dfn[v]) {
                    stack[top++] = e; branch[e] = 1;
28
                   DFS(v, e);
29
30
                    low[u] = std::min(low[v], low[u]);
31
                    if (low[v] >= dfn[u]) {
                        if (!cut[u]) {
32
                            cut[u] = 1;
33
                            compress_to[u] = forest.new_node();
34
                            compress_cut[compress_to[u]] = 1;
35
                        }
36
37
                        int cc = forest.new_node();
38
                        forest.bi_ins(compress_to[u], cc);
                        compress_cut[cc] = 0;
39
                        //BCC_component[cc].clear();
40
41
                            int cur_e = stack[--top];
42
43
                            compress_to[expand_to[cur_e]] = cc;
                            compress_to[expand_to[cur_e^1]] = cc;
44
                            if (branch[cur_e]) {
45
                                int v = g->v[cur_e];
46
                                if (cut[v])
47
```

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```
forest.bi_ins(cc, compress_to[v]);
48
                                  else {
49
                                      //BCC_component[cc].push_back(v);
50
                                      compress to[v] = cc;
51
52
53
                         } while (stack[top] != e);
54
                    }
55
               }
56
           }
57
       }
58
59
       void solve() {
           forest.init(g->base);
60
           int n = g->n;
61
           for (int i = 0; i < g \rightarrow e; i + +) {
62
                expand_to[i] = g->new_node();
63
64
           memset(branch, 0, sizeof(*branch) * g->e);
65
           memset(dfn + g->base, 0, sizeof(*dfn) * n); DFN = 0;
66
           for (int i = 0; i < n; i++)
67
                if (!dfn[i + g->base]) {
68
                     top = 0;
69
                    DFS(i + g -> base, -1);
70
71
                }
72
73
  } bcc;
74
75
  bcc.init(&raw_graph);
76 bcc.solve();
  // Do something with bcc.forest ...
```

#### 5.4 边双连通分量

```
1
   struct BCC {
2
       Graph *g, forest;
       int dfn[N], low[N], stack[N], tot[N], belong[N], vis[N], top, dfs_clock;
3
       // tot[] is the size of each BCC, belong[] is the BCC that each node belongs to
4
       pair<int, int > ori[M]; // bridge in raw_graph(raw node)
5
       bool is_bridge[M];
6
       __inline void init(Graph *raw_graph) {
7
8
           g = raw_graph;
9
           memset(is_bridge, false, sizeof(*is_bridge) * g -> e);
10
           memset(vis + g \rightarrow base, 0, sizeof(*vis) * g \rightarrow n);
11
       void tarjan(int u, int from) {
12
           dfn[u] = low[u] = ++dfs\_clock; vis[u] = 1; stack[++top] = u;
13
            for (int p = g \rightarrow adj[u]; \sim p; p = g \rightarrow nxt[p]) {
14
                if ((p ^ 1) == from) continue;
15
16
                int v = g \rightarrow v[p];
                if (vis[v]) {
17
                     if (vis[v] == 1) low[u] = min(low[u], dfn[v]);
18
                } else {
19
                    tarjan(v, p);
20
```

5.4. 边双连通分量 23

```
21
                    low[u] = min(low[u], low[v]);
                     if (low[v] > dfn[u]) is_bridge[p / 2] = true;
22
                }
23
           }
24
           if (dfn[u] != low[u]) return;
25
           tot[forest.new_node()] = 0;
26
           do {
27
                belong[stack[top]] = forest.n;
28
                vis[stack[top]] = 2;
29
                tot[forest.n]++;
30
31
                --top;
           } while (stack[top + 1] != u);
32
       }
33
       void solve() {
34
           forest.init(g -> base);
35
            int n = g \rightarrow n;
36
            for (int i = 0; i < n; ++i)
37
                if (!vis[i + g -> base]) {
38
                    top = dfs_clock = 0;
39
                    tarjan(i + g \rightarrow base, -1);
40
                }
41
            for (int i = 0; i < g -> e / 2; ++i)
42
                if (is_bridge[i]) {
43
44
                    int e = forest.e;
                    forest.bi_ins(belong[g \rightarrow v[i * 2]], belong[g \rightarrow v[i * 2 + 1]], g \rightarrow w[i * 2]);
45
                    ori[e] = make_pair(g -> v[i * 2 + 1], g -> v[i * 2]);
46
                    ori[e + 1] = make_pair(g -> v[i * 2], g -> v[i * 2 + 1]);
47
48
49
50
  } bcc;
```

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# 其他

### 6.1 Dancing Links

```
struct Node {
2
       Node *1, *r, *u, *d, *col;
       int size, line_no;
3
4
       Node() {
5
           size = 0; line_no = -1;
6
           l = r = u = d = col = NULL;
7
       }
8
  } *root;
9
10
   void cover(Node *c) {
       c->l->r = c->r; c->r->l = c->l;
11
       for (Node *u = c->d; u != c; u = u->d)
12
13
           for (Node *v = u->r; v != u; v = v->r) {
                v\rightarrow d\rightarrow u = v\rightarrow u;
14
                v->u->d = v->d;
15
                -- v->col->size;
16
17
           }
18
19
   void uncover(Node *c) {
20
       for (Node *u = c->u; u != c; u = u->u) {
21
           for (Node *v = u->l; v != u; v = v->l) {
22
                ++ v->col->size;
23
                v->u->d = v;
24
25
                v->d->u = v;
26
           }
27
       c->l->r = c; c->r->l = c;
28
  }
29
30
   std::vector<int> answer;
32
   bool search(int k) {
       if (root->r == root) return true;
33
       Node *r = NULL;
34
       for (Node *u = root->r; u != root; u = u->r)
35
           if (r == NULL || u->size < r->size)
36
```

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```
37
                r = u;
       if (r == NULL || r->size == 0) return false;
38
       else {
39
           cover(r);
40
           bool succ = false;
41
           for (Node *u = r->d; u != r \&\& !succ; u = u->d) {
42
                answer.push_back(u->line_no);
43
                for (Node *v = u->r; v != u; v = v->r) // Cover row
44
                    cover(v->col);
45
                succ |= search(k + 1);
46
47
                for (Node *v = u->1; v != u; v = v->1)
48
                    uncover(v->col);
                if (!succ) answer.pop_back();
49
           }
50
           uncover(r);
51
52
           return succ;
53
54 }
55
56 bool entry[CR][CC];
57 Node *who[CR][CC];
58 int cr, cc;
59
60
   void construct() {
       root = new Node();
61
       Node *last = root;
62
       for (int i = 0; i < cc; ++ i) {
63
           Node *u = new Node();
64
           last->r = u; u->l = last;
65
66
           Node *v = u; u->line_no = i;
           last = u;
67
           for (int j = 0; j < cr; ++ j)
68
                if (entry[j][i]) {
69
70
                    ++ u->size;
71
                    Node *cur = new Node();
72
                    who[j][i] = cur;
73
                    cur->line_no = j;
                    cur->col = u;
74
                    cur->u = v; v->d = cur;
75
                    v = cur;
76
77
78
           v->d = u; u->u = v;
79
       last->r = root; root->l = last;
80
       for (int j = 0; j < cr; ++ j) {
81
           Node *last = NULL;
82
           for (int i = cc - 1; i >= 0; -- i)
83
84
                if (entry[j][i]) {
                    last = who[j][i];
85
86
                    break;
87
           for (int i = 0; i < cc; ++ i)
88
                if (init[j][i]) {
89
90
                    last->r = who[j][i];
```

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```
who[j][i]->l = last;
91
                    last = who[j][i];
92
                }
93
94
   }
95
96
   void destruct() {
97
       for (Node *u = root->r; u != root; ) {
98
            for (Node *v = u->d; v != u; ) {
99
                Node *nxt = v->d;
100
                delete(v);
101
                v = nxt;
102
            }
103
            Node *nxt = u->r;
104
105
            delete(u); u = nxt;
106
       delete root;
107
108 }
```

28 CHAPTER 6. 其他

# 技巧

### 7.1 真正的释放 STL 容器内存空间

```
template <typename T>
__inline void clear(T& container) {
    container.clear(); // 或者删除了一堆元素
    T(container).swap(container);
}
```

#### 7.2 无敌的大整数相乘取模

Time complexity O(1).

```
// 需要保证 x 和 y 非负
long long mult(long long x, long long y, long long MODN) {
    long long t = (x * y - (long long)((long double)x / MODN * y + 1e-3) * MODN) % MODN;
    return t < 0 ? t + MODN : t;
}
```

### 7.3 无敌的读入优化

```
1 // getchar() 读入优化 << 关同步 cin << 此优化
2 // 用 isdigit() 会小幅变慢
3 // 返回 false 表示读到文件尾
  namespace Reader {
      const int L = (1 << 15) + 5;
      char buffer[L], *S, *T;
6
      __inline bool getchar(char &ch) {
7
          if (S == T) {
8
              T = (S = buffer) + fread(buffer, 1, L, stdin);
9
10
              if (S == T) {
                  ch = EOF;
11
                  return false;
12
              }
13
          }
14
          ch = *S++;
15
```

30 CHAPTER 7. 技巧

```
return true;
16
17
      __inline bool getint(int &x) {
18
19
           char ch; bool neg = 0;
           for (; getchar(ch) && (ch < '0' || ch > '9'); ) neg ^= ch == '-';
20
           if (ch == EOF) return false;
21
           x = ch - '0';
22
           for (; getchar(ch), ch >= '0' && ch <= '9'; )
23
               x = x * 10 + ch - '0';
24
           if (neg) x = -x;
25
26
           return true;
      }
27
28 }
```

### 7.4 控制 cout 输出实数精度

```
std::cout << std::fixed << std::setprecision(5);
```

# 提示

### 8.1 线性规划转对偶

$$\begin{array}{l} \text{maximize } \mathbf{c}^T \mathbf{x} \\ \text{subject to } \mathbf{A} \mathbf{x} \leq \mathbf{b}, \mathbf{x} \geq 0 \\ \end{array} \Longleftrightarrow \begin{array}{l} \text{minimize } \mathbf{y}^T \mathbf{b} \\ \text{subject to } \mathbf{y}^T \mathbf{A} \geq \mathbf{c}^T, \mathbf{y} \geq 0 \end{array}$$

### 8.2 32-bit/64-bit 随机素数

32-bit	64-bit
73550053	1249292846855685773
148898719	1701750434419805569
189560747	3605499878424114901
459874703	5648316673387803781
1202316001	6125342570814357977
1431183547	6215155308775851301
1438011109	6294606778040623451
1538762023	6347330550446020547
1557944263	7429632924303725207
1981315913	8524720079480389849

### 8.3 NTT 素数及其原根

Prime	Primitive root
1053818881	7
1051721729	6
1045430273	3
1012924417	5
1007681537	3