Integrals of Rational Functions

$$\int \frac{1}{1+x^2}dx = \tan^{-1} x \tag{1}$$

$$\int \frac{1}{a^2+x^2}dx = \frac{1}{a}\tan^{-1}\frac{x}{a} \tag{2}$$

$$\int \frac{x}{a^2+x^2}dx = \frac{1}{2}\ln|a^2+x^2| \tag{3}$$

$$\int \frac{x^2}{a^2+x^2}dx = x - a\tan^{-1}\frac{x}{a} \tag{4}$$

$$\int \frac{x^3}{a^2+x^2}dx = \frac{1}{2}x^2 - \frac{1}{2}a^2\ln|a^2+x^2| \tag{5}$$

$$\int \frac{1}{ax^2+bx+c}dx = \frac{2}{\sqrt{4ac-b^2}}\tan^{-1}\frac{2ax+b}{\sqrt{4ac-b^2}} \tag{6}$$

$$\int \frac{1}{(x+a)(x+b)}dx = \frac{1}{b-a}\ln\frac{a+x}{b+x},\ a\neq b \tag{7}$$

$$\int \frac{x}{(x+a)^2}dx = \frac{a}{a+x}+\ln|a+x| \tag{8}$$

$$\int \frac{x}{ax^2+bx+c}dx = \frac{1}{2a}\ln|ax^2+bx+c|$$
$$- \frac{b}{a\sqrt{4ac-b^2}}\tan^{-1}\frac{2ax+b}{\sqrt{4ac-b^2}} \tag{9}$$

Integrals with Roots

$$\int \frac{x}{\sqrt{x\pm a}}dx = \frac{2}{3}(x\mp 2a)\sqrt{x\pm a} \tag{10}$$

$$\int \sqrt{\frac{x}{a-x}}dx = -\sqrt{x(a-x)} - a\tan^{-1}\frac{\sqrt{x(a-x)}}{x-a} \tag{11}$$

$$\int \sqrt{\frac{x}{a+x}}dx = \sqrt{x(a+x)} - a\ln\left[\sqrt{x}+\sqrt{x+a}\right] \tag{12}$$

$$\int x\sqrt{ax+b}dx = \frac{2}{15a^2}(-2b^2+abx+3a^2x^2)\sqrt{ax+b} \tag{13}$$

$$\int \sqrt{x(ax+b)}dx = \frac{1}{4a^{3/2}}\left[(2ax+b)\sqrt{ax(ax+b)}\right.$$
$$\left. -b^2\ln\left|a\sqrt{x}+\sqrt{a(ax+b)}\right|\right] \tag{14}$$

$$\int \sqrt{x^3(ax+b)}dx = \left[\frac{b}{12a}-\frac{b^2}{8a^2x}+\frac{x}{3}\right]\sqrt{x^3(ax+b)}$$
$$+\frac{b^3}{\ \ \ }\ln\left|a\sqrt{x}+\sqrt{a(ax+b)}\right| \tag{15}$$

$$\int \sqrt{x^2\pm a^2}dx = \frac{1}{2}x\sqrt{x^2\pm a^2}\pm\frac{1}{2}a^2\ln\left|x+\sqrt{x^2\pm a^2}\right| \tag{16}$$

$$\int \sqrt{a^2-x^2}dx = \frac{1}{2}x\sqrt{a^2-x^2}+\frac{1}{2}a^2\tan^{-1}\frac{x}{\sqrt{a^2-x^2}} \tag{17}$$

$$\int x\sqrt{x^2\pm a^2}dx = \frac{1}{3}\left(x^2\pm a^2\right)^{3/2} \tag{18}$$

$$\int \frac{1}{\sqrt{x^2\pm a^2}}dx = \ln\left|x+\sqrt{x^2\pm a^2}\right| \tag{19}$$

$$\int \frac{1}{\sqrt{a^2-x^2}}dx = \sin^{-1}\frac{x}{a} \tag{20}$$

$$\int \frac{x}{\sqrt{x^2\pm a^2}}dx = \sqrt{x^2\pm a^2} \tag{21}$$

$$\int \frac{x}{\sqrt{a^2-x^2}}dx = -\sqrt{a^2-x^2} \tag{22}$$

$$\int \frac{x^2}{\sqrt{x^2\pm a^2}}dx = \frac{1}{2}x\sqrt{x^2\pm a^2}\mp\frac{1}{2}a^2\ln\left|x+\sqrt{x^2\pm a^2}\right| \tag{23}$$

$$\int \sqrt{ax^2+bx+c}dx = \frac{b+2ax}{4a}\sqrt{ax^2+bx+c}$$
$$+\frac{4ac-b^2}{8a^{3/2}}\ln\left|2ax+b+2\sqrt{a(ax^2+bx^+c)}\right| \tag{24}$$

$$\int x\sqrt{ax^2+bx+c} = \frac{1}{48a^{5/2}}\left(2\sqrt{a}\sqrt{ax^2+bx+c}\right.$$
$$\times\left(-3b^2+2abx+8a(c+ax^2)\right)$$
$$\left.+3(b^3-4abc)\ln\left|b+2ax+2\sqrt{a}\sqrt{ax^2+bx+c}\right|\right) \tag{25}$$

$$\int \frac{1}{\sqrt{ax^2+bx+c}}dx = \frac{1}{\sqrt{a}}\ln\left|2ax+b+2\sqrt{a(ax^2+bx+c)}\right| \tag{26}$$

$$\int \frac{x}{\sqrt{ax^2+bx+c}}dx = \frac{1}{a}\sqrt{ax^2+bx+c}$$
$$-\frac{b}{2a^{3/2}}\ln\left|2ax+b+2\sqrt{a(ax^2+bx+c)}\right| \tag{27}$$

$$\int \frac{dx}{(a^2+x^2)^{3/2}} = \frac{x}{a^2\sqrt{a^2+x^2}} \tag{28}$$

Integrals with Logarithms

$$\int \frac{\ln ax}{x}dx = \frac{1}{2}\left(\ln ax\right)^2 \tag{29}$$

$$\int \ln(ax+b)dx = \left(x+\frac{b}{a}\right)\ln(ax+b)-x, a\neq 0 \tag{30}$$

$$\int \ln(x^2+a^2)\,dx = x\ln(x^2+a^2)+2a\tan^{-1}\frac{x}{a}-2x \tag{31}$$

$$\int \ln(x^2-a^2)\,dx = x\ln(x^2-a^2)+a\ln\frac{x+a}{x-a}-2x \tag{32}$$

$$\int \ln\left(ax^2+bx+c\right)dx = \frac{1}{a}\sqrt{4ac-b^2}\tan^{-1}\frac{2ax+b}{\sqrt{4ac-b^2}}$$
$$-2x+\left(\frac{b}{2a}+x\right)\ln\left(ax^2+bx+c\right) \tag{33}$$

$$\int x\ln(ax+b)dx = \frac{bx}{2a}-\frac{1}{4}x^2$$
$$+\frac{1}{2}\left(x^2-\frac{b^2}{a^2}\right)\ln(ax+b) \tag{34}$$

$$\int x\ln\left(a^2-b^2x^2\right)dx = -\frac{1}{2}x^2+$$
$$\frac{1}{2}\left(x^2-\frac{a^2}{b^2}\right)\ln\left(a^2-b^2x^2\right) \tag{35}$$

Integrals with Exponentials

$$\int x^ne^{ax}\,dx = \frac{x^ne^{ax}}{a}-\frac{n}{a}\int x^{n-1}e^{ax}\,dx \tag{36}$$

$$\int xe^{-ax^2}\,dx = -\frac{1}{2a}e^{-ax^2} \tag{37}$$

Integrals with Trigonometric Functions

$$\int \sin^3 axdx = -\frac{3\cos ax}{4a}+\frac{\cos 3ax}{12a} \tag{38}$$

$$\int \cos^2 axdx = \frac{x}{2}+\frac{\sin 2ax}{4a} \tag{39}$$

$$\int \cos^3 axdx = \frac{3\sin ax}{4a}+\frac{\sin 3ax}{12a} \tag{40}$$

$$\int \cos ax\sin bxdx = \frac{\cos[(a-b)x]}{2(a-b)}-\frac{\cos[(a+b)x]}{2(a+b)}, a\neq b \tag{41}$$

$$\int \sin^2 ax\cos bxdx = -\frac{\sin[(2a-b)x]}{4(2a-b)}$$
$$+\frac{\sin bx}{2b}-\frac{\sin[(2a+b)x]}{4(2a+b)} \tag{42}$$

$$\int \sin^2 x\cos xdx = \frac{1}{3}\sin^3 x \tag{43}$$

$$\int \cos^2 ax \sin bx \, dx = \frac{\cos[(2a-b)x]}{4(2a-b)} - \frac{\cos bx}{2b}$$
$$- \frac{\cos[(2a+b)x]}{4(2a+b)} \tag{44}$$

$$\int \cos^2 ax \sin ax \, dx = -\frac{1}{3a} \cos^3 ax \tag{45}$$

$$\int \sin^2 ax \cos^2 bx \, dx = \frac{x}{4} - \frac{\sin 2ax}{8a} - \frac{\sin[2(a-b)x]}{16(a-b)}$$
$$+ \frac{\sin 2bx}{8b} - \frac{\sin[2(a+b)x]}{16(a+b)} \tag{46}$$

$$\int \sin^2 ax \cos^2 ax \, dx = \frac{x}{8} - \frac{\sin 4ax}{32a} \tag{47}$$

$$\int \tan ax \, dx = -\frac{1}{a} \ln \cos ax \tag{48}$$

$$\int \tan^2 ax \, dx = -x + \frac{1}{a} \tan ax \tag{49}$$

$$\int \tan^3 ax \, dx = \frac{1}{a} \ln \cos ax + \frac{1}{2a} \sec^2 ax \tag{50}$$

$$\int \sec x \, dx = \ln|\sec x + \tan x| = 2 \tanh^{-1}\left(\tan \frac{x}{2}\right) \tag{51}$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax \tag{52}$$

$$\int \sec^3 x \, dx = \frac{1}{2} \sec x \tan x + \frac{1}{2} \ln|\sec x + \tan x| \tag{53}$$

$$\int \sec x \tan x \, dx = \sec x \tag{54}$$

$$\int \sec^2 x \tan x \, dx = \frac{1}{2} \sec^2 x \tag{55}$$

$$\int \sec^n x \tan x \, dx = \frac{1}{n} \sec^n x, n \neq 0 \tag{56}$$

$$\int \csc x \, dx = \ln\left|\tan \frac{x}{2}\right| = \ln|\csc x - \cot x| + C \tag{57}$$

$$\int \csc^2 ax \, dx = -\frac{1}{a} \cot ax \tag{58}$$

$$\int \csc^3 x \, dx = -\frac{1}{2} \cot x \csc x + \frac{1}{2} \ln|\csc x - \cot x| \tag{59}$$

$$\int \csc^n x \cot x \, dx = -\frac{1}{n} \csc^n x, n \neq 0 \tag{60}$$

$$\int \sec x \csc x \, dx = \ln|\tan x| \tag{61}$$

Products of Trigonometric Functions and Monomials

$$\int x \cos x \, dx = \cos x + x \sin x \tag{62}$$

$$\int x \cos ax \, dx = \frac{1}{a^2} \cos ax + \frac{x}{a} \sin ax \tag{63}$$

$$\int x^2 \cos x \, dx = 2x \cos x + \left(x^2 - 2\right) \sin x \tag{64}$$

$$\int x^2 \cos ax \, dx = \frac{2x \cos ax}{a^2} + \frac{a^2 x^2 - 2}{a^3} \sin ax \tag{65}$$

$$\int x \sin x \, dx = -x \cos x + \sin x \tag{66}$$

$$\int x \sin ax \, dx = -\frac{x \cos ax}{a} + \frac{\sin ax}{a^2} \tag{67}$$

$$\int x^2 \sin x \, dx = \left(2 - x^2\right) \cos x + 2x \sin x \tag{68}$$

$$\int x^2 \sin ax \, dx = \frac{2 - a^2 x^2}{a^3} \cos ax + \frac{2x \sin ax}{a^2} \tag{69}$$

Products of Trigonometric Functions and Exponentials

$$\int e^x \sin x \, dx = \frac{1}{2} e^x (\sin x - \cos x) \tag{70}$$

$$\int e^{bx} \sin ax \, dx = \frac{1}{a^2 + b^2} e^{bx} (b \sin ax - a \cos ax) \tag{71}$$

$$\int e^x \cos x \, dx = \frac{1}{2} e^x (\sin x + \cos x) \tag{72}$$

$$\int e^{bx} \cos ax \, dx = \frac{1}{a^2 + b^2} e^{bx} (a \sin ax + b \cos ax) \tag{73}$$

$$\int x e^x \sin x \, dx = \frac{1}{2} e^x (\cos x - x \cos x + x \sin x) \tag{74}$$

$$\int x e^x \cos x \, dx = \frac{1}{2} e^x (x \cos x - \sin x + x \sin x) \tag{75}$$