

Gungnir's Standard Code Library

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Dated: October 13, 2016

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Chapter 1 计算几何

1.1 二维

1.1.1 基础

```
1 typedef double DB;
2 const DB eps = 1e-8;
3
4 int sign(DB x) {
5     return x < -eps ? -1 : (x > eps ? 1 : 0);
6 }
7 DB msqrt(DB x) {
8     return sign(x) > 0 ? sqrt(x) : 0;
9 }
10
11 struct Point {
12     DB x, y;
13     Point(): x(0), y(0) {}
14     Point(DB x, DB y): x(x), y(y) {}
15     Point operator+(const Point &rhs) const {
16         return Point(x + rhs.x, y + rhs.y);
17     }
18     Point operator-(const Point &rhs) const {
19         return Point(x - rhs.x, y - rhs.y);
20     }
21     Point operator*(DB k) const {
22         return Point(x * k, y * k);
23     }
24     Point operator/(DB k) const {
25         assert(sign(k));
26         return Point(x / k, y / k);
27     }
28     Point rotate(DB ang) const { // 逆时针旋转 ang 弧度
29         return Point(cos(ang) * x - sin(ang) * y,
30             cos(ang) * y + sin(ang) * x);
31     }
32     Point turn90() const { // 逆时针旋转 90 度
33         return Point(-y, x);
34     }
35     Point unit() const {
36         return *this / len();
37     }
38 };
39 DB dot(const Point& a, const Point& b) {
40     return a.x * b.x + a.y * b.y;
41 }
42 DB det(const Point& a, const Point& b) {
43     return a.x * b.y - a.y * b.x;
44 }
45 #define cross(p1,p2,p3) ((p2.x-p1.x)*(p3.y-p1.y)-(p3.x-p1.x)*(p2.y-p1.y))
46 #define crossOp(p1,p2,p3) sign(cross(p1,p2,p3))
47 bool isLL(const Line& l1, const Line& l2, Point& p) { // 直线与直线交点
48     DB s1 = det(l2.b - l2.a, l1.a - l2.a);
49     s2 = -det(l2.b - l2.a, l1.b - l2.a);
50     if (!sign(s1 + s2)) return false;
51     p = (l1.a * s2 + l1.b * s1) / (s1 + s2);
52     return true;
53 }
54 bool onSeg(const Line& l, const Point& p) { // 点在线段上
55     return sign(det(p - l.a, l.b - l.a)) == 0 && sign(dot(p - l.a, p - l.b)) <= 0;
56 }
57 Point projection(const Line & l, const Point& p) {
58     return l.a + (l.b - l.a) * (dot(p - l.a, l.b - l.a) / (l.b - l.a).len2());
59 }
60 DB disToLine(const Line& l, const Point& p) { // 点到 * 直线 * 距离
61     return fabs(det(p - l.a, l.b - l.a) / (l.b - l.a).len());
62 }
63 DB disToSeg(const Line& l, const Point& p) { // 点到线段距离
64     return sign(dot(p - l.a, l.b - l.a)) * sign(dot(p - l.b, l.a - l.b)) == 1 ?
        disToLine(l, p) : std::min((p - l.a).len(), (p - l.b).len());
```

```
65 }
66 // 圆与直线交点
67 bool isCL(Circle a, Line l, Point& p1, Point& p2) {
68     DB x = dot(l.a - a.o, l.b - l.a);
69     y = (l.b - l.a).len2(),
70     d = x * x - y * ((l.a - a.o).len2() - a.r * a.r);
71     if (sign(d) < 0) return false;
72     Point p = l.a - ((l.b - l.a) * (x / y)), delta = (l.b - l.a) * (msqrt(d) / y);
73     p1 = p + delta; p2 = p - delta;
74     return true;
75 }
76 // 圆与圆的交面积
77 DB areaCC(const Circle& c1, const Circle& c2) {
78     DB d = (c1.o - c2.o).len();
79     if (sign(d - (c1.r + c2.r)) >= 0) return 0;
80     if (sign(d - std::abs(c1.r - c2.r)) <= 0) {
81         DB r = std::min(c1.r, c2.r);
82         return r * r * PI;
83     }
84     DB x = (d * d + c1.r * c1.r - c2.r * c2.r) / (2 * d);
85     t1 = acos(x / c1.r), t2 = acos((d - x) / c2.r);
86     return c1.r * c1.r * t1 + c2.r * c2.r * t2 - d * c1.r * sin(t1);
87 }
88 // 圆与圆交点
89 bool isCC(Circle a, Circle b, P& p1, P& p2) {
90     DB s1 = (a.o - b.o).len();
91     if (sign(s1 - a.r - b.r) > 0 || sign(s1 - std::abs(a.r - b.r)) < 0) return false;
92     DB s2 = (a.r * a.r - b.r * b.r) / s1;
93     DB aa = (s1 + s2) * 0.5, bb = (s1 - s2) * 0.5;
94     P o = (b.o - a.o) * (aa / (aa + bb)) + a.o;
95     P delta = (b.o - a.o).unit().turn90() * msqrt(a.r * a.r - aa * aa);
96     p1 = o + delta, p2 = o - delta;
97     return true;
98 }
99 // 求点到圆的切点, 按关于点的顺时针方向返回两个点
100 bool tanCP(const Circle &c, const Point &p0, Point &p1, Point &p2) {
101     double x = (p0 - c.o).len2(), d = x - c.r * c.r;
102     if (d < eps) return false; // 点在圆上认为没有切点
103     Point p = (p0 - c.o) * (c.r * c.r / x);
104     Point delta = ((p0 - c.o) * (-c.r * sqrt(d) / x)).turn90();
105     p1 = c.o + p + delta;
106     p2 = c.o + p - delta;
107     return true;
108 }
109 // 求圆到圆的内共切线, 按关于 c1.o 的顺时针方向返回两条线
110 std::vector<Line> intanCC(const Circle &c1, const Circle &c2) {
111     std::vector<Line> ret;
112     Point p = (c1.o * c2.r + c2.o * c1.r) / (c1.r + c2.r);
113     Point p1, p2, q1, q2;
114     if (tanCP(c1, p, p1, p2) && tanCP(c2, p, q1, q2)) { // 两圆相切认为没有切线
115         ret.push_back(Line(p1, q1));
116         ret.push_back(Line(p2, q2));
117     }
118     return ret;
119 }
120 // 点在多边形内
121 bool inPolygon(const Point& p, const std::vector<Point>& poly) {
122     int n = poly.size();
123     int counter = 0;
124     for (int i = 0; i < n; ++i) {
125         P a = poly[i], b = poly[(i + 1) % n];
126         if (onSeg(Line(a, b), p)) return false; // 边界上不算
127         int x = sign(det(p - a, b - a));
128         int y = sign(a.y - p.y);
129         int z = sign(b.y - p.y);
130         if (x > 0 && y <= 0 && z > 0) ++ counter;
131         if (x < 0 && z <= 0 && y > 0) -- counter;
132     }
133     return counter != 0;
134 }
```

```

135 // 用半平面 (q1,q2) 的逆时针方向去切凸多边形
136 std::vector<Point> convexCut(const std::vector<Point>&ps, Point q1, Point q2) {
137     std::vector<Point> qs; int n = ps.size();
138     for (int i = 0; i < n; ++i) {
139         Point p1 = ps[i], p2 = ps[(i + 1) % n];
140         int d1 = crossOp(q1,q2,p1), d2 = crossOp(q1,q2,p2);
141         if (d1 >= 0) qs.push_back(p1);
142         if (d1 * d2 < 0) qs.push_back(isSS(p1, p2, q1, q2));
143     }
144     return qs;
145 }
146 // 求凸包
147 std::vector<Point> convexHull(std::vector<Point> ps) {
148     int n = ps.size(); if (n <= 1) return ps;
149     std::sort(ps.begin(), ps.end());
150     std::vector<Point> qs;
151     for (int i = 0; i < n; qs.push_back(ps[i++]))
152         while (qs.size() > 1 && sign(det(qs[qs.size() - 2], qs.back(), ps[i])) <= 0)
153             qs.pop_back();
154     for (int i = n - 2, t = qs.size(); i >= 0; qs.push_back(ps[i--]))
155         while ((int)qs.size() > t && sign(det(qs[qs.size() - 2], qs.back(), ps[i])) <=
156             0)
157             qs.pop_back();
158     return qs;

```

1.1.2 凸包

```

1 // 凸包中的点按逆时针方向
2 struct Convex {
3     int n;
4     std::vector<Point> a, upper, lower;
5     void make_shell(const std::vector<Point>& p,
6                     std::vector<Point>& shell) { // p needs to be sorted.
7         clear(shell); int n = p.size();
8         for (int i = 0, j = 0; i < n; i++, j++) {
9             for (; j >= 2 && sign(det(shell[j-1] - shell[j-2],
10                                     p[i] - shell[j-2])) <= 0; --j) shell.pop_back();
11             shell.push_back(p[i]);
12         }
13     }
14     void make_convex() {
15         std::sort(a.begin(), a.end());
16         make_shell(a, lower);
17         std::reverse(a.begin(), a.end());
18         make_shell(a, upper);
19         a = lower; a.pop_back();
20         a.insert(a.end(), upper.begin(), upper.end());
21         if ((int)a.size() >= 2) a.pop_back();
22         n = a.size();
23     }
24     void init(const std::vector<Point>& _a) {
25         clear(a); a = _a; n = a.size();
26         make_convex();
27     }
28     void read(int _n) { // Won't make convex.
29         clear(a); n = _n; a.resize(n);
30         for (int i = 0; i < n; i++)
31             a[i].read();
32     }
33     std::pair<DB, int> get_tangent(
34         const std::vector<Point>& convex, const Point& vec) {
35         int l = 0, r = (int)convex.size() - 2;
36         assert(r >= 0);
37         for (; l + 1 < r; ) {
38             int mid = (l + r) / 2;
39             if (sign(det(convex[mid + 1] - convex[mid], vec)) > 0)
40                 r = mid;
41             else l = mid;
42         }
43         return std::max(std::make_pair(det(vec, convex[r]), r),
44             std::make_pair(det(vec, convex[0]), 0));

```

```

45     }
46     int binary_search(Point u, Point v, int l, int r) {
47         int s1 = sign(det(v - u, a[l % n] - u));
48         for (; l + 1 < r; ) {
49             int mid = (l + r) / 2;
50             int smid = sign(det(v - u, a[mid % n] - u));
51             if (smid == s1) l = mid;
52             else r = mid;
53         }
54         return l % n;
55     }
56     // 求凸包上和向量 vec 叉积最大的点, 返回编号, 共线的多个切点返回任意一个
57     int get_tangent(Point vec) {
58         std::pair<DB, int> ret = get_tangent(upper, vec);
59         ret.second = (ret.second + (int)lower.size() - 1) % n;
60         ret = std::max(ret, get_tangent(lower, vec));
61         return ret.second;
62     }
63     // 求凸包和直线 u, v 的交点, 如果不相交返回 false, 如果有则是和 (i, next(i)) 的
64     // 交点, 交在点上不确定返回前后两条边其中之一
65     bool get_intersection(Point u, Point v, int &i0, int &i1) {
66         int p0 = get_tangent(u - v), p1 = get_tangent(v - u);
67         if (sign(det(v - u, a[p0] - u)) * sign(det(v - u, a[p1] - u)) <= 0) {
68             if (p0 > p1) std::swap(p0, p1);
69             i0 = binary_search(u, v, p0, p1);
70             i1 = binary_search(u, v, p1, p0 + n);
71             return true;
72         }
73         else return false;
74     }

```

1.2 三维

1.2.1 基础

```

1 // 三维绕轴旋转, 大拇指指向 axis 向量方向, 四指弯曲方向转 w 弧度
2 Point rotate(const Point& s, const Point& axis, DB w) {
3     DB x = axis.x, y = axis.y, z = axis.z;
4     DB s1 = x * x + y * y + z * z, ss1 = msqrt(s1),
5         cosw = cos(w), sinw = sin(w);
6     DB a[4][4];
7     memset(a, 0, sizeof a);
8     a[3][3] = 1;
9     a[0][0] = ((y * y + z * z) * cosw + x * x) / s1;
10    a[0][1] = x * y * (1 - cosw) / s1 + z * sinw / ss1;
11    a[0][2] = x * z * (1 - cosw) / s1 - y * sinw / ss1;
12    a[1][0] = x * y * (1 - cosw) / s1 - z * sinw / ss1;
13    a[1][1] = ((x * x + z * z) * cosw + y * y) / s1;
14    a[1][2] = y * z * (1 - cosw) / s1 + x * sinw / ss1;
15    a[2][0] = x * z * (1 - cosw) / s1 + y * sinw / ss1;
16    a[2][1] = y * z * (1 - cosw) / s1 - x * sinw / ss1;
17    a[2][2] = ((x * x + y * y) * cosw + z * z) / s1;
18    DB ans[4] = {0, 0, 0, 0}, c[4] = {s.x, s.y, s.z, 1};
19    for (int i = 0; i < 4; ++i)
20        for (int j = 0; j < 4; ++j)
21            ans[i] += a[j][i] * c[j];
22    return Point(ans[0], ans[1], ans[2]);
23 }

```

Chapter 2 数论

2.1 $O(m^2 \log n)$ 求线性递推数列第 n 项

Given a_0, a_1, \dots, a_{m-1}
 $a_n = c_0 \times a_{n-m} + \dots + c_{m-1} \times a_{n-1}$
 Solve for $a_n = v_0 \times a_0 + v_1 \times a_1 + \dots + v_{m-1} \times a_{m-1}$

```

1 void linear_recurrence(long long n, int m, int a[], int c[], int p) {
2     long long v[M] = {1 % p}, u[M << 1], msk = !n;

```

```

3  for(long long i(n); i > 1; i >= 1) {
4      msk <= 1;
5  }
6  for(long long x(0); msk; msk >= 1, x <= 1) {
7      fill_n(u, m < 1, 0);
8      int b(!!(n & msk));
9      x |= b;
10     if(x < m) {
11         u[x] = 1 % p;
12     } else {
13         for(int i(0); i < m; i++) {
14             for(int j(0), t(i + b); j < m; j++, t++) {
15                 u[t] = (u[t] + v[i] * v[j]) % p;
16             }
17         }
18         for(int i((m < 1) - 1); i >= m; i--) {
19             for(int j(0), t(i - m); j < m; j++, t++) {
20                 u[t] = (u[t] + c[j] * u[i]) % p;
21             }
22         }
23     }
24     copy(u, u + m, v);
25 }
26 //a[n] = v[0] * a[0] + v[1] * a[1] + ... + v[m - 1] * a[m - 1].
27 for(int i(m); i < 2 * m; i++) {
28     a[i] = 0;
29     for(int j(0); j < m; j++) {
30         a[i] = (a[i] + (long long)c[j] * a[i + j - m]) % p;
31     }
32 }
33 for(int j(0); j < m; j++) {
34     b[j] = 0;
35     for(int i(0); i < m; i++) {
36         b[j] = (b[j] + v[i] * a[i + j]) % p;
37     }
38 }
39 for(int j(0); j < m; j++) {
40     a[j] = b[j];
41 }
42 }

```

2.2 求逆元

```

1 void ex_gcd(long long a, long long b, long long &x, long long &y) {
2     if (!b) {
3         x = 1;
4         y = 0;
5         return;
6     }
7     long long xx, yy;
8     ex_gcd(b, a % b, xx, yy);
9     y = xx - a / b * yy;
10    x = yy;
11 }
12
13 long long inv(long long x, long long MODN) {
14     long long inv_x, y;
15     ex_gcd(x, MODN, inv_x, y);
16     return (inv_x % MODN + MODN) % MODN;
17 }

```

2.3 中国剩余定理

```

1 // 返回 (ans, M), 其中 ans 是模 M 意义下的解
2 std::pair<long long, long long> CRT(const std::vector<long long>& m, const
3     std::vector<long long>& a) {
4     long long M = 1, ans = 0;
5     int n = m.size();
6     for (int i = 0; i < n; i++) M *= m[i];
7     for (int i = 0; i < n; i++) {

```

```

7         ans = (ans + (M / m[i]) * a[i] % M * inv(M / m[i], m[i])) % M; // 可能需要大
8         整数相乘取模
9     }
10    return std::make_pair(ans, M);

```

Chapter 3 代数

3.1 快速傅里叶变换

```

1 // n 必须是 2 的次幂
2 void fft(Complex a[], int n, int f) {
3     for (int i = 0; i < n; ++i)
4         if (R[i] < i) swap(a[i], a[R[i]]);
5     for (int i = 1, h = 0; i < n; i <= 1, h++) {
6         Complex wn = Complex(cos(pi / i), f * sin(pi / i));
7         Complex w = Complex(1, 0);
8         for (int k = 0; k < i; ++k, w = w * wn) tmp[k] = w;
9         for (int p = i < 1, j = 0; j < n; j += p) {
10             for (int k = 0; k < i; ++k) {
11                 Complex x = a[j + k], y = a[j + k + i] * tmp[k];
12                 a[j + k] = x + y; a[j + k + i] = x - y;
13             }
14         }
15     }
16 }

```

Chapter 4 字符串

4.1 后缀数组

```

1 const int MAXN = MAXL * 2 + 1;
2 int a[MAXN], x[MAXN], y[MAXN], c[MAXN], sa[MAXN], rank[MAXN], height[MAXN];
3 void calc_sa(int n) {
4     int m = alphabet, k = 1;
5     memset(c, 0, sizeof(*c) * (m + 1));
6     for (int i = 1; i <= n; ++i) c[x[i]] = a[i]++;
7     for (int i = 1; i <= m; ++i) c[i] += c[i - 1];
8     for (int i = n; i; --i) sa[c[x[i]]--] = i;
9     for (; k <= n; k <= 1) {
10         int tot = k;
11         for (int i = n - k + 1; i <= n; ++i) y[i - n + k] = i;
12         for (int i = 1; i <= n; ++i)
13             if (sa[i] > k) y[++tot] = sa[i] - k;
14         memset(c, 0, sizeof(*c) * (m + 1));
15         for (int i = 1; i <= n; ++i) c[x[i]]++;
16         for (int i = 1; i <= m; ++i) c[i] += c[i - 1];
17         for (int i = n; i; --i) sa[c[x[y[i]]]--] = y[i];
18         for (int i = 1; i <= n; ++i) y[i] = x[i];
19         tot = 1; x[sa[1]] = 1;
20         for (int i = 2; i <= n; ++i) {
21             if (max(sa[i], sa[i - 1]) + k > n || y[sa[i]] != y[sa[i - 1]] || y[sa[i] +
22                 k] != y[sa[i - 1] + k]) ++tot;
23             x[sa[i]] = tot;
24         }
25         if (tot == n) break; else m = tot;
26     }
27 }
28 void calc_height(int n) {
29     for (int i = 1; i <= n; ++i) rank[sa[i]] = i;
30     for (int i = 1; i <= n; ++i) {
31         height[rank[i]] = max(0, height[rank[i - 1]] - 1);
32         if (rank[i] == 1) continue;
33         int j = sa[rank[i] - 1];
34         while (max(i, j) + height[rank[i]] <= n && a[i + height[rank[i]]] == a[j +
35             height[rank[i]]] ++height[rank[i]];

```

4.2 后缀自动机

```
1 static const int MAXL = MAXN * 2; // MAXN is original length
2 static const int alphabet = 26; // sometimes need changing
3 int l, last, cnt, trans[MAXN][alphabet], par[MAXN], sum[MAXN], seq[MAXN], mxl[MAXN],
   size[MAXN]; // mxl is maxlength, size is the size of right
4 char str[MAXN];
5 inline void init() {
6     l = strlen(str + 1); cnt = last = 1;
7     for (int i = 0; i <= l * 2; ++i) memset(trans[i], 0, sizeof(trans[i]));
8     memset(par, 0, sizeof(*par) * (l * 2 + 1));
9     memset(mxl, 0, sizeof(*mxl) * (l * 2 + 1));
10    memset(size, 0, sizeof(*size) * (l * 2 + 1));
11 }
12 inline void extend(int pos, int c) {
13     int p = last, np = last = ++cnt;
14     mxl[np] = mxl[p] + 1; size[np] = 1;
15     for (; p && !trans[p][c]; p = par[p]) trans[p][c] = np;
16     if (!p) par[np] = 1;
17     else {
18         int q = trans[p][c];
19         if (mxl[p] + 1 == mxl[q]) par[np] = q;
20         else {
21             int nq = ++cnt;
22             mxl[nq] = mxl[p] + 1;
23             memcpy(trans[nq], trans[q], sizeof(trans[nq]));
24             par[nq] = par[q];
25             par[np] = par[q] = nq;
26             for (; trans[p][c] == q; p = par[p]) trans[p][c] = nq;
27         }
28     }
29 }
30 inline void buildsam() {
31     for (int i = 1; i <= l; ++i) extend(i, str[i] - 'a');
32     memset(sum, 0, sizeof(*sum) * (l * 2 + 1));
33     for (int i = 1; i <= cnt; ++i) sum[mxl[i]]++;
34     for (int i = 1; i <= l; ++i) sum[i] += sum[i - 1];
35     for (int i = cnt; i; --i) seq[sum[mxl[i]] - 1] = i;
36     for (int i = cnt; i; --i) size[par[seq[i]]] += size[seq[i]];
37 }
```

4.3 回文自动机

```
1 int nT, nStr, last, c[MAXT][26], fail[MAXT], r[MAXN], l[MAXN], s[MAXN];
2 int allocate(int len) {
3     l[nT] = len;
4     r[nT] = 0;
5     fail[nT] = 0;
6     memset(c[nT], 0, sizeof(c[nT]));
7     return nT++;
8 }
9 void init() {
10    nT = nStr = 0;
11    int newE = allocate(0);
12    int new0 = allocate(-1);
13    last = newE;
14    fail[newE] = new0;
15    fail[new0] = newE;
16    s[0] = -1;
17 }
18 void add(int x) {
19     s[++nStr] = x;
20     int now = last;
21     while (s[nStr - l[now] - 1] != s[nStr]) now = fail[now];
22     if (!c[now][x]) {
23         int newnode = allocate(l[now] + 2), &newfail = fail[newnode];
24         newfail = fail[now];
25         while (s[nStr - l[newfail] - 1] != s[nStr]) newfail = fail[newfail];
26         newfail = c[newfail][x];
27         c[now][x] = newnode;
28     }
29     last = c[now][x];
30 }
```

```
30     r[last]++;
31 }
32 void count() {
33     for (int i = nT - 1; i >= 0; i--) {
34         r[fail[i]] += r[i];
35     }
36 }
```

Chapter 5 数据结构

5.1 KD-Tree

```
1 long long norm(const long long &x) {
2     // For manhattan distance
3     return std::abs(x);
4     // For euclid distance
5     return x * x;
6 }
7
8 struct Point {
9     int x, y, id;
10
11     const int& operator [] (int index) const {
12         if (index == 0) {
13             return x;
14         } else {
15             return y;
16         }
17     }
18
19     friend long long dist(const Point &a, const Point &b) {
20         long long result = 0;
21         for (int i = 0; i < 2; ++i) {
22             result += norm(a[i] - b[i]);
23         }
24         return result;
25     }
26 } point[N];
27
28 struct Rectangle {
29     int min[2], max[2];
30
31     Rectangle() {
32         min[0] = min[1] = INT_MAX; // sometimes int is not enough
33         max[0] = max[1] = INT_MIN;
34     }
35
36     void add(const Point &p) {
37         for (int i = 0; i < 2; ++i) {
38             min[i] = std::min(min[i], p[i]);
39             max[i] = std::max(max[i], p[i]);
40         }
41     }
42
43     long long dist(const Point &p) {
44         long long result = 0;
45         for (int i = 0; i < 2; ++i) {
46             // For minimum distance
47             result += norm(std::min(std::max(p[i], min[i]), max[i]) - p[i]);
48             // For maximum distance
49             result += std::max(norm(max[i] - p[i]), norm(min[i] - p[i]));
50         }
51         return result;
52     }
53 };
54
55 struct Node {
56     Point separator;
57     Rectangle rectangle;
58     int child[2];
59
60     void reset(const Point &p) {
```

```

61     seperator = p;
62     rectangle = Rectangle();
63     rectangle.add(p);
64     child[0] = child[1] = 0;
65 }
66 } tree[N << 1];
67
68 int size, pivot;
69
70 bool compare(const Point &a, const Point &b) {
71     if (a[pivot] != b[pivot]) {
72         return a[pivot] < b[pivot];
73     }
74     return a.id < b.id;
75 }
76
77 // 左閉右開: build(1, n + 1)
78 int build(int l, int r, int type = 1) {
79     pivot = type;
80     if (l >= r) {
81         return 0;
82     }
83     int x = ++size;
84     int mid = l + r >> 1;
85     std::nth_element(point + l, point + mid, point + r, compare);
86     tree[x].reset(point[mid]);
87     for (int i = l; i < r; ++i) {
88         tree[x].rectangle.add(point[i]);
89     }
90     tree[x].child[0] = build(l, mid, type ^ 1);
91     tree[x].child[1] = build(mid + 1, r, type ^ 1);
92     return x;
93 }
94
95 int insert(int x, const Point &p, int type = 1) {
96     pivot = type;
97     if (x == 0) {
98         tree[++size].reset(p);
99         return size;
100     }
101     tree[x].rectangle.add(p);
102     if (compare(p, tree[x].seperator)) {
103         tree[x].child[0] = insert(tree[x].child[0], p, type ^ 1);
104     } else {
105         tree[x].child[1] = insert(tree[x].child[1], p, type ^ 1);
106     }
107     return x;
108 }
109
110 // For minimum distance
111 // For maximum: 下面递归 query 时 0, 1 换顺序; < and >; min and max
112 void query(int x, const Point &p, std::pair<long long, int> &answer, int type = 1) {
113     pivot = type;
114     if (x == 0 || tree[x].rectangle.dist(p) > answer.first) {
115         return;
116     }
117     answer = std::min(answer,
118         std::make_pair(dist(tree[x].seperator, p), tree[x].seperator.id));
119     if (compare(p, tree[x].seperator)) {
120         query(tree[x].child[0], p, answer, type ^ 1);
121     } else {
122         query(tree[x].child[1], p, answer, type ^ 1);
123     }
124     query(tree[x].child[0], p, answer, type ^ 1);
125 }
126
127 std::priority_queue<std::pair<long long, int> > answer;
128
129 void query(int x, const Point &p, int k, int type = 1) {
130     pivot = type;
131     if (x == 0 || (int)answer.size() == k && tree[x].rectangle.dist(p) >
132         answer.top().first) {

```

```

133         return;
134     }
135     answer.push(std::make_pair(dist(tree[x].seperator, p), tree[x].seperator.id));
136     if ((int)answer.size() > k) {
137         answer.pop();
138     }
139     if (compare(p, tree[x].seperator)) {
140         query(tree[x].child[0], p, k, type ^ 1);
141     } else {
142         query(tree[x].child[1], p, k, type ^ 1);
143     }
144     query(tree[x].child[0], p, k, type ^ 1);
145 }
146 }

```

5.2 Treap

```

1 struct Node{
2     int mn, key, size, tag;
3     bool rev;
4     Node* ch[2];
5     Node(int mn, int key, int size): mn(mn), key(key), size(size), rev(0), tag(0){}
6     void dntag();
7     Node* update(){
8         mn = min(ch[0] -> mn, min(key, ch[1] -> mn));
9         size = ch[0] -> size + 1 + ch[1] -> size;
10        return this;
11    }
12 };
13 typedef pair<Node*, Node*> Pair;
14 Node *null, *root;
15 void Node::dntag(){
16     if(rev){
17         for(int i = 0; i < 2; i++){
18             if(ch[i] != null){
19                 ch[i] -> rev ^= 1;
20                 swap(ch[i] -> ch[0], ch[i] -> ch[1]);
21             }
22             rev = 0;
23         }
24     }
25     if(tag){
26         for(int i = 0; i < 2; i++){
27             if(ch[i] != null){
28                 ch[i] -> key += tag;
29                 ch[i] -> mn += tag;
30                 ch[i] -> tag += tag;
31             }
32             tag = 0;
33         }
34     }
35 }
36 int r(){
37     static int s = 3023192386;
38     return (s += (s << 3) + 1) & (~0u >> 1);
39 }
40 bool random(int x, int y){
41     return r() % (x + y) < x;
42 }
43 Node* merge(Node *p, Node *q){
44     if(p == null) return q;
45     if(q == null) return p;
46     p -> dntag();
47     q -> dntag();
48     if(random(p -> size, q -> size)){
49         p -> ch[1] = merge(p -> ch[1], q);
50         return p -> update();
51     }else{
52         q -> ch[0] = merge(p, q -> ch[0]);
53         return q -> update();
54     }
55 }

```



```

54 Pair split(Node *x, int n){
55     if(x == null) return make_pair(null, null);
56     x -> downtag();
57     if(n <= x -> ch[0] -> size){
58         Pair ret = split(x -> ch[0], n);
59         x -> ch[0] = ret.second;
60         return make_pair(ret.first, x -> update());
61     }
62     Pair ret = split(x -> ch[1], n - x -> ch[0] -> size - 1);
63     x -> ch[1] = ret.first;
64     return make_pair(x -> update(), ret.second);
65 }
66 pair<Node*, Pair> get_segment(int l, int r){
67     Pair ret = split(root, l - 1);
68     return make_pair(ret.first, split(ret.second, r - l + 1));
69 }
70 int main(){
71     null = new Node(INF, INF, 0);
72     null -> ch[0] = null -> ch[1] = null;
73     root = null;
74 }

```

5.3 Link/cut Tree

```

1  inline void reverse(int x) {
2      tr[x].rev ^= 1; swap(tr[x].c[0], tr[x].c[1]);
3  }
4
5  inline void rotate(int x, int k) {
6      int y = tr[x].fa, z = tr[y].fa;
7      tr[x].fa = z; tr[z].c[tr[z].c[1] == y] = x;
8      tr[tr[x].c[k ^ 1]].fa = y; tr[y].c[k] = tr[x].c[k ^ 1];
9      tr[x].c[k ^ 1] = y; tr[y].fa = x;
10 }
11
12 inline void splay(int x, int w) {
13     int z = x; pushdown(x);
14     while (tr[x].fa != w) {
15         int y = tr[x].fa; z = tr[y].fa;
16         if (z == w) {
17             pushdown(z = y); pushdown(x);
18             rotate(x, tr[y].c[1] == x);
19             update(y); update(x);
20         } else {
21             pushdown(z); pushdown(y); pushdown(x);
22             int t1 = tr[y].c[1] == x, t2 = tr[z].c[1] == y;
23             if (t1 == t2) rotate(y, t2), rotate(x, t1);
24             else rotate(x, t1), rotate(x, t2);
25             update(z); update(y); update(x);
26         }
27     }
28     update(x);
29     if (x != z) par[x] = par[z], par[z] = 0;
30 }
31
32 inline void access(int x) {
33     for (int y = 0; x; y = x, x = par[x]) {
34         splay(x, 0);
35         if (tr[x].c[1]) par[tr[x].c[1]] = x, tr[tr[x].c[1]].fa = 0;
36         tr[x].c[1] = y; par[y] = 0; tr[y].fa = x; update(x);
37     }
38 }
39
40 inline void makeroot(int x) {
41     access(x); splay(x, 0); reverse(x);
42 }
43
44 inline void link(int x, int y) {
45     makeroot(x); par[x] = y;
46 }
47
48 inline void cut(int x, int y) {

```

```

49     access(x); splay(y, 0);
50     if (par[y] != x) swap(x, y), access(x), splay(y, 0);
51     par[y] = 0;
52 }
53
54 inline void split(int x, int y) { // x will be the root of the tree
55     makeroot(y); access(x); splay(x, 0);
56 }

```

Chapter 6 图论

6.1 基础

```

1  struct Graph { // Remember to call .init()!
2      int e, nxt[M], v[M], adj[N], n;
3      bool base;
4      __inline void init(bool _base, int _n = 0) {
5          assert(n < N);
6          n = _n; base = _base;
7          e = 0; memset(adj + base, -1, sizeof(*adj) * n);
8      }
9      __inline int new_node() {
10         adj[n + base] = -1;
11         assert(n + base + 1 < N);
12         return n++ + base;
13     }
14     __inline void ins(int u0, int v0) { // directional
15         assert(u0 < n + base && v0 < n + base);
16         v[e] = v0; nxt[e] = adj[u0]; adj[u0] = e++;
17         assert(e < M);
18     }
19     __inline void bi_ins(int u0, int v0) { // bi-directional
20         ins(u0, v0); ins(v0, u0);
21     }
22 };

```

6.2 KM

```

1  struct KM {
2      // Truly O(n^3)
3      // 邻接矩阵，不能连的边设为 -INF，求最小权匹配时边权取负，但不能连的还是 -INF，
4      // 使用时先对 1 -> n 调用 hungary()，再 get_ans() 求值
5      int w[N][N];
6      int lx[N], ly[N], match[N], way[N], slack[N];
7      bool used[N];
8      void init() {
9          for (int i = 1; i <= n; i++) {
10             match[i] = 0;
11             lx[i] = 0;
12             ly[i] = 0;
13             way[i] = 0;
14         }
15     }
16     void hungary(int x) {
17         match[0] = x;
18         int j0 = 0;
19         for (int j = 0; j <= n; j++) {
20             slack[j] = INF;
21             used[j] = false;
22         }
23         do {
24             used[j0] = true;
25             int i0 = match[j0], delta = INF, j1 = 0;
26             for (int j = 1; j <= n; j++) {
27                 if (used[j] == false) {
28                     int cur = -w[i0][j] - lx[i0] - ly[j];
29                     if (cur < slack[j]) {
30                         slack[j] = cur;

```



```

31         way[j] = j0;
32     }
33     if (slack[j] < delta) {
34         delta = slack[j];
35         j1 = j;
36     }
37 }
38 }
39 for (int j = 0; j <= n; j++) {
40     if (used[j]) {
41         lx[match[j]] += delta;
42         ly[j] -= delta;
43     }
44     else slack[j] -= delta;
45 }
46 j0 = j1;
47 } while (match[j0] != 0);
48
49 do {
50     int j1 = way[j0];
51     match[j0] = match[j1];
52     j0 = j1;
53 } while (j0);
54 }
55
56 int get_ans() {
57     int sum = 0;
58     for (int i = 1; i <= n; i++) {
59         if (w[match[i]][i] == -INF) ; // 无解
60         if (match[i] > 0) sum += w[match[i]][i];
61     }
62     return sum;
63 }
64 } km;

```

6.3 点双连通分量

bcc.forest is a set of connected tree whose vertices are chequered with cut-vertex and BCC.

```

1 const bool BCC_VERTEX = 0, BCC_EDGE = 1;
2 struct BCC { // N = N0 + M0. Remember to call init(&raw_graph).
3     Graph *g, forest; // g is raw graph ptr.
4     int dfn[N], DFN, low[N];
5     int stack[N], top;
6     int expand_to[N]; // Where edge i is expanded to in expaned graph.
7     // Vertex i expanded to i.
8     int compress_to[N]; // Where vertex i is compressed to.
9     bool vertex_type[N], cut[N], compress_cut[N], branch[M];
10    //std::vector<int> BCC_component[N]; // Cut vertex belongs to none.
11    __inline void init(Graph *raw_graph) {
12        g = raw_graph;
13    }
14    void DFS(int u, int pe) {
15        dfn[u] = low[u] = ++DFN; cut[u] = false;
16        if (!g->adj[u]) {
17            cut[u] = 1;
18            compress_to[u] = forest.new_node();
19            compress_cut[compress_to[u]] = 1;
20        }
21        for (int e = g->adj[u]; ~e; e = g->nxt[e]) {
22            int v = g->v[e];
23            if ((e ^ pe) > 1 && dfn[v] > 0 && dfn[v] < dfn[u]) {
24                stack[top++] = e;
25                low[u] = std::min(low[u], dfn[v]);
26            }
27            else if (!dfn[v]) {
28                stack[top++] = e; branch[e] = 1;
29                DFS(v, e);
30                low[u] = std::min(low[v], low[u]);
31                if (low[v] >= dfn[u]) {
32                    if (!cut[u]) {

```

```

33            cut[u] = 1;
34            compress_to[u] = forest.new_node();
35            compress_cut[compress_to[u]] = 1;
36        }
37        int cc = forest.new_node();
38        forest.bi_ins(compress_to[u], cc);
39        compress_cut[cc] = 0;
40        //BCC_component[cc].clear();
41        do {
42            int cur_e = stack[--top];
43            compress_to[expand_to[cur_e]] = cc;
44            compress_to[expand_to[cur_e^1]] = cc;
45            if (branch[cur_e]) {
46                int v = g->v[cur_e];
47                if (cut[v])
48                    forest.bi_ins(cc, compress_to[v]);
49            }
50            else {
51                //BCC_component[cc].push_back(v);
52                compress_to[v] = cc;
53            }
54        } while (stack[top] != e);
55    }
56 }
57
58 void solve() {
59     forest.init(g->base);
60     int n = g->n;
61     for (int i = 0; i < g->e; i++) {
62         expand_to[i] = g->new_node();
63     }
64     memset(branch, 0, sizeof(*branch) * g->e);
65     memset(dfn + g->base, 0, sizeof(*dfn) * n); DFN = 0;
66     for (int i = 0; i < n; i++)
67         if (!dfn[i + g->base]) {
68             top = 0;
69             DFS(i + g->base, -1);
70         }
71 }
72 }
73 } bcc;
74
75 bcc.init(&raw_graph);
76 bcc.solve();
77 // Do something with bcc.forest ...

```

6.4 边双连通分量

```

1 struct BCC {
2     Graph *g, forest;
3     int dfn[N], low[N], stack[N], tot[N], belong[N], vis[N], top, dfs_clock;
4     // tot[] is the size of each BCC, belong[] is the BCC that each node belongs to
5     pair<int, int> ori[M]; // bridge in raw_graph(raw node)
6     bool is_bridge[M];
7     __inline void init(Graph *raw_graph) {
8         g = raw_graph;
9         memset(is_bridge, false, sizeof(*is_bridge) * g->e);
10        memset(vis + g->base, 0, sizeof(*vis) * g->n);
11    }
12    void tarjan(int u, int from) {
13        dfn[u] = low[u] = ++dfs_clock; vis[u] = 1; stack[++top] = u;
14        for (int p = g->adj[u]; ~p; p = g->nxt[p]) {
15            if ((p ^ 1) == from) continue;
16            int v = g->v[p];
17            if (vis[v]) {
18                if (vis[v] == 1) low[u] = min(low[u], dfn[v]);
19            }
20            else {
21                tarjan(v, p);
22                low[u] = min(low[u], low[v]);
23                if (low[v] > dfn[u]) is_bridge[p / 2] = true;

```

```

24     }
25     if (dfn[u] != low[u]) return;
26     tot[forest.new_node()] = 0;
27     do {
28         belong[stack[top]] = forest.n;
29         vis[stack[top]] = 2;
30         tot[forest.n]++;
31         --top;
32     } while (stack[top + 1] != u);
33 }
34 void solve() {
35     forest.init(g -> base);
36     int n = g -> n;
37     for (int i = 0; i < n; ++i)
38         if (!vis[i + g -> base]) {
39             top = dfs_clock = 0;
40             tarjan(i + g -> base, -1);
41         }
42     for (int i = 0; i < g -> e / 2; ++i)
43         if (is_bridge[i]) {
44             int e = forest.e;
45             forest.bi_ins(belong[g -> v[i * 2]], belong[g -> v[i * 2 + 1]], g ->
46                 w[i * 2]);
47             ori[e] = make_pair(g -> v[i * 2 + 1], g -> v[i * 2]);
48             ori[e + 1] = make_pair(g -> v[i * 2], g -> v[i * 2 + 1]);
49         }
50 } bcc;

```

Chapter 7 其他

7.1 Dancing Links

```

1 struct Node {
2     Node *l, *r, *u, *d, *col;
3     int size, line_no;
4     Node() {
5         size = 0; line_no = -1;
6         l = r = u = d = col = NULL;
7     }
8 } *root;
9
10 void cover(Node *c) {
11     c->l->r = c->r; c->r->l = c->l;
12     for (Node *u = c->d; u != c; u = u->d)
13         for (Node *v = u->r; v != u; v = v->r) {
14             v->d->u = v->u;
15             v->u->d = v->d;
16             -- v->col->size;
17         }
18 }
19
20 void uncover(Node *c) {
21     for (Node *u = c->u; u != c; u = u->u) {
22         for (Node *v = u->l; v != u; v = v->l) {
23             ++ v->col->size;
24             v->u->d = v;
25             v->d->u = v;
26         }
27     }
28     c->l->r = c; c->r->l = c;
29 }
30
31 std::vector<int> answer;
32 bool search(int k) {
33     if (root->r == root) return true;
34     Node *r = NULL;
35     for (Node *u = root->r; u != root; u = u->r)
36         if (r == NULL || u->size < r->size)
37             r = u;
38     if (r == NULL || r->size == 0) return false;
39     else {

```

```

40         cover(r);
41         bool succ = false;
42         for (Node *u = r->d; u != r && !succ; u = u->d) {
43             answer.push_back(u->line_no);
44             for (Node *v = u->r; v != u; v = v->r) // Cover row
45                 cover(v->col);
46             succ |= search(k + 1);
47             for (Node *v = u->l; v != u; v = v->l)
48                 uncover(v->col);
49             if (!succ) answer.pop_back();
50         }
51         uncover(r);
52         return succ;
53     }
54 }
55
56 bool entry[CR][CC];
57 Node *who[CR][CC];
58 int cr, cc;
59
60 void construct() {
61     root = new Node();
62     Node *last = root;
63     for (int i = 0; i < cc; ++ i) {
64         Node *u = new Node();
65         last->r = u; u->l = last;
66         Node *v = u; u->line_no = i;
67         last = u;
68         for (int j = 0; j < cr; ++ j)
69             if (entry[j][i]) {
70                 ++ u->size;
71                 Node *cur = new Node();
72                 who[j][i] = cur;
73                 cur->line_no = j;
74                 cur->col = u;
75                 cur->u = v; v->d = cur;
76                 v = cur;
77             }
78         v->d = u; u->u = v;
79     }
80     last->r = root; root->l = last;
81     for (int j = 0; j < cr; ++ j) {
82         Node *last = NULL;
83         for (int i = cc - 1; i >= 0; -- i)
84             if (entry[j][i]) {
85                 last = who[j][i];
86                 break;
87             }
88         for (int i = 0; i < cc; ++ i)
89             if (init[j][i]) {
90                 last->r = who[j][i];
91                 who[j][i]->l = last;
92                 last = who[j][i];
93             }
94     }
95 }
96
97 void destruct() {
98     for (Node *u = root->r; u != root; ) {
99         for (Node *v = u->d; v != u; ) {
100             Node *nxt = v->d;
101             delete(v);
102             v = nxt;
103         }
104         Node *nxt = u->r;
105         delete(u); u = nxt;
106     }
107     delete root;
108 }

```

Chapter 8 技巧

8.1 真正的释放 STL 容器内存空间

```
1 template <typename T>
2 __inline void clear(T& container) {
3     container.clear(); // 或者删除了一堆元素
4     T(container).swap(container);
5 }
```

8.2 无敌的大整数相乘取模

Time complexity $O(1)$.

```
1 // 需要保证 x 和 y 非负
2 long long mult(long long x, long long y, long long MODN) {
3     long long t = (x * y - (long long)((long double)x / MODN * y + 1e-3) * MODN) %
4     ↪ MODN;
5     return t < 0 ? t + MODN : t;
6 }
```

8.3 无敌的读入优化

```
1 // getchar() 读入优化 << 关同步 cin << 此优化
2 // 用 isdigit() 会小幅变慢
3 // 返回 false 表示读到文件尾
4 namespace Reader {
5     const int L = (1 << 15) + 5;
6     char buffer[L], *S, *T;
7     __inline bool getChar(char &ch) {
8         if (S == T) {
9             T = (S = buffer) + fread(buffer, 1, L, stdin);
10            if (S == T) {
11                ch = EOF;
12                return false;
13            }
14        }
15        ch = *S++;
16        return true;
17    }
18    __inline bool getInt(int &x) {
```

```
19     char ch; bool neg = 0;
20     for (; getChar(ch) && (ch < '0' || ch > '9'); ) neg ^= ch == '-';
21     if (ch == EOF) return false;
22     x = ch - '0';
23     for (; getChar(ch), ch >= '0' && ch <= '9'; )
24         x = x * 10 + ch - '0';
25     if (neg) x = -x;
26     return true;
27 }
28 }
```

8.4 控制 cout 输出实数精度

```
1 std::cout << std::fixed << std::setprecision(5);
```

Chapter 9 提示

9.1 线性规划转对偶

$$\begin{aligned} &\text{maximize } \mathbf{c}^T \mathbf{x} \\ &\text{subject to } \mathbf{A} \mathbf{x} \leq \mathbf{b}, \mathbf{x} \geq 0 \end{aligned} \iff \begin{aligned} &\text{minimize } \mathbf{y}^T \mathbf{b} \\ &\text{subject to } \mathbf{y}^T \mathbf{A} \geq \mathbf{c}^T, \mathbf{y} \geq 0 \end{aligned}$$

9.2 32-bit/64-bit 随机素数

32-bit	64-bit
73550053	1249292846855685773
148898719	1701750434419805569
189560747	3605499878424114901
459874703	5648316673387803781
1202316001	6125342570814357977
1431183547	6215155308775851301
1438011109	6294606778040623451
1538762023	6347330550446020547
1557944263	7429632924303725207
1981315913	8524720079480389849

9.3 NTT 素数及其原根

Prime	Primitive root
1053818881	7
1051721729	6
1045430273	3
1012924417	5
1007681537	3