Gungnir'l Standard Code Library*

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 $^{{\}rm *https://github.com/footoredo/Gungnir}$

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Chapter 1 计算几何

1.1 二维

1.1.1 基础

```
typedef double DB;
   const DB eps = 1e-8;
   int sign(DB x) {
       return x < -eps ? -1 : (x > eps ? 1 : 0);
       return sign(x) > 0 ? sgrt(x) : 0;
10
11
   struct Point {
       DB x, y;
Point(): x(0), y(0) {}
12
13
       Point(DB x, DB y): x(x), y(y) {}
14
15
       Point operator+(const Point &rhs) const {
16
            return Point(x + rhs.x, y + rhs.y);
17
18
       Point operator-(const Point &rhs) const {
            return Point(x - rhs.x, y - rhs.y);
19
20
21
       Point operator*(DB k) const {
22
            return Point(x * k, y * k);
23
24
25
26
       Point operator/(DB k) const {
           assert(sign(k)):
            return Point(x / k, y / k);
27
28
       Point rotate(DB ang) const { // 逆时针旋转 ang 弧度
29
            return Point(cos(ang) *x - sin(ang) *y,
30
                    cos(ang) * y + sin(ang) * x);
31
       }
32
       Point turn90() const { // 逆时针旋转 90 度
33
            return Point(-y, x);
34
35
       Point unit() const {
36
           return *this / len();
37
38
39
   DB dot(const Point& a, const Point& b) {
       return a.x * b.x + a.y * b.y;
41
42
   DB det(const Point& a, const Point& b) {
43
       return a.x * b.y - a.y * b.x;
44
45 | #define cross(p1,p2,p3) ((p2.x-p1.x)*(p3.y-p1.y)-(p3.x-p1.x)*(p2.y-p1.y))
   #define crossOp(p1,p2,p3) sign(cross(p1,p2,p3))
   bool isLL(const Line& l1, const Line& l2, Point& p) { // 直线与直线交点 DB s1 = det(l2.b - l2.a, l1.a - l2.a), s2 = -det(l2.b - l2.a, l1.b - l2.a);
48
49
50
       if (!sign(s1 + s2)) return false;
51
52
53
       p = (l1.a * s2 + l1.b * s1) / (s1 + s2);
       return true:
54
   bool onSeg(const Line& l, const Point& p) { // 点在线段上
55
       return sign(det(p - l.a, l.b - l.a)) == 0 && sign(dot(p - l.a, p - l.b)) <= 0;
56
57
   Point projection(const Line & l, const Point& p) {
       return l.a + (l.b - l.a) * (dot(p - l.a, l.b - l.a) / (l.b - l.a).len2());
58
59
   DB disToLine(const Line& l, const Point& p) { // 点到 * 直线 * 距离
       return fabs(det(p - l.a, l.b - l.a) / (l.b - l.a).len());
61
62 }
63 | DB disToSeg(const Line& l, const Point& p) { // 点到线段距离
       return sign(dot(p - l.a, l.b - l.a)) * sign(dot(p - l.b, l.a - l.b)) == 1 ?
      \rightarrow disToLine(l, p) : std::min((p - l.a).len(), (p - l.b).len());
```

```
65 | }
    // 圆与直线交点
    bool isCL(Circle a, Line l, Point& p1, Point& p2) {
    DB x = dot(l.a - a.o, l.b - l.a),
        y = (l.b - l.a).len2(),
         d = x * x - y * ((l.a - a.o).len2() - a.r * a.r);
if (sign(d) < 0) return false;</pre>
 70
 71
 72
73
74
         Point p = l.a - ((l.b - l.a) * (x / y)), delta = (l.b - l.a) * (msqrt(d) / y);
         p1 = p + delta; p2 = p - delta;
 75 }
 76 //圆与圆的交面积
 77 DB areaCC(const Circle& c1, const Circle& c2) {
        DB d = (c1.o - c2.o).len();
if (sign(d - (c1.r + c2.r)) >= 0) return 0;
if (sign(d - std::abs(c1.r - c2.r)) <= 0) {
 80
 81
             DB r = std::min(c1.r, c2.r);
             return r * r * PI:
 82
 83
 84
         DB x = (d * d + c1.r * c1.r - c2.r * c2.r) / (2 * d),
 85
             t1 = acos(x / c1.r), t2 = acos((d - x) / c2.r);
 86
         return c1.r * c1.r * t1 + c2.r * c2.r * t2 - d * c1.r * sin(t1);
 87
    // 圆与圆交点
 88
    | DB s1 = (a.o - b.o).len();
 89
         if (sign(s1 - a.r - b.r) > 0 \mid | sign(s1 - std::abs(a.r - b.r)) < 0) return false;
         DB s2 = (a.r * a.r - b.r * b.r) / s1;
        DB aa = (s1 + s2) * 0.5, bb = (s1 - s2) * 0.5;
P o = (b.o - a.o) * (aa / (aa + bb)) + a.o;
 94
         P delta = (b.o - a.o).unit().turn90() * msqrt(a.r * a.r - aa * aa);
 95
 96
97
         p1 = o + delta, p2 = o - delta;
         return true:
 98 }
    // 求点到圆的切点,按关于点的顺时针方向返回两个点
 99
    bool tanCP(const Circle &c, const Point &p0, Point &p1, Point &p2) {
100
         double x = (p0 - c.o) \cdot len2(), d = x - c.r * c.r;
         if (d < eps) return false; // 点在圆上认为没有切点
         Point p = (p0 - c.o) * (c.r * c.r / x);
Point delta = ((p0 - c.o) * (-c.r * sqrt(d) / x)).turn90();
103
104
105
         p1 = c.o + p + delta;
         p2 = c.o + p - delta;
106
107
         return true;
108 }
    // 求圆到圆的内共切线,按关于 cl.o 的顺时针方向返回两条线
std::vector<Line> intanCC(const Circle &c1, const Circle &c2) {
111
         std::vector<Line> ret;
         Point p = (c1.0 * c2.r + c2.0 * c1.r) / (c1.r + c2.r);
112
113
         Point p1, p2, q1, q2;
114
         if (tanCP(c1, p, p1, p2) && tanCP(c2, p, q1, q2)) { // 两圆相切认为没有切线
115
             ret.push_back(Line(p1, q1));
116
             ret_push_back(Line(p2, q2));
117
118
         return ret:
119
    // 点在多边形内
120
    bool inPolygon(const Point& p, const std::vector<Point>& poly) {
         int n = polv.size():
123
         int counter = 0;
124
         for (int i = 0; i < n; ++ i) {
125
             P = poly[i], b = poly[(i + 1) % n];
126
             if (onSeg(Line(a, b), p)) return false; // 边界上不算
127
             int x = sign(det(p - a, b - a));
             int y = sign(a.y - p.y);
128
             int z = sign(b.y - p.y);
129
             if (x < 0 & & z < 0 & & y > 0) ++ counter;
if (x < 0 & & z <= 0 & & y > 0) -- counter;
130
131
132
133
         return counter != 0;
134
```

CHAPTER 1. 计算几何 3

```
135 // 用半平面 (q1,q2) 的逆时针方向去切凸多边形
    std::vector<Point> convexCut(const std::vector<Point>&ps, Point q1, Point q2) {
137
        std::vector<Point> qs; int n = ps.size();
        for (int i = 0; i < n; ++i) {
   Point p1 = ps[i], p2 = ps[(i + 1) % n];
   int d1 = crossOp(q1,q2,p1), d2 = crossOp(q1,q2,p2);
138
139
140
141
             if (d1 \ge 0) qs.push_back(p1);
142
             if (d1 * d2 < 0) qs_push_back(isSS(p1, p2, q1, q2));
143
144
        return qs;
145 }
    // 求凸包
146
147
    std::vector<Point> convexHull(std::vector<Point> ps) {
148
        int n = ps.size(); if (n <= 1) return ps;</pre>
149
        std::sort(ps.begin(), ps.end());
150
        std::vector<Point> qs;
151
        for (int i = 0; i < n; qs.push_back(ps[i ++]))</pre>
152
             while (qs.size() > 1 \&\& siqn(det(qs[qs.size() - 2], qs.back(), ps[i])) \le 0)
153
154
        for (int i = n - 2, t = qs.size(); i >= 0; qs.push_back(ps[i --]))
155
             while ((int)qs.size() > t && sign(det(qs[qs.size() - 2], qs.back(), ps[i])) <=
       → 0)
156
                 qs.pop_back();
        return qs;
```

1.1.2 凸包

```
// 凸包中的点按逆时针方向
   struct Convex {
        std::vector<Point> a, upper, lower;
        void make_shell(const std::vector<Point>& p,
                 std::vector<Point>& shell) { // p needs to be sorted.
            clear(shell); int n = p.size();
            shell.push_back(p[i]);
11
12
13
14
        void make convex() {
15
            std::sort(a.begin(), a.end());
make_shell(a, lower);
16
17
            std::reverse(a.begin(), a.end());
            ake_shell(a, upper);
a = lower; a.pop_back();
a.insert(a.end(), upper.begin(), upper.end());
if ((int)a.size() >= 2) a.pop_back();
18
19
20
21
22
23
            n = a.size();
24
25
26
27
28
29
        void init(const std::vector<Point>& _a) {
            clear(a); a = _a; n = a.size();
make_convex();
        void read(int _n) { // Won't make convex.
            clear(a); n = _n; a.resize(n);
for (int i = 0; i < n; i++)
30
31
32
33
                 a[i].read();
        std::pair<DB, int> get_tangent(
34
35
36
                 const std::vecTor<Point>& convex, const Point& vec) {
            int l = 0, r = (int)convex.size() - 2;
            assert(r >= 0);
37
            for (; l + 1 < r; ) {
38
39
40
                 int mid = (l + r) / 2;
                 if (sign(det(convex[mid + 1] - convex[mid], vec)) > 0)
                     r = mid;
41
                 else l = mid;
42
43
            return std::max(std::make_pair(det(vec, convex[r]), r),
                     std::make_pair(det(vec, convex[0]), 0));
```

```
int binary_search(Point u, Point v, int l, int r) {
46
47
              int s1 = sign(det(v - u, a[l % n] - u));
48
              for (; l + 1 < r; ) {
49
                    int mid = (l + r) / 2;
                    int smid = sign(det(v - u, a[mid % n] - u));
50
                   if (smid == s1) l = mid;
51
52
                   else r = mid;
53
54
              return 1 % n:
55
         }
         // 求凸包上和向量 vec 叉积最大的点,返回编号,共线的多个切点返回任意一个 int get_tangent(Point vec) {
56
57
              std::pair<DB, int> ret = get_tangent(upper, vec);
ret.second = (ret.second + (int)lower.size() - 1) % n;
58
59
60
              ret = std::max(ret, get_tangent(lower, vec));
61
              return ret.second;
62
         // 求凸包和直线 u, v 的交点, 如果不相交返回 false, 如果有则是和 (i, next(i)) 的
63
       → 交点, 交在点上不确定返回前后两条边其中之一
        → 文献、文社無工不確定返目前海内赤色共平之
bool get_intersection(Point u, Point v, int &i0, int &i1) {
    int p0 = get_tangent(u - v), p1 = get_tangent(v - u);
    if (sign(det(v - u, a[p0] - u)) * sign(det(v - u, a[p1] - u)) <= 0) {
        if (p0 > p1) std::swap(p0, p1);
        if (p0 > p1) std::swap(p0, p1);
65
66
67
68
                    i0 = binary_search(u, v, p0, p1);
69
                   i1 = binary_search(u, v, p1, p0 + n);
70
                   return true:
71
72
              else return false;
73
74 }:
```

1.2 三维

1.2.1 基础

```
// 三维绕轴旋转, 大拇指指向 axis 向量方向, 四指弯曲方向转 w 弧度
      Point rotate(const Point& s, const Point& axis, DB w) {
             DB x = axis.x, y = axis.y, z = axis.z;
             DB s1 = x * x + y * y + z * z, ss1 = msqrt(s1),

cosw = cos(w), sinw = sin(w);
             DB a[4][4];
             memset(a, 0, sizeof a);
             a[0][0] = ((y * y + z * z) * cosw + x * x) / s1;

a[0][1] = x * y * (1 - cosw) / s1 + z * sinw / ss1;
10
            a[0][2] = x * y * (1 - cosw) / s1 - y * sinw / ss1;
a[1][0] = x * y * (1 - cosw) / s1 - y * sinw / ss1;
a[1][1] = ((x * x + z * z) * cosw + y * y) / s1;
a[1][2] = y * z * (1 - cosw) / s1 + x * sinw / ss1;
11
12
13
14
15
             a[2][0] = x * z * (1 - cosw) / s1 + y * sinw / ss1;

a[2][1] = y * z * (1 - cosw) / s1 - x * sinw / ss1;
16
            \begin{array}{ll} \text{d2}[2] = ((x * x + y * y) * \cos(w) + z * z) / \text{s1}; \\ \text{DB ans}[4] = \{0, 0, 0, 0\}, c[4] = \{\text{s.x, s.y, s.z, 1}\}; \\ \text{for (int i = 0; i < 4; ++ i)} \end{array}
17
18
19
20
                    for (int j = 0; j < 4; ++ j)
ans[i] += a[j][i] * c[j];
21
22
23 }
             return Point(ans[0], ans[1], ans[2]);
```

1.2.2 凸包

```
__inline P cross(const P& a, const P& b) {
    return P(
        a.y * b.z - a.z * b.y,
        a.z * b.x - a.x * b.z,
        a.x * b.y - a.y * b.x
    }
}
```

```
__inline DB mix(const P& a, const P& b, const P& c) {
         return dot(cross(a, b), c);
11 }
12
13
     __inline DB volume(const P& a, const P& b, const P& c, const P& d) {
14
          return mix(b - a, c - a, d - a);
15 }
16 }
    struct Face {
17 l
          int a, b, c;
__inline Face() {}
18
19
         __inline Face(int _a, int _b, int _c):
    a(_a), b(_b), c(_c) {}
    _inline DB area() const {
20
21
22
23
               return 0.5 * cross(p[b] - p[a], p[c] - p[a]).len();
24
25
26
          __inline P normal() const {
               return cross(p[b] - p[a], p[c] - p[a]).unit();
27
28
          __inline DB dis(const P& p0) const {
29
30
               return dot(normal(), p0 - p[a]);
31
32 };
    std::vector<Face> face, tmp; // Should be O(n).
34
35
    int mark[N][N], Time, n;
36
       _inline void add(int v) {
37
          ++ Time;
38
39
          for (int i = 0; i < (int)face.size(); ++ i) {
               int a = face[i].a, b = face[i].b, c = face[i].c;
if (sign(volume(p[v], p[a], p[b], p[c])) > 0) {
    mark[a][b] = mark[b][a] = mark[a][c] =
40
41
42
                           mark[c][a] = mark[b][c] = mark[c][b] = Time;
43
44
45
               else {
46
                     tmp.push_back(face[i]);
47
48
49
          clear(face); face = tmp;
         for (int i = 0; i < (int)tmp.size(); ++ i) {
  int a = face[i].a, b = face[i].b, c = face[i].c;
  if (mark[a][b] == Time) face.emplace_back(v, b, a);
  if (mark[b][c] == Time) face.emplace_back(v, c, b);
  if (mark[c][a] == Time) face.emplace_back(v, a, c);</pre>
50
51
52
53
54
55
               assert(face.size() < 500u);
56
          }
57
58
59
     void reorder() {
          for (int i = 2; i < n; ++ i) {
   P tmp = cross(p[i] - p[0], p[i] - p[1]);</pre>
60
61
                if (sign(tmp.len())) {
62
                     std::swap(p[i], p[2]);
for (int j = 3; j < n; ++ j)
    if (sign(volume(p[0], p[1], p[2], p[j]))) {</pre>
63
64
65
66
                                 std::swap(p[j], p[3]);
67
                                 return;
68
69
70
          }
71
72
73
    void build_convex() {
74
          reorder();
75
          clear(face);
          face emplace_back(0, 1, 2);
76
77
          face emplace_back(0, 2, 1);
78
          for (int i = 3; i < n; ++ i)
79
               add(i);
80 }
```

Chapter 2 数论

$2.1 \quad O(m^2 \log n)$ 求线性递推数列第 n 项

Given $a_0, a_1, \ldots, a_{m-1}$ $a_n = c_0 \times a_{n-m} + \cdots + c_{m-1} \times a_{n-1}$ Solve for $a_n = v_0 \times a_0 + v_1 \times a_1 + \cdots + v_{m-1} \times a_{m-1}$

```
void linear_recurrence(long long n, int m, int a[], int c[], int p) {
         long long v[M] = \{1 \% \tilde{p}\}, u[M \ll 1], msk = !!n;
        for(long long i(n); i > 1; i >>= 1) {
4
             msk <<= 1:
5
        for(long long x(0); msk; msk >>= 1, x <<= 1) {
6
             fill_n(u, m << 1, 0);
             int \overline{b}(!!(n \& msk));
             x \mid = b;
10
             if(x < m) {
                  u[x] = 1 % p;
11
12
             }else {
13
                  for(int i(0); i < m; i++) {
   for(int j(0), t(i + b); j < m; j++, t++) {
      u[t] = (u[t] + v[i] * v[j]) % p;</pre>
14
15
16
17
                  for(int i((m << 1) - 1); i >= m; i--) {
18
19
                       for(int j(0), t(i - m); j < m; j++, t++) {
20
                           u[t] = (u[t] + c[j] * u[i]) % p;
21
22
23
24
             copy(u, u + m, v);
25
26
27
        //a[n] = v[0] * a[0] + v[1] * a[1] + ... + v[m-1] * a[m-1].
        for(int i(m); i < 2 * m; i++) {
28
             a[i] = 0;
             for(int j(0); j < m; j++) {
    a[i] = (a[i] + (long long)c[j] * a[i + j - m]) % p;
29
30
31
32
33
        for(int j(0); j < m; j++) {
             b[j] = 0;
34
35
             for(int i(0); i < m; i++) {
36
                  b[j] = (b[j] + v[i] * a[i + j]) % p;
37
38
        for(int j(0); j < m; j++) {
   a[j] = b[j];</pre>
39
40
41
42
```

2.2 求逆元

```
void ex_gcd(long long a, long long b, long long &x, long long &y) {
       if (b == 0) {
            x = 1;
            y = 0;
5
            return:
6
       long long xx, yy;
       ex_gcd(b, a % b, xx, yy);
       y = xx - a / b * yy;
10
       \dot{x} = yy;
11
12
13
   long_long_inv(long long x, long long MODN) {
        long long inv_x, y;
15
       ex_gcd(x, MOD\overline{N}, inv_x, y);
16
       return (inv_x % MODN + MODN) % MODN;
17
```

2.3 中国剩余定理

2.4 素性测试

```
int strong_pseudo_primetest(long long n,int base) {
        long long n2=n-1, res;
        while(n2%2==0) n2>>=1,s++;
        res=powmod(base,n2,n);
        if((res==1)||(res==n-1)) return 1;
        while(s \ge 0) {
            res=mulmod(res,res,n);
            if(res==n-1) return 1;
11
12
13
        return 0; // n is not a strong pseudo prime
14
15
   int isprime(long long n) {
       static LL testNum[]={2,3,5,7,11,13,17,19,23,29,31,37};
static LL lim[]={4,0,1373653LL,25326001LL,25000000000LL,2152302898747LL,
16
17
      → 3474749660383LL,341550071728321LL,0,0,0,0);
       if(n<2||n==3215031751LL) return 0;
18
        for(int i=0;i<12;++i){
19
            if(n<lim[i]) return 1;</pre>
21
            if(strong_pseudo_primetest(n,testNum[i])==0) return 0;
22
23
        return 1;
24 }
```

2.5 质因数分解

```
int ansn; LL ans[1000];
   LL func(LL x,LL n) { return(mod_mul(x,x,n)+1)%n; }
   LL Pollard(LL n){
       LL i,x,y,p;
if(Rabin_Miller(n)) return n;
       if(!(n&1)) return 2;
       for(i=1;i<20;i++){
           x=i; y=func(x,n); p=gcd(y-x,n);
           while(p==1) {x=func(x,n); y=func(func(y,n),n); p=gcd((y-x+n)%n,n)%n;}
           if(p==0||p==n) continue;
11
           return p;
12
13 }
   void factor(LL n){
14
15
16
       x=Pollard(n);
       if(x==n){ ans[ansn++]=x; return; }
17
18
       factor(x), factor(n/x);
```

2.6 线下整点

```
 \begin{array}{c|c} 1 \\ // & \sum_{i=0}^{n-1} \lfloor \frac{a+bi}{m} \rfloor \text{, } n,m,a,b>0 \\ 2 \\ \text{LL solve(LL n,LL a,LL b,LL m)} \\ \end{array}
```

```
if(b==0) return n*(a/m);
if(a>=m) return n*(a/m)+solve(n,a%m,b,m);
if(b>=m) return (n-1)*n/2*(b/m)+solve(n,a,b%m,m);
return solve((a+b*n)/m,(a+b*n)%m,m,b);
}
```

Chapter 3 代数

3.1 快速傅里叶变换

```
// n 必须是 2 的次幂
    void fft(Complex a[], int n, int f) {
         for (int i = 0; i < n; ++i)
               if (R[i] < i) swap(a[i], a[R[i]]);</pre>
         for (int i = 1, h = 0; i < n; i <<= 1, h++) {
              Complex wn = Complex(cos(pi / i), f * sin(pi / i));
              Complex w = Complex(1, 0);
              for (int k = 0; k < i; ++k, w = w * wn) tmp[k] = w;
for (int p = i << 1, j = 0; j < n; j += p) {
  for (int k = 0; k < i; ++k) {
8
10
                         Complex x = a[j + k], y = a[j + k + i] * tmp[k];

a[j + k] = x + y; a[j + k + i] = x - y;
11
12
13
14
              }
15
         }
16 }
```

Chapter 4 字符串

4.1 后缀数组

```
const int MAXN = MAXL * 2 + 1;
   int a[MAXN], x[MAXN], y[MAXN], c[MAXN], sa[MAXN], rank[MAXN], height[MAXN];
   void calc_sa(int n) {
        int m = alphabet, k = 1;
        memset(c, 0, sizeof(*c) * (m + 1));
        for (int i = 1; i \le n; ++i) c[x[i] = a[i]]++;
        for (int i = 1; i \le m; ++i) c[i] += c[i - 1];
        for (int i = n; i; --i) sa[c[x[i]]--] = i;
        for (; k <= n; k <<= 1) {
10
             int tot = k;
             for (int i = n - k + 1; i \le n; ++i) y[i - n + k] = i;
11
             for (int i = 1; i \le n; ++i)
                  if (sa[i] > k) y[++tot] = sa[i] - k;
13
             memset(c, 0, sizeof(*c) * (m + 1));
14
            for (int i = 1; i <= n; ++i) c[x[i]]++;

for (int i = 1; i <= m; ++i) c[i] += c[i - 1];

for (int i = n; i; --i) sa[c[x[y[i]]]--] = y[i];

for (int i = 1; i <= n; ++i) y[i] = x[i];

tot = 1; x[sa[1]] = 1;
15
16
17
18
19
20
             for (int i = 2; i \le n; ++i) {
                  if (max(sa[i], sa[i-1]) + k > n || y[sa[i]] != y[sa[i-1]] || y[sa[i] +
21
      \rightarrow k] != y[sa[i - 1] + k]) ++tot;
                 x[sa[i]] = tot;
23
24
             if (tot == n) break; else m = tot;
25
26
27
   void calc_height(int n) {
        for (int i = 1; i \le n; ++i) rank[sa[i]] = i;
        for (int i = 1; i <= n; ++i) {
29
30
             height[rank[i]] = max(0, height[rank[i-1]] - 1);
31
             if (rank[i] == 1) continue;
32
             int j = sa[rank[i] - 1];
33
             while (\max(i, j) + \text{height}[\text{rank}[i]] \le n \& a[i + \text{height}[\text{rank}[i]]] == a[j + n]

    height[rank[i]]]) ++height[rank[i]];
34
35
```

4.2 后缀自动机

```
static const int MAXL = MAXN * 2; // MAXN is original length
static const int alphabet = 26; // sometimes need changing
 3 int l, last, cnt, trans[MAXL][alphabet], par[MAXL], sum[MAXL], seq[MAXL], mxl[MAXL],

    size[MAXL]; // mxl is maxlength, size is the size of right

    char str[MAXL];
    inline void init() {
         l = strlen(str + 1); cnt = last = 1;
         for (int i = 0; i \le l * 2; ++i) memset(trans[i], 0, sizeof(trans[i]));
         memset(par, 0, sizeof(*par) * (l * 2 + 1));
        memset(mxl, 0, sizeof(*mxl) * (l * 2 + 1));
memset(size, 0, sizeof(*size) * (l * 2 + 1));
9
10
11 | }
   inline void extend(int pos, int c) {
   int p = last, np = last = ++cnt;
   mxl[np] = mxl[p] + 1; size[np] = 1;
12
13
14
         for (; p && !trans[p][c]; p = par[p]) trans[p][c] = np;
15
         if (!p) par[np] = 1;
16
17
18
              int q = trans[p][c];
19
              if (mxl[p] + 1 == mxl[q]) par[np] = q;
20
              else {
21
                    int ng = ++cnt;
22
23
                   mxl[nq] = mxl[p] + 1;
                    memcpy(trans[nq], trans[q], sizeof(trans[nq]));
24
25
26
                    par[nq] = par[q];
                    par[np] = par[q] = nq;
for (; trans[p][c] == q; p = par[p]) trans[p][c] = nq;
27
28
         }
29
30
    inline void buildsam() {
         for (int i = 1; i <= l; ++i) extend(i, str[i] - 'a');
31
         memset(sum, 0, sizeof(*sum) * (l * 2 + 1));
32
33
         for (int i = 1; i <= cnt; ++i) sum[mxl[i]]++;</pre>
         for (int i = 1; i <= l; ++i) sum[i] += sum[i - 1];
for (int i = cnt; i; --i) seq[sum[mxl[i]]--] = i;
for (int i = cnt; i; --i) size[par[seq[i]]] += size[seq[i]];
34
35
36
```

4.3 EX 后缀自动机

```
inline void add node(int x, int &last) {
        int lastnode = last;
       if (c[lastnode][x]) {
            int nownode = c[lastnode][x];
            if (l[nownode] == l[lastnode] + 1) last = nownode:
            else {
                 int auxnode = ++cnt; l[auxnode] = l[lastnode] + 1;
                for (int i = 0; i < alphabet; ++i) c[auxnode][i] = c[nownode][i];</pre>
                 par[auxnode] = par[nownode]; par[nownode] = auxnode;
                 for (; lastnode && c[lastnode][x] == nownode; lastnode = par[lastnode]) {
                     c[lastnode][x] = auxnode;
12
13
                 last = auxnode;
14
15
       } else {
16
            int newnode = ++cnt; l[newnode] = l[lastnode] + 1;
            for (: lastnode && !c[lastnode][x]: lastnode = par[lastnode]) c[lastnode][x] =

→ newnode;

            if (!lastnode) par[newnode] = 1;
18
19
            else {
20
                int nownode = c[lastnode][x];
                if (l[lastnode] + 1 == l[nownode]) par[newnode] = nownode;
21
22
23
                     int auxnode = ++cnt; l[auxnode] = l[lastnode] + 1;
24
                     for (int i = 0; i < alphabet; ++i) c[auxnode][i] = c[nownode][i];</pre>
25
                     par[auxnode] = par[nownode]; par[nownode] = par[newnode] = auxnode;
for (; lastnode && c[lastnode][x] == nownode; lastnode =
      → par[lastnodel) {
```

4.4 回文自动机

```
int nT, nStr, last, c[MAXT][26], fail[MAXT], r[MAXN], l[MAXN], s[MAXN]; int allocate(int len) {
       l[nT] = len;
       r[nT] = 0;
5
       fail[nT] = 0;
       memset(c[nT], 0, sizeof(c[nT]));
       return nT++:
8
9
   void init() {
10
       nT = nStr = 0;
11
       int newE = allocate(0);
12
       int new0 = allocate(-1);
13
       last = newE:
14
       fail[newE] = new0;
       fail[new0] = newE;
15
16
       s[0] = -1;
17 }
18 | void add(int x) {
19
       s[++nStr] = x;
       int now = last;
20
       while (s[nStr - l[now] - 1] != s[nStr]) now = fail[now]; if (!c[now][x]) {
21
22
23
            int newnode = allocate(l[now] + 2), &newfail = fail[newnode];
24
           newfail = fail[now];
25
           while (s[nStr - l[newfail] - 1] != s[nStr]) newfail = fail[newfail]:
26
           newfail = c[newfail][x]:
27
           c[now][x] = newnode:
28
29
       last = c[now][x]:
30
       r[last]++:
31
32
   void count() {
33
       for (int i = nT - 1; i \ge 0; i--) {
           r[fail[i]] += r[i];
34
35
36
```

Chapter 5 数据结构

5.1 KD-Tree

```
long long norm(const long long &x) {
            For manhattan distance
       return std::abs(x);
           For euclid distance
       return x * x;
6
8
   struct Point {
       int x, y, id;
10
       const int& operator [] (int index) const {
11
12
           if (index == 0) {
13
               return x;
14
           } else {
15
               return y;
16
       }
17
18
19
       friend long long dist(const Point &a, const Point &b) {
20
           long long result = 0:
```

CHAPTER 5. 数据结构

```
7
```

```
for (int i = 0; i < 2; ++i) {
                result += norm(a[i] - b[i]);
22
23
24
25
26
27
28
            return result;
   } point[N];
   struct Rectangle {
29
30
        int min[2], max[2];
31
        Rectangle() {
32
33
            min[0] = min[1] = INT_MAX; // sometimes int is not enough
            \max[0] = \max[1] = INT MIN;
34
35
36
        void add(const Point &p) {
37
            for (int i = 0; i < 2; ++i) {
38
                min[i] = std::min(min[i], p[i]);
39
                max[i] = std::max(max[i], p[i]);
40
        }
41
42
43
        long long dist(const Point &p) {
44
            long long result = 0;
45
            for (int i = 0; i < 2; ++i) {
46
                // For minimum distance
47
48
                result += norm(std::min(std::max(p[i], min[i]), max[i]) - p[i]);
                // For maximum distance
49
                result += std::max(norm(max[i] - p[i]), norm(min[i] - p[i]));
50
51
            return result:
52
53
54
55
   };
   struct Node {
56
57
        Point seperator;
        Rectangle rectangle;
58
59
        int child[2];
60
        void reset(const Point &p) {
61
            seperator = p;
62
            rectangle = Rectangle();
63
            rectangle.add(p);
64
            child[0] = child[1] = 0;
   } tree[N << 1];</pre>
68
   int size, pivot;
   bool compare(const Point &a, const Point &b) {
   if (a[pivot] != b[pivot]) {
70
71
72
            return a[pivot] < b[pivot]:</pre>
73
74
75
76
        return a.id < b.id;</pre>
   // 左閉右開: build(1, n + 1)
78
   int build(int l, int r, int type = 1) {
79
        pivot = type;
80
        if (l >= r) {
81
            return 0;
82
83
84
        int mid = l + r \gg 1;
        std::nth_element(point + l, point + mid, point + r, compare);
85
86
        tree[x].reset(point[mid]);
87
        for (int i = l; i < r; ++i) {
88
            tree[x].rectangle.add(point[i]);
89
90
        tree[x].child[0] = build(l, mid, type ^ 1);
91
92
93
        tree[x].child[1] = build(mid + 1, r, type ^ 1);
        return x;
```

```
95
    int insert(int x, const Point &p, int type = 1) {
 96
        pivot = type;
 97
        if (x == 0)
 98
            tree[++size].reset(p);
 99
             return size;
100
101
        tree[x].rectangle.add(p);
        if (compare(p, tree[x] seperator)) {
102
103
             tree[x].child[0] = insert(tree[x].child[0], p, type ^ 1);
104
105
            tree[x].child[1] = insert(tree[x].child[1], p, type ^ 1);
106
107
        return x;
108
109
110 // For minimum distance
    // For maximum: 下面递归 query 时 0, 1 换顺序;< and >;min and max
    void query(int x, const Point &p, std::pair<long long, int> &answer, int type = 1) {
        if (x == 0 | | tree[x].rectangle.dist(p) > answer.first) {
114
115
             return;
116
117
        answer = std::min(answer,
118
                  std::make_pair(dist(tree[x].seperator, p), tree[x].seperator.id));
         if (compare(p, tree[x].seperator)) {
119
            query(tree[x].child[0], p, answer, type ^ 1);
120
121
             query(tree[x].child[1], p, answer, type ^ 1);
122
        } else {
123
             query(tree[x].child[1], p, answer, type ^ 1);
124
             query(tree[x].child[0], p, answer, type ^ 1);
125
126
128
129
    std::priority_queue<std::pair<long long, int> > answer;
130
    void query(int x, const Point &p, int k, int type = 1) {
131
         if (x == 0 \mid | (int)answer.size() == k && tree[x].rectangle.dist(p) >
132
       → answer.top().first) {
133
134
135
        answer.push(std::make_pair(dist(tree[x].seperator, p), tree[x].seperator.id));
        if ((int)answer size(\overline{)} > k) {
136
137
            answer.pop();
138
        if (compare(p, tree[x].seperator)) {
139
            query(tree[x] child[0], p, k, type ^ 1);
140
141
             query(tree[x].child[1], p, k, type ^ 1);
142
        } else {
143
            query(tree[x].child[1], p, k, type ^ 1);
query(tree[x].child[0], p, k, type ^ 1);
144
145
146 }
```

5.2 Treap

```
struct Node{
         int mn, key, size, tag;
         bool rev;
        Node* ch[2];
        Node(int mn, int key, int size): mn(mn), key(key), size(size), rev(0), tag(0){}
         void downtag():
         Node* update(){
             mn = min(ch[0] \rightarrow mn, min(key, ch[1] \rightarrow mn));

size = ch[0] \rightarrow size + 1 + ch[1] \rightarrow size;
9
10
             return this:
11
12
13 typedef pair<Node*, Node*> Pair;
14 Node *null, *root;
15 | void Node::downtag(){
```

```
16
        if(rev){
17
             for(int i = 0; i < 2; i++)
18
                  if(ch[i] != null){
                      ch[i] -> rev ^= 1;
19
20
                      swap(ch[i] \rightarrow ch[0], ch[i] \rightarrow ch[1]);
21
22
23
             rev = 0:
24
        if(tag){
25
             for(int i = 0; i < 2; i++)
26
                 if(ch[i] != null){
                      ch[i] -> key += tag;
27
28
                      ch[i] -> mn += tag;
29
                      ch[i] \rightarrow tag += tag;
30
31
32
             tag = 0;
33
34
   int r(){
35
        static int s = 3023192386;
36
        return (s += (s << 3) + 1) & (\sim0u >> 1);
37
38
   bool random(int x, int y){
39
        return r() % (x + y) < x;
40
41
   Node* merge(Node *p, Node *q){
42
        if(p == null) return q;
43
        if(q == null) return p;
44
        p -> downtag();
45
        q -> downtag();
46
        if(random(p -> size, q -> size)){
             p \rightarrow ch[1] = merge(p \rightarrow ch[1], q);
47
48
             return p -> update();
49
50
             q \rightarrow ch[0] = merge(p, q \rightarrow ch[0]);
51
             return q -> update();
52
53
54 | Pair split(Node *x, int n){
55
        if(x == null) return make_pair(null, null);
56
        x -> downtag();
        if(n <= x -> ch[0] -> size){
    Pair ret = split(x -> ch[0], n);
57
58
             x \rightarrow ch[0] = ret.second;
59
             return make_pair(ret.first, x -> update());
60
61
62
        Pair ret = split(x \rightarrow ch[1], n - x \rightarrow ch[0] \rightarrow size - 1);
63
        x \rightarrow ch[1] = ret.first;
64
        return make_pair(x -> update(), ret.second);
65
66
   pair<Node*, Pair> get_segment(int l, int r){
67
        Pair ret = split(root, l - 1);
68
        return make_pair(ret.first, split(ret.second, r - l + 1));
69
70
        null = new Node(INF, INF, 0);
null -> ch[0] = null -> ch[1] = null;
71
72
73
        root = null:
74 | }
```

5.3 Link/cut Tree

```
inline void reverse(int x) {
    tr[x].rev ^= 1; swap(tr[x].c[0], tr[x].c[1]);
}
inline void rotate(int x, int k) {
    int y = tr[x].fa, z = tr[y].fa;
    tr[x].fa = z; tr[z].c[tr[z].c[1] == y] = x;
    tr[tr[x].c[k ^ 1]].fa = y; tr[y].c[k] = tr[x].c[k ^ 1];
    tr[x].c[k ^ 1] = y; tr[y].fa = x;
```

```
10 | }
11
12
   inline void splay(int x, int w) {
13
        int z = x; pushdown(x);
        while (tr[x] fa != w) {
14
15
            int y = tr[x].fa; z = tr[y].fa;
16
            if (z == w)
                pushdown(z = y); pushdown(x);
rotate(x, tr[y].c[1] == x);
update(y); update(x);
17
18
19
20
            } else {
                pushdown(z); pushdown(y); pushdown(x);
21
22
                 int t1 = tr[y] c[1] == x, t2 = tr[z] c[1] == y;
23
                if (t1 == t2) rotate(y, t2), rotate(x, t1);
24
                else rotate(x, t1), rotate(x, t2);
25
                update(z); update(x);
26
27
28
       update(x);
29
        if (x != z) par[x] = par[z], par[z] = 0;
30 }
31
32
   inline void access(int x) {
33
        for (int y = 0; x; y = x, x = par[x]) {
            splay(x, 0);
            if (tr[x].c[1]) par[tr[x].c[1]] = x, tr[tr[x].c[1]].fa = 0;
            tr[x].c[1] = y; par[y] = 0; tr[y].fa = x; update(x);
37
38
39
40
   inline void makeroot(int x) {
41
42
        access(x); splay(x, 0); reverse(x);
43
   inline void link(int x, int y) {
45
        makeroot(x); par[x] = y;
46
47
48
   inline void cut(int x, int y) {
       access(x); splay(y, 0);
if (par[y] != x) swap(x, y), access(x), splay(y, 0);
49
51
        par[y] = 0;
52
53
54
   inline void split(int x, int y) { // x will be the root of the tree
55
        makeroot(y); access(x); splay(x, 0);
```

Chapter 6 图论

6.1 基础

```
struct Graph { // Remember to call .init()!
       int e, nxt[M], v[M], adj[N], n;
       bool base;
       __inline void init(bool _base, int _n = 0) {
           assert(n < N);</pre>
           n = n; base = _base;
e = 0; memset(adj + base, -1, sizeof(*adj) * n);
       __inline int new_node() {
10
           adj[n + base] = -1;
11
           assert(n + base + \dot{1} < N);
12
13
           return n++ + base;
14
       __inline void ins(int u0, int v0) { // directional
15
           assert(u0 < n + base && v0 < n + base);
16
           v[e] = v0; nxt[e] = adj[u0]; adj[u0] = e++;
17
           assert(e < M);
18
19
       __inline void bi_ins(int u0, int v0) { // bi-directional
```

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```
20 | ins(u0, v0); ins(v0, u0); 21 | };
```

6.2 KM

```
struct KM {
       // Trulv 0(n^3)
        // 邻接矩阵,不能连的边设为 -INF, 求最小权匹配时边权取负, 但不能连的还是 -INF,
      → 使用时先对 1 -> n 调用 hungary() , 再 get_ans() 求值
       int w[N][N];
       int lx[N], ly[N], match[N], way[N], slack[N];
bool used[N];
       void init() {
            for (int i = 1; i <= n; i++) {
                match[i] = 0;
10
                 lx[i] = 0;
11
                 lv[i] = 0;
12
                wav[i] = 0;
13
14
15
        void hungary(int x) {
            match[0] = x;
16
17
            int j0 = 0;
18
            for (int j = 0; j <= n; j++) {
                 slack[j] = INF;
19
20
                used[j] = false;
21
22
23
            do {
24
                 used[i0] = true:
25
                 int i0 = match[j0], delta = INF, j1 = 0;
                for (int j = 1; j <= n; j++) {
   if (used[j] == false) {</pre>
26
27
                         int cur = -w[i0][j] - lx[i0] - ly[j];
if (cur < slack[j]) {
28
29
30
                              slack[j] = cur;
31
                              way[j] = j0;
32
33
                         if (slack[j] < delta) {</pre>
34
                              delta = slack[j];
35
                              j1 = j;
36
37
                     }
38
                for (int j = 0; j <= n; j++) {
   if (used[j]) {</pre>
39
40
                         lx[match[j]] += delta;
41
42
                         lv[i] -= delta;
43
44
                     else slack[j] -= delta;
45
46
                 i0 = j1;
47
48
            } while (match[j0] != 0);
49
                 int j1 = way[j0];
50
51
                 match[j0] = match[j1];
52
                 i0 = i1;
53
            } while (j0);
54
55
56
57
       int get_ans() {
            int sum = 0;
58
            for(int i = 1; i <= n; i++) {
59
                 if (w[match[i]][i] == -INF); // 无解
60
                 if (match[i] > 0) sum += w[match[i]][i];
61
62
            return sum;
       }
63
```

64|} km;

6.3 点双连通分量

bcc.forest is a set of connected tree whose vertices are chequered with cut-vertex and BCC.

```
const bool BCC_VERTEX = 0, BCC_EDGE = 1;
   struct BCC {    // N = N0 + M0. Remember to call init(&raw_graph).
    Graph *g, forest; // g is raw graph ptr.
    int dfn[N], DFN, low[N];
        int stack[N], top;
        int expand to[N];
                                     // Where edge i is expanded to in expaned graph.
        // Vertex \bar{i} expaned to i.
        int compress_to[N]; // Where vertex i is compressed to.
bool vertex_type[N], cut[N], compress_cut[N], branch[M];
//std::vector<int> BCC_component[N]; // Cut vertex belongs to none.
__inline void init(Graph *raw_graph) {
8
9
10
11
12
             g = raw_graph;
13
14
15
        void DFS(int u, int pe) {
   dfn[u] = low[u] = ++DFN; cut[u] = false;
16
             if (!~g->adj[u]) {
17
                  cut[u] = 1;
18
                  compress_to[u] = forest.new_node();
19
                  compress_cut[compress_to[u]] = 1;
20
21
             for (int e = g\rightarrow adj[u]; \sim e; e = g\rightarrow nxt[e]) {
                   int v = g->v[e];
22
23
                  if ((e^pe) > 1 && dfn[v] > 0 && dfn[v] < dfn[u]) {
24
                       stack[top++] = e;
25
                       low[u] = std::min(low[u], dfn[v]);
26
27
                  else if (!dfn[v]) {
28
                       stack[top++] = e; branch[e] = 1;
                       DFS(v, e);
29
30
                       low[u] = std::min(low[v], low[u]);
                       if (low[v] >= dfn[u]) {
31
32
                            if (!cut[u]) {
33
                                 cut[u] = 1;
                                 compress_to[u] = forest_new_node();
34
35
                                 compress_cut[compress_to[u]] = 1;
36
37
                            int cc = forest.new_node();
38
                            forest.bi ins(compress to[u], cc);
39
                            compress cut[cc] = 0;
40
                            //BCC_component[cc].clear();
41
                            do {
42
                                 int cur_e = stack[--top];
43
                                 compress_to[expand_to[cur_e]] = cc;
44
                                 compress_to[expand_to[cur_e^1]] = cc;
45
                                 if (branch[cur e]) {
46
                                      int v = g - v[cur_e];
                                      if (cut[v])
47
48
                                           forest.bi_ins(cc, compress_to[v]);
49
                                      else {
50
                                           //BCC_component[cc].push_back(v);
51
                                           compress to[v] = cc:
52
53
54
                            } while (stack[top] != e);
55
56
                  }
57
             }
58
59
        void solve() {
             forest.init(q->base):
60
61
             int n = g -> n;
             for (int i = 0; i < g->e; i++)
62
63
                  expand_to[i] = g->new_node();
64
65
             memset(branch, 0, sizeof(*branch) * g->e);
```

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```
66
            memset(dfn + g->base, 0, sizeof(*dfn) * n); DFN = 0;
67
            for (int i = 0; i < n; i++)
68
                if (!dfn[i + g->base]) {
69
70
                    top = 0;
                    DFS(i + q->base, -1);
71
72
73
74
   } bcc;
75
   bcc.init(&raw_graph);
76 | bcc.solve();
  // Do something with bcc.forest ...
```

6.4 边双连通分量

```
struct BCC {
         Graph *q, forest;
         int dfn[N], low[N], stack[N], tot[N], belong[N], vis[N], top, dfs\_clock; // tot[] is the size of each BCC, belong[] is the BCC that each node belongs to
         pair<int, int > ori[M]; // bridge in raw_graph(raw node)
         bool is_bridge[M];
         __inline void init(Graph *raw_graph) {
              g = raw_graph;
              memset(is_bridge, false, sizeof(*is_bridge) * g -> e);
10
              memset(vis + g \rightarrow base, 0, sizeof(*vis) * g \rightarrow n);
11
12
         void tarjan(int u, int from)
13
              dfn[u] = low[u] = ++dfs\_clock; vis[u] = 1; stack[++top] = u;
14
              for (int p = g -> adj[u]; ~p; p = g -> nxt[p]) {
   if ((p ^ 1) == from) continue;
15
                   int v = g \rightarrow v[p];
16
17
                   if (vis[v]) {
18
                        if (vis[v] == 1) low[u] = min(low[u], dfn[v]);
19
                   } else {
20
                        tarjan(v, p);
                        low[u] = min(low[u], low[v]);
if (low[v] > dfn[u]) is_bridge[p / 2] = true;
21
22
23
24
25
              if (dfn[u] != low[u]) return;
26
27
28
              tot[forest.new node()] = 0:
                   belong[stack[top]] = forest.n;
vis[stack[top]] = 2;
29
30
31
                   tot[forest.n]++;
                   --top;
              } while (stack[top + 1] != u);
32
33
34
35
         void solve() {
              forest.init(g -> base);
              int n = g \rightarrow n;
36
37
              for (int i = 0; i < n; ++i)
38
                   if (!vis[i + g -> base]) {
39
                        top = dfs_clock = 0;
40
                        tarjan(i + g \rightarrow base, -1);
41
42
              for (int i = 0; i < g -> e / 2; ++i)
    if (is_bridge[i]) {
43
44
                        int e = forest.e;
45
                        forest.bi_ins(belong[g \rightarrow v[i * 2]], belong[g \rightarrow v[i * 2 + 1]], g \rightarrow
       \hookrightarrow w[i * 2]);
46
                        ori[e] = make_pair(g -> v[i * 2 + 1], g -> v[i * 2]);
                        ori[e + 1] = make_pair(g \rightarrow v[i * 2], g \rightarrow v[i * 2 + 1]);
47
48
49
   } bcc;
```

6.5 最小树形图

```
const int MAXN,INF;// INF >= sum( W_ij )
int from[MAXN + 10][MAXN * 2 + 10], n, m, edge[MAXN + 10][MAXN * 2 + 10];
```

```
3 | int sel[MAXN * 2 + 10], fa[MAXN * 2 + 10], vis[MAXN * 2 + 10];
   int getfa(int x){if(x == fa[x]) return x; return fa[x] = getfa(fa[x]);}
   void liuzhu(){ // 1-base: root is 1, answer = (sel[i], i) for i in [2..n]
        for(int i = 2; i \le n; ++i){
            sel[i] = 1: fa[i] = i:
            for(int j = 1; j <= n; ++j) if(fa[j] != i)
9
                 if(from[j][i] = i, edge[sel[i]][i] > edge[j][i]) sel[i] = j;
10
11
12
        int limit = n;
13
        while(1){
            int prelimit = limit; memset(vis, 0, sizeof(vis)); vis[1] = 1;
for(int i = 2; i <= prelimit; ++i) if(fa[i] == i && !vis[i]){</pre>
14
15
                 int j = i; while(!vis[j]) vis[j] = i, j = getfa(sel[j]);
if(j == 1 || vis[j] != i) continue; vector<int> C; int k = j;
16
17
                 do C.push_back(k), k = getfa(sel[k]); while(k != j);
18
19
20
                 for(int i = 1; i <= n; ++i){
21
                      edge[i][limit] = INF, from[i][limit] = limit;
22
23
                 fa[limit] = vis[limit] = limit;
24
                 for(int i = 0; i < int(C.size()); ++i){
25
                      int x = C[i], fa[x] = limit;
26
                      for(int j = 1; j <= n; ++j)
27
                           if(edge[j][x] != INF && edge[j][limit] > edge[j][x] -
      \hookrightarrow edge[sel[x]][x]){
28
29
                               edge[j][limit] = edge[j][x] - edge[sel[x]][x];
                               from[j][limit] = x;
30
31
32
                 for(int j=1;j<=n;++j) if(getfa(j)==limit) edge[j][limit] = INF;</pre>
33
                 sel[limit] = 1;
34
                 for(int j = 1; j <= n; ++j)
                      if(edge[sel[limit]][limit] > edge[j][limit]) sel[limit] = j;
35
36
37
38
            if(prelimit == limit) break;
39
        for(int i = limit; i > 1; --i) sel[from[sel[i]][i]] = sel[i];
40
```

6.6 带花树

```
vector<int> link[maxn];
   int n,match[maxn],Queue[maxn],head,tail;
   int pred[maxn], base[maxn], start, finish, newbase;
   bool InQueue[maxn], InBlossom[maxn];
   void push(int u){ Queue[tail++]=u;InQueue[u]=true; }
   int pop(){ return Queue[head++]; }
   int FindCommonAncestor(int u,int v){
       bool InPath[maxn];
       for(int i=0;i<n;i++) InPath[i]=0;</pre>
       while(true){ u=base[u];InPath[u]=true;if(u==start) break;u=pred[match[u]]; }
10
       while(true){ v=base[v];if(InPath[v]) break;v=pred[match[v]]; }
11
12
       return v;
13
14
   void ResetTrace(int u){
15
16
       while(base[u]!=newbase){
17
           v=match[u];
18
           InBlossom[base[u]]=InBlossom[base[v]]=true;
19
           u=pred[v];
20
           if(base[u]!=newbase) pred[u]=v;
21
22
23
   void BlossomContract(int u,int v){
24
25
       newbase=FindCommonAncestor(u,v);
       for (int i=0;i<n;i++)</pre>
26
       InBlossom[i]=0;
27
       ResetTrace(u); ResetTrace(v)
28
       if(base[u]!=newbase) pred[u]=v;
```

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```
29
         if(base[v]!=newbase) pred[v]=u;
30
         for(int i=0;i<n;++i</pre>
31
         if(InBlossom[base[i]]){
32
              base[i]=newbase;
33
              if(!InOueue[i]) push(i):
34
35
36
    bool FindAugmentingPath(int u){
         bool found=false;
37
38
        for(int i=0;i<n;++i) pred[i]=-1,base[i]=i;
for (int i=0;i<n;i++) InQueue[i]=0;</pre>
39
40
         start=u;finish=-1; head=tail=0; push(start);
41
         while(head<tail){</pre>
             int u=pop();
for(int i=link[u].size()-1;i>=0;i--){
    int v=link[u][i];
    int v=link[u][i];
42
43
44
                   if(base[u]!=base[v]&&match[u]!=v)
   if(v==start||(match[v]>=0&&pred[match[v]]>=0))
        BlossomContract(u,v);
45
46
47
                        else if(pred[v]==-1){
48
                             pred[v]=u:
49
50
                              if(match[v]>=0) push(match[v]);
51
                              else{ finish=v; return true; }
52
53
54
55
56
         return found;
57
58
    void AugmentPath(){
         int u=finish,v,w;
59
         while(u>=0){ v=pred[u];w=match[v];match[v]=u;match[u]=v;u=w; }
60
61
    void FindMaxMatching(){
         for(int i=0:i<n:++i) match[i]=-1:
62
         for(int i=0;i<n;++i) if(match[i]==-1) if(FindAugmentingPath(i)) AugmentPath();</pre>
63
```

6.7 Dominator Tree

```
vector<int> prec[N], succ[N];
 vector<int> ord;
 3 int stamp, vis[N];
   int num[N);
   int fa[N];
   void dfs(int u) {
       vis[u] = stamp;
num[u] = ord.size();
        ord.push_back(u);
10
        for (int^i = 0; i < (int)succ[u].size(); ++i) {
11
            int v = succ[u][i];
12
            if (vis[v] != stamp) {
13
                 fa[v] = u;
14
                dfs(v);
15
16
17
18
   int fs[N], mins[N], dom[N], sem[N];
int find(int u) {
19
20
        if (u != fs[u])
21
            int v = fs[u];
22
            fs[u] = find(fs[u]);
23
            if (\min[v] != -1 \&\& num[sem[mins[v]]] < num[sem[mins[u]]]) {
24
                mins[u] = mins[v];
25
26
27
        return fs[u];
28
29 void merge(int u, int v) { fs[u] = v; }
30 vector<int> buf[N];
31 int buf2[N];
32 void mark(int source) {
```

```
ord.clear():
34
       ++stamp;
35
       dfs(source):
36
        for (int i = 0: i < (int) \text{ ord.size}(): ++i) {
37
            int u = ord[i];
38
            fs[u] = u, mins[u] = -1, buf2[u] = -1;
39
40
        for (int i = (int) ord.size() - 1; i > 0; --i) {
            int u = ord[i], p = fa[u];
41
            sem[u] = p;
42
43
            for (int j = 0; j < (int)prec[u].size(); ++j) {</pre>
                int v = prec[u][j];
44
45
                 if (use[v] != stamp) continue;
46
                if (num[v] > num[u]) {
47
                     find(v); v = sem[mins[v]];
48
49
                if (num[v] < num[sem[u]]) {</pre>
50
                     sem[u] = v;
51
52
53
            buf[sem[u]].push_back(u);
54
            mins[u] = u;
55
            merge(u, p);
            while (buf[p] size()) {
56
57
58
                int v = buf[p].back();
buf[p].pop_back();
                find(v);
59
60
                   (sem[v] == sem[mins[v]]) {
                     dom[v] = sem[v];
61
62
                } else {
63
                    buf2[v] = mins[v];
64
65
            }
66
67
       dom[ord[0]] = ord[0];
       for (int i = 0; i < (int)ord.size(); ++i) {
68
            int u = ord[i];
69
            if (~buf2[u]) {
70
                dom[u] = dom[buf2[u]];
71
72
73
       }
74
```

无向图最小割

```
int cost[maxn] [maxn], seq[maxn], len[maxn], n, m, pop, ans;
    bool used[maxn];
 3
    void Init(){
         int i, j, a, b, c;
         for(i=0;i<n;i++) for(j=0;j<n;j++) cost[i][j]=0;
         for(i=0;i<m;i++){
              scanf("%d %d %d",&a,&b,&c); cost[a][b]+=c; cost[b][a]+=c;
 9
         pop=n; for(i=0;i<n;i++) seq[i]=i;
10 }
11
12
   void Work(){
        ans=inf; int i,j,k,l,mm,sum,pk;
while(pop > 1){
13
14
              for(i=1;i<pop;i++) used[seq[i]]=0; used[seq[0]]=1;</pre>
15
              for(i=1;i<pop;i++) len[seq[i]]=cost[seq[0]][seq[i]];
pk=0; mm=-inf; k=-1;</pre>
16
17
              for(i=1;i<pop;i++) if(len[seq[i]] > mm){ mm=len[seq[i]]; k=i; }
18
              for(i=1;i<pop;i++){
    used[seq[l=k]]=1;</pre>
19
                   if(i==pop-2) pk=k;
if(i==pop-1) break;
20
21
22
                   mm=-inf;
                   for(j=1;j<pop;j++) if(!used[seq[j]])
    if((len[seq[j]]+=cost[seq[l]][seq[j]]) > mm)
23
24
25
                             mm=len[seq[j]], k=j;
```

Chapter 7 其他

7.1 Dancing Links

```
1 | struct Node {
        Node *1, *r, *u, *d, *col;
         int size, line_no;
        Node() {
              size = 0; line_no = -1;
              l = r = \dot{u} = d = col = \dot{N}ULL:
   } *root;
   void cover(Node *c) {
10
11
        c -> l -> r = c -> r; c -> r -> l = c -> l;
         for (Node *u = c->d: u != c: u = u->d)
12
13
             for (Node *v = u->r; v != u; v = v->r) {
14
                  v\rightarrow d\rightarrow u = v\rightarrow u;
15
                  v->u->d = v->d;
16
                  -- v->col->size;
17
18
19
    void uncover(Node *c) {
21
         for (Node *u = c->u; u != c; u = u->u) {
22
23
24
25
26
27
28
              for (Node *v = u -> 1; v != u; v = v -> 1) {
                  ++ v->col->size;
                  v->u->d = v:
                  v\rightarrow d\rightarrow u = v;
        c -> l -> r = c; c -> r -> l = c;
29
30
31
   std::vector<int> answer:
   bool search(int k) {
33
         if (root->r == root) return true;
34
35
        Node *r = NULL;
        for (Node *u = root->r; u != root; u = u->r)
36
             if (r == NULL || u->size < r->size)
37
38
        if (r == NULL || r->size == 0) return false;
39
        else {
40
              cover(r);
41
             bool succ = false;
             for (Node *u = r->d; u != r && !succ; u = u->d) {
   answer.push_back(u->line_no);
42
43
44
                  for (Node *\overline{v} = u \rightarrow r; v != u; v = v \rightarrow r) // Cover row
45
                       cover(v->col);
                  succ |= search(k + 1);
46
                  for (Node *v = u -> 1; v != u; v = v -> 1)
47
48
                       uncover(v->col);
49
                  if (!succ) answer pop back();
50
51
52
             uncover(r);
             return succ;
53
54
55
56 bool entry[CR][CC];
57 | Node *who[CR][CC];
58 int cr, cc;
```

```
void construct() {
 60
         root = new Node();
 61
         Node *last = root;
 62
 63
         for (int i = 0; i < cc; ++ i) {
             Node *u = new Node();
 64
 65
              last->r = u; u->l = last;
 66
             Node *v = u; u \rightarrow line_no = i;
 67
              last = u;
             for (int j = 0; j < cr; ++ j)
    if (entry[j][i]) {</pre>
 68
 69
 70
                       ++ u->size;
 71
                      Node *cur = new Node();
 72
73
                       who[j][i] = cur;
                       cur->line_no = j;
 74
                       cur->col = u:
 75
76
                       cur->u = v; v->d = cur;
                       v = cur:
 77
 78
             v->d = u; u->u = v;
 79
 80
         last->r = root; root->l = last;
 81
         for (int j = 0; j < cr; ++ j) {
             Node *last = NULL;
 82
 83
             for (int i = cc - 1; i >= 0; -- i)
   if (entry[j][i]) {
 84
 85
                       last = who[j][i];
                       break:
 87
 88
              for (int i = 0; i < cc; ++ i)
                  if (entry[j][i]) {
 89
                       last->r = who[j][i];
who[j][i]->l = last;
 90
 91
 92
                       last = who[j][i];
 93
 94
         }
 95
 96
 97
     void destruct() {
         for (Node *u = root->r; u != root; ) {
              for (Node *v = u->d; v != u; ) {
100
                  Node *nxt = v->d:
101
                  delete(v);
102
                  v = nxt;
103
104
             Node *nxt = u->r;
105
             delete(u); u = nxt;
106
107
         delete root;
108
```

7.2 蔡勒公式

```
int zeller(int y,int m,int d) {
   if (m<=2) y--,m+=12; int c=y/100; y%=100;
   int w=((c>>2)-(c<<1)+y+(y>>2)+(13*(m+1)/5)+d-1)%7;
   if (w<0) w+=7; return(w);
}</pre>
```

Chapter 8 技巧

8.1 真正的释放 STL 容器内存空间

```
template <typename T>
__inline void clear(T& container) {
    container.clear(); // 或者删除了一堆元素
    T(container).swap(container);
}
```

8.2 无敌的大整数相乘取模

Time complexity O(1).

```
1 // 需要保证 x 和 y 非负
2 long long mult(long long x, long long y, long long MODN) {
3 long long t = (x * y - (long long)((long double)x / MODN * y + 1e-3) * MODN) %

→ MODN;
return t < 0 ? t + MODN : t;
}
```

8.3 无敌的读入优化

```
1 // getchar() 读入优化 << 关同步 cin << 此优化
 2 // 用 isdigit() 会小幅变慢
 3 // 返回 false 表示读到文件尾
    namespace Reader {
        const int L = (1 << 15) + 5;
        char buffer[L], *S, *T;
        __inline bool getchar(char &ch) {
    if (S == T) {
                  T = (S = buffer) + fread(buffer, 1, L, stdin);
                  if (S == T) {
                       ch = EOF:
                       return false;
12
13
14
15
             ch = *S++;
16
             return true;
17
        __inline bool getint(int &x) {
    char ch; bool neg = 0;
    for (; getchar(ch) && (ch < '0' || ch > '9'); ) neg ^= ch == '-';
18
19
20
21
             if (ch == EOF) return false;
             x = ch - '0';
22
23
24
25
26
             for (; getchar(ch), ch >= '0' && ch <= '9'; )
    x = x * 10 + ch - '0';
             if (neg) x = -x;
return true;
27
```

Chapter 9 提示

9.1 控制 cout 输出实数精度

```
std::cout << std::fixed << std::setprecision(5);</pre>
```

9.2 vimrc

```
set nu
set sw=4
set sts=4
set ts=4
syntax on
set cindent
```

9.3 让 make 支持 c ++ 11

In .bashrc or whatever:

export CXXFLAGS='-std=c++11 -Wall'

9.4 线性规划转对偶

$$\begin{array}{l} \text{maximize } \mathbf{c}^T \mathbf{x} \\ \text{subject to } \mathbf{A} \mathbf{x} \leq \mathbf{b}, \mathbf{x} \geq 0 \\ \end{array} & \text{minimize } \mathbf{y}^T \mathbf{b} \\ \text{subject to } \mathbf{y}^T \mathbf{A} \geq \mathbf{c}^T, \mathbf{y} \geq 0 \end{array}$$

9.5 32-bit/64-bit 随机素数

| 32-bit | 64-bit |
|------------|---------------------|
| 73550053 | 1249292846855685773 |
| 148898719 | 1701750434419805569 |
| 189560747 | 3605499878424114901 |
| 459874703 | 5648316673387803781 |
| 1202316001 | 6125342570814357977 |
| 1431183547 | 6215155308775851301 |
| 1438011109 | 6294606778040623451 |
| 1538762023 | 6347330550446020547 |
| 1557944263 | 7429632924303725207 |
| 1981315913 | 8524720079480389849 |

9.6 NTT 素数及其原根

| Prime | Primitive root |
|------------|----------------|
| 1053818881 | 7 |
| 1051721729 | 6 |
| 1045430273 | 3 |
| 1012924417 | 5 |
| 1007681537 | 3 |