Gungnir's Standard Code Library

Shanghai Jiao Tong University

Dated: November 6, 2016

Contents

Chapter 1

计算几何

1.1 二维

1.1.1 基础

```cpp
typedef double DB;
const DB eps = 1e-8;

int sign(DB x) {
    return x < -eps ? -1 : ( x > eps ? 1 : 0 );
}
DB msqrt(DB x) {
    return sign(x) > 0 ? sqrt(x) : 0;
}

struct Point {
    DB x, y;
    Point(): x(0), y(0) {}
    Point(DB x, DB y): x(x), y(y) {}
    Point operator+(const Point &rhs) const {
        return Point(x + rhs.x, y + rhs.y);
    }
    Point operator-(const Point &rhs) const {
        return Point(x - rhs.x, y - rhs.y);
    }
    Point operator*(DB k) const {
        return Point(x * k, y * k);
    }
    Point operator/(DB k) const {
        assert(sign(k));
        return Point(x / k, y / k);
    }
    Point rotate(DB ang) const {  // 逆时针旋转 ang 弧度
        return Point(cos(ang) * x - sin(ang) * y,
                cos(ang) * y + sin(ang) * x);
    }
```

```
32      Point turn90() const {  // 逆时针旋转 90 度
33          return Point(-y, x);
34      }
35      Point unit() const {
36          return *this / len();
37      }
38  };
39  DB dot(const Point& a, const Point& b) {
40      return a.x * b.x + a.y * b.y;
41  }
42  DB det(const Point& a, const Point& b) {
43      return a.x * b.y - a.y * b.x;
44  }
45  #define cross(p1,p2,p3) ((p2.x-p1.x)*(p3.y-p1.y)-(p3.x-p1.x)*(p2.y-p1.y))
46  #define crossOp(p1,p2,p3) sign(cross(p1,p2,p3))
47  bool isLL(const Line& l1, const Line& l2, Point& p) {  // 直线与直线交点
48      DB s1 = det(l2.b - l2.a, l1.a - l2.a),
49         s2 = -det(l2.b - l2.a, l1.b - l2.a);
50      if (!sign(s1 + s2)) return false;
51      p = (l1.a * s2 + l1.b * s1) / (s1 + s2);
52      return true;
53  }
54  bool onSeg(const Line& l, const Point& p) {  // 点在线段上
55      return sign(det(p - l.a, l.b - l.a)) == 0 && sign(dot(p - l.a, p - l.b)) <= 0;
56  }
57  Point projection(const Line & l, const Point& p) {
58      return l.a + (l.b - l.a) * (dot(p - l.a, l.b - l.a) / (l.b - l.a).len2());
59  }
60  DB disToLine(const Line& l, const Point& p) {  // 点到 * 直线 * 距离
61      return fabs(det(p - l.a, l.b - l.a) / (l.b - l.a).len());
62  }
63  DB disToSeg(const Line& l, const Point& p) {  // 点到线段距离
64      return sign(dot(p - l.a, l.b - l.a)) * sign(dot(p - l.b, l.a - l.b)) == 1 ?
    ↪disToLine(l, p) : std::min((p - l.a).len(), (p - l.b).len());
65  }
66  // 圆与直线交点
67  bool isCL(Circle a, Line l, Point& p1, Point& p2) {
68      DB x = dot(l.a - a.o, l.b - l.a),
69         y = (l.b - l.a).len2(),
70         d = x * x - y * ((l.a - a.o).len2() - a.r * a.r);
71      if (sign(d) < 0) return false;
72      Point p = l.a - ((l.b - l.a) * (x / y)), delta = (l.b - l.a) * (msqrt(d) / y);
73      p1 = p + delta; p2 = p - delta;
74      return true;
75  }
76  //圆与圆的交面积
77  DB areaCC(const Circle& c1, const Circle& c2) {
78      DB d = (c1.o - c2.o).len();
79      if (sign(d - (c1.r + c2.r)) >= 0) return 0;
```

```
80      if (sign(d - std::abs(c1.r - c2.r)) <= 0) {
81          DB r = std::min(c1.r, c2.r);
82          return r * r * PI;
83      }
84      DB x = (d * d + c1.r * c1.r - c2.r * c2.r) / (2 * d),
85          t1 = acos(x / c1.r), t2 = acos((d - x) / c2.r);
86      return c1.r * c1.r * t1 + c2.r * c2.r * t2 - d * c1.r * sin(t1);
87  }
88  // 圆与圆交点
89  bool isCC(Circle a, Circle b, P& p1, P& p2) {
90      DB s1 = (a.o - b.o).len();
91      if (sign(s1 - a.r - b.r) > 0 || sign(s1 - std::abs(a.r - b.r)) < 0) return
      ↪ false;
92      DB s2 = (a.r * a.r - b.r * b.r) / s1;
93      DB aa = (s1 + s2) * 0.5, bb = (s1 - s2) * 0.5;
94      P o = (b.o - a.o) * (aa / (aa + bb)) + a.o;
95      P delta = (b.o - a.o).unit().turn90() * msqrt(a.r * a.r - aa * aa);
96      p1 = o + delta, p2 = o - delta;
97      return true;
98  }
99  // 求点到圆的切点，按关于点的顺时针方向返回两个点
100 bool tanCP(const Circle &c, const Point &p0, Point &p1, Point &p2) {
101     double x = (p0 - c.o).len2(), d = x - c.r * c.r;
102     if (d < eps) return false; // 点在圆上认为没有切点
103     Point p = (p0 - c.o) * (c.r * c.r / x);
104     Point delta = ((p0 - c.o) * (-c.r * sqrt(d) / x)).turn90();
105     p1 = c.o + p + delta;
106     p2 = c.o + p - delta;
107     return true;
108 }
109 // 求圆到圆的内共切线，按关于 c1.o 的顺时针方向返回两条线
110 std::vector<Line> intanCC(const Circle &c1, const Circle &c2) {
111     std::vector<Line> ret;
112     Point p = (c1.o * c2.r + c2.o * c1.r) / (c1.r + c2.r);
113     Point p1, p2, q1, q2;
114     if (tanCP(c1, p, p1, p2) && tanCP(c2, p, q1, q2)) { // 两圆相切认为没有切线
115         ret.push_back(Line(p1, q1));
116         ret.push_back(Line(p2, q2));
117     }
118     return ret;
119 }
120 // 点在多边形内
121 bool inPolygon(const Point& p, const std::vector<Point>& poly) {
122     int n = poly.size();
123     int counter = 0;
124     for (int i = 0; i < n; ++ i) {
125         P a = poly[i], b = poly[(i + 1) % n];
126         if (onSeg(Line(a, b), p)) return false; // 边界上不算
127         int x = sign(det(p - a, b - a));
```

```
        int y = sign(a.y - p.y);
        int z = sign(b.y - p.y);
        if (x > 0 && y <= 0 && z > 0) ++ counter;
        if (x < 0 && z <= 0 && y > 0) -- counter;
    }
    return counter != 0;
}
// 用半平面 (q1,q2) 的逆时针方向去切凸多边形
std::vector<Point> convexCut(const std::vector<Point>&ps, Point q1, Point q2) {
    std::vector<Point> qs; int n = ps.size();
    for (int i = 0; i < n; ++i) {
        Point p1 = ps[i], p2 = ps[(i + 1) % n];
        int d1 = crossOp(q1,q2,p1), d2 = crossOp(q1,q2,p2);
        if (d1 >= 0) qs.push_back(p1);
        if (d1 * d2 < 0) qs.push_back(isSS(p1, p2, q1, q2));
    }
    return qs;
}
// 求凸包
std::vector<Point> convexHull(std::vector<Point> ps) {
    int n = ps.size(); if (n <= 1) return ps;
    std::sort(ps.begin(), ps.end());
    std::vector<Point> qs;
    for (int i = 0; i < n; qs.push_back(ps[i ++]))
        while (qs.size() > 1 && sign(det(qs[qs.size() - 2], qs.back(), ps[i])) <= 0)
            qs.pop_back();
    for (int i = n - 2, t = qs.size(); i >= 0; qs.push_back(ps[i --]))
        while ((int)qs.size() > t && sign(det(qs[qs.size() - 2], qs.back(), ps[i]))
  ↪ <= 0)
            qs.pop_back();
    return qs;
```

1.1.2 凸包

```
// 凸包中的点按逆时针方向
struct Convex {
    int n;
    std::vector<Point> a, upper, lower;
    void make_shell(const std::vector<Point>& p,
            std::vector<Point>& shell) {  // p needs to be sorted.
        clear(shell); int n = p.size();
        for (int i = 0, j = 0; i < n; i++, j++) {
            for (; j >= 2 && sign(det(shell[j-1] - shell[j-2],
                                p[i] - shell[j-2])) <= 0; --j) shell.pop_back();
            shell.push_back(p[i]);
        }
    }
    void make_convex() {
        std::sort(a.begin(), a.end());
```

```
        make_shell(a, lower);
        std::reverse(a.begin(), a.end());
        make_shell(a, upper);
        a = lower; a.pop_back();
        a.insert(a.end(), upper.begin(), upper.end());
        if ((int)a.size() >= 2) a.pop_back();
        n = a.size();
    }
    void init(const std::vector<Point>& _a) {
        clear(a); a = _a; n = a.size();
        make_convex();
    }
    void read(int _n) {  // Won't make convex.
        clear(a); n = _n; a.resize(n);
        for (int i = 0; i < n; i++)
            a[i].read();
    }
    std::pair<DB, int> get_tangent(
            const std::vector<Point>& convex, const Point& vec) {
        int l = 0, r = (int)convex.size() - 2;
        assert(r >= 0);
        for (; l + 1 < r; ) {
            int mid = (l + r) / 2;
            if (sign(det(convex[mid + 1] - convex[mid], vec)) > 0)
                r = mid;
            else l = mid;
        }
        return std::max(std::make_pair(det(vec, convex[r]), r),
                std::make_pair(det(vec, convex[0]), 0));
    }
    int binary_search(Point u, Point v, int l, int r) {
        int s1 = sign(det(v - u, a[l % n] - u));
        for (; l + 1 < r; ) {
            int mid = (l + r) / 2;
            int smid = sign(det(v - u, a[mid % n] - u));
            if (smid == s1) l = mid;
            else r = mid;
        }
        return l % n;
    }
    // 求凸包上和向量 vec 叉积最大的点，返回编号，共线的多个切点返回任意一个
    int get_tangent(Point vec) {
        std::pair<DB, int> ret = get_tangent(upper, vec);
        ret.second = (ret.second + (int)lower.size() - 1) % n;
        ret = std::max(ret, get_tangent(lower, vec));
        return ret.second;
    }
    // 求凸包和直线 u, v 的交点，如果不相交返回 false，如果有则是和 (i, next(i)) 的交点，交在
↪ 点上不确定返回前后两条边其中之一
```

```
    bool get_intersection(Point u, Point v, int &i0, int &i1) {
        int p0 = get_tangent(u - v), p1 = get_tangent(v - u);
        if (sign(det(v - u, a[p0] - u)) * sign(det(v - u, a[p1] - u)) <= 0) {
            if (p0 > p1) std::swap(p0, p1);
            i0 = binary_search(u, v, p0, p1);
            i1 = binary_search(u, v, p1, p0 + n);
            return true;
        }
        else return false;
    }
};
```

1.2 三维

1.2.1 基础

```
// 三维绕轴旋转，大拇指指向 axis 向量方向，四指弯曲方向转 w 弧度
Point rotate(const Point& s, const Point& axis, DB w) {
    DB x = axis.x, y = axis.y, z = axis.z;
    DB s1 = x * x + y * y + z * z, ss1 = msqrt(s1),
       cosw = cos(w), sinw = sin(w);
    DB a[4][4];
    memset(a, 0, sizeof a);
    a[3][3] = 1;
    a[0][0] = ((y * y + z * z) * cosw + x * x) / s1;
    a[0][1] = x * y * (1 - cosw) / s1 + z * sinw / ss1;
    a[0][2] = x * z * (1 - cosw) / s1 - y * sinw / ss1;
    a[1][0] = x * y * (1 - cosw) / s1 - z * sinw / ss1;
    a[1][1] = ((x * x + z * z) * cosw + y * y) / s1;
    a[1][2] = y * z * (1 - cosw) / s1 + x * sinw / ss1;
    a[2][0] = x * z * (1 - cosw) / s1 + y * sinw / ss1;
    a[2][1] = y * z * (1 - cosw) / s1 - x * sinw / ss1;
    a[2][2] = ((x * x + y * y) * cos(w) + z * z) / s1;
    DB ans[4] = {0, 0, 0, 0}, c[4] = {s.x, s.y, s.z, 1};
    for (int i = 0; i < 4; ++ i)
        for (int j = 0; j < 4; ++ j)
            ans[i] += a[j][i] * c[j];
    return Point(ans[0], ans[1], ans[2]);
}
```

1.2.2 凸包

```
__inline P cross(const P& a, const P& b) {
    return P(
            a.y * b.z - a.z * b.y,
            a.z * b.x - a.x * b.z,
            a.x * b.y - a.y * b.x
        );
```

```
}

__inline DB mix(const P& a, const P& b, const P& c) {
    return dot(cross(a, b), c);
}

__inline DB volume(const P& a, const P& b, const P& c, const P& d) {
    return mix(b - a, c - a, d - a);
}

struct Face {
    int a, b, c;
    __inline Face() {}
    __inline Face(int _a, int _b, int _c):
        a(_a), b(_b), c(_c) {}
    __inline DB area() const {
        return 0.5 * cross(p[b] - p[a], p[c] - p[a]).len();
    }
    __inline P normal() const {
        return cross(p[b] - p[a], p[c] - p[a]).unit();
    }
    __inline DB dis(const P& p0) const {
        return dot(normal(), p0 - p[a]);
    }
};

std::vector<Face> face, tmp;  // Should be O(n).
int mark[N][N], Time, n;

__inline void add(int v) {
    ++ Time;
    clear(tmp);
    for (int i = 0; i < (int)face.size(); ++ i) {
        int a = face[i].a, b = face[i].b, c = face[i].c;
        if (sign(volume(p[v], p[a], p[b], p[c])) > 0) {
            mark[a][b] = mark[b][a] = mark[a][c] =
                mark[c][a] = mark[b][c] = mark[c][b] = Time;
        }
        else {
            tmp.push_back(face[i]);
        }
    }
    clear(face); face = tmp;
    for (int i = 0; i < (int)tmp.size(); ++ i) {
        int a = face[i].a, b = face[i].b, c = face[i].c;
        if (mark[a][b] == Time) face.emplace_back(v, b, a);
        if (mark[b][c] == Time) face.emplace_back(v, c, b);
        if (mark[c][a] == Time) face.emplace_back(v, a, c);
        assert(face.size() < 500u);
```

```
    }
}

void reorder() {
    for (int i = 2; i < n; ++ i) {
        P tmp = cross(p[i] - p[0], p[i] - p[1]);
        if (sign(tmp.len())) {
            std::swap(p[i], p[2]);
            for (int j = 3; j < n; ++ j)
                if (sign(volume(p[0], p[1], p[2], p[j]))) {
                    std::swap(p[j], p[3]);
                    return;
                }
        }
    }
}

void build_convex() {
    reorder();
    clear(face);
    face.emplace_back(0, 1, 2);
    face.emplace_back(0, 2, 1);
    for (int i = 3; i < n; ++ i)
        add(i);
}
```

Chapter 2

数论

2.1 $O(m^2 \log n)$ 求线性递推数列第 n 项

Given $a_0, a_1, \ldots, a_{m-1}$
$a_n = c_0 \times a_{n-m} + \cdots + c_{m-1} \times a_{n-1}$
Solve for $a_n = v_0 \times a_0 + v_1 \times a_1 + \cdots + v_{m-1} \times a_{m-1}$

```
void linear_recurrence(long long n, int m, int a[], int c[], int p) {
    long long v[M] = {1 % p}, u[M << 1], msk = !!n;
    for(long long i(n); i > 1; i >>= 1) {
        msk <<= 1;
    }
    for(long long x(0); msk; msk >>= 1, x <<= 1) {
        fill_n(u, m << 1, 0);
        int b(!!(n & msk));
        x |= b;
        if(x < m) {
            u[x] = 1 % p;
        }else {
            for(int i(0); i < m; i++) {
                for(int j(0), t(i + b); j < m; j++, t++) {
                    u[t] = (u[t] + v[i] * v[j]) % p;
                }
            }
            for(int i((m << 1) - 1); i >= m; i--) {
                for(int j(0), t(i - m); j < m; j++, t++) {
                    u[t] = (u[t] + c[j] * u[i]) % p;
                }
            }
        }
        copy(u, u + m, v);
    }
    //a[n] = v[0] * a[0] + v[1] * a[1] + ... + v[m - 1] * a[m - 1].
    for(int i(m); i < 2 * m; i++) {
        a[i] = 0;
        for(int j(0); j < m; j++) {
```

```
            a[i] = (a[i] + (long long)c[j] * a[i + j - m]) % p;
        }
    }
    for(int j(0); j < m; j++) {
        b[j] = 0;
        for(int i(0); i < m; i++) {
            b[j] = (b[j] + v[i] * a[i + j]) % p;
        }
    }
    for(int j(0); j < m; j++) {
        a[j] = b[j];
    }
}
```

2.2 求逆元

```
void ex_gcd(long long a, long long b, long long &x, long long &y) {
    if (b == 0) {
        x = 1;
        y = 0;
        return;
    }
    long long xx, yy;
    ex_gcd(b, a % b, xx, yy);
    y = xx - a / b * yy;
    x = yy;
}

long long inv(long long x, long long MODN) {
    long long inv_x, y;
    ex_gcd(x, MODN, inv_x, y);
    return (inv_x % MODN + MODN) % MODN;
}
```

2.3 中国剩余定理

```
// 返回 (ans, M)，其中 ans 是模 M 意义下的解
std::pair<long long, long long> CRT(const std::vector<long long>& m, const
  ↪ std::vector<long long>& a) {
    long long M = 1, ans = 0;
    int n = m.size();
    for (int i = 0; i < n; i++) M *= m[i];
    for (int i = 0; i < n; i++) {
        ans = (ans + (M / m[i]) * a[i] % M * inv(M / m[i], m[i])) % M;  // 可能需要大
  ↪ 整数相乘取模
    }
```

```
9       return std::make_pair(ans, M);
10  }
```

Chapter 3

代数

3.1 快速傅里叶变换

```
// n 必须是 2 的次幂
void fft(Complex a[], int n, int f) {
    for (int i = 0; i < n; ++i)
        if (R[i] < i) swap(a[i], a[R[i]]);
    for (int i = 1, h = 0; i < n; i <<= 1, h++) {
        Complex wn = Complex(cos(pi / i), f * sin(pi / i));
        Complex w = Complex(1, 0);
        for (int k = 0; k < i; ++k, w = w * wn) tmp[k] = w;
        for (int p = i << 1, j = 0; j < n; j += p) {
            for (int k = 0; k < i; ++k) {
                Complex x = a[j + k], y = a[j + k + i] * tmp[k];
                a[j + k] = x + y; a[j + k + i] = x - y;
            }
        }
    }
}
```

Chapter 4

字符串

4.1 后缀数组

```cpp
const int MAXN = MAXL * 2 + 1;
int a[MAXN], x[MAXN], y[MAXN], c[MAXN], sa[MAXN], rank[MAXN], height[MAXN];
void calc_sa(int n) {
    int m = alphabet, k = 1;
    memset(c, 0, sizeof(*c) * (m + 1));
    for (int i = 1; i <= n; ++i) c[x[i] = a[i]]++;
    for (int i = 1; i <= m; ++i) c[i] += c[i - 1];
    for (int i = n; i; --i) sa[c[x[i]]--] = i;
    for (; k <= n; k <<= 1) {
        int tot = k;
        for (int i = n - k + 1; i <= n; ++i) y[i - n + k] = i;
        for (int i = 1; i <= n; ++i)
            if (sa[i] > k) y[++tot] = sa[i] - k;
        memset(c, 0, sizeof(*c) * (m + 1));
        for (int i = 1; i <= n; ++i) c[x[i]]++;
        for (int i = 1; i <= m; ++i) c[i] += c[i - 1];
        for (int i = n; i; --i) sa[c[x[y[i]]]--] = y[i];
        for (int i = 1; i <= n; ++i) y[i] = x[i];
        tot = 1; x[sa[1]] = 1;
        for (int i = 2; i <= n; ++i) {
            if (max(sa[i], sa[i - 1]) + k > n || y[sa[i]] != y[sa[i - 1]] || y[sa[i]
  ↪+ k] != y[sa[i - 1] + k]) ++tot;
            x[sa[i]] = tot;
        }
        if (tot == n) break; else m = tot;
    }
}
void calc_height(int n) {
    for (int i = 1; i <= n; ++i) rank[sa[i]] = i;
    for (int i = 1; i <= n; ++i) {
        height[rank[i]] = max(0, height[rank[i - 1]] - 1);
        if (rank[i] == 1) continue;
        int j = sa[rank[i] - 1];
```

```
        while (max(i, j) + height[rank[i]] <= n && a[i + height[rank[i]]] == a[j + height[rank[i]]]) ++height[rank[i]];
    }
}
```

4.2 后缀自动机

```
static const int MAXL = MAXN * 2;  // MAXN is original length
static const int alphabet = 26;  // sometimes need changing
int l, last, cnt, trans[MAXL][alphabet], par[MAXL], sum[MAXL], seq[MAXL], mxl[MAXL], size[MAXL];  // mxl is maxlength, size is the size of right
char str[MAXL];
inline void init() {
    l = strlen(str + 1); cnt = last = 1;
    for (int i = 0; i <= l * 2; ++i) memset(trans[i], 0, sizeof(trans[i]));
    memset(par, 0, sizeof(*par) * (l * 2 + 1));
    memset(mxl, 0, sizeof(*mxl) * (l * 2 + 1));
    memset(size, 0, sizeof(*size) * (l * 2 + 1));
}
inline void extend(int pos, int c) {
    int p = last, np = last = ++cnt;
    mxl[np] = mxl[p] + 1; size[np] = 1;
    for (; p && !trans[p][c]; p = par[p]) trans[p][c] = np;
    if (!p) par[np] = 1;
    else {
        int q = trans[p][c];
        if (mxl[p] + 1 == mxl[q]) par[np] = q;
        else {
            int nq = ++cnt;
            mxl[nq] = mxl[p] + 1;
            memcpy(trans[nq], trans[q], sizeof(trans[nq]));
            par[nq] = par[q];
            par[np] = par[q] = nq;
            for (; trans[p][c] == q; p = par[p]) trans[p][c] = nq;
        }
    }
}
inline void buildsam() {
    for (int i = 1; i <= l; ++i) extend(i, str[i] - 'a');
    memset(sum, 0, sizeof(*sum) * (l * 2 + 1));
    for (int i = 1; i <= cnt; ++i) sum[mxl[i]]++;
    for (int i = 1; i <= l; ++i) sum[i] += sum[i - 1];
    for (int i = cnt; i; --i) seq[sum[mxl[i]]--] = i;
    for (int i = cnt; i; --i) size[par[seq[i]]] += size[seq[i]];
}
```

4.3 回文自动机

```
int nT, nStr, last, c[MAXT][26], fail[MAXT], r[MAXN], l[MAXN], s[MAXN];
int allocate(int len) {
    l[nT] = len;
    r[nT] = 0;
    fail[nT] = 0;
    memset(c[nT], 0, sizeof(c[nT]));
    return nT++;
}
void init() {
    nT = nStr = 0;
    int newE = allocate(0);
    int newO = allocate(-1);
    last = newE;
    fail[newE] = newO;
    fail[newO] = newE;
    s[0] = -1;
}
void add(int x) {
    s[++nStr] = x;
    int now = last;
    while (s[nStr - l[now] - 1] != s[nStr]) now = fail[now];
    if (!c[now][x]) {
        int newnode = allocate(l[now] + 2), &newfail = fail[newnode];
        newfail = fail[now];
        while (s[nStr - l[newfail] - 1] != s[nStr]) newfail = fail[newfail];
        newfail = c[newfail][x];
        c[now][x] = newnode;
    }
    last = c[now][x];
    r[last]++;
}
void count() {
    for (int i = nT - 1; i >= 0; i--) {
        r[fail[i]] += r[i];
    }
}
```

Chapter 5

数据结构

5.1 KD-Tree

```
long long norm(const long long &x) {
    //    For manhattan distance
    return std::abs(x);
    //    For euclid distance
    return x * x;
}

struct Point {
    int x, y, id;

    const int& operator [] (int index) const {
        if (index == 0) {
            return x;
        } else {
            return y;
        }
    }

    friend long long dist(const Point &a, const Point &b) {
        long long result = 0;
        for (int i = 0; i < 2; ++i) {
            result += norm(a[i] - b[i]);
        }
        return result;
    }
} point[N];

struct Rectangle {
    int min[2], max[2];

    Rectangle() {
        min[0] = min[1] = INT_MAX;  // sometimes int is not enough
        max[0] = max[1] = INT_MIN;
```

```
34      }
35
36      void add(const Point &p) {
37          for (int i = 0; i < 2; ++i) {
38              min[i] = std::min(min[i], p[i]);
39              max[i] = std::max(max[i], p[i]);
40          }
41      }
42
43      long long dist(const Point &p) {
44          long long result = 0;
45          for (int i = 0; i < 2; ++i) {
46              //    For minimum distance
47              result += norm(std::min(std::max(p[i], min[i]), max[i]) - p[i]);
48              //    For maximum distance
49              result += std::max(norm(max[i] - p[i]), norm(min[i] - p[i]));
50          }
51          return result;
52      }
53  };
54
55  struct Node {
56      Point seperator;
57      Rectangle rectangle;
58      int child[2];
59
60      void reset(const Point &p) {
61          seperator = p;
62          rectangle = Rectangle();
63          rectangle.add(p);
64          child[0] = child[1] = 0;
65      }
66  } tree[N << 1];
67
68  int size, pivot;
69
70  bool compare(const Point &a, const Point &b) {
71      if (a[pivot] != b[pivot]) {
72          return a[pivot] < b[pivot];
73      }
74      return a.id < b.id;
75  }
76
77  // 左閉右開: build(1, n + 1)
78  int build(int l, int r, int type = 1) {
79      pivot = type;
80      if (l >= r) {
81          return 0;
82      }
```

```
83      int x = ++size;
84      int mid = l + r >> 1;
85      std::nth_element(point + l, point + mid, point + r, compare);
86      tree[x].reset(point[mid]);
87      for (int i = l; i < r; ++i) {
88          tree[x].rectangle.add(point[i]);
89      }
90      tree[x].child[0] = build(l, mid, type ^ 1);
91      tree[x].child[1] = build(mid + 1, r, type ^ 1);
92      return x;
93  }
94
95  int insert(int x, const Point &p, int type = 1) {
96      pivot = type;
97      if (x == 0) {
98          tree[++size].reset(p);
99          return size;
100     }
101     tree[x].rectangle.add(p);
102     if (compare(p, tree[x].seperator)) {
103         tree[x].child[0] = insert(tree[x].child[0], p, type ^ 1);
104     } else {
105         tree[x].child[1] = insert(tree[x].child[1], p, type ^ 1);
106     }
107     return x;
108 }
109
110 // For minimum distance
111 // For maximum: 下面递归 query 时 0, 1 换顺序;< and >;min and max
112 void query(int x, const Point &p, std::pair<long long, int> &answer, int type = 1) {
113     pivot = type;
114     if (x == 0 || tree[x].rectangle.dist(p) > answer.first) {
115         return;
116     }
117     answer = std::min(answer,
118               std::make_pair(dist(tree[x].seperator, p), tree[x].seperator.id));
119     if (compare(p, tree[x].seperator)) {
120         query(tree[x].child[0], p, answer, type ^ 1);
121         query(tree[x].child[1], p, answer, type ^ 1);
122     } else {
123         query(tree[x].child[1], p, answer, type ^ 1);
124         query(tree[x].child[0], p, answer, type ^ 1);
125     }
126 }
127
128 std::priority_queue<std::pair<long long, int> > answer;
129
130 void query(int x, const Point &p, int k, int type = 1) {
131     pivot = type;
```

```
    if (x == 0 || (int)answer.size() == k && tree[x].rectangle.dist(p) > answer.top().first) {
        return;
    }
    answer.push(std::make_pair(dist(tree[x].seperator, p), tree[x].seperator.id));
    if ((int)answer.size() > k) {
        answer.pop();
    }
    if (compare(p, tree[x].seperator)) {
        query(tree[x].child[0], p, k, type ^ 1);
        query(tree[x].child[1], p, k, type ^ 1);
    } else {
        query(tree[x].child[1], p, k, type ^ 1);
        query(tree[x].child[0], p, k, type ^ 1);
    }
}
```

5.2 Treap

```
struct Node{
    int mn, key, size, tag;
    bool rev;
    Node* ch[2];
    Node(int mn, int key, int size): mn(mn), key(key), size(size), rev(0), tag(0){}
    void downtag();
    Node* update(){
        mn = min(ch[0] -> mn, min(key, ch[1] -> mn));
        size = ch[0] -> size + 1 + ch[1] -> size;
        return this;
    }
};
typedef pair<Node*, Node*> Pair;
Node *null, *root;
void Node::downtag(){
    if(rev){
        for(int i = 0; i < 2; i++)
            if(ch[i] != null){
                ch[i] -> rev ^= 1;
                swap(ch[i] -> ch[0], ch[i] -> ch[1]);
            }
        rev = 0;
    }
    if(tag){
        for(int i = 0; i < 2; i++)
            if(ch[i] != null){
                ch[i] -> key += tag;
                ch[i] -> mn += tag;
                ch[i] -> tag += tag;
```

```
            }
        tag = 0;
    }
}
int r(){
    static int s = 3023192386;
    return (s += (s << 3) + 1) & (~0u >> 1);
}
bool random(int x, int y){
    return r() % (x + y) < x;
}
Node* merge(Node *p, Node *q){
    if(p == null) return q;
    if(q == null) return p;
    p -> downtag();
    q -> downtag();
    if(random(p -> size, q -> size)){
        p -> ch[1] = merge(p -> ch[1], q);
        return p -> update();
    }else{
        q -> ch[0] = merge(p, q -> ch[0]);
        return q -> update();
    }
}
Pair split(Node *x, int n){
    if(x == null) return make_pair(null, null);
    x -> downtag();
    if(n <= x -> ch[0] -> size){
        Pair ret = split(x -> ch[0], n);
        x -> ch[0] = ret.second;
        return make_pair(ret.first, x -> update());
    }
    Pair ret = split(x -> ch[1], n - x -> ch[0] -> size - 1);
    x -> ch[1] = ret.first;
    return make_pair(x -> update(), ret.second);
}
pair<Node*, Pair> get_segment(int l, int r){
    Pair ret = split(root, l - 1);
    return make_pair(ret.first, split(ret.second, r - l + 1));
}
int main(){
    null = new Node(INF, INF, 0);
    null -> ch[0] = null -> ch[1] = null;
    root = null;
}
```

5.3 Link/cut Tree

```
inline void reverse(int x) {
    tr[x].rev ^= 1; swap(tr[x].c[0], tr[x].c[1]);
}

inline void rotate(int x, int k) {
    int y = tr[x].fa, z = tr[y].fa;
    tr[x].fa = z; tr[z].c[tr[z].c[1] == y] = x;
    tr[tr[x].c[k ^ 1]].fa = y; tr[y].c[k] = tr[x].c[k ^ 1];
    tr[x].c[k ^ 1] = y; tr[y].fa = x;
}

inline void splay(int x, int w) {
    int z = x; pushdown(x);
    while (tr[x].fa != w) {
        int y = tr[x].fa; z = tr[y].fa;
        if (z == w) {
            pushdown(z = y); pushdown(x);
            rotate(x, tr[y].c[1] == x);
            update(y); update(x);
        } else {
            pushdown(z); pushdown(y); pushdown(x);
            int t1 = tr[y].c[1] == x, t2 = tr[z].c[1] == y;
            if (t1 == t2) rotate(y, t2), rotate(x, t1);
            else rotate(x, t1), rotate(x, t2);
            update(z); update(y); update(x);
        }
    }
    update(x);
    if (x != z) par[x] = par[z], par[z] = 0;
}

inline void access(int x) {
    for (int y = 0; x; y = x, x = par[x]) {
        splay(x, 0);
        if (tr[x].c[1]) par[tr[x].c[1]] = x, tr[tr[x].c[1]].fa = 0;
        tr[x].c[1] = y; par[y] = 0; tr[y].fa = x; update(x);
    }
}

inline void makeroot(int x) {
    access(x); splay(x, 0); reverse(x);
}

inline void link(int x, int y) {
    makeroot(x); par[x] = y;
}

```

```
48 inline void cut(int x, int y) {
49     access(x); splay(y, 0);
50     if (par[y] != x) swap(x, y), access(x), splay(y, 0);
51     par[y] = 0;
52 }
53
54 inline void split(int x, int y) {  // x will be the root of the tree
55     makeroot(y); access(x); splay(x, 0);
56 }
```

Chapter 6

图论

6.1 基础

```cpp
struct Graph {  // Remember to call .init()!
    int e, nxt[M], v[M], adj[N], n;
    bool base;
    __inline void init(bool _base, int _n = 0) {
        assert(n < N);
        n = _n; base = _base;
        e = 0; memset(adj + base, -1, sizeof(*adj) * n);
    }
    __inline int new_node() {
        adj[n + base] = -1;
        assert(n + base + 1 < N);
        return n++ + base;
    }
    __inline void ins(int u0, int v0) {  // directional
        assert(u0 < n + base && v0 < n + base);
        v[e] = v0; nxt[e] = adj[u0]; adj[u0] = e++;
        assert(e < M);
    }
    __inline void bi_ins(int u0, int v0) {  // bi-directional
        ins(u0, v0); ins(v0, u0);
    }
};
```

6.2 KM

```cpp
struct KM {
    // Truly O(n^3)
    // 邻接矩阵，不能连的边设为 -INF，求最小权匹配时边权取负，但不能连的还是 -INF，使用时先对 1
    ↪-> n 调用 hungary() ，再 get_ans() 求值
    int w[N][N];
    int lx[N], ly[N], match[N], way[N], slack[N];
    bool used[N];
```

```
    void init() {
        for (int i = 1; i <= n; i++) {
            match[i] = 0;
            lx[i] = 0;
            ly[i] = 0;
            way[i] = 0;
        }
    }
    void hungary(int x) {
        match[0] = x;
        int j0 = 0;
        for (int j = 0; j <= n; j++) {
            slack[j] = INF;
            used[j] = false;
        }

        do {
            used[j0] = true;
            int i0 = match[j0], delta = INF, j1 = 0;
            for (int j = 1; j <= n; j++) {
                if (used[j] == false) {
                    int cur = -w[i0][j] - lx[i0] - ly[j];
                    if (cur < slack[j]) {
                        slack[j] = cur;
                        way[j] = j0;
                    }
                    if (slack[j] < delta) {
                        delta = slack[j];
                        j1 = j;
                    }
                }
            }
            for (int j = 0; j <= n; j++) {
                if (used[j]) {
                    lx[match[j]] += delta;
                    ly[j] -= delta;
                }
                else slack[j] -= delta;
            }
            j0 = j1;
        } while (match[j0] != 0);

        do {
            int j1 = way[j0];
            match[j0] = match[j1];
            j0 = j1;
        } while (j0);
    }

```

```
    int get_ans() {
        int sum = 0;
        for(int i = 1; i <= n; i++) {
            if (w[match[i]][i] == -INF) ; // 无解
            if (match[i] > 0) sum += w[match[i]][i];
        }
        return sum;
    }
} km;
```

6.3 点双连通分量

`bcc.forest` is a set of connected tree whose vertices are chequered with cut-vertex and BCC.

```
const bool BCC_VERTEX = 0, BCC_EDGE = 1;
struct BCC {  // N = N0 + M0. Remember to call init(&raw_graph).
    Graph *g, forest; // g is raw graph ptr.
    int dfn[N], DFN, low[N];
    int stack[N], top;
    int expand_to[N];       // Where edge i is expanded to in expaned graph.
    // Vertex i expaned to i.
    int compress_to[N];  // Where vertex i is compressed to.
    bool vertex_type[N], cut[N], compress_cut[N], branch[M];
    //std::vector<int> BCC_component[N];  // Cut vertex belongs to none.
    __inline void init(Graph *raw_graph) {
        g = raw_graph;
    }
    void DFS(int u, int pe) {
        dfn[u] = low[u] = ++DFN; cut[u] = false;
        if (!~g->adj[u]) {
            cut[u] = 1;
            compress_to[u] = forest.new_node();
            compress_cut[compress_to[u]] = 1;
        }
        for (int e = g->adj[u]; ~e; e = g->nxt[e]) {
            int v = g->v[e];
            if ((e ^ pe) > 1 && dfn[v] > 0 && dfn[v] < dfn[u]) {
                stack[top++] = e;
                low[u] = std::min(low[u], dfn[v]);
            }
            else if (!dfn[v]) {
                stack[top++] = e; branch[e] = 1;
                DFS(v, e);
                low[u] = std::min(low[v], low[u]);
                if (low[v] >= dfn[u]) {
                    if (!cut[u]) {
                        cut[u] = 1;
                        compress_to[u] = forest.new_node();
                        compress_cut[compress_to[u]] = 1;
```

```
                    }
                    int cc = forest.new_node();
                    forest.bi_ins(compress_to[u], cc);
                    compress_cut[cc] = 0;
                    //BCC_component[cc].clear();
                    do {
                        int cur_e = stack[--top];
                        compress_to[expand_to[cur_e]] = cc;
                        compress_to[expand_to[cur_e^1]] = cc;
                        if (branch[cur_e]) {
                            int v = g->v[cur_e];
                            if (cut[v])
                                forest.bi_ins(cc, compress_to[v]);
                            else {
                                //BCC_component[cc].push_back(v);
                                compress_to[v] = cc;
                            }
                        }
                    } while (stack[top] != e);
                }
            }
        }
    }
    void solve() {
        forest.init(g->base);
        int n = g->n;
        for (int i = 0; i < g->e; i++) {
            expand_to[i] = g->new_node();
        }
        memset(branch, 0, sizeof(*branch) * g->e);
        memset(dfn + g->base, 0, sizeof(*dfn) * n); DFN = 0;
        for (int i = 0; i < n; i++)
            if (!dfn[i + g->base]) {
                top = 0;
                DFS(i + g->base, -1);
            }
    }
} bcc;

bcc.init(&raw_graph);
bcc.solve();
// Do something with bcc.forest ...
```

6.4 边双连通分量

```
struct BCC {
    Graph *g, forest;
    int dfn[N], low[N], stack[N], tot[N], belong[N], vis[N], top, dfs_clock;
```

```
    // tot[] is the size of each BCC, belong[] is the BCC that each node belongs to
    pair<int, int > ori[M]; // bridge in raw_graph(raw node)
    bool is_bridge[M];
    __inline void init(Graph *raw_graph) {
        g = raw_graph;
        memset(is_bridge, false, sizeof(*is_bridge) * g -> e);
        memset(vis + g -> base, 0, sizeof(*vis) * g -> n);
    }
    void tarjan(int u, int from) {
        dfn[u] = low[u] = ++dfs_clock; vis[u] = 1; stack[++top] = u;
        for (int p = g -> adj[u]; ~p; p = g -> nxt[p]) {
            if ((p ^ 1) == from) continue;
            int v = g -> v[p];
            if (vis[v]) {
                if (vis[v] == 1) low[u] = min(low[u], dfn[v]);
            } else {
                tarjan(v, p);
                low[u] = min(low[u], low[v]);
                if (low[v] > dfn[u]) is_bridge[p / 2] = true;
            }
        }
        if (dfn[u] != low[u]) return;
        tot[forest.new_node()] = 0;
        do {
            belong[stack[top]] = forest.n;
            vis[stack[top]] = 2;
            tot[forest.n]++;
            --top;
        } while (stack[top + 1] != u);
    }
    void solve() {
        forest.init(g -> base);
        int n = g -> n;
        for (int i = 0; i < n; ++i)
            if (!vis[i + g -> base]) {
                top = dfs_clock = 0;
                tarjan(i + g -> base, -1);
            }
        for (int i = 0; i < g -> e / 2; ++i)
            if (is_bridge[i]) {
                int e = forest.e;
                forest.bi_ins(belong[g -> v[i * 2]], belong[g -> v[i * 2 + 1]], g ->
  ↪w[i * 2]);
                ori[e] = make_pair(g -> v[i * 2 + 1], g -> v[i * 2]);
                ori[e + 1] = make_pair(g -> v[i * 2], g -> v[i * 2 + 1]);
            }
    }
} bcc;
```

Chapter 7

其他

7.1 Dancing Links

```
struct Node {
    Node *l, *r, *u, *d, *col;
    int size, line_no;
    Node() {
        size = 0; line_no = -1;
        l = r = u = d = col = NULL;
    }
} *root;

void cover(Node *c) {
    c->l->r = c->r; c->r->l = c->l;
    for (Node *u = c->d; u != c; u = u->d)
        for (Node *v = u->r; v != u; v = v->r) {
            v->d->u = v->u;
            v->u->d = v->d;
            -- v->col->size;
        }
}

void uncover(Node *c) {
    for (Node *u = c->u; u != c; u = u->u) {
        for (Node *v = u->l; v != u; v = v->l) {
            ++ v->col->size;
            v->u->d = v;
            v->d->u = v;
        }
    }
    c->l->r = c; c->r->l = c;
}

std::vector<int> answer;
bool search(int k) {
    if (root->r == root) return true;
```

```
    Node *r = NULL;
    for (Node *u = root->r; u != root; u = u->r)
        if (r == NULL || u->size < r->size)
            r = u;
    if (r == NULL || r->size == 0) return false;
    else {
        cover(r);
        bool succ = false;
        for (Node *u = r->d; u != r && !succ; u = u->d) {
            answer.push_back(u->line_no);
            for (Node *v = u->r; v != u; v = v->r)  // Cover row
                cover(v->col);
            succ |= search(k + 1);
            for (Node *v = u->l; v != u; v = v->l)
                uncover(v->col);
            if (!succ) answer.pop_back();
        }
        uncover(r);
        return succ;
    }
}

bool entry[CR][CC];
Node *who[CR][CC];
int cr, cc;

void construct() {
    root = new Node();
    Node *last = root;
    for (int i = 0; i < cc; ++ i) {
        Node *u = new Node();
        last->r = u; u->l = last;
        Node *v = u; u->line_no = i;
        last = u;
        for (int j = 0; j < cr; ++ j)
            if (entry[j][i]) {
                ++ u->size;
                Node *cur = new Node();
                who[j][i] = cur;
                cur->line_no = j;
                cur->col = u;
                cur->u = v; v->d = cur;
                v = cur;
            }
        v->d = u; u->u = v;
    }
    last->r = root; root->l = last;
    for (int j = 0; j < cr; ++ j) {
        Node *last = NULL;
```

```
        for (int i = cc - 1; i >= 0; -- i)
            if (entry[j][i]) {
                last = who[j][i];
                break;
            }
        for (int i = 0; i < cc; ++ i)
            if (entry[j][i]) {
                last->r = who[j][i];
                who[j][i]->l = last;
                last = who[j][i];
            }
    }
}

void destruct() {
    for (Node *u = root->r; u != root; ) {
        for (Node *v = u->d; v != u; ) {
            Node *nxt = v->d;
            delete(v);
            v = nxt;
        }
        Node *nxt = u->r;
        delete(u); u = nxt;
    }
    delete root;
}
```

Chapter 8

技巧

8.1 真正的释放 STL 容器内存空间

```cpp
template <typename T>
__inline void clear(T& container) {
    container.clear();  // 或者删除了一堆元素
    T(container).swap(container);
}
```

8.2 无敌的大整数相乘取模

Time complexity $O(1)$.

```cpp
// 需要保证 x 和 y 非负
long long mult(long long x, long long y, long long MODN) {
    long long t = (x * y - (long long)((long double)x / MODN * y + 1e-3) * MODN) %
    ↪MODN;
    return t < 0 ? t + MODN : t;
}
```

8.3 无敌的读入优化

```cpp
// getchar() 读入优化 << 关同步 cin << 此优化
// 用 isdigit() 会小幅变慢
// 返回 false 表示读到文件尾
namespace Reader {
    const int L = (1 << 15) + 5;
    char buffer[L], *S, *T;
    __inline bool getchar(char &ch) {
        if (S == T) {
            T = (S = buffer) + fread(buffer, 1, L, stdin);
            if (S == T) {
                ch = EOF;
                return false;
```

```
            }
        }
        ch = *S++;
        return true;
    }
    __inline bool getint(int &x) {
        char ch; bool neg = 0;
        for (; getchar(ch) && (ch < '0' || ch > '9'); ) neg ^= ch == '-';
        if (ch == EOF) return false;
        x = ch - '0';
        for (; getchar(ch), ch >= '0' && ch <= '9'; )
            x = x * 10 + ch - '0';
        if (neg) x = -x;
        return true;
    }
}
```

8.4 控制 cout 输出实数精度

```
std::cout << std::fixed << std::setprecision(5);
```

Chapter 9

提示

9.1 线性规划转对偶

$$\begin{array}{l}\text{maximize } \mathbf{c}^T\mathbf{x} \\ \text{subject to } \mathbf{A}\mathbf{x} \leq \mathbf{b}, \mathbf{x} \geq 0\end{array} \iff \begin{array}{l}\text{minimize } \mathbf{y}^T\mathbf{b} \\ \text{subject to } \mathbf{y}^T\mathbf{A} \geq \mathbf{c}^T, \mathbf{y} \geq 0\end{array}$$

9.2 32-bit/64-bit 随机素数

32-bit	64-bit
73550053	1249292846855685773
148898719	1701750434419805569
189560747	3605499878424114901
459874703	5648316673387803781
1202316001	6125342570814357977
1431183547	6215155308775851301
1438011109	6294606778040623451
1538762023	6347330550446020547
1557944263	7429632924303725207
1981315913	8524720079480389849

9.3 NTT 素数及其原根

Prime	Primitive root
1053818881	7
1051721729	6
1045430273	3
1012924417	5
1007681537	3