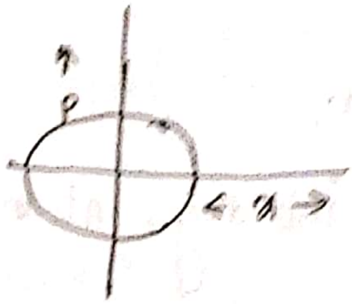
phase space

$$E = \underbrace{\frac{p^2}{2m}}_{K.E} + \underbrace{\frac{1}{2}Kx^2}_{P.E}$$

$p=0, \; x = \sqrt{\frac{2E}{K}}$



$x=0, \; p = \sqrt{2mE}$

$$\frac{p^2}{(\sqrt{2mE})^2} + \frac{x^2}{\left(\sqrt{\frac{2E}{K}}\right)^2} = 1$$

Def:- A point in this space describes both the position and the momentum of the particle at particular time. This combined position and momentum of space for a single particle is called phase space.

comparision of maxwell - boltzmann statistics and Fermi Dirac statistics

Modes	M.B	B-E	F-D
Nature of particle:-	Particle is distinguishable	indistinguishable	=
spin of particle:-	No spin	0 or integral value	spin is half or $\frac{3}{2}$ or $\frac{5}{2}$ ....
No. of particles for energy system	:- No upper limit to the no. of particle for quantum state.	Boson particle don't obey paulis exclusion principle	Boson particle obey paulis exclusion principle.
Nature of application	:- It's applicable for ideal gas	applicable to photon with symmetric wave function	It is applicable to the electron, proton with antisymmetric wave function.

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Statistical Mechanics

13.1.20

Macrostate and Microstate

Macrostate:- A collection of non-intracting particles which are identical in nature, in a isolated system with a fixed number of particles 'N' having a fixed internal energy 'E' and occupying a fixed volume 'v' is called a Macrostate.

Microstate:- The different number of meaningful ways in which total energy of a system can be distributed among the constituent particles called Microstate.

Macrostate

Total energy = 5ϵ

Total No. of particles = 3

ϵ	2ϵ	3ϵ	
a	bc		— Macrostate - 1
ab		c	— Macrostate - 2

Ensemble

Is a collection of macroscopic systems which can interact with each other. There are three different types of ensembles

(i) canonical ensembles: when macroscopic system interact with each other so that these can exchange energy but not particle (matter).

(ii) Micro canonical ensembles: A collection of macroscopic system which ~~do not exch~~ neither exchange energy nor matter during their interaction are called Micro canonical ensembles.

(iii) Grand canonical ensembles: A collection of macroscopic systems which exchange both energy and matter during their interaction are called Grand canonical ensembles.

Phase space :-

~~The six dimensional~~

In describing the dynamical state of a system of particles a six dimensional space considering or taking 3 position coordinates (x, y, z) and 3 momentum coordinates (P_x, P_y, P_z) is known as phase space.

Phase space of harmonic oscillator

Let us consider a harmonic oscillator of mass 'm' and spring constant 'K'. For a displacement of x the total energy of the oscillator $E = \frac{1}{2}mv^2 + \frac{1}{2}Kx^2$ — (1)

$$= \frac{1}{2}m\dot{x}^2 + \frac{1}{2}Kx^2$$

$$= \frac{(m\dot{x})^2}{2m} + \frac{x^2}{\frac{2}{K}}$$

$$= \frac{P_x^2}{2m} + \frac{x^2}{\frac{2}{K}}$$

$$\Rightarrow \frac{P_x^2}{2mE} + \frac{x^2}{\frac{2E}{K}} = 1 \quad \text{— (2)}$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad \text{— (3)}$$

comparing eq (2) and (3) we can conclude that the dynamical state of an 1D harmonic oscillator describes an ellipse in phase space.

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Density of states :-

The DOS represents the no. of quantum states available per unit energy range per unit volume.

Let us consider an elementary volume of particles in a system, given by

$$d\tau = dx\,dy\,dz\,dp_x\,dp_y\,dp_z$$

so, the finite volume in the free space,

$$\int d\tau = \tau = \iiiint\!\!\iint dx\,dy\,dz\,dp_x\,dp_y\,dp_z$$

$$= \iiint dx\,dy\,dz \iiint dp_x\,dp_y\,dp_z$$

This volume in the phase space can be divided into a large no. of equal elementary cells.

For quantum mechanical system, we can use heisenberg's uncertainty principle to obtain minimum value of $dx\,dp_x$ so that $\boxed{dx\,dp_x \geq h}$

Therefore, min value is $\boxed{dx\,dp_x = h}$

$$d\tau = dx\,dp_x\,dy\,dp_y\,dz\,dp_z$$

$d\tau = h^3$, represents min^{m} elementary value of cell.

Therefore the no. of possible quantum states corresponding to momentum p to $p+dp$ is

$$N = \frac{d\tau}{h^3} = \frac{\iiint dx\,dy\,dz \iiint dp_x\,dp_y\,dp_z}{h^3}$$

$$\boxed{\Rightarrow N = \frac{V\iiint dp_x\,dp_y\,dp_z}{h^3}} \quad -(2)$$

In order to find $\iiint dP_x \, dP_y \, dP_z$ let us consider to concentric spheres of radius 'p' and 'p+dp' in the momentum space. For this sphere $p^2 = P_x^2 + P_y^2 + P_z^2$

$$= 2ME$$

Now we can find the volume of the spherical shell enclosed by the sphere of radius p and p+dp

as $\int dP_x \, dP_y \, dP_z = 4\pi p^2 dp$

Maxwell Boltzmann statistics (M.B) 27.1.20

Basic postulates of M.B statistics

1. M.B statistics is applicable for distinguishable unidentical particles.

2. The particles have integral spin.

3. The particles obeying M.B statistics can occupy an energy level with any number.

4. These particles donot obey pauli's exclusion principle.

Boltzon

The particles which obey maxwell boltzmann statistics are called Boltzon. Ex:- Gas molecules

Maxwell Boltzmann distribution function

M.B distribution function gives the probability of an energy state to be occupied with a boltzon.

The M.B distribution function can be obtained as:

$$\cancel{N_i} = \frac{g_i}{e^{(\alpha+\beta)E_i}}$$

$$N_i = \frac{g_i}{e^{\alpha+\beta E_i}}$$, where α and β are constants dependendent on the physical system.

N_i is the number of the particles occupied by the energy state E_i.

g_i = Total number of states.

Limitations of M.B statistics

(i) M.B statistics is applicable only for distinguishable identical particles. Therefore it is not valid for indistinguishable identical particles like electrons, protons;

(ii) According to MB statistics any number of particles can be occupied in a single energy state. But quantum particles like electron proton which obey pauli's exclusion principle only one particle can occupy a single quantum state.

Quantum statistics

The need of development of new statistics are based on the limitations of M.B statistics. Consequently two new statistical distribution were developed

(i) ~~Bohr's~~ Bose eistein statistics :

(ii) Fermi - Dirac statistics

$\frac{Ni}{gi} >> 1$ strongly degenerate

$\frac{Ni}{gi} > 1$ degenerate

$\frac{Ni}{gi} < 1$ Non-degenerate

Basic postulates of Bose - Einstein statistics

indistinguishable identical particle, integral spin, donot obey pauli's exclusion principle, have symmetric wave function

Boson

The particles which obey bose einstein statistics

Ex - photon, Graviton, meson

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Bose-Einstein distribution function

$$\frac{N_i}{g_i} = \frac{1}{e^{(E_i-\mu)/KT}-1}$$

$$f(E) = \frac{1}{e^{(E_i-\mu)/KT}-1}$$

μ = chemical potential which is equal to the energy for including a new particle inside the system.

Plank's Radiation law from B.E statistics

A black body during radiation emmits photons. photons are Particles which obey bose-eistein statistics. photons ~~as in~~ have integral spin and a given quantum State can be occupied with any number of photons.

Radiation in an enclosed volume can be regarded as photon gas. Therefore Bose eistein statistics can be applied for deriving planks radiation law.

Bose einstein distribution function is given by $\frac{N_i}{g_i} = \frac{1}{e^{(E_i-\mu)/KT}-1}$ — (1)

photons can be absorbed by the walls of the encloser and photons of new energy or frequency can be created.

$$\sum N_i \neq \text{constant}$$

$$\sum_i dN_i \neq 0.$$

This imposes that $\mu = 0$ for photon gas.

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$$N_i = \frac{g_i}{e^{E_i/KT}-1} \quad - (2)$$

The number of photons in the frequency range ν to $\nu + d\nu$ can be obtained by substituting g_i with ~~$g(\nu)d\nu$~~

$g(\nu)\,d\nu$

$$N(\nu)\,d\nu = \frac{g(\nu)\,d\nu}{(e^{E_i/KT}-1)}$$

where $N(\nu)\,d\nu$ represents the number of photons in the frequency range $\nu + d\nu$.

For a given volume 'V' for the phase space of photon gas the density of state corresponding to momentum $p + dp$ is given by $g(p)\,dp = \frac{8\pi V p^2 dp}{h^3}$ — (4)

For photons $E = h\nu$

and momentum $p = \frac{E}{c}$

$$p = \frac{h\nu}{c} \quad - (5)$$

$$dp = \frac{h}{c}\,d\nu \quad - (6)$$

substituting eq (5) and eq 6 in eq 4

$$g(p)\,dp = \frac{8\pi}{\cancel{h^3}} \times \frac{\cancel{h^2}\nu^2 V}{c^2} \times \frac{\cancel{h}}{c}\,d\nu$$

$$= \frac{8\pi\nu^2 V}{c^3}\,d\nu \quad - (7)$$

using eq (7) in eq (3)

$$N(\nu)\,d\nu = \frac{8\pi V \nu^2}{c^3} \times \frac{1}{e^{h\nu/KT}-1}\,d\nu$$

All the photons $N(\nu)\,d\nu$ has frequency ν. Each of these photons has energy $h\nu$. Therefore energy due to all the photons within the frequency range $\nu + d\nu$ can be obtained as $E_\nu\,d\nu = h\nu\,N(\nu)\,d\nu$

$$\boxed{E_\nu\,d\nu = \frac{8\pi V h \nu^3}{c^3} \times \frac{1}{e^{h\nu/KT}-1}\,d\nu} \quad \text{—} (8)$$

Takin $\nu = \frac{c}{\lambda}$ and $d\nu = -\frac{c}{\lambda^2}\,d\lambda$

$$E_\nu\,d\nu = \cancel{\frac{8\pi V h}{c^3}} \times \cancel{\frac{c^3}{\lambda^3}} \times \frac{1}{e^{\frac{hc}{\lambda KT}}-1} \times \left(-\frac{c}{\lambda^2}\right) d\lambda$$

$$\boxed{E_\lambda\,d\lambda = -\frac{8\pi V c h}{\lambda^5} \times \frac{1}{\left(e^{\frac{hc}{\lambda KT}}-1\right)}\,d\lambda}$$

Energy density = Energy per unit volume

$$\Rightarrow \frac{E_\lambda\,d\lambda}{V} = \frac{8\pi h c}{\lambda^5} \times \frac{1}{\left(e^{\frac{hc}{\lambda KT}}-1\right)}\,d\lambda$$

Entropy

Is the measure of disordness (degree of) of a system which is thermodynamic in nature.

Boltzmann obtained entropy ~~as~~

$S = K \ln \Omega$ ($w \rightarrow$ small omega, $\Omega \rightarrow$ Big omega)