

Chapter 1

About Condensed Matter Physics

This chapter is just my personal take on why this topic is interesting. It seems unlikely to me that any exam would ask you why you study this topic, so you should probably consider this section to be not examinable. Nonetheless, you might want to read it to figure out why you should think this course is interesting if that isn't otherwise obvious.

1.1 What is Condensed Matter Physics

Quoting Wikipedia:

Condensed matter physics is the field of physics that deals with the macroscopic and microscopic physical properties of matter. In particular, it is concerned with the “condensed” phases that appear whenever the number of constituents in a system is extremely large and the interactions between the constituents are strong. The most familiar examples of condensed phases are solids and liquids, which arise from the electromagnetic forces between atoms.

The use of the term “Condensed Matter” being more general than just solid state was coined and promoted by Nobel-Laureate Philip W. Anderson.

1.2 Why Do We Study Condensed Matter Physics?

There are several very good answers to this question

1. Because it is the world around us

Almost all of the physical world that we see is in fact condensed matter. Questions such as

- why are metals shiny and why do they feel cold?
- why is glass transparent?

- why is water a fluid, and why does fluid feel wet?
- why is rubber soft and stretchy?

These questions are all in the domain of condensed matter physics. In fact almost every question you might ask about the world around you, short of asking about the sun or stars, is probably related to condensed matter physics in some way.

2. Because it is useful

Over the last century our command of condensed matter physics has enabled us humans to do remarkable things. We have used our knowledge of physics to engineer new materials and exploit their properties to change our world and our society completely. Perhaps the most remarkable example is how our understanding of solid state physics enabled new inventions exploiting semiconductor technology, which enabled the electronics industry, which enabled computers, iPhones, and everything else we now take for granted.

3. Because it is deep

The questions that arise in condensed matter physics are as deep as those you might find anywhere. In fact, many of the ideas that are now used in other fields of physics can trace their origins to condensed matter physics.

A few examples for fun:

- The famous Higgs boson, which the LHC is searching for, is no different from a phenomenon that occurs in superconductors (the domain of condensed matter physicists). The Higgs mechanism, which gives mass to elementary particles is frequently called the “Anderson-Higgs” mechanism, after the condensed matter physicist Phil Anderson (the same guy who coined the term “condensed matter”) who described much of the same physics before Peter Higgs, the high energy theorist.
- The ideas of the renormalization group (Nobel prize to Kenneth Wilson in 1982) was developed simultaneously in both high-energy and condensed matter physics.
- The ideas of topological quantum field theories, while invented by string theorists as theories of quantum gravity, have been discovered in the laboratory by condensed matter physicists!
- In the last few years there has been a mass exodus of string theorists applying black-hole physics (in N -dimensions!) to phase transitions in real materials. The very same structures exist in the lab that are (maybe!) somewhere out in the cosmos!

That this type of physics is deep is not just my opinion. The Nobel committee agrees with me. During this course we will discuss the work of no fewer than 50 Nobel laureates! (See the index of scientists at the end of this set of notes).

4. Because reductionism doesn't work

begin{rant} People frequently have the feeling that if you continually ask “what is it made of” you learn more about something. This approach to knowledge is known as *reductionism*. For example, asking what water is made of, someone may tell you it is made from molecules, then molecules are made of atoms, atoms of electrons and protons, protons of quarks, and quarks are made of who-knows-what. But none of this information tells you anything about why water is wet, about why protons and neutrons bind to form nuclei, why the atoms bind to form water, and so forth. Understanding physics inevitably involves understanding how many objects all interact with each other. And this is where things get difficult very

quickly. We understand the Schroedinger equation extremely well for one particle, but the Schroedinger equations for four or more particles, while in principle solvable, in practice are never solved because they are too difficult — even for the world’s biggest computers. Physics involves figuring out what to do then. How are we to understand how many quarks form a nucleus, or how many electrons and protons form an atom if we cannot solve the many particle Schroedinger equation?

Even more interesting is the possibility that we understand very well the microscopic theory of a system, but then we discover that macroscopic properties *emerge* from the system that we did not expect. My personal favorite example is when one puts together many electrons (each with charge $-e$) one can sometimes find new particles emerging, each having one third the charge of an electron!¹ Reductionism would never uncover this — it misses the point completely. `end{rant}`

5. Because it is a Laboratory

Condensed matter physics is perhaps the best laboratory we have for studying quantum physics and statistical physics. Those of us who are fascinated by what quantum mechanics and statistical mechanics can do often end up studying condensed matter physics which is deeply grounded in both of these topics. Condensed matter is an infinitely varied playground for physicists to test strange quantum and statistical effects.

I view this entire course as an extension of what you have already learned in quantum and statistical physics. If you enjoyed those courses, you will likely enjoy this as well. If you did not do well in those courses, you might want to go back and study them again because many of the same ideas will arise here.

¹Yes, this truly happens. The Nobel prize in 1998 was awarded to Dan Tsui, Horst Stormer and Bob Laughlin, for discovery of this phenomenon known as the fractional quantum Hall effect.

Part I

Physics of Solids without Considering Microscopic Structure: The Early Days of Solid State

Chapter 2

Specific Heat of Solids: Boltzmann, Einstein, and Debye

Our story of condensed matter physics starts around the turn of the last century. It was well known (and you should remember from last year) that the heat capacity¹ of a monatomic (ideal) gas is $C_v = 3k_B/2$ per atom with k_B being Boltzmann's constant. The statistical theory of gases described why this is so.

As far back as 1819, however, it had also been known that for many solids the heat capacity is given by²

$$\begin{aligned} C &= 3k_B \text{ per atom} \\ \text{or} \quad C &= 3R \end{aligned}$$

which is known as the *Law of Dulong-Petit*³. While this law is not always correct, it frequently is close to true. For example, at room temperature we have

With the exception of diamond, the law $C/R = 3$ seems to hold extremely well at room temperature, although at lower temperatures all materials start to deviate from this law, and typically

¹We will almost always be concerned with the heat capacity C per atom of a material. Multiplying by Avogadro's number gives the *molar heat capacity* or heat capacity per mole. The *specific heat* (denoted often as c rather than C) is the heat capacity per unit mass. However, the phrase "specific heat" is also used loosely to describe the molar heat capacity since they are both intensive quantities (as compared to the total heat capacity which is extensive — i.e., proportional to the amount of mass in the system). We will try to be precise with our language but one should be aware that frequently things are written in non-precise ways and you are left to figure out what is meant. For example, Really we should say C_v per atom $= 3k_B/2$ rather than $C_v = 3k_B/2$ per atom, and similarly we should say C per mole $= 3R$. To be more precise I really would have liked to title this chapter "Heat Capacity Per Atom of Solids" rather than "Specific Heat of Solids". However, for over a century people have talked about the "Einstein Theory of Specific Heat" and "Debye Theory of Specific Heat" and it would have been almost scandalous to not use this wording.

²Here I do not distinguish between C_p and C_v because they are very close to the same. Recall that $C_p - C_v = VT\alpha^2/\beta_T$ where β_T is the isothermal compressibility and α is the coefficient of thermal expansion. For a solid α is relatively small.

³Both Pierre Dulong and Alexis Petit were French chemists. Neither is remembered for much else besides this law.

Material	C/R
Aluminum	2.91
Antimony	3.03
Copper	2.94
Gold	3.05
Silver	2.99
Diamond	0.735

Table 2.1: Heat Capacities of Some Solids

C drops rapidly below some temperature. (And for diamond when the temperature is raised, the heat capacity increases towards $3R$ as well, see Fig. 2.2 below).

In 1896 Boltzmann constructed a model that accounted for this law fairly well. In his model, each atom in the solid is bound to neighboring atoms. Focusing on a single particular atom, we imagine that atom as being in a harmonic well formed by the interaction with its neighbors. In such a classical statistical mechanical model, the heat capacity of the vibration of the atom is $3k_B$ per atom, in agreement with Dulong-Petit. (Proving this is a good homework assignment that you should be able to answer with your knowledge of statistical mechanics and/or the equipartition theorem).

Several years later in 1907, Einstein started wondering about why this law does not hold at low temperatures (for diamond, “low” temperature appears to be room temperature!). What he realized is that quantum mechanics is important!

Einstein’s assumption was similar to that of Boltzmann. He assumed that every atom is in a harmonic well created by the interaction with its neighbors. Further he assumed that every atom is in an identical harmonic well and has an oscillation frequency ω (known as the “Einstein” frequency).

The quantum mechanical problem of a simple harmonic oscillator is one whose solution we know. We will now use that knowledge to determine the heat capacity of a single one dimensional harmonic oscillator. This entire calculation should look familiar from your statistical physics course.

2.1 Einstein’s Calculation

In one dimension, the eigenstates of a single harmonic oscillator are

$$E_n = \hbar\omega(n + 1/2)$$

with ω the frequency of the harmonic oscillator (the “Einstein frequency”). The partition function is then⁴

$$\begin{aligned} Z_{1D} &= \sum_{n \geq 0} e^{-\beta \hbar \omega (n + 1/2)} \\ &= \frac{e^{-\beta \hbar \omega / 2}}{1 - e^{-\beta \hbar \omega}} = \frac{1}{2 \sinh(\beta \hbar \omega / 2)} \end{aligned}$$

⁴We will very frequently use the standard notation $\beta = 1/(k_B T)$.

The expectation of energy is then

$$\langle E \rangle = -\frac{1}{Z} \frac{\partial Z}{\partial \beta} = \frac{\hbar\omega}{2} \coth\left(\frac{\beta\hbar\omega}{2}\right) = \hbar\omega \left(n_B(\beta\hbar\omega) + \frac{1}{2} \right) \quad (2.1)$$

where n_B is the Bose⁵ occupation factor

$$n_B(x) = \frac{1}{e^x - 1}$$

This result is easy to interpret: the mode ω is an excitation that is excited on average n_B times, or equivalently there is a “boson” orbital which is “occupied” by n_B bosons.

Differentiating the expression for energy we obtain the heat capacity for a single oscillator,

$$C = \frac{\partial \langle E \rangle}{\partial T} = k_B (\beta\hbar\omega)^2 \frac{e^{\beta\hbar\omega}}{(e^{\beta\hbar\omega} - 1)^2}$$

Note that the high temperature limit of this expression gives $C = k_B$ (check this if it is not obvious!).

Generalizing to the three-dimensional case,

$$E_{n_x, n_y, n_z} = \hbar\omega[(n_x + 1/2) + (n_y + 1/2) + (n_z + 1/2)]$$

and

$$Z_{3D} = \sum_{n_x, n_y, n_z \geq 0} e^{-\beta E_{n_x, n_y, n_z}} = [Z_{1D}]^3$$

resulting in $\langle E_{3D} \rangle = 3\langle E_{1D} \rangle$, so correspondingly we obtain

$$C = 3k_B (\beta\hbar\omega)^2 \frac{e^{\beta\hbar\omega}}{(e^{\beta\hbar\omega} - 1)^2}$$

Plotted this looks like Fig. 2.1.

⁵Satyendra Bose worked out the idea of Bose statistics in 1924, but could not get it published until Einstein lent his support to the idea.

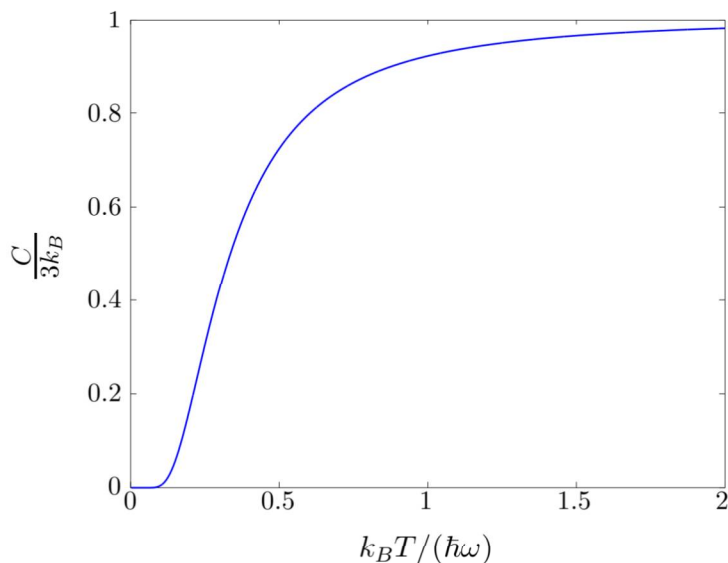


Figure 2.1: Einstein Heat Capacity Per Atom in Three Dimensions

Note that in the high temperature limit $k_B T \gg \hbar \omega$ recover the law of Dulong-Petit — $3k_B$ heat capacity per atom. However, at low temperature ($T \ll \hbar \omega / k_B$) the degrees of freedom “freeze out”, the system gets stuck in only the ground state eigenstate, and the heat capacity vanishes rapidly.

Einstein’s theory reasonably accurately explained the behavior of the the heat capacity as a function of temperature with only a single fitting parameter, the Einstein frequency ω . (Sometimes this frequency is quoted in terms of the Einstein temperature $\hbar \omega = k_B T_{\text{Einstein}}$). In Fig. 2.2 we show Einstein’s original comparison to the heat capacity of diamond.

For most materials, the Einstein frequency ω is low compared to room temperature, so the Dulong-Petit law hold fairly well (being relatively high temperature compared to the Einstein frequency). However, for diamond, ω is high compared to room temperature, so the heat capacity is lower than $3R$ at room temperature. The reason diamond has such a high Einstein frequency is that the bonding between atoms in diamond is very strong and its mass is relatively low (hence a high $\omega = \sqrt{\kappa/m}$ oscillation frequency with κ a spring constant and m the mass). These strong bonds also result in diamond being an exceptionally hard material.

Einstein’s result was remarkable, not only in that it explained the temperature dependence