Chapter-4

Interference of Light

4.1 Introduction

According to wave theory of light, a source of light spreads its disturbances in all directions. In vacuum or in a homogeneous medium, the points those are situated at equal distance from the source of light will vibrate in the same phase. **At any moment, the continuous locus of all such points having the same phase of vibration is called a wavefront.**

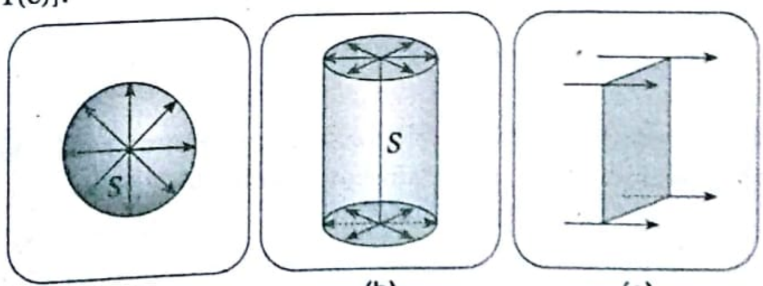
Depending upon the shape of the source of light, wavefronts are of three types. They are—

① **Spherical wavefront** This is produced by a point source of light [Fig. 1(a)]. It is because the locus of all such points, which are equidistant from the point source lie on the surface of a sphere.

② **Cylindrical wavefront** When the source of light is linear in shape, such as a slit, the cylindrical wavefront is produced. Here, all the points equidistant from a linear source lie on the surface of a cylinder [Fig. 1(b)].

③ **Plane wavefront** A small part of a spherical or cylindrical wavefront originating from a distant source will appear as a plane and hence it is called a plane wavefront [Fig. 1(c)]. However, in further discussion, it will be represented by simply a straight line.

The line drawn normal to the wavefront represents the path or a ray of light [Fig. 1(c)].



(a) (b) (c)

Fig. 1 ▷ (a) Spherical wavefront, (b) Cylindrical wavefront, (c) Plane wavefront

Fig. 2(a), 2(b) and 2(c) represent a plane wavefront, converging spherical wavefront and a diverging spherical wavefront respectively and in all cases, the thick arrows represent rays of light.

According to **Huygen's principle**, *each point on the primary wavefront acts as a secondary source (wavelets) of disturbances and spread their disturbances in all directions in a similar manner as the original source of light does. At this moment, the new position of the wavefront (called secondary wavefront), will be the envelope of the secondary wavelets and they again spread their disturbances in all directions. In this way, the light waves are propagated through the medium.*

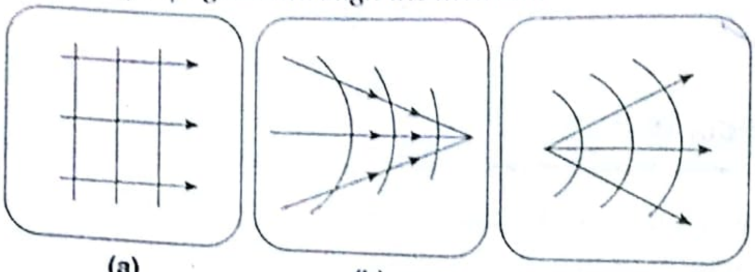


Fig. 2 ▷ (a) Plane wavefront, (b) Converging spherical wavefront, (c) Diverging spherical wavefront

A wave train implies the continuous transfer of energy in the medium from the source of light energy. So a simple harmonic wave train is represented by the equation of the displacement y at a point x from the source at a time t as

$$y = a \sin 2\pi \left(\frac{t}{T} - \frac{x}{\lambda} \right) \qquad \cdots (4.1)$$

where a = amplitude, T = time period and λ = wavelength.

4.2. Coherent Sources

Two sources of light emitting waves of the same frequency, nearly same amplitude but with a constant phase difference are called coherent sources.

In practice, it *is not possible to have two independent coherent sources as the phase difference of the two independent sources does not remain constant. If the phase difference changes rapidly in an irregular way, the resulting waves are non-coherent and the corresponding sources are called non-coherent sources.*

Also, two coherent sources must emit waves of same colour (*i.e.* wavelength) of light.

To get coherent sources, we usually use a single source and divide it into two parts by **division of wavefront** or by **division of amplitude.**

In case of division of wavefront, light from a single source is divided into two beams by the use of apertures or optical components which separate neighbouring parts of the wavefront and the separated beams are subsequently brought together. This method is applicable in *Young's experiment, Fresnel's biprism experiment* and *Lloyd's single mirror experiment* etc.

Similarly in case of division of amplitude, a single beam is divided by means of partial reflection in a thin film or semi-transparent mirror and the separated beams are superposed subsequently. This method is applicable in *Michelson interferometer, Newton's rings* and *thin film* etc.

4.2.1. Temporal and Spatial Coherence

There are two independent concepts of coherence.

These are —**1.** temporal coherence, **2.** spatial coherence.

① **Temporal coherence** *When the phase difference of any two points (or at a single point in the space) lying along the direction of propagation of the wave does not change in between a fixed interval of time, the beam of light is called temporal coherence.*

Actually, light emitted by a source is sinusoidal only for a very small interval of time.

- ***Examples :*** The temporal coherent source is produced in *Michelson-Morley experiment or in Newton's rings experiment.*

Coherence time The time interval for which the phase difference remains constant is known as coherence time (τ_c). Therefore, coherence length $l_c = c\tau_c$, where c is the velocity of light.

So, for a perfectly coherent monochromatic plane wave, coherence time $\tau_c = \infty$, while for a completely incoherent wave (radiation from a black body source), $\tau_c = 0$.

② **Spatial coherence** *When the phase difference between any two points on the two waves lying on a plane perpendicular to the direction of wave propagation does not change with time, the beam of light is called spatial coherence.*

For example, the phase difference between any two points P and Q [Fig. 3] lying on the same wavefront is zero (*i.e.*, constant) at all times. The wave is then said to exhibit perfect spatial coherence.

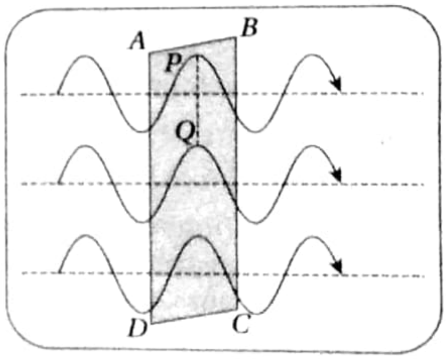


Fig 3. ▷ Spatial coherence is seen at the points P and Q in the plane $ABCD$

Problem 1

A He-Ne laser giving light at 6330 Å has a coherence length of 20km. Determine (i) its coherence time and (ii) the number of waves per wavetrain.

Solution Here, coherence length $l_c = 20\text{km} = 2 \times 10^4\text{m}$,

wavelength of light $\lambda = 6330 \times 10^{-10}\text{m}$

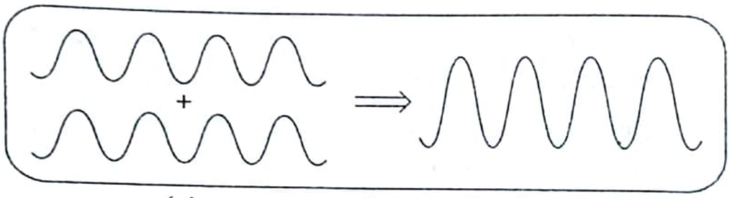
$\therefore$ (i) coherence time $\tau_c = \dfrac{l_c}{c} = \dfrac{2 \times 10^4\text{m}}{3 \times 10^8\text{m}\cdot\text{s}^{-1}} = 6.7 \times 10^{-5}\text{s}$

(ii) number of waves per wavetrain $N = \dfrac{l_c}{\lambda} = \dfrac{2 \times 10^4}{6330 \times 10^{-10}} = 3.2 \times 10^{10}$ waves.

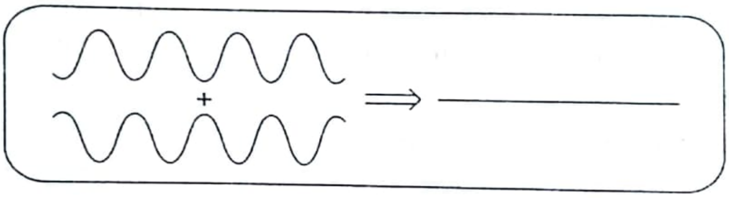
4.3. Interference of Light

The phenomenon, in which alternate bright and dark bands (fringes) are produced as a result of superposition of two monochromatic light waves having same wavelength but a constant phase difference proceeding in the same direction, is called interference of light.

According to the **principle of superposition** of waves, we know that *the resultant intensity of the light at any point is the algebraic sum of the intensities of two superimposed lightwaves travelling simultaneously in a medium. [Fig. 4]*



(a) waves superimpose in phase



(b) waves superimpose out of phase

Fig 4. ▷ Principle of superposition

If the two interfering waves are of the same amplitude, it can be shown that at the points of constructive interference (with maximum intensity of light), the intensity of light is four times the intensity due to any single interfering wave [as I (intensity) $\propto A^2$ (amplitude)]. Similarly, at the points of destructive interference, the intensity of light is zero (*i.e.* complete darkness).

So, *energy is neither created nor destroyed due to interference. The energy that disappears at the points of destructive interference reappears at the points of constructive interference and vice versa. So, in interference of light, redistribution of light energy takes place.*

and dark interference fringes on the screen, where they superpose. At the centre of the screen, the intensity of light is maximum and it is called *central maximum*.

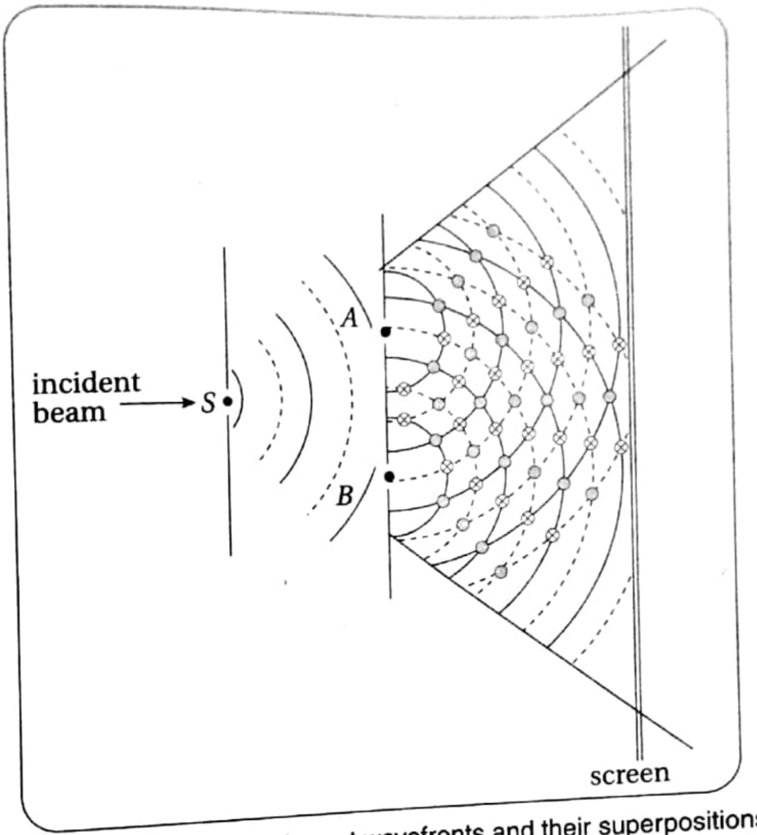


Fig. 5 ▷ Formation of coherent wavefronts and their superpositions

Explanation The wavefront from A and B superpose on the screen. At points where a crest (or trough) due to one falls on a crest (or trough) due to other [those points are seen by ○ sign in Fig. 5], the resultant amplitude and hence resultant intensity of light at these points becomes maximum due to constructive interference of light. These points correspond to the position of bright fringe on the screen. At points where a crest due to one wave falls on a trough due to the other [those points are seen by ⊗ in Fig. 4], the resultant amplitude is zero and hence the resultant intensity is zero due to destructive interference of light. Thus a symmetrical pattern of alternate bright and dark fringes is obtained on the screen.

Conditions for sustained or permanent interference

1. The two sources of light must be coherent *i.e.* the two light waves emitted by them must have a constant phase difference or in the same phase.
2. The two sources must emit light of the same wavelength but the amplitudes between them should differ as little as possible. The emitted waves should be preferably of the same amplitude to get completely dark fringes.
3. The two sources should be very narrow. Otherwise with the increase of slit width, the coherence property will be lost. Hence, no interference pattern will be obtained.
4. The two sources must lie very close to each other. Otherwise overlapping of bright and dark points will hinder interference.

⑤ At maxima, the path difference between two light waves is always an even multiple of $\frac{\lambda}{2}$ and at minima, it is an odd multiple of $\frac{\lambda}{2}$.

Shape of interference fringes Let us consider a co-ordinate system whose origin O [Fig. 6] is situated at the midpoint between two coherent sources S_1 and S_2 (lying along the Y-axis). $P(x, y)$ is any point on the screen along a straight line parallel to Y-axis. Suppose the co-ordinates of sources S_1 and S_2 are $(0, d)$ and $(0, -d)$ respectively.

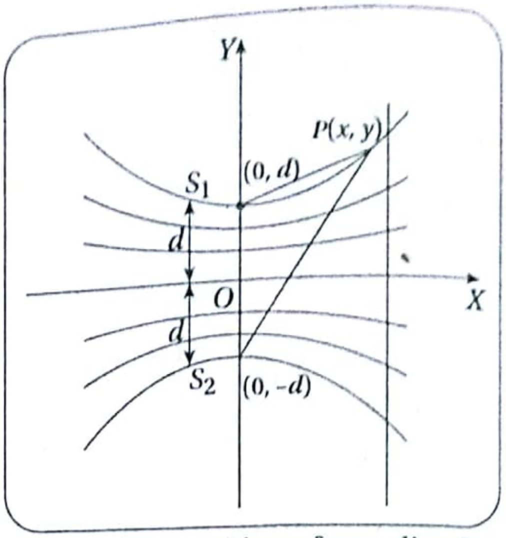


Fig 6. ▷ The position of two slits S_1 and S_2 and the corresponding co-ordinate system.

If Δ is the path difference between two interfering waves S_1P and S_2P, then $\Delta = S_2P - S_1P$.

Now, $S_2P = \sqrt{(x-0)^2 + [y-(-d)]^2}$

$= \sqrt{x^2 + (y+d)^2}$ and

$$S_1P = \sqrt{(x-0)^2 + (y-d)^2} = \sqrt{x^2 + (y-d)^2}$$

$$\therefore \quad S_2P^2 - S_1P^2 = x^2 + (y+d)^2 - [x^2 + (y-d)^2] = 4yd$$

Again, $\Delta = S_2P - S_1P$

or, $S_2P^2 = (\Delta + S_1P)^2$

or, $S_2P^2 = \Delta^2 + S_1P^2 + 2\Delta \cdot S_1P$

or, $(S_2P^2 - S_1P^2) - \Delta^2 = 2\Delta S_1P$

or, $4yd - \Delta^2 = 2\Delta S_1P \quad [\because S_2P^2 - S_1P^2 = 4yd]$

Squaring both sides, we get,

$$(4yd - \Delta^2)^2 = 4\Delta^2 S_1P^2$$

or, $16y^2d^2 + \Delta^4 - 8\Delta^2 yd = 4\Delta^2[x^2 + (y-d)^2]$

or, $16y^2d^2 + \Delta^4 - 8\Delta^2 yd = 4\Delta^2x^2 + 4\Delta^2y^2 + 4\Delta^2d^2 - 8\Delta^2 yd$

or, $y^2(16d^2 - 4\Delta^2) - 4x^2\Delta^2 = 4\Delta^2d^2 - \Delta^4$

or, $16y^2\left(d^2 - \frac{\Delta^2}{4}\right) - 4x^2\Delta^2 = 4\Delta^2d^2 - \Delta^4$

or, $4y^2\left(d^2 - \frac{\Delta^2}{4}\right) - x^2\Delta^2 = \Delta^2d^2 - \frac{\Delta^4}{4}$

or, $4y^2\left(d^2 - \frac{\Delta^2}{4}\right) - x^2\Delta^2 = \Delta^2\left(d^2 - \frac{\Delta^2}{4}\right)$

Dividing both sides by $4\Delta^2\left(d^2 - \frac{\Delta^2}{4}\right)$, we can write $\frac{4y^2}{\Delta^2} - \frac{4x^2}{4d^2 - \Delta^2} = 1$

or, $$\frac{y^2}{\frac{\Delta^2}{4}} - \frac{x^2}{\left(d^2 - \frac{\Delta^2}{4}\right)} = 1 \qquad \ldots 4.2$$

which is the equation of hyperbola.

So, the shapes of interference fringes obtained in Young's double slit experiment are hyperbola.

4.3.2. Analytical Treatment of Interference of Light, Conditions for Constructive and Destructive Interference

A source of monochromatic light S illuminates two narrow slits A and B [Fig. 7]. A and B are equidistant from S and so act as two coherent sources. Let a be the amplitude of each wave to get ideal interference fringes[1] and δ be the constant phase difference between two waves for their path difference to reach at P on the screen at any instant t.

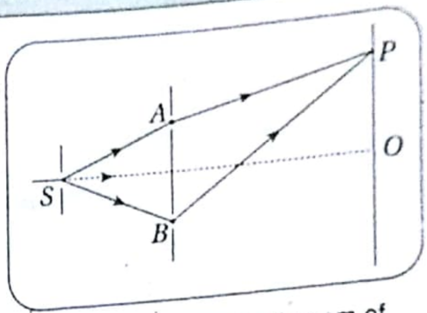


Fig. 7 ▷ Schematic diagram of interference

If y_1 and y_2 are the displacements of the two interfering waves coming from the two coherent sources A and B at any point P then,

$$y_1 = a\sin\omega t \quad \ldots(4.3a) \quad \text{and} \quad y_2 = a\sin(\omega t + \delta) \quad \ldots(4.3b)$$

where δ = phase difference at the point P for the two coherent sources A and B.

By *the superposition principle*, the resultant displacement at P is given by,

$$y = y_1 + y_2 = a\sin\omega t + a\sin(\omega t + \delta)$$
$$= a\sin\omega t + a\sin\omega t\cos\delta + a\cos\omega t\sin\delta$$
$$= a(1 + \cos\delta)\sin\omega t + a\sin\delta\cos\omega t \qquad \ldots(4.4)$$

Let, $$a(1 + \cos\delta) = A\cos\theta \qquad \ldots(4.5)$$

and $$a\sin\delta = A\sin\theta \qquad \ldots(4.6)$$

Then, equation (4.4) becomes

$$y = A\cos\theta\sin\omega t + A\sin\theta\cos\omega t = A\sin(\omega t + \theta) \qquad \ldots(4.7)$$

Hence, the *resultant displacement at point P is a simple harmonic wave of amplitude A*. The amplitude (A) can be obtained by squaring and adding equations (4.5) and (4.6),

$$A^2 = a^2(1 + \cos\delta)^2 + a^2\sin^2\delta$$
$$= a^2(1 + 2\cos\delta + \cos^2\delta) + a^2\sin^2\delta$$
$$= a^2 + 2a^2\cos\delta + (a^2\cos^2\delta + a^2\sin^2\delta)$$
$$= 2a^2 + 2a^2\cos\delta = 2a^2(1 + \cos\delta) = 2a^2 \cdot 2\cos^2\frac{\delta}{2} = 4a^2\cos^2\frac{\delta}{2} \qquad \ldots(4.8)$$

[1] **Ideal Interference fringe :** In ideal interference phenomenon, the amplitudes of two interfering waves are equal.

The intensity of light is proportional to the square of amplitude of the wave. For the sake of simplicity, we assume that the intensity of light is equal to the square of amplitude.

Hence, the intensity (I) at P due to these superposed waves,

$$I = A^2 = 4a^2 \cos^2 \frac{\delta}{2} \qquad \cdots(4.9)$$

Conditions for constructive and destructive interference

① For **constructive interference**, the intensity of light at P will be maximum, if

$$\cos^2 \frac{\delta}{2} = 1 \quad \text{or,} \quad \cos \frac{\delta}{2} = 1$$

It is possible only when $\delta = 2n\pi$, where $n = 0, 1, 2, \ldots$

So, $\delta = 0, 2\pi, 4\pi, \ldots$ $\cdots(4.10)$

In terms of path difference, the condition is

$$x = \frac{2n\lambda}{2} \quad \left[\because \delta = \frac{2\pi}{\lambda} x\right] \qquad \cdots(4.11)$$

Hence, $I = 4a^2$ [from equation (4.9)].

Therefore, for constructive interference (maximum intensity) the phase difference is an even integral multiple of π (or the path difference is an even integral multiple of $\frac{\lambda}{2}$).

Note that, at the centre 'O' of the screen [Fig. 5], the intensity of light is maximum and it is called **central maximum.**

② For **destructive interference**, the intensity of light (I) at P will be minimum (*i.e.* $I = 0$), if

$$\cos \frac{\delta}{2} = 0$$

It is possible only when

$$\delta = (2n+1)\pi, \text{ where } n = 0, 1, 2, \ldots$$

So, $\delta = \pi, 3\pi, 5\pi, \ldots$ $\cdots(4.12)$

In terms of path difference, the corresponding condition is

$$x = (2n+1)\frac{\lambda}{2} \quad \left[\because \delta = \frac{2\pi}{\lambda} x\right] \qquad \cdots(4.13)$$

Hence, for destructive interference (minimum intensity), the phase difference is an odd integral multiple of π (or the path difference is an odd integral multiple of $\frac{\lambda}{2}$).

Special Note :

If the amplitudes of the two interfering waves are different (as small as possible), then from the principle of superposition we get,

$$y = a_1 \sin\omega t + a_2 \sin(\omega t + \delta)$$

$$= (a_1 + a_2 \cos\delta) \sin\omega t + a_2 \sin\delta \cos\omega t$$

$$= A \sin(\omega t + \theta)$$

Here $A\cos\theta = a_1 + a_2\cos\delta$ and $A\sin\theta = a_2 \sin\delta$

So, $A^2 = a_1^2 + a_2^2 + 2a_1 a_2 \cos\delta$

Hence, $I = a_1^2 + a_2^2 + 2a_1 a_2 \cos\delta$ ...(1)

So, for constructive interference, $\cos\delta = 1$

$\therefore$ $\delta = 2n\pi$, $[n = 0, 1, 2, \cdots]$ *i.e.* $\delta = 0, 2\pi, 4\pi, \cdots$

Similarly, for destructive interference, $\cos\delta = -1$

$\therefore$ $\delta = (2n+1)\pi$, $[n = 0, 1, 2, 3]$ *i.e.* $\delta = \pi, 3\pi, 5\pi$

So, the maximum and minimum intensity of the interference pattern due to interference effect are given by,

$I_{max} = (a_1 + a_2)^2$ and $I_{min} = (a_1 - a_2)^2$ [from equation (1)]

Problem 1

Two waves each of equal amplitude and equal frequency pass through a point in the medium in the same direction with phase difference of 60°. Calculate the amplitude of the resultant wave at this point.

Solution The resultant amplitude of two interfering waves can be written from equation 4.8.

$$A = 2a\cos\frac{\delta}{2}$$ (here, δ = phase difference = 60°)

$$\therefore \quad A = 2a\cos 30° = 2a \times \frac{\sqrt{3}}{2} = \sqrt{3}a$$

Problem 2

The ratio of intensity of bright fringe to dark fringe in a interference pattern due to superposition of two coherent sources is 25 : 1. Find the ratio of amplitude of two coherent sources.

Solution If a_1 and a_2 are the amplitudes of two coherent sources, the ratio of maximum intensity to minimum intensity after interference is

$$\frac{(a_1 + a_2)^2}{(a_1 - a_2)^2} = \frac{I_{max}}{I_{min}}$$

or, $\frac{(a_1+a_2)^2}{(a_1-a_2)^2} = \frac{25}{1}$

or, $\frac{a_1+a_2}{a_1-a_2} = \frac{5}{1}$ or, $5a_1 - 5a_2 = a_1 + a_2$ or, $4a_1 = 6a_2$ or, $\frac{a_1}{a_2} = \frac{3}{2}$

Problem 3

Two waves having intensities in the ratio of 9 : 1 produce interference. Find the ratio of maximum intensity to minimum intensity.

Solution If a_1 and a_2 are the amplitudes of two interfering waves,

$$\frac{a_1^2}{a_2^2} = \frac{9}{1} \quad \text{or,} \quad \frac{a_1}{a_2} = \frac{3}{1} \quad \text{or,} \quad a_1 = 3a_2$$

After interference, the ratio of maximum intensity to minimum intensity is

$$\frac{I_{max}}{I_{min}} = \frac{(a_1+a_2)^2}{(a_1-a_2)^2} = \frac{(3a_2+a_2)^2}{(3a_2-a_2)^2} = \left(\frac{4}{2}\right)^2 = 4:1.$$

4.3.3. Energy Conservation in Interference

In absence of interference phenomenon the resultant intensity due to the two waves having same amplitude 'a',

$$I = I_1 + I_2 = a^2 + a^2 = 2a^2 \quad \cdots(4.14)$$

$$[\because y_1 = a \sin\omega t,\ y_2 = a \sin(\omega t + \delta)]$$

When the two light waves from coherent sources interfere for constructive interference, the maximum intensity will be

$$I_{max} = (a+a)^2 = 4a^2 \quad \cdots(4.15)$$

Similarly, for destructive interference, the minimum intensity will be

$$I_{min} = (a-a)^2 = 0 \quad \cdots(4.16)$$

But from equation (4.9), $I = 4a^2 \cos^2\frac{\delta}{2}$. So, the variation of I with phase difference δ can be shown below.

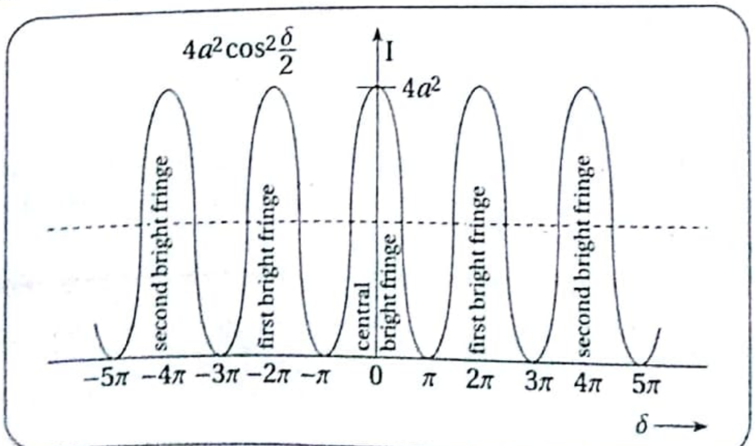


Fig. 8 ▷ Intensity distribution in interference

Special Note :

We know, when the amplitude of two interfering waves are different (as small as possible), the maximum and minimum intensity of resultant disturbance after interference,

$$I_{max} = (a_1 + a_2)^2$$

and $I_{min} = (a_1 - a_2)^2$ respectively.

So, average intensity $= \dfrac{I_{max} + I_{min}}{2} = a_1^2 + a_2^2$

Again, when there is no interference,

$$I = I_1 + I_2 = a_1^2 + a_2^2$$

So, the *law of conservation of energy is,proved.*

4.3.4. Theory of Interference Fringes

Hence, $BP^2 = BB_1^2 + B_1P^2$ [from right angle ΔBB_1P]

and $AP^2 = AA_1^2 + A_1P^2$ [from right angle ΔAA_1P]

So, $BP^2 - AP^2 = \left[D^2 + \left(x+\frac{d}{2}\right)^2\right] - \left[D^2 + \left(x-\frac{d}{2}\right)^2\right]$

or, $(BP+AP)(BP-AP) = 2xd$ or, $(BP-AP) = \dfrac{2xd}{BP+AP}$

In practice, point P lies very close to O_1. So, $BP \simeq AP \simeq D$.

$$\therefore \quad BP - AP = \frac{2xd}{2D} \quad \text{or,} \quad \boxed{BP - AP = \frac{xd}{D}} \qquad \cdots (4.18)$$

Position of maximum and minimum on the screen

Condition for constructive interference at P The path difference $(BP-AP)$ between two interfering waves to reach at P on the screen is an even integral multiple of $\frac{\lambda}{2}$.

So, $BP - AP = 2n\frac{\lambda}{2}$, where $n = 0, 1, 2, \cdots$

$$\text{or,} \quad \frac{xd}{D} = n\lambda \quad \text{or,} \quad \boxed{x = \frac{D}{d}n\lambda} \qquad \cdots (4.19)$$

Here, $n = 0$ is for central bright fringe, $n = 1$ for 1st order bright fringe and so on.

If x_n and x_{n+1} denote the distance of nth and $(n+1)$th bright fringes respectively,

then $x_n = \frac{D}{d}n\lambda$ and $x_{n+1} = \frac{D}{d}(n+1)\lambda$

So, spacing between two consecutive bright fringes or *fringe width is*

$$\beta = x_{n+1} - x_n = \frac{D}{d}(n+1)\lambda - \frac{D}{d}n\lambda$$

$$\text{or,} \quad \boxed{\beta = \frac{D\lambda}{d}} \qquad \cdots (4.20)$$

As β is independent of n, the bright fringes are equispaced.

Condition for destructive interference at P The path difference $(BP-AP)$, is an odd integral multiple of $\frac{\lambda}{2}$.

So, $BP - AP = (2n+1)\frac{\lambda}{2}$, where $n = 0, 1, 2, 3 \ldots$

or, $\frac{x'd}{D} = (2n+1)\frac{\lambda}{2}$

$$\text{or,} \quad \boxed{x' = \frac{D}{d}(2n+1)\frac{\lambda}{2}} \qquad \cdots (4.21)$$

Here, $n = 0, 1, 2, \cdots$ are for 1st order, 2nd order and 3rd order dark fringes and so on.

Hence, we can write from equation (4.21), the spacing (β') between two consecutive dark fringes [say nth and $(n+1)$th order] is,

$$\beta' = x'_{n+1} - x'_n = \frac{D}{d}(2n+3)\frac{\lambda}{2} - \frac{D}{d}(2n+1)\frac{\lambda}{2}$$

or $$\beta' = \frac{D\lambda}{d} \qquad \cdots(4.22)$$

Hence, the dark fringes are also equispaced.

It is also seen from equation (4.20) and equation (4.22), *fringe width for bright and dark fringes are equal* (*i.e.* $\beta = \beta'$).

Discussions

① For appreciable fringe width β, d should be small and D large.

② The position of $(n+1)$th dark fringe and nth bright fringe are given by,

$$x'_{n+1} = \frac{(2n+3)D\lambda}{2d} \text{ and } x_n = \frac{nD\lambda}{d} \text{ respectively, where } n = 0, 1, 2, \cdots$$

So, the dark or bright fringes will be distinct, if x'_{n+1} or x_n are large *i.e.* the distance of separation between two slits 'd' is small. For this reason, the two slits should be very close to each other.

③ If the source emits white light, the inside edge of each band which is towards the central maximum of the fringe will be violet and the outside edge is red. But the central band will be white as all the coloured waves of white light reach in same phase at the central band. So, the fringes will appear coloured.

Problem 1

When two narrow slits separated by a small distance are illuminated by a light of wavelength 5×10^{-7} m, interference fringes of width 0.5mm are found on a screen. What should be the wavelength of light source to obtain fringes of width 0.3mm, if the distance between the screen and the slit is reduced to half of its initial value?

Solution Here, $\lambda = 5 \times 10^{-7}$ m, $\beta = 0.5$ mm $= 0.5 \times 10^{-3}$ m

Now fringe width, $\beta = \frac{D\lambda}{d}$

or, $$\frac{d}{D} = \frac{\lambda}{\beta} = \frac{5 \times 10^{-7}}{0.5 \times 10^{-3}} = 10^{-3} \qquad \cdots(1)$$

Again, when the distance between the screen and the slit is reduced to half of its initial value, then

$$d' = d;\ D' = \frac{D}{2},\ \beta' = 0.3 \text{ mm} = 0.3 \times 10^{-3} \text{ m}$$

Let, λ' be the wavelength of the light source.

Now, fringe width, $\beta' = \frac{D'\lambda'}{d'}$

or, $\lambda' = \frac{d'}{D'}\beta' = \frac{d}{\frac{D}{2}} \times 0.3 \times 10^{-3} = 0.6 \times 10^{-3} \times \frac{d}{D}$ ...(2)

From equation (1) and (2) we get

$$\lambda' = 0.6 \times 10^{-3} \times 10^{-3} = 6 \times 10^{-7} \text{ m}$$

Problem 2

In Young's double slit experiment, red light of wavelength 620 nm is used and the two slits are 0.3 mm apart. Interference fringes of width 1.3 mm are observed on a screen. Calculate the distance of the slits from the screen.

Solution Here, $\lambda = 620 \text{ nm} = 620 \times 10^{-9} \text{ m}$,

d = distance between the two slits $= 0.3 \text{ mm} = 0.3 \times 10^{-3} \text{ m}$

β = fringe width $= 1.3 \times 10^{-3} \text{ m}$

$\therefore$ Fringe width, $\beta = \frac{D\lambda}{d}$

or, $D = \frac{\beta d}{\lambda} = \frac{1.3 \times 10^{-3} \times 0.3 \times 10^{-3}}{620 \times 10^{-9}}$

$= 0.629 \text{ m}$

Problem 3

In Young's double slit experiment, the distance between the two slits is 0.5 mm. The wavelength of the light used is 5000 Å and the separation between the source and the screen is 50 cm. Calculate the fringe width in this case.

[Kalyani University Engg. Exam 1997]

Solution Here, d = distance between two slits = 0.5 mm = 0.05 cm

λ = wavelength = 5000×10^{-8} cm

D = distance between the source and screen = 50 cm

So, fringe width $\beta = \frac{D\lambda}{d} = \frac{50 \times 5000 \times 10^{-8}}{0.05} = 0.05$ cm

Problem 4

Here, $n = 4$, $D = 1.6\text{ m} = 1.6 \times 100\text{ cm} = 160\text{ cm}$

$d = 1.3\text{mm} = 0.13\text{ cm}$, $\lambda = 6000 \times 10^{-8}\text{ cm}$

$$\therefore \quad x_4 = \frac{4 \times 160 \times 6000 \times 10^{-8}}{0.13} = 0.29\text{ cm}$$

Problem 5

In a Young's double slit experiment, the slits are 1.2 mm apart. The slits are illuminated by light of wavelength 5890 Å. The distance of the third dark fringe from the central maximum is 1.1 cm. Calculate the distance between the screen and the slit.

Solution The distance of the nth dark fringe from the central maximum,

$$x'_n = \frac{D}{d}(2n+1)\frac{\lambda}{2}$$

Here, $n = 2$, $d = 1.2\text{mm} = 0.12\text{ cm}$, $\lambda = 5890 \times 10^{-8}\text{ cm}$, $x'_n = 1.1\text{ cm}$

$$D = \frac{2dx'_n}{(2n+1)\lambda} = \frac{2 \times 0.12 \times 1.1}{(2 \times 2 + 1) \times 5890 \times 10^{-8}} = 896.43\text{ cm}$$

4.4. Fresnel's Biprism❷

Scientist Fresnel, obtained interference pattern with the help of a biprism. The wavelength of monochromatic light can be determined from the interference fringes produced by Fresnel's biprism.

The biprism consists of two acute angled prisms placed base to base. In practice, *it is constructed as a single isosceles glass prism EFG with obtuse angle* $\angle EFG = 179°$ *and the other two acute angles are about* $30'$ *each.* A vertical narrow slit S is mounted on an optical bench such that the refracting edge of the biprism is parallel to the source of light S [Fig. 10]. The slit is illuminated by monochromatic light and is allowed to fall

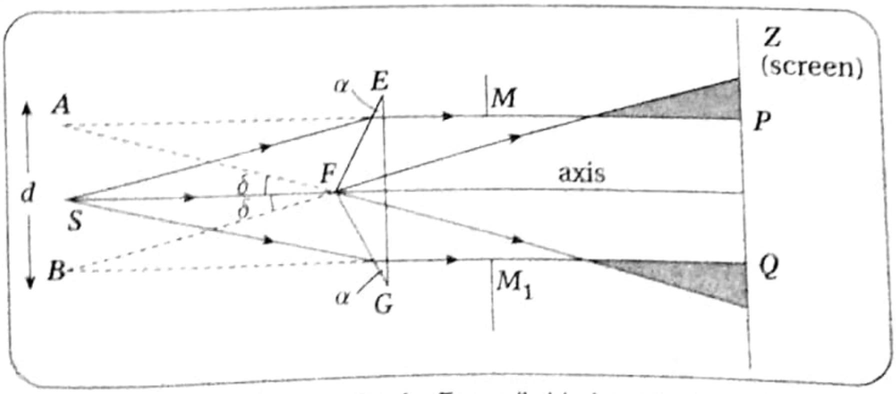


Fig. 10 ▷ Arrangement for Fresnel's biprism experiment

on the biprism. Then the refracting edge divides the incident light into two portions. The light emerging after refraction on faces EF and GF of the biprism appears to

❷ Not included in WBUT syllabus.

come from points A and B. Therefore, A and B act as two virtual coherent sources in the two halves of the prism. As the prism EFG has a large obtuse angle, A and B are near to each other and they produce an interference pattern at the distant screen Z.

The pattern of interference can also be seen through an eyepiece in the place of screen and the fringes are formed in the focal plane of eyepiece. However, the interference pattern is seen between the region PQ of the screen Z where the waves overlap. But beyond P and Q, the fringes of large width are formed due to diffraction which is limited by using stop MM_1.

4.4.1. Theory of Interference Fringes of Fresnel's Biprism

The point 'O' is equidistant from two virtual coherent sources A and B [Fig. 11]. So, the central maximum is seen at O. The alternately bright and dark fringes are produced on both sides of O. Now the fringe width 'β' between any two consecutive fringes is given by

$$\beta = \frac{D\lambda}{d} \; \bullet$$

where D is the distance between the source and the screen and d is the distance between two coherent sources. So, the wavelength of a given monochromatic source of light

$$\lambda = \frac{\beta d}{D} \qquad \cdots (4.23)$$

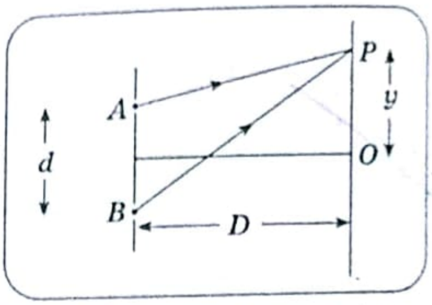


Fig. 11 ▷ Geometrical drawing for the position of interference fringes

Experiment / Procedure

The optical bench used for biprism experiment consists of three uprights. The first upright carries an adjustable slit, the second a biprism and the third a micrometer eyepiece. The source is illuminated by monochromatic light, whose wavelength is to be determined. As discussed earlier, the two virtual coherent sources produce interference pattern of alternate bright and dark fringes at the eyepiece (or on the screen).

At first, to measure the fringe width (β), the distance between 15 fringes (say) is measured by moving the micrometer eyepiece across the optical bench and then the fringe width β is obtained by dividing the measured distance by 15.

Secondly, to measure the distance D, the distance between the eyepiece and the source is noted from the scale attached to the optical bench.

Finally, the distance 'd' between two virtual coherent sources A and B is measured by introducing a convex lens of small focal length between the eyepiece and the biprism. By moving the lens along the length of the optical bench, two positions L_1 and L_2 of the convex lens are obtained such that images of A and B are obtained in the plane of the cross-wires of the eyepiece. The distance between the images of A and B in each position of the lens is measured by the micrometer screw of the eyepiece.

● The proof of this equation is already given in Article 4.3.4

Let d_1 and d_2 be the separation between the two images (of A and B) as seen in the eyepiece for L_1 and L_2 positions of lens respectively (Fig. 12). We have,

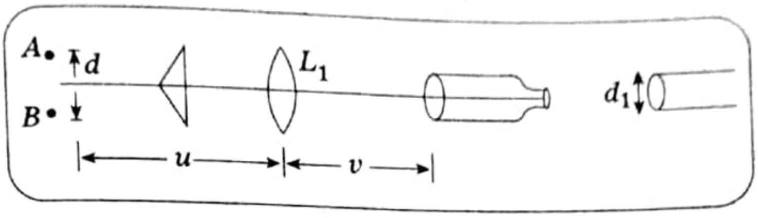


Fig. 12(a)

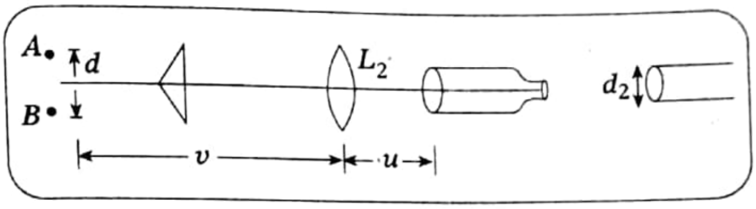


Fig. 12(b)

$$\frac{d_1}{d} = \frac{v}{u} \text{ (for the position } L_1\text{)} \qquad \cdots(4.24)$$

and
$$\frac{d_2}{d} = \frac{u}{v} \text{ (for the position } L_2\text{)} \qquad \cdots(4.25)$$

as the two positions of the lens are conjugate. Here, u and v are the distances of the slits and the eyepiece from the lens L_1 respectively for the first position and vice versa.

$$\therefore \qquad \frac{d_1 d_2}{d^2} = 1 \quad \text{or,} \quad d = \sqrt{d_1 d_2} \qquad \cdots(4.26)$$

Separate sets of reading of d_1 and d_2 are to be taken and the mean value of d is then calculated. Hence, by measuring β, d and D the wavelength of a given source of monochromatic light can be calculated from the equation,

$$\boldsymbol{\lambda = \frac{\beta d}{D}}$$

Alternative method for measuring *d*

If the small base angle (α) of the biprism is known, the total deviation of the two rays of light from the two faces EF and FG of the prism [Fig. 10] is

$$2\delta = 2(\mu - 1)\alpha \qquad \cdots(4.27)$$

[$\because$ in Fig 10, the deviation $\angle SFA (= \angle SFB)$, *i.e.* $\delta = (\mu - 1)\alpha$]

If x be the distance between the slit and biprism, we get from Fig. 10,

$$\frac{d}{x} = 2(\mu - 1)\alpha \quad \text{or,} \quad \boldsymbol{d = 2(\mu - 1)\alpha x} \qquad \cdots(4.28)$$

Therefore, d can be calculated and this helps to find the wavelength of unknown monochromatic source of light.

Problem 1

A biprism is placed at a distance of 5 cm in front of a narrow slit illuminated by sodium light and virtual images of the slit formed by the prism are 0.5 mm apart. Find the width of the fringes formed on a screen placed 75 cm in front of the biprism. The wavelength of light used is 5.89×10^{-5} cm. **[C.U. 1968]**

Solution Here, the wavelength $\lambda = 5.89 \times 10^{-5}$ cm

d = distance between the two coherent sources = 0.5 mm = 0.05 cm

D = distance between the slit and the screen = (5 + 75) = 80 cm

So, fringe width $\beta = \frac{D\lambda}{d} = \frac{80 \times 5.89 \times 10^{-5}}{0.05} = 0.094$ cm

Problem 2

Calculate the separation between the coherent sources formed by a prism, whose inclined faces make angles of 2° with its base. The slit source being 10 cm away from the biprism ($\mu = 1.50$). **[Delhi University, 1974, 1977]**

Solution Separation between two coherent sources $d = 2(\mu - 1)\alpha x$.

Here, μ = refractive index of the material of biprism = 1.5

α = base angle = $2^\circ = \frac{2 \times \pi}{180}$ radian = $\frac{\pi}{90}$ radian

x = distance between slit source and the biprism = 10 cm

$\therefore \quad d = \frac{2 \times (1.5 - 1) \times \pi \times 10}{90} = 0.35$ cm

Problem 3

Using sodium light with a Fresnel's biprism, the fringes were found to have a width of 0.0196 cm, when observed at a distance of 100 cm from the slit. When a convex lens was placed between the biprism and the observer to give an image of the source at 100 cm from the slit, the distance apart of the images was found to be 0.70 cm. Calculate the wavelength, given the distance from the slit to the lens is 30 cm. **[Lucknow University]**

Solution Here, d_1 = distance between two images of the slit as seen in eyepiece = 0.7 cm

β = fringe width = 0.0196 cm

D = distance between source and screen = 100 cm

u = distance between the source and convex lens = 30 cm

v = distance between the convex lens and screen = (100 − 30) = 70 cm

Now, d = distance between two coherent sources which is to be calculated.

$$\therefore \quad \frac{v}{u} = \frac{d_1}{d} \quad \text{or,} \quad d = d_1\frac{u}{v} = \frac{0.7 \times 30}{70} = 0.3 \text{ cm}$$

Now, the wavelength

$$\lambda = \frac{\beta d}{D} = \frac{0.0196 \times 0.3}{100} = 5880 \times 10^{-8} \text{ cm} = 5880 \text{ Å}$$

Problem 4

In an experiment with a biprism, a convex lens is kept between the eyepiece and the biprism. The distance between the source and the eyepiece is 100 cm. At two different positions of the lens, the distance between the images as seen in the eyepiece are 0.42 mm and 1.21 mm. If the wavelength of used light is 5892 Å, find the fringe width.

Solution Here, $D = 100 \text{ cm}, \lambda = 5892 \times 10^{-8} \text{ cm}$.

$$d_1 = 0.42 \text{ mm} = 0.042 \text{ cm}, \; d_2 = 1.21 \text{mm} = 0.121 \text{cm}$$

$$\therefore \quad d = \sqrt{d_1 d_2} = 0.0712 \text{ cm}$$

$$\therefore \quad \beta = \frac{D\lambda}{d} = \frac{100 \times 5892 \times 10^{-8}}{0.0712} = \frac{5892}{712} \times 10^{-2} = 0.08275 \text{ cm}$$

4.4.2. Displacement of Fringes by a Thin Plate (or Sheet)

Let A and B be two coherent virtual sources. Since the point O is equidistant from A and B, the central fringe is formed at O.

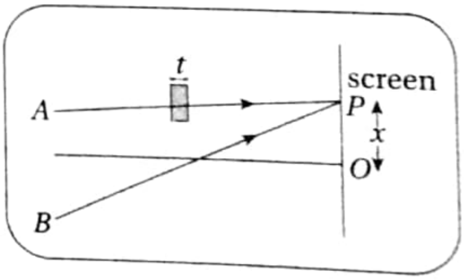


Fig. 13 ▷ Displacement of interference fringes due to thin plate

Now, if a thin sheet (or plate) of transparent material such as glass or mica G of refractive index μ and thickness 't' is introduced in the path of one of the interfering beams [Fig. 13], the entire fringe system (including the central maximum) is found to move through a constant distance towards the path of the beam in which the plate is introduced *i.e.* moves upward.

Let P be the shifted position of any order maximum (say central maximum). Let $OP = x$, the displaced position of the maximum after introduction of sheet.

Hence, the time taken by source A to reach at P

$$= \frac{AP - t}{c} + \frac{t}{c_m}; \quad [c \text{ and } c_m \text{ are the velocities of light in air and in the plate respectively.}]$$

$$= \frac{AP - t}{c} + \frac{t}{c/\mu} \quad \left[\because c_m = \frac{c}{\mu}\right]$$

$$= \frac{AP + (\mu - 1)t}{c}$$

Path AP is equivalent to air path $AP + (\mu - 1)t$.

So, *in the presence of a thin plate*, the path difference at P due to two interfering waves coming from A and B is given by

$$\Delta = BP - [AP + (\mu - 1)t]; \; [BP = \text{path } BP \text{ in air}]$$

$$= (BP - AP) - (\mu - 1)t \qquad \cdots(4.29)$$

But, *in absence of thin plate* the path difference of the waves to reach at P is

$$BP - AP = \frac{dx}{D} \qquad \cdots(4.30)$$

where d is the distance between two coherent sources A and B, and D is distance between coherent sources to screen.

So, from equations (4.29) and (4.30) we get,

$$\Delta = \frac{dx}{D} - (\mu - 1)t \qquad \cdots(4.31)$$

For nth maximum at P, the path difference $\Delta = n\lambda$ $\qquad \cdots(4.32)$

or, $\quad \frac{dx_n}{D} - (\mu - 1)t = n\lambda \quad$ [from equation (4.31)]

or, $$x_n = \frac{D}{d}[n\lambda + (\mu - 1)t] \qquad \cdots(4.33)$$

where, x_n denotes the position of nth maximum from central maximum.

In *the absence of sheet* (*i.e.*, $t = 0$), the *n th maximum* is obtained at a distance $\frac{nD\lambda}{d}$.

So, *the displacement of nth maximum due to introduction of the sheet of thickness t*

$$x_d = \frac{D}{d}[n\lambda + (\mu - 1)t] - \frac{nD\lambda}{d} = \frac{D}{d}(\mu - 1)t$$

$$\therefore \quad \boldsymbol{x_d = \frac{\beta}{\lambda}(\mu - 1)t} \quad \left[\because \beta = \frac{D\lambda}{d}\right] \qquad \cdots(4.34)$$

The calculation of this fringe shift helps us to find **either thickness of the plate** $t\left[= \frac{x_d\lambda}{\beta(\mu - 1)}\right]$ or **refractive index μ** of the sheet.

Problem 1

A mica sheet of refractive index 1.58 is introduced in one of the interfering beams and the central fringe gets shifted by 0.2 cm. The distance between the coherent sources is 0.1 cm and the screen is placed at a distance of 50 cm from the sources. Determine the thickness of the mica sheet. **[W.B.U.T 2000]**

Solution The thickness of the mica sheet

$$t = \frac{x_d\lambda}{\beta(\mu - 1)}, \text{ where fringe width } \beta = \frac{D\lambda}{d}$$

Here, $\mu = 1.58$, $x_d = 0.2\,\text{cm}$, $d = 0.1\,\text{cm}$, $D = 50\,\text{cm}$

Now $$t = \frac{x_d \lambda}{(D\lambda/d)(\mu - 1)} = \frac{x_d d}{D(\mu - 1)}$$

$$= \frac{0.2 \times 0.1}{50(1.58 - 1)} = \frac{0.02}{50 \times 0.58} = 6896 \times 10^{-7}\,\text{cm}$$

Problem 2

On placing a thin sheet of mica of thickness 12×10^{-5} cm in the path of one of the interfering beams in a biprism arrangement, it is seen that the central fringe shifts to a position, which is equal to the width of a bright fringe. Calculate the refractive index of mica ($\lambda = 6 \times 10^{-5}$ cm).

Solution Here, the displacement of fringe $x_d = \frac{\beta}{\lambda}(\mu - 1)t$

Here, $x_d = \beta$ (bright fringe width), $t = 12 \times 10^{-5}$ cm, $\lambda = 6 \times 10^{-5}$ cm

$\therefore$ $$\beta = \frac{\beta}{\lambda}(\mu - 1)t$$

or, $\lambda = (\mu - 1)t$ or, $$\mu = 1 + \frac{\lambda}{t} = 1 + \frac{6 \times 10^{-5}}{12 \times 10^{-5}} = 1.5$$

4.5. Interference Based on Amplitude Division

4.5.1. Stokes' Law : Change of Phase on Reflection

Stokes' law states that **the light wave reflecting from the surface of denser medium suffers a phase change of π (or a path difference of $\frac{\lambda}{2}$).**

We consider a wave AO of light falls on the denser medium from a rarer medium. Let, a = amplitude of incident light wave AO. If r and t are the reflection coefficient (fraction of unit amplitude reflected into rarer medium) and transmission coefficient (fraction of unit amplitude transmitted from rarer to denser medium) respectively, the amplitude of reflected wave OB is ar and that transmitted wave OC is at.

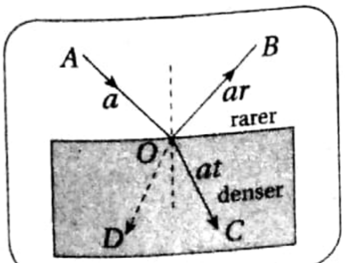


Fig. 14 (a) ▷ Reflection and refraction from rarer to denser medium

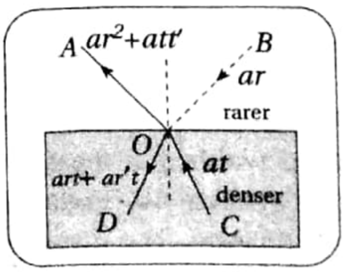


Fig. 14 (b) ▷ Reflection and refraction from denser to rarer medium

Now consider that the directions of reflected and transmitted wave are reversed. So, on reversing the reflected wave OB, we get reflected wave of amplitude ar^2 along OA and transmitted wave of amplitude art along OD. Now on reversing the refracted wave CO, we get reflected wave OD of amplitude $ar't$ and transmitted wave OA of amplitude att', where r' and t' are the fraction of unit amplitude reflected into denser medium and transmitted from denser to rarer medium respectively.

As the reversal of light must generate a wave of amplitude a along OA, there is no wave along OD.

$$\therefore \quad att' + ar^2 = a \qquad \cdots (4.35) \quad \text{and} \quad art + ar't = 0 \qquad (4.36)$$

From equation (4.35) we get, $\cdots (4.37)$

$$tt' = 1 - r^2$$

And from equation (4.36), we have $\cdots (4.38)$

$$r' = -r$$

The negative sign in equation (4.38) implies a displacement in opposite direction which is equivalent to a phase change of π or a path difference of $\frac{\lambda}{2}$.

Hence, a phase change of π will be introduced when the reflection takes place from the surface of a denser medium.

4.6. Lloyd's Mirror Experiment[●]

The experimental arrangement is seen in figure [Fig. 15]. We can prove that phase change of π will occur by reflection from the denser medium.

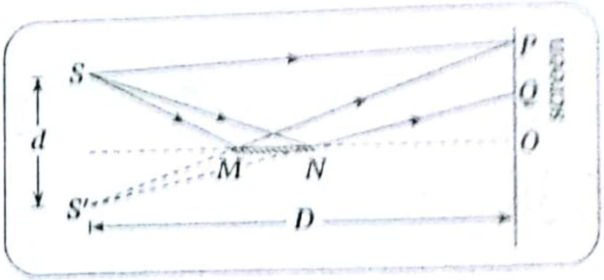


Fig. 15 ▷ Arrangement for Lloyd's mirror experiment

Here, two coherent sources are the monochromatic source S and its virtual image S' formed by the plane mirror MN. Here, the interference pattern is obtained in the region PQ of the screen. The expected position of central fringe lies at O. But, the point O is outside the region of interference and so the central fringe is not usually seen. To observe the central fringe (zero order fringe), it is necessary to move the screen to be nearly in contact with the end N of the mirror. This zero order fringe is found dark instead of bright. The only explanation of this fact is that one of the beams producing interference fringes has undergone a phase change of π. But the direct beam from S can not undergo any phase change. Therefore, only the reflected beam undergoes a phase change of π. So, this confirms experimentally that a light beam after reflection from an optically denser medium will undergo a phase change of π.

The fringe width β is measured with the help of the micrometer eyepiece. The distance 'd' between two coherent sources is measured by a convex lens, using displacement method. The distance D between the slit and screen is measured by metre scale. Hence, the wavelength of monochromatic light is calculated from $\lambda = \frac{\beta d}{D}$.

● Not included in the syllabus of W.B.U.T.

4.7. Interference in Thin Film

A thin film of any transparent substance, for example, oil spread over water surface (film of oil) when viewed in white light, brilliant colours are observed. *These brilliant colours are produced by the interference of light waves coming from the lower and upper surface of the film.*

We consider a plane parallel thin film of thickness t as seen in Fig. 16. Let the light be incident at B. A part of light is reflected towards BC and the other part BD is refracted into the film towards D. This second part BD is reflected and emerges from D. The reflected light DE is produced backward to meet BL at L.

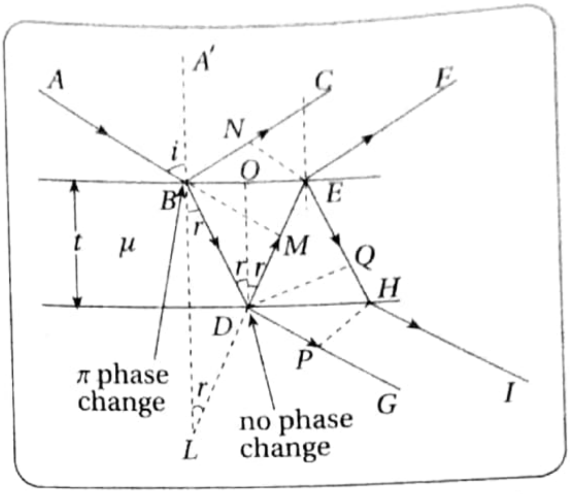


Fig. 16 ▷ Interference in thin film for reflected system

EN and BM are drawn perpendicular to BC and DE respectively. Now, at the upper surface of the film, the angle of incidence $\angle ABA' = i$ and the angle of refraction $\angle LBD = r$ [Fig. 16]. The ray of light will suffer similar reflections and refractions at points D, E and H etc.

Reflected system The reflected rays BC and EF will interfere in reflected system. The path difference between BC and EF is

$$\Delta' = \text{Path } (BD + DE) \text{ in film} - \text{path } BN \text{ in air}$$

[$\because$ B and E are very close, so $NC = EF$]

$$= \mu(BD + DM + ME) - BN$$
$$= \mu(LD + DM + ME) - BN$$
$$= \mu(LM + ME) - BN$$
$$= \mu LM \quad [\because \mu ME = BN]$$
$$= \mu LB \cos r = 2\mu t \cos r \quad \left[\because LB = 2t \;\&\; \cos r = \frac{LM}{LB}\right] \qquad \cdots(4.39)$$

But when the light is reflected from the surface of denser medium, there is a phase change of π. So, there is an additional path difference of $\frac{\lambda}{2}$ is introduced for reflection from the surface of denser medium *i.e.* at B.

Hence, the effective path difference between the two reflected rays

$$\Delta = 2\mu t \cos r \pm \frac{\lambda}{2} \qquad \cdots(4.40)$$

Hence, the film will appear bright (*i.e.* for constructive interference) when

$$2\mu t \cos r \pm \frac{\lambda}{2} = n\lambda$$

[$\because$ Δ = even integral multiple of $\frac{\lambda}{2}$ for constructive interference] or, $$2\mu t\cos r = (2n \pm 1)\frac{\lambda}{2}, \text{ where } n = 0, 1, 2, \cdots \quad \cdots(4.41)$$

The film will appear dark (*i.e.* for destructive interference) in the reflected light when

$$2\mu t\cos r \pm \frac{\lambda}{2} = (2n \pm 1)\frac{\lambda}{2}$$

$$\left[\because \text{ for destructive interference } \Delta = (2n \pm 1)\frac{\lambda}{2}\right]$$

or, $$2\mu t\cos r = n\lambda, \text{ where } n = 0, 1, 2, \cdots \quad \cdots(4.42)$$

In case, if film is of negligible thickness *i.e.* $t \ll \lambda$, the net path difference will be $\frac{\lambda}{2}$ and therefore film will appear dark.

Transmitted system Here, the transmitted rays DG and HI which are derived from the same point source A will interfere [Fig. 17]. As the two rays DG and HI come after refraction at rarer medium (air) at points D and E, the additional path difference between the two rays is zero.

Now, the path difference Δ between the transmitted rays DG and HI can be calculated in similar way for reflected rays as

$$\Delta = \mu(DE + EH) - DP$$

$$= \mu(DE + EQ + QH) - \mu QH$$

$$\left[\because \mu = \frac{\sin i}{\sin r} = \frac{DP/DH}{QH/DH} = \frac{DP}{QH}\right]$$

$$= \mu(DE + EQ)$$

$$= \mu(RE + EQ) = \mu\, QR$$

$$= \mu RD\cos r$$

$$= 2\mu t\cos r \quad \cdots(4.43)$$

$$\left[\because RD = 2t \;\&\; \cos r = \frac{QR}{RD}\right]$$

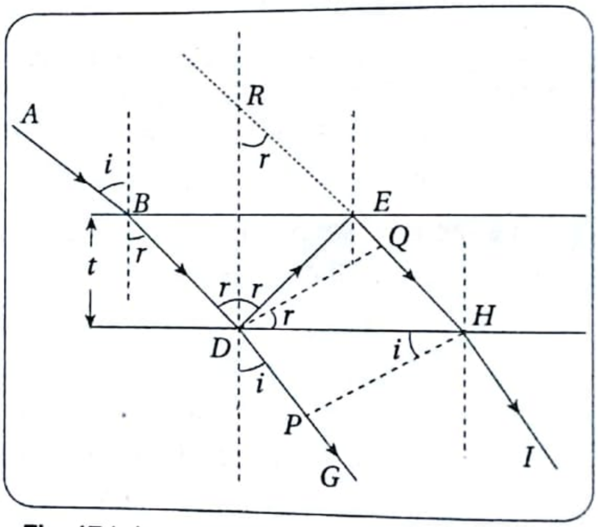


Fig. 17 ▷ Interference in thin film for transmitted system

Therefore, the film will appear bright, when

$$2\mu t\cos r = n\lambda \quad [n = 1, 2, 3, \cdots] \quad \cdots(4.44)$$

And the film will appear dark when

$$2\mu t\cos r = (2n \pm 1)\frac{\lambda}{2} \quad [n = 1, 2, \cdots] \quad \cdots(4.45)$$

Thus, the condition for bright or dark films for transmitted rays are just reverse of the conditions for reflected light.

4.8. Interference in Wedge Shaped Film

A thin wedge shaped film is bounded by two plane surfaces PQ and RQ inclined at an angle θ. If the film is illuminated by a monochromatic light (say AB), then the reflected ray (BF) from the front surface and the other emergent ray (DF) reflected from the back surface (RQ) of the film will interfere [Fig. 18]. As these two rays are derived from the same ray AB, they are coherent and hence they will produce interference pattern.

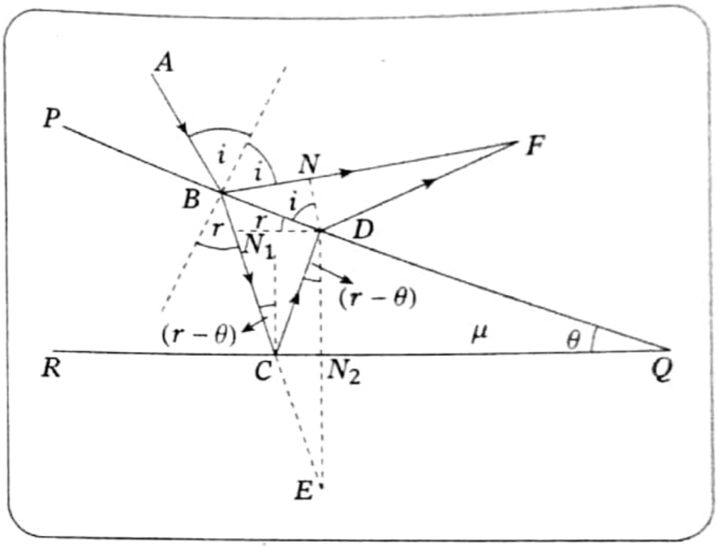


Fig. 18 ▷ Interference in wedge shaped film

Now to find the condition for bright or dark fringes, we shall have to find the path difference between the rays. Let μ be the refractive index of the material of the film.

Now, the path difference between the two rays BF and DF,

$$d = \text{path } (BC + CD) \text{ in the film} - \text{path } BN \text{ in air}$$

[$\because$ B and D are very close, so $NF = DF$]

$$= \mu(BN_1 + N_1C + CD) - BN$$

$$= \mu(N_1C + CD) \quad [\because BN = \mu BN_1] \qquad \cdots(4.46)$$

Special Note :

$\angle NDB = i$ and $\angle BDN_1 = r$

$\therefore$ $\sin i = \mu \sin r$ or, $\dfrac{BN}{BD} = \mu \dfrac{BN_1}{BD}$, then $BN = \mu BN_1$

Now a perpendicular DN_2 is drawn on RQ. Let DN_2 is produced and meets with produced BC at point E. Then from geometry, it can be proved that

$$\angle CDN_2 = (r - \theta),$$

$$DN_2 = N_2E = t \text{ (say), thickness of the wedge shaped film at } D$$

and $$CD = CE$$

$$\therefore \quad d = \mu(N_1C + CE) = \mu N_1E = 2\mu t \cos(r - \theta) \qquad \cdots(4.47)$$

Due to reflection from the denser medium *i.e.* at B, an additional path difference of $\frac{\lambda}{2}$ is to be introduced.

or, $$\mathbf{2\mu t\cos(r-\theta) = (2n+1)\frac{\lambda}{2}}, \text{ where } n = 0, 1, 2, 3\cdots$$

Similarly, for **destructive interference** at F,

$$2\mu t\cos(r-\theta)+\frac{\lambda}{2} = \text{odd multiple of } \frac{\lambda}{2}$$

or, $$2\mu t\cos(r-\theta) = \text{even multiple of } \frac{\lambda}{2}$$

or, $$\mathbf{2\mu t\cos(r-\theta) = n\lambda}, \text{ where } n = 0, 1, 2, 3\cdots \quad \cdots(4.50)$$

[It is to be noted that for a parallel film, $\theta = 0$].

Fringe width

If x_n be the distance of nth order bright fringe from the thin edge of the wedge shaped film of thickness t,

$$\frac{t}{x_n} = \tan\theta \quad i.e. \quad t = x_n\tan\theta \quad \cdots(4.51)$$

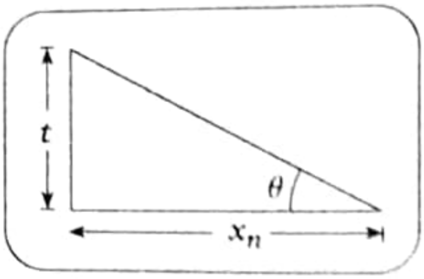


Fig. 19 ▷ Position of n th order bright fringe in a wedge shaped film

For normal incidence ($i.e.$ $r = 0$), we have from equation (4.49)

$$2\mu t\cos\theta = (2n+1)\frac{\lambda}{2}$$

or, $$2\mu x_n\tan\theta\cos\theta = (2n+1)\frac{\lambda}{2} \quad \text{or,} \quad 2\mu x_n\sin\theta = (2n+1)\frac{\lambda}{2}$$

or, $$x_n = (2n+1)\frac{\lambda}{4\mu\sin\theta} \quad \cdots(4.52)$$

So, fringe spacing $$\boldsymbol{\beta = x_{n+1}-x_n = \frac{\lambda}{2\mu\sin\theta}} \quad \cdots(4.53)$$

For small value of θ, $$\boldsymbol{\beta = \frac{\lambda}{2\mu\theta}} \quad \cdots(4.54)$$

This equation is also true for *dark fringes.*

Conclusions

① When two surface are parallel to each other and each of thickness t, then $\theta = 0$. Then the condition for constructive interference is

$$2\mu t\cos r = (2n+1)\frac{\lambda}{2} \quad \cdots(4.55)$$

and the condition for destructive interference $2\mu t\cos r = n\lambda$ $\cdots(4.56)$

② When the surface is very thin *i.e.* t is almost zero, then the path difference between the two rays $= \frac{\lambda}{2}$.

So, the film surface would appear perfectly dark even with white light.

③ When white light is incident on the film, λ, r, μ will be different for different colours of light. So, at a particular point all the wavelengths may not satisfy the condition for maxima and minima. Hence, some of the colours may be absent in the reflected ray from the coloured thin film.

Transmitted system

A similar interference pattern will be observed for the transmitted rays from the back surface RQ [Fig. 18]. For transmitted system it can be easily seen that

$$2\mu t \cos r = n\lambda \text{ (for bright fringe)} \qquad \cdots(4.57)$$

and $$2\mu t \cos r = (2n+1)\frac{\lambda}{2} \text{ (for dark fringe)} \qquad \cdots(4.58)$$

So, we can conclude from equation (4.49), (4.50), (4.57) and (4.58) that fringes of the reflected and transmitted system are complementary to each other.

Problem 1

A wedge shaped film of air between two glass plates gives equally spaced bright fringes for normal incidence of sodium light, of wavelength 589 nm. The two consecutive bright fringes are 0.23 nm apart from each other. When monochromatic light of another wavelength is used the fringes are 0.25 nm apart from each other. Calculate the wavelength of second source of light.

Solution We know, fringe width $\beta = \frac{\lambda}{2\mu\theta}$

Let, β_1 and β_2 be the fringe widths for two monochromatic lights of wavelength λ_1 and λ_2 respectively.

$$\therefore \quad \beta_1 = \frac{\lambda_1}{2\mu\theta} \quad \text{and} \quad \beta_2 = \frac{\lambda_2}{2\mu\theta}$$

$$\therefore \quad \frac{\lambda_2}{\lambda_1} = \frac{\beta_2}{\beta_1} \quad \text{or,} \quad \lambda_2 = \lambda_1 \frac{\beta_2}{\beta_1}$$

Here, $\beta_1 = 0.23$ nm, $\beta_2 = 0.25$ nm and $\lambda_1 = 589$nm

$$\therefore \quad \lambda_2 = 589 \times \frac{0.25}{0.23} = 640.21\text{nm}$$

Problem 2

A wedge shaped film of air between two glass plates is illuminated normally by sodium light (λ = 5893 Å). Dark and bright fringes are formed, with 8 of each per cm in length of the wedge, measured normal to the edge in contact. Calculate the angle of the wedge.

Solution Fringe width $\beta = \dfrac{\lambda}{2\mu\theta}$

Here, $\mu = 1$, $\beta = \frac{1}{8}$ cm, $\lambda = 5893 \times 10^{-8}$ cm

$$\therefore \quad \theta = \frac{\lambda}{2\beta} = \frac{5893 \times 10^{-8}}{2 \times \frac{1}{8}} \text{ rad}$$

or, $\theta = 23572 \times 10^{-8}$ rad $= 2.3572 \times 10^{-4}$ rad

4.9. Newton's Rings

When the convex surface of a plano-convex lens (L) with long focal length is placed on a plane glass plate (G) [Fig. 20], a very thin air film of varying thickness is formed between the lower surface of the lens and the upper surface of the glass plate. The thickness of the air film increases from their point of contact towards the periphery of the lens. Moreover, *the points of constant thickness in this air film will lie on circles with point of contact (O) as the centre due to the circular symmetry of the air film.*

Suppose, light from the monochromatic source S is made parallel by placing it at the focus of lens L_1 [Fig. 20]. The parallel rays are reflected by a glass plate P (inclined at angle 45° to the horizontal plane) and so incident normally on the air film between the plano-convex lens L and the glass plate G viewed as shown in Fig. 21. A part of the incident ray will be reflected at B. The other part enters into the air film and gets reflected at the glass plate at the point C and finally emerges from the lens.

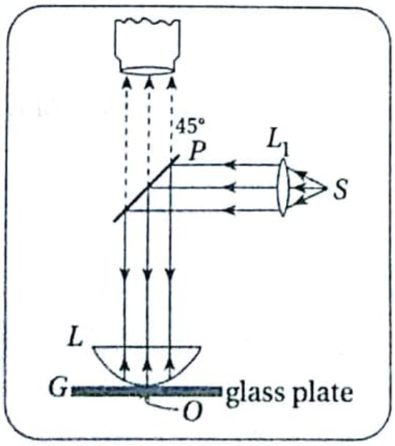


Fig. 20 ▷ Experimental set up for Newton's rings

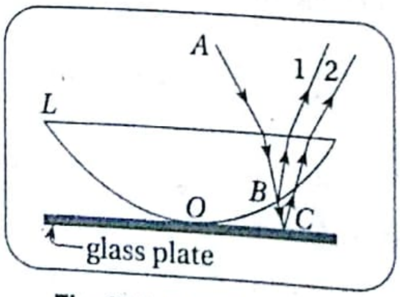


Fig. 21 ▷ Ray diagram of interference beams coming from lower & upper surface of thin film

Newton's rings for reflected light Here, the incident ray is reflected from the upper surface and lower surface of the air film (*i.e.* from the convex side of the lens and from the glass plate). These two reflected rays interfere and produce alternate bright and dark concentric rings.

Since there will be an additional path difference of $\pm\frac{\lambda}{2}$ due to reflection from the surface of the glass plate (denser medium), the path difference (d) for normal incidence of the two reflected interfering waves at a distance r_n from O and height 't' [Fig. 22] from the glass plate is,

$$d = 2\mu t \pm \frac{\lambda}{2} \quad \cdots (4.59)$$

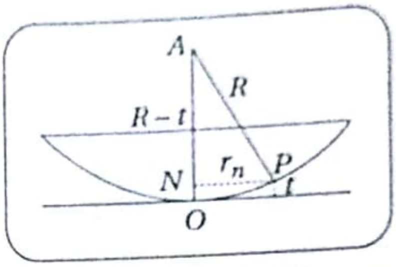


Fig. 22 ▷ Geometrical diagram for the measurement of thickness of film

where μ is refractive index of the medium between the lens and the plate and $\cos r = 1$ for normal incidence.

Now at the point of contact $t = 0$, the path difference is $\frac{\lambda}{2}$. Hence, **the central spot is dark.**

So, for **bright rings,**

$$2\mu t \pm \frac{\lambda}{2} = \text{even multiple of } \frac{\lambda}{2} \quad \text{or,} \quad 2\mu t = \text{odd multiple of } \frac{\lambda}{2}$$

or, $$\mathbf{2\mu t = (2n+1)\frac{\lambda}{2}}, \text{ where } n = 1, 2, 3 \cdots \quad \cdots (4.60)$$

and for **dark rings,**

$$2\mu t \pm \frac{\lambda}{2} = \text{odd multiple of } \frac{\lambda}{2} \quad \text{or,} \quad 2\mu t = \text{even multiple of } \frac{\lambda}{2}$$

or, $$\mathbf{2\mu t = n\lambda}, \text{ where } n = 0, 1, 2, 3, \cdots \quad \cdots (4.61)$$

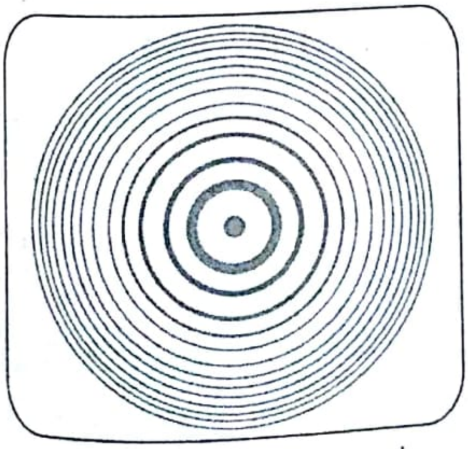


Fig. 23 (a) ▷ Newton's rings in reflected light

Fig. 23 (b) ▷ Newton's rings in transmitted light

Again fringe of given order (n) will be along the loci of points of film of equal thickness (t) and so the fringe will be circular. If the point P fulfills the condition of brightness (or darkness), then all points on the circumference of a circle of radius $r_n (= PN)$ will be bright (or dark) ring.

Fig. 22 shows that the thickness t of the film at any point is related to the radius of the circular ring NP on which it lies and the radius of curvature R of the lens.

Now from geometry, We have

$$R^2 = r_n^2 + (R - t)^2 \text{ or, } r_n^2 = 2Rt, \text{ as } t \text{ is very small in comparison to } R.$$

or, $$t = \frac{r_n^2}{2R} \qquad \cdots (4.62)$$

Substituting the value of t in equations (4.60) and (4.61), we get the radius of nth ring as given below

$$2\mu\left(\frac{r_n^2}{2R}\right) = (2n+1)\frac{\lambda}{2} \quad \text{(for bright rings)}$$

$$\therefore \quad r_n^2 = R(2n+1)\frac{\lambda}{2\mu} \qquad \cdots (4.63)$$

and $$2\mu\left(\frac{r_n^2}{2R}\right) = n\lambda \quad \text{(for dark rings)}$$

$$\therefore \quad r_n^2 = R\frac{n\lambda}{\mu} \qquad \cdots (4.64)$$

Hence, the diameter of nth ring

$$D_n^2 = 2R(2n+1)\frac{\lambda}{\mu} \quad \text{(for bright rings)} \qquad \cdots (4.65)$$

and $$D_n^2 = 4R\frac{n\lambda}{\mu} \quad \text{(for dark rings)} \qquad \cdots (4.66)$$

So the diameter of $(n+m)$th ring for air film $(\mu = 1)$, is given by

$$D_{n+m}^2 = 2R(2n+2m+1)\lambda \quad \text{(for bright rings)} \qquad \cdots (4.67)$$

$$D_{n+m}^2 = 4R(n+m)\lambda \quad \text{(for dark rings)} \qquad \cdots (4.68)$$

Subtracting either (4.65) from equation (4.67) or equation (4.66) from equation (4.68), for air film, we have

$$D_{n+m}^2 - D_n^2 = 4Rm\lambda \qquad \cdots (4.69)$$

or, $$\lambda = \frac{D_{n+m}^2 - D_n^2}{4mR} \qquad \cdots (4.70)$$ ●

This equation will give the **wavelength of unknown light** by using Newton's rings method.

● In W.B.U.T syllabus, deduction is not required.

Determination of unknown wavelength from the graph If we draw a graph between the order number of rings along X-axis and the corresponding square of diameters along Y-axis, the nature of the graph will be a straight line passing through the origin [Fig. 24].

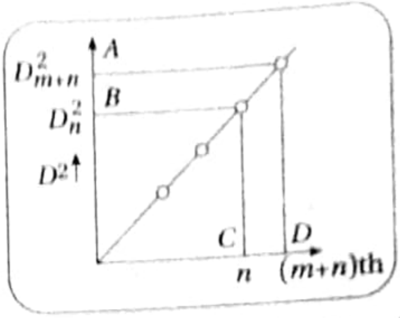


Fig. 24 ▷ The graph between D^2 and n (order number of Newton's rings)

From the slope (p) of the above, we can find the wavelength of unknown light as given bellow :

$$\lambda = \frac{1}{4R}p \text{ where}$$

$$p = \frac{AB}{CD} = \frac{D^2_{m+n} - D^2_n}{m}$$

4.9.1. Refractive Index of a Liquid by Newton's Rings Method

At first, we will perform the above experiment taking air film in between the lens and glass plate. Now, the diameter of the $(n+m)$th and nth order dark rings are to be determined. So from equation (4.69) we can write

$$D^2_{n+m} - D^2_n = 4m\lambda R \quad \cdots (4.71)$$

Now a few drops of the liquid, whose refractive index is to be determined is poured into the container without disturbing the entire arrangement [Fig. 25]. Again, the diameter of nth and $(n+m)$th order dark fringes are determined for the liquid film and hence we may write,

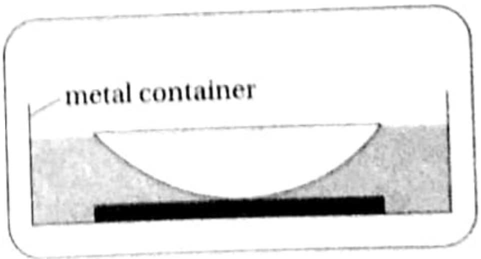


Fig. 25 ▷ Position of a unknown liquid in place of air film

$$D'^2_{n+m} - D'^2_n = \frac{4m\lambda R}{\mu} \quad \cdots (4.72)$$

Hence, from equations (4.71) and (4.72), the refractive index (μ) of unknown liquid is given by,

$$\mu = \frac{D^2_{n+m} - D^2_n}{D'^2_{n+m} - D'^2_n} \quad \cdots (4.73)$$

4.9.2. Fringe Width β

If D_{n+1} and D_n are the diameters of two successive rings, the fringe width

$$\beta = \frac{D_{n+1} - D_n}{2} \quad \cdots (4.74)$$

But, we have from equation (4.69) for $m = 1$,

$$D^2_{n+1} - D^2_n = 4R\lambda$$

or, $$D_{n+1} - D_n = \frac{4R\lambda}{D_{n+1} + D_n} = \frac{4R\lambda}{2D_n} \qquad [\because D_{n+1} \simeq D_n]$$

$$\therefore \quad \beta = \frac{D_{n+1} - D_n}{2} = \frac{R\lambda}{D_n} \qquad \cdots(4.75)$$

So, the fringe width β decreases with the increase of diameter (D_n) of the ring.

4.9.3. Newton's Rings for Transmitted Light

For transmitted light [Fig. 26],

$$2\mu t = n\lambda \quad \text{(for bright rings)} \qquad \cdots(4.76)$$

and $$2\mu t = (2n-1)\frac{\lambda}{2} \quad \text{(for dark rings)} \qquad \cdots(4.77)$$

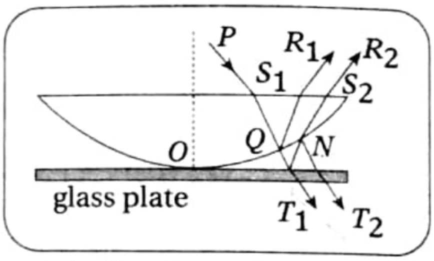


Fig. 26 ▷ Newton's rings for transmitted light

So, the above equations written for the conditions of maxima and minima in refracted light are just reverse of conditions for reflected light.

Hence, for *bright rings*

$$2\mu\left(\frac{r_n^2}{2R}\right) = n\lambda,\ n = 0, 1, 2, \cdots \qquad \left[\because t = \frac{r_n^2}{2R}\right]$$

or, $$\boldsymbol{r_n^2 = \frac{n\lambda R}{\mu}} \qquad \cdots(4.78)$$

Similarly, for *dark rings* $2\mu\left(\frac{r_n^2}{2R}\right) = (2n-1)\frac{\lambda}{2}; \quad n = 1, 2, 3, \cdots$

or, $$\boldsymbol{r_n^2 = (2n-1)\frac{\lambda R}{2\mu}} \qquad \cdots(4.79)$$

So, when $n = 0$ for bright rings, the radius of the ring $r = 0$.

Hence, the **central ring is bright** for transmitted light.

Thus, by comparing the conditions of Newton's rings for reflected and transmitted system, **we can conclude that the rings obtained in reflected system are exactly complementary to those seen with transmitted system.**

But the Newton's rings for transmitted system are not clearly distinct as seen with reflected system. This is observed as the dark rings for transmitted system are not completely dark. Due to this reason, **Newton's rings are usually studied for reflected light.**

4.9.4. Newton's Rings with White Light

If we use white light as incident light then the *central spot will be black surrounded by a few* (8-10) *coloured rings and beyond this the general illuminations due to overlapping of different coloured rings will be observed.*

Problem 1

In Newton's ring experiment, the diameters of 5 th and 15th dark rings are 0.336 cm and 0.590 cm respectively. If the radius of curvature of the curved surface of the plano-convex lens used be 100 cm, find the wavelength of light used. **[C.U. 1971]**

Solution If d_n and d_{n+m} are the diameters of n th and $(n+m)$ th rings, then

$$\lambda = \frac{d_{n+m}^2 - d_n^2}{4mR} = \frac{(0.590)^2 - (0.336)^2}{4 \times (15-5) \times 100} = 5880 \times 10^{-8}\,\text{cm} = 5880\,\text{Å}$$

Problem 2

In Newton's ring experiment a source of light having two wavelengths 6000 Å and 4500 Å is used. It is found that n th dark ring due to 6000 Å coincides with $(n+1)$ th dark ring due to 4500 Å. Calculate the radii of n th ring due to 6000 Å and 4500 Å if radii of curvature of plano-convex lens is 100 cm.

Solution The square of the diameter of n th order dark ring

$$D_n^2 = 4n\lambda R, \text{ where } R = \text{radius of plano-convex lens.}$$

From the condition, we can write

$$(4n)6000R = 4(n+1)4500R \quad \text{or,} \quad n = 3$$

$\therefore$ The radius of 3rd order ring due to 6000Å light

$$r_3 = \sqrt{n\lambda R} = \sqrt{3 \times 6000 \times 10^{-8} \times 100} = 0.13\,\text{cm}$$

Similarly, the radius of 3rd order ring due to 4500Å light

$$r'_3 = \sqrt{3 \times 4000 \times 10^{-8} \times 100}\,\text{cm} = 0.10\,\text{cm}.$$

Problem 3

Newton's rings are observed with reflected light of wavelength 6000 Å. The diameter of the 10 th dark ring is 0.52 cm. Calculate the radius of curvature of the lens and the fringe width.

Solution The wavelength $\lambda = \dfrac{d_{n+m}^2 - d_n^2}{4mR}$

Here $\quad m = 5,\ d_{n+m} = (2 \times 0.6)\ \text{cm},\ d_n = (2 \times 0.4)\ \text{cm}$

$R = 10\ \text{m} = 1000\ \text{cm}$

$$\therefore \quad \lambda = \frac{(2 \times 0.6)^2 - (2 \times 0.4)^2}{4 \times 5 \times 1000} = 4 \times 10^{-5} = 4000 \times 10^{-8}\ \text{cm} = 4000\ \text{Å}$$

Problem 5

In a Newton's rings experiment the diameter of the 12 th ring changes from 1.50 cm to 1.35 cm, when a liquid is introduced between the lens and the plate. Calculate the refractive index of liquid. [Delhi University 1990]

Solution Now for air medium, the diameter of the 12 th ring

$$D_1^2 = 4n\lambda R \qquad \cdots(1)$$

Again for liquid medium, the diameter of the 12 th ring

$$D_2^2 = \frac{4n\lambda R}{\mu} \qquad \cdots(2)$$

Dividing (1) by (2)

$$\frac{D_1^2}{D_2^2} = \mu \quad \text{or,} \quad \mu = \left(\frac{1.50}{1.35}\right)^2 \quad [\because D_1 = 1.50\ \text{cm and } D_2 = 1.35\ \text{cm}]$$

$$= 1.235$$

Problem 6

In Newton's rings experiment, the rings are observed due to reflected light of wavelength 6000 Å. The diameter of the 8 th dark ring is 0.6 cm. Find the thickness of the air film.

Solution Let 't' be the thickness of the air film.

So, the path difference between the two reflected interfering waves for normal incidence for dark rings,

$$2t = n\lambda \quad \text{or,} \quad t = \frac{n\lambda}{2}$$

$$\therefore \quad t = \frac{8 \times 6000 \times 10^{-8}}{2} = 24 \times 10^3 \times 10^{-8} = 24 \times 10^{-5}\ \text{cm}$$