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Appendix A
Construction of Boxplots

The upper and lower limits of the central box are defined using either quartiles or hinges. These definitions are clarified below. Then the influence of each definition on the position of the whiskers is demonstrated. Definitions used by commercial software packages are listed, including one non-conventional form called a "box graph".

Quartiles

Quartiles are the 25th, 50th and 75th percentiles of a data set, as defined in chapter 1. Consider a data set X_i, i=1,...n. Computation of percentiles follows the equation

$$\boxed{p_j = X_{(n+1)\bullet j}}$$

where n is the sample size of X_i,

j is the fraction of data less than or equal to the percentile value (for the 3 quartiles, j= .25, .50, and .75).

Non-integer values of $(n+1)\bullet j$ imply linear interpolation between adjacent values of X. Computation of quartiles for two small example data sets is illustrated in Table 1.

Hinges

Tukey (1977) used values for the ends of the box which, along with the median, divided the data into four equal parts. These "fourths" or "hinges" are defined as:

Lower hinge h_L = median of all observations less than or equal to the sample median.

Upper hinge h_U = median of all observations equal to or greater than the overall sample median.

They may also be defined as:

$$\text{Lower hinge } h_L = \quad X_L, \text{ where } L = \frac{\text{integer}\,[\,(n+3)/2\,]}{2}, \text{ and}$$

$$\text{Upper hinge } h_U = \quad X_U, \text{ where } U = (n+1) - L.$$

where "integer [ ]" is the integer portion of the number in brackets. For example, integer [5.7] = 5. Again, non-integer values of L and U imply interpolation. With hinges, however, this will always be halfway between adjacent data points. Therefore, hinges are always either data values themselves, or averages of two data points, and so are easier to compute by hand than are percentiles. Hinges will generally be similar to quartiles for large (n> 30) sample sizes. For smaller data sets, differences will be more apparent. For example, when n=12 the lower hinge is halfway between the 3rd and 4th data points, while the lower quartile is one-quarter of the way between the two points (see Table 1) . Both measures split the data into one-fourth below and three fourths above their value. Either are acceptable for use in boxplots.

<u>Table A1</u>

A. For the following data X_i of sample size n=11:

2 3 5 45 46 47 48 50 90 151 208

$p_{.25}$ = lower quartile = $X_{(n+1)\bullet.25}$ $= X_3$ = 5.
$p_{.75}$ = upper quartile = $X_{(n+1)\bullet.75}$ $= X_9$ = 90.
$p_{.50}$ = median = $X_{(n+1)\bullet.50}$ $= X_6$ = 47.
h_l = lower hinge = median [2 3 5 45 46 47] = 25.
h_u = upper hinge = median [47 48 50 90 151 208] = 70.

B. For sample size n=12, and data X_i, i=1,...n equal to:

2 3 5 45 46 47 48 49 50 90 151 208

$p_{.25}$ = lower quartile = $X_{(n+1)\bullet.25}$ $= X_{3.25} = X_3 + 0.25\bullet(X_4 - X_3)$ = 15.
$p_{.75}$ = upper quartile = $X_{(n+1)\bullet.75}$ $= X_{9.75} = X_9 + 0.75\bullet(X_{10} - X_9)$ = 80.
$p_{.50}$ = median = $X_{(n+1)\bullet.50}$ $= X_{6.5} = X_6 + 0.50\bullet(X_7 - X_6)$ = 47.5.
h_l = lower hinge = median [2 3 5 45 46 47] = 25.
h_u = upper hinge = median [48 49 50 90 151 208] = 70.

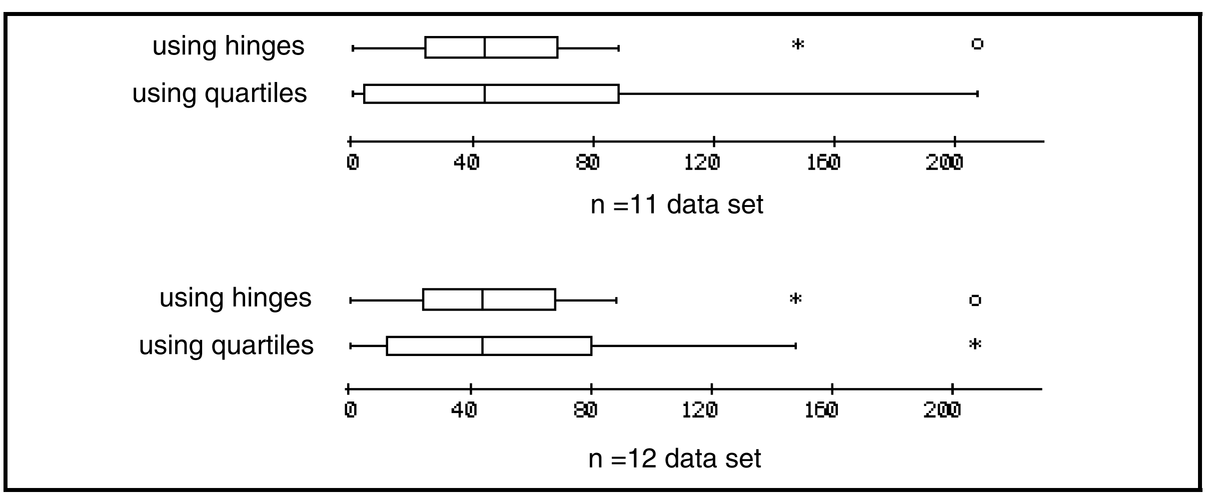


Figure A1. Boxplots for the Table A1 data

Figure A1 shows standard boxplots for the Table 1 data using both percentiles and hinges. Data in Table 1 were designed to maximize differences between the two measures. Real data, and larger sample sizes, will evidence much smaller differences. Note that the definitions of the box boundaries directly affect whisker lengths, and also determines which data are plotted as "outside" values.

It would be ideal if all software used the same conventions for drawing boxplots. However, that has not happened. Software written by developers who stick to the original definitions prefer hinges; those who want box boundaries to agree with tabled percentiles use quartiles. The Table 1 data can be used to determine which convention is used to produce boxplots.

<u>Non-conventional definitions</u>

Other statistical software use another (non-conventional) value for the box boundaries (Frigge and others, 1989). They use the next highest data value for the lower box boundary whenever n/4 is not an integer. This avoids all interpolation. Note that n, not n+1, is used.

StatView uses a percentile-type boxplot similar to the truncated boxplot, except that the upper and lower 10 percent of data are plotted as individual points. The weakness of this scheme is that 10 percent of the data will always be plotted individually at each end of the plot, and so it is less effective for defining and emphasizing unusual values. Also important is that StatView uses yet another definition for the box boundaries, $X_{(n+2)\bullet j}$, in calculating the quartiles. This non-conventional boxplot was called a "box graph" by Cleveland (1985).

Therefore some statistical software will produce boxes differing from conventional boxplots, particularly for small data sets.

<u>Boxplots for Censored Data</u>

Data sets whose values include some observations known only to be below (or above) a limit or threshold can also be effectively displayed by boxplots. First set all values below the threshold to some value less than (not equal to) the reporting limit. The actual value is not important, and could be 0, one-half the reporting limit, etc. Produce the boxplot. Then draw a line across the graph at the value of the threshold, and erase all lines below this value from the graph.

This procedure was used for data in figure A2. If less than 25 percent of the data are below the threshold, this procedure will affect at most only the lower whisker (as in the Hoover Dam through Morelos Dam boxplots). If between 25 and 75 percent are below the threshold, the box will be partially hidden below the threshold (as in the CO-UT Line and Cisco boxes). If more than 75 percent of the data are below the threshold, part of the upper whisker and outside values will be visible above the threshold, as in the Lees Ferry box. In each case, these boxplots accurately and fairly illustrate both the distribution of data above the threshold, and the percentage of data below the threshold.

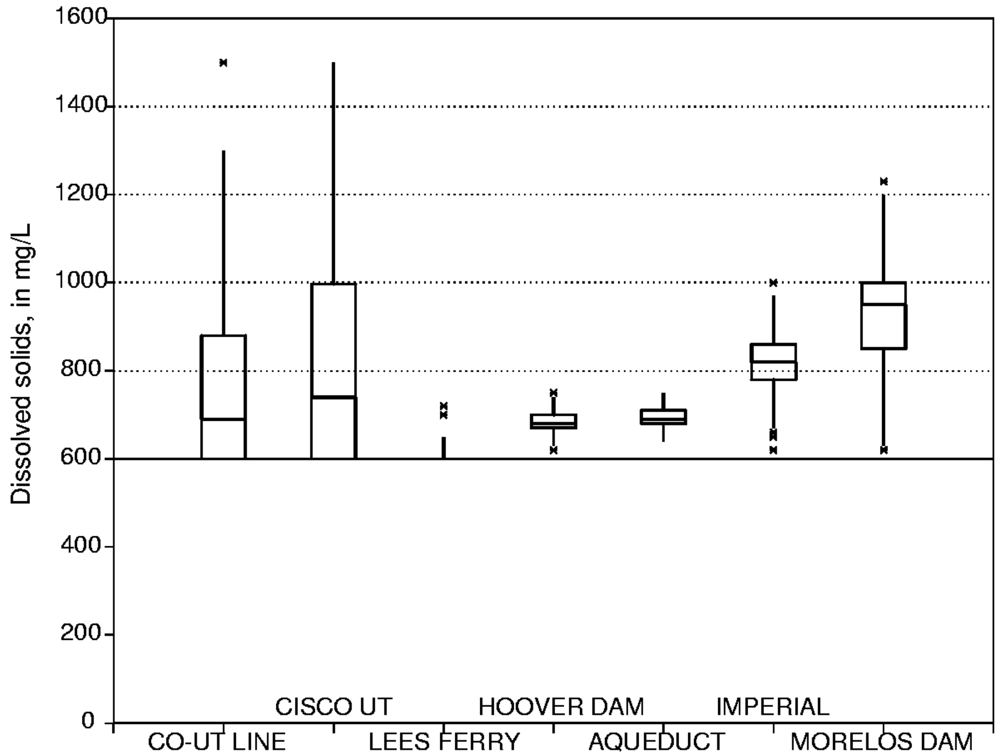


Figure A2. Dissolved solids concentrations along the Colorado River, artificially censored at a threshold of 600 mg/L.

A second alternative for boxplots of censored data is to estimate the percentiles falling below the threshold, and drawing dashed portions of the box below the threshold using these estimates. Helsel and Cohn (1988) have compared methods for estimating these percentiles. When multiple thresholds occur, such as thresholds which have changed over time or between laboratories, a solid line can be drawn across the plot at the highest threshold. Portions of the boxes above the highest threshold will be correct as long as each censored observation is assigned some value below its threshold. Quartiles falling below the highest threshold should be determined by using the methods recommended by Helsel and Cohn (1988). All lines below the highest threshold are only estimates, and should be drawn as dashed lines on the plot.

Displaying confidence intervals

As an aid for displaying whether two groups of data have different medians, confidence intervals for the median as defined in chapter 3 can be added to boxplots. When boxplots are placed side by side, their medians are significantly different if the confidence intervals do not overlap. Three methods of displaying these intervals are shown in figure A3. In the first method (A), the box is "notched" at both upper and lower limits, making the box narrower for all values within the interval. In the second (B), parentheses are drawn within the box at each limit. Shading is used in (C) to illustrate interval width. If displaying differences in medians is not of primary interest, these methods add visual confusion to boxplots and are probably best avoided. Confusion is compounded when the interval width falls beyond the 25th or 75th percentiles. Of the three, shading seems the easiest to visualize and least confusing.

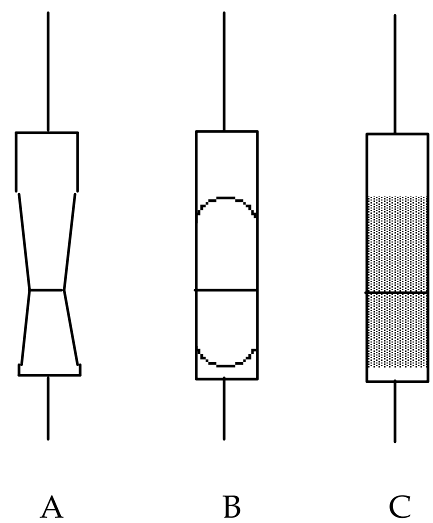


Figure A3. Methods for displaying confidence interval of median using a boxplot.
A. Notched boxplots B. Parentheses C. Shaded boxplot

Appendix B
Tables

Table B1. Cunnane plotting positions for sample sizes n = 1 to 20

	i																			
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
N= 5	.12	.31	.50	.69	.88															
N= 6	.10	.26	.42	.58	.74	.90														
N= 7	.08	.22	.36	.50	.64	.78	.92													
N= 8	.07	.20	.32	.44	.56	.68	.80	.93												
N= 9	.07	.17	.28	.39	.50	.61	.72	.83	.93											
N= 10	.06	.16	.25	.35	.45	.55	.65	.75	.84	.94										
N= 11	.05	.14	.23	.32	.41	.50	.59	.68	.77	.86	.95									
N= 12	.05	.13	.21	.30	.38	.46	.54	.62	.70	.79	.87	.95								
N= 13	.05	.12	.20	.27	.35	.42	.50	.58	.65	.73	.80	.88	.95							
N= 14	.04	.11	.18	.25	.32	.39	.46	.54	.61	.68	.75	.82	.89	.96						
N= 15	.04	.11	.17	.24	.30	.37	.43	.50	.57	.63	.70	.76	.83	.89	.96					
N= 16	.04	.10	.16	.22	.28	.35	.41	.47	.53	.59	.65	.72	.78	.84	.90	.96				
N= 17	.03	.09	.15	.21	.27	.33	.38	.44	.50	.56	.62	.67	.73	.79	.85	.91	.97			
N= 18	.03	.09	.14	.20	.25	.31	.36	.42	.47	.53	.58	.64	.69	.75	.80	.86	.91	.97		
N- 19	.03	.08	.14	.19	.24	.29	.34	.40	.45	.50	.55	.60	.66	.71	.76	.81	.86	.92	.97	
N= 20	.03	.08	.13	.18	.23	.28	.33	.38	.43	.48	.52	.57	.62	.67	.72	.77	.82	.87	.92	.97

Table B2. Upper tail normal quantiles for the plotting positions of Table B1

(for lower tail quantiles, multiply all nonzero quantiles by −1)

N= 5

0.000	0.502	1.198

N= 6

0.203	0.649	1.300

N= 7

0.000	0.355	0.765	1.383

N= 8

0.153	0.475	0.859	1.453

N= 9

0.000	0.276	0.575	0.939	1.513

N= 10

0.123	0.377	0.659	1.007	1.565

N= 11

0.000	0.225	0.463	0.732	1.067	1.611

N= 12

0.103	0.313	0.538	0.796	1.121	1.653

N= 13

0.000	0.191	0.389	0.604	0.852	1.169	1.691

N= 14

0.088	0.267	0.456	0.663	0.904	1.212	1.725

N= 15

0.000	0.165	0.336	0.517	0.716	0.950	1.252	1.757

N= 16

0.077	0.234	0.397	0.571	0.765	0.992	1.289	1.787

N= 17

0.000	0.146	0.295	0.452	0.620	0.809	1.031	1.323	1.814

N= 18

0.069	0.208	0.351	0.502	0.666	0.849	1.067	1.354	1.839

N= 19

0.000	0.131	0.264	0.402	0.548	0.707	0.887	1.101	1.383	1.864

N= 20

0.062	0.187	0.315	0.449	0.591	0.746	0.922	1.133	1.411	1.886

Table B3. Critical r* values for the probability plot correlation coefficient test of normality (from Looney and Gulledge, 1985a)

[reject H_0: data are normal when PPCC $r < r^*$]

	α-level					
n	.005	.010	.025	.050	.100	.250
3	.867	.869	.872	.879	.891	.924
4	.813	.824	.846	.868	.894	.931
5	.807	.826	.856	.880	.903	.934
6	.820	.838	.866	.888	.910	.939
7	.828	.850	.877	.898	.918	.944
8	.840	.861	.887	.906	.924	.948
9	.854	.871	.894	.912	.930	.952
10	.862	.879	.901	.918	.934	.954
11	.870	.886	.907	.923	.938	.957
12	.876	.892	.912	.928	.942	.960
13	.885	.899	.918	.932	.945	.962
14	.890	.905	.923	.935	.948	.964
15	.896	.910	.927	.939	.951	.965
16	.899	.913	.929	.941	.953	.967
17	.905	.917	.932	.944	.954	.968
18	.908	.920	.935	.946	.957	.970
19	.914	.924	.938	.949	.958	.971
20	.916	.926	.940	.951	.960	.972
21	.918	.930	.943	.952	.961	.973
22	.923	.933	.945	.954	.963	.974
23	.925	.935	.947	.956	.964	.975
24	.927	.937	.949	.957	.965	.976
25	.929	.939	.951	.959	.966	.976
26	.932	.941	.952	.960	.967	.977
27	.934	.943	.953	.961	.968	.978
28	.936	.944	.955	.962	.969	.978
29	.939	.946	.956	.963	.970	.979
30	.939	.947	.957	.964	.971	.979
31	.942	.950	.958	.965	.972	.980
32	.943	.950	.959	.966	.972	.980
33	.944	.951	.961	.967	.973	.981
34	.946	.953	.962	.968	.974	.981
35	.947	.954	.962	.969	.974	.982
36	.948	.955	.963	.969	.975	.982
37	.950	.956	.964	.970	.976	.983
38	.951	.957	.965	.971	.976	.983
39	.951	.958	.966	.971	.977	.983
40	.953	.959	.966	.972	.977	.984

Table B3. Cont.

	α-level					
n	.005	.010	.025	.050	.100	.250
41	.953	.960	.967	.973	.977	.984
42	.954	.961	.968	.973	.978	.984
43	.956	.961	.968	.974	.978	.984
44	.957	.962	.969	.974	.979	.985
45	.957	.963	.969	.974	.979	.985
46	.958	.963	.970	.975	.980	.985
47	.959	.965	.971	.976	.980	.986
48	.959	.965	.971	.976	.980	.986
49	.961	.966	.972	.976	.981	.986
50	.961	.966	.972	.977	.981	.986
55	.965	.969	.974	.979	.982	.987
60	.967	.971	.976	.980	.984	.988
65	.969	.973	.978	.981	.985	.989
70	.971	.975	.979	.983	.986	.990
75	.973	.976	.981	.984	.987	.990
80	.975	.978	.982	.985	.987	.991
85	.976	.979	.983	.985	.988	.991
90	.977	.980	.984	.986	.988	.992
95	.979	.981	.984	.987	.989	.992
100	.979	.982	.985	.987	.989	.992

Table B4. Quantiles (p-values) for the rank-sum test statistic W_{rs}

$p = \text{Prob}\,[W_{rs} \geq x] = \text{Prob}\,[W_{rs} \leq x^*]$

n [smaller sample size] = 3

m=4			m=5			m=6			m=7			m=8			m=9			m=10		
x	p	x*	x	p	x*	x	p	x*	x	p	x*	x	p	x*	x	p	x*	x	p	x*
16	.114	8	18	.125	9	20	.131	10	22	.133	11	24	.139	12	27	.105	12	29	.108	13
17	.057	7	19	.071	8	21	.083	9	23	.092	10	25	.097	11	28	.073	11	30	.080	12
18	.029	6	20	.036	7	22	.048	8	24	.058	9	26	.067	10	29	.050	10	31	.056	11
19	0	5	21	.018	6	23	.024	7	25	.033	8	27	.042	9	30	.032	9	32	.038	10
			22	0	5	24	.012	6	26	.017	7	28	.024	8	31	.018	8	33	.024	9
						25	0	5	27	.008	6	29	.012	7	32	.009	7	34	.014	8
									28	0	5	30	.006	6	33	.005	6	35	.007	7

n [smaller sample size] = 4

m=4			m=5			m=6			m=7			m=8			m=9			m=10		
x	p	x*	x	p	x*	x	p	x*	x	p	x*	x	p	x*	x	p	x*	x	p	x*
22	.171	14	25	.143	15	28	.129	16	31	.115	17	34	.107	18	36	.130	20	39	.120	21
23	.100	13	26	.095	14	29	.086	15	32	.082	16	35	.077	17	37	.099	19	40	.094	20
24	.057	12	27	.056	13	30	.057	14	33	.055	15	36	.055	16	38	.074	18	41	.071	19
25	.029	11	28	.032	12	31	.033	13	34	.036	14	37	.036	15	39	.053	17	42	.053	18
26	.014	10	29	.016	11	32	.019	12	35	.021	13	38	.024	14	40	.038	16	43	.038	17
27	0	9	30	.008	10	33	.010	11	36	.012	12	39	.014	13	41	.025	15	44	.027	16
			31	0	9	34	.005	10	37	.006	11	40	.008	12	42	.017	14	45	.018	15
									38	.003	10	41	.004	11	43	.010	13	46	.012	14
															44	.006	12	47	.007	13

n [smaller sample size] = 5

m=5			m=6			m=7			m=8			m=9			m=10		
x	p	x*	x	p	x*	x	p	x*	x	p	x*	x	p	x*	x	p	x*
34	.111	21	37	.123	23	41	.101	24	44	.111	26	47	.120	28	51	.103	29
35	.075	20	38	.089	22	42	.074	23	45	.085	25	48	.095	27	52	.082	28
36	.048	19	39	.063	21	43	.053	22	46	.064	24	49	.073	26	53	.065	27
37	.028	18	40	.041	20	44	.037	21	47	.047	23	50	.056	25	54	.050	26
38	.016	17	41	.026	19	45	.024	20	48	.033	22	51	.041	24	55	.038	25
39	.008	16	42	.015	18	46	.015	19	49	.023	21	52	.030	23	56	.028	24
40	.004	15	43	.009	17	47	.009	18	50	.015	20	53	.021	22	57	.020	23
			44	.004	18	48	.005	17	51	.009	19	54	.014	21	58	.014	22
									52	.005	18	55	.009	20	59	.010	21
												56	.006	19	60	.006	20

n [smaller sample size] = 6

m=6			m=7			m=8			m=9			m=10		
x	p	x*	x	p	x*	x	p	x*	x	p	x*	x	p	x*
47	.120	31	51	.117	33	55	.114	35	59	.112	37	63	.110	39
48	.090	30	52	.090	32	56	.091	34	60	.091	36	64	.090	38
49	.066	29	53	.069	31	57	.071	33	61	.072	35	65	.074	37
50	.047	28	54	.051	30	58	.054	32	62	.057	34	66	.059	36
51	.032	27	55	.037	29	59	.041	31	63	.044	33	67	.047	35
52	.021	26	56	.026	28	60	.030	30	64	.033	32	68	.036	34
53	.013	25	57	.017	27	61	.021	29	65	.025	31	69	.028	33
54	.008	24	58	.011	26	62	.015	28	66	.018	30	70	.021	32
			59	.007	25	63	.010	27	67	.013	29	71	.016	31
									68	.009	28	72	.011	30
												73	.008	29

TABLE B4 continued

n [smaller sample size] = 7

m=7			m=8			m=9			m=10		
x	p	x*	x	p	x*	x	p	x*	x	p	x*
61	.159	44	65	.168	47	70	.150	49	74	.15752	
62	.130	43	66	.140	46	71	.126	48	75	.13551	
63	.104	42	67	.116	45	72	.105	47	76	.11550	
64	.082	41	68	.095	44	73	.087	46	77	.09749	
65	.064	40	69	.076	43	74	.071	45	78	.08148	
66	.049	39	70	.060	42	75	.057	44	79	.06747	
67	.036	38	71	.047	41	76	.045	43	80	.05446	
68	.027	37	72	.036	40	77	.036	42	81	.04445	
69	.019	36	73	.027	39	78	.027	41	82	.03544	
70	.013	35	74	.020	38	79	.021	40	83	.02843	
71	.009	34	75	.014	37	80	.016	39	84	.02242	
72	.006	33	76	.010	36	81	.011	38	85	.01741	
			77	.007	35	82	.008	37	86	.01240	
						83	.006	36	87	.00939	

n [smaller sample size] = 9

m=9			m=10		
x	p	x*	x	p	x*
98	.149	73	104	.13976	
99	.129	72	105	.12175	
100	.111	71	106	.10674	
101	.095	70	107	.09173	
102	.081	69	108	.07872	
103	.068	68	109	.06771	
104	.057	67	110	.05670	
105	.047	66	111	.04769	
106	.039	65	112	.03968	
107	.031	64	113	.03367	
108	.025	63	114	.02766	
109	.020	62	115	.02265	
110	.016	61	116	.01764	
111	.012	60	117	.01463	
112	.009	59	118	.01162	
113	.007	58	119	.00961	

n [smaller sample size] = 8

m=8			m=9			m=10		
x	p	x*	x	p	x*	x	p	x*
79	.13957		84	.138	60	89	.137	63
80	.11756		85	.118	59	90	.118	62
81	.09755		86	.100	58	91	.102	61
82	.08054		87	.084	57	92	.086	60
83	.06553		88	.069	56	93	.073	59
84	.05252		89	.057	55	94	.061	58
85	.04151		90	.046	54	95	.051	57
86	.03250		91	.037	53	96	.042	56
87	.02549		92	.030	52	97	.034	55
88	.01948		93	.023	51	98	.027	54
89	.01447		84	.018	50	99	.022	53
90	.01046		95	.014	49	100	.017	52
91	.00745		96	.010	48	101	.013	51
92	.00544		97	.008	47	102	.010	50

n [smaller sample size] = 10

m=10		
x	p	x*
119	.157	91
120	.140	90
121	.124	89
122	.109	88
123	.095	87
124	.083	86
125	.072	85
126	.062	84
127	.053	83
128	.045	82
129	.038	81
130	.032	80
131	.026	79
132	.022	78
133	.018	77
134	.014	76
135	.012	75
136	.009	74
137	.007	73
138	.006	72

Table generated by D. Helsel

Table B5. -- Quantiles (p-values) for the sign test statistic S^+

Quantiles for the sign test are identical to quantiles of the binomial distribution with percentile p=0.5. The approximation given in chapter 6 and used by most statistical software packages can be used for $n \geq 20$. Statistics textbooks that contain a table of exact quantiles for the binomial distribution for sizes below 20 include Hollander and Wolfe (1999) and Zar (1999).

An online table of exact quantiles for the binomial distribution can be found as of 5/2002 at: http://faculty.vassar.edu/lowry/binomial01.html

An example of using this online table:
Enter n (the number of data pairs) and p (=0.5). An exact table will be printed. P-values are cumulative probabilities, or values of the cumulative distribution function (cdf). For small values of the test statistic S^+ (called k in the online table) – values below n/2, use the "Down" column to read off a one-sided p-value for the sign test. For S^+ larger than n/2, use the "Up" column. The example output below is for n=13. A one-sided p-value for $S^+ = 4$ (the probability of getting an $S^+ \leq 4$) is 0.133. The p-value for $S^+ = 9$ (the probability of getting an $S^+ \geq 9$) also equals 0.133. For a two-sided test, p = 0.266.

		Cumulative Probability	
k	**Exact Probability**	**Down**	**Up**
0	0.000122070313	0.000122070313	1.0
1	0.001586914063	0.001708984375	0.999877929688
2	0.009521484375	0.01123046875	0.998291015625
3	0.034912109375	0.046142578125	0.98876953125
4	0.087280273438	0.133422851563	0.953857421875
5	0.157104492188	0.29052734375	0.866577148438
6	0.20947265625	0.5	0.70947265625
7	0.20947265625	0.70947265625	0.5
8	0.157104492188	0.866577148438	0.29052734375
9	0.087280273438	0.953857421875	0.133422851563
10	0.034912109375	0.98876953125	0.046142578125
11	0.009521484375	0.998291015625	0.01123046875
12	0.001586914063	0.999877929688	0.001708984375
13	0.000122070313	1.0	
	0.000122070313		

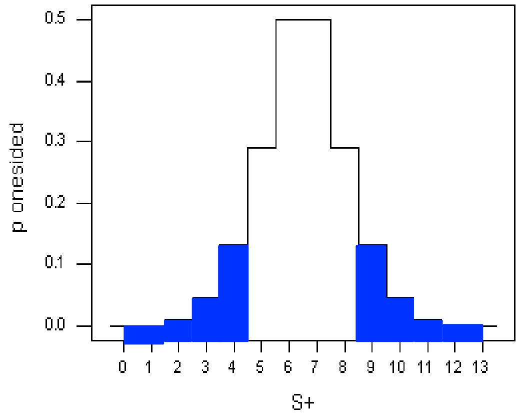


Figure B1. Two-sided critical region (p-values), shaded, for the sign test. n=13, S^{+}= 4 or 9.

Table B6 – Critical test statistic values for the signed-rank statistic W^+

(from McCornack, 1965)

The approximation given in chapter 6, used by most statistics software packages, can be used for $n > 15$ and $\alpha \geq 0.025$. For $\alpha < 0.025$, see exact tables in the McCornack paper or a textbook such as Hollander and Wolfe (1999), even for large sample sizes.

[reject H_0: at one-sided α when $W+ \leq$ table entry (small W)]

	α-level			
n	**.005**	**.010**	**.025**	**.050**
5				0
6			0	2
7		0	2	3
8	0	1	3	5
9	1	3	5	8
10	3	5	8	10
11	5	7	10	13
12	7	9	13	17
13	9	12	17	21
14	12	15	21	25
15	15	19	25	30
16	19	23	29	35
17	23	27	34	41
18	27	32	40	47
19	32	37	46	53
20	37	43	52	60

[reject H_0: at one-sided α when $W+ \geq$ table entry (large W)]

	α-level			
n	**.005**	**.010**	**.025**	**.050**
5				15
6			21	19
7		28	26	25
8	36	35	33	31
9	44	42	40	37
10	52	50	47	45
11	61	59	56	53
12	71	69	65	61
13	82	79	74	70
14	93	90	84	80
15	105	101	95	90
16	117	113	107	101
17	130	126	119	112
18	144	139	131	124
19	158	153	144	137
20	173	167	158	150

Table B7 – Critical test statistic values for the Friedman statistic Xf (from Martin, Leblanc and Toan, 1993)

The chi-square approximation given in chapter 7 is used by most statistics software packages. For comparing 3 to 5 groups of data with sample sizes (blocks) n <10 in each group, an exact table should be used.

[reject H_0: at α when Xf $\geq$ table entry]

k = 3 groups

n	α-level .005	.010	.025	.050	.10
3				6.00	6.00
4	8.00	8.00	8.00	6.50	6.00
5	10.00	8.40	7.60	6.40	5.20
6	10.33	9.00	8.33	7.00	5.33
7	10.29	8.86	7.71	7.14	5.43
8	9.75	9.00	7.75	6.25	5.25
9	10.67	8.67	8.00	6.22	5.56
10	10.40	9.60	7.80	6.20	5.00

k = 4 groups

n	α-level .005	.010	.025	.050	.10
2				6.00	6.00
3	9.00	9.00	8.20	7.40	6.60
4	10.20	9.60	8.40	7.80	6.30
5	10.92	9.96	8.76	7.80	6.36
6	11.40	10.20	8.80	7.60	6.40
7	11.40	10.37	9.00	7.80	6.43
8	11.85	10.50	9.00	7.65	6.30
9	12.07	10.87	9.13	7.80	6.47
10	12.00	10.80	9.12	7.80	6.36

k = 5 groups

n	α-level .005	.010	.025	.050	.10
2		8.00	8.00	7.60	7.20
3	10.67	10.13	9.60	8.53	7.47
4	12.00	11.20	9.80	8.80	7.60
5	12.48	11.68	10.24	8.96	7.68
6	13.07	11.87	10.40	9.07	7.73
7	13.26	12.11	10.51	9.14	7.77
8	13.50	12.30	10.60	9.30	7.80
9	13.69	12.44	10.67	9.24	7.73
10	13.84	12.48	10.72	9.28	7.76

Table B8 -- Quantiles (p-values) for Kendall's S statistic and tau correlation coefficient

For N>10 use the approximation given in section 8.2.2

One-sided p = Prob [S ≥ x] = Prob [S ≤ −x]

	N = Number of data pairs			
x	4	5	8	9
0	0.625	0.592	0.548	0.540
2	0.375	0.408	0.452	0.460
4	0.167	0.242	0.360	0.381
6	0.042	0.117	0.274	0.306
8		0.042	0.199	0.238
10		0.0083	0.138	0.179
12			0.089	0.130
14			0.054	0.090
16			0.031	0.060
18			0.0156	0.038
20			0.0071	0.022
22			0.0028	0.0124
24			0.0009	0.0063
26			0.0002	0.0029
28			<0.0001	0.0012
30				0.0004
32				0.0001
34				<0.0001
36				<0.0001

	N = Number of data pairs			
x	3	6	7	10
1	0.500	0.500	0.500	0.500
3	0.167	0.360	0.386	0.431
5		0.235	0.281	0.364
7		0.136	0.191	0.300
9		0.068	0.119	0.242
11		0.028	0.068	0.190
13		0.0083	0.035	0.146
15		0.0014	0.015	0.108
17			0.0054	0.078
19			0.0014	0.054
21			0.0002	0.036
23				0.023
25				0.0143
27				0.0083
29				0.0046
31				0.0023
33				0.0011
35				0.0005
37				0.0002
39				<0.0001
41				<0.0001
43				<0.0001
45				<0.0001

Table generated by D. Helsel

Appendix C
Data Sets

		Chapter cited
Data Set C1	Annual peak discharges for the Saddle River, NJ	2, 3
Data Set C2	Annual streamflows for the Conecuh River, AL	3, 6
Data Set C3	Daily streamflow for the Potomac River, Wash. D.C.	3
Data Set C4	Atrazine concentrations	6
Data Set C5	Subset of iron concentrations at low flow	7
Data Set C6	Complete set of iron concentrations	7
Data Set C7	Specific capacities of wells in Pennsylvania	7
Data Set C8	Corbicula on the Tennessee River	7
Data Set C9	TDS concentrations for the Cuyahoga River, Ohio	9
Data Set C10	Phosphorus transport, Illinois River at Marseilles	9
Data Set C11	Grain size and permeability of alluvial aquifers	9
Data Set C12	ROE and TDS data,	
	Rappahannock R. near Fredericksburg, Virginia	10
Data Set C13	Streamflow data used for record extension	10
Data Set C14	Mean annual runoff and basin characteristics	11
Data Set C15	Urban total nitrogen loads	11
Data Set C16	Uranium and TDS in groundwaters	11
Data Set C17	Green River, Kentucky sediment transport data	12
Data Set C18	Maumee River, Ohio total P trends data	12
Data Set C19	Water levels, P-R-M system middle aquifer, NJ	12
Data Set C20	Factors affecting contamination from impoundments	15

Data sets are available in both ASCII and MS Excel formats. See the online location from which you obtained this book for the data files HhappC.dat and HhappC.xls .

Appendix D

Answers to Selected Exercises

Chapter 1

1.1 For the well yield data:

a) mean = 0.19
b) trimmed mean = 0.05
c) geometric mean = 0.04
d) median = 0.04
e) They differ because the data are skewed. The estimates which are more robust are similar, while the mean is larger.

1.2

a) standard deviation = 0.31
b) interquartile range = 0.36
c) MAD = 0.04
d) skew = 2.07. quartile skew = 0.83.

Because the data are asymmetric, the median difference is small, but the IQR and standard deviation are not.

1.3

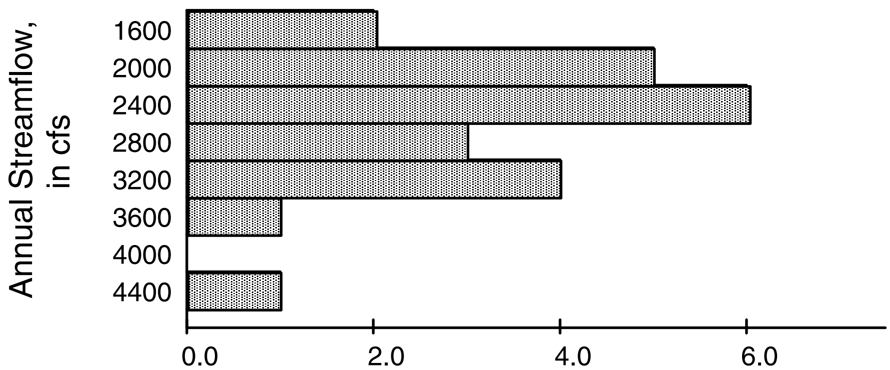
mean = 1.64	std. dev. = 2.85
median = 0.80	IQR = 0.61
geometric mean = 0.04	MAD = 0.25
skew = 3.09	quartile skew = −0.10

The largest observation is an outlier. Though the skew appears to be strongly positive, and the standard deviation large, this is due only to the effect of that one point. The majority of the data are not skewed, as shown by the more resistant quartile skew coefficient.

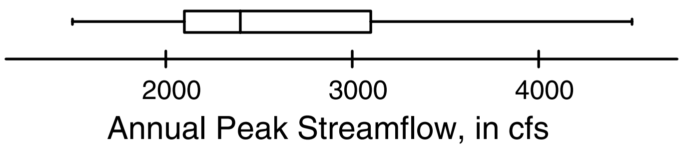
a) assuming the outlying observation is accurate, representing some high-nitrogen location which is important to have sampled, the mean must be used to compute the mass of nitrogen falling per square mile. It would probably be computed by weighting concentrations by the surface area represented by each environment. The median would under-represent this mass loading.
b) the median would be a better "typical" concentration, and the IQR a better "typical" variability, than the mean and standard deviation. This is due to the strong effect of the one unusual point on these traditional measures.

Chapter 2

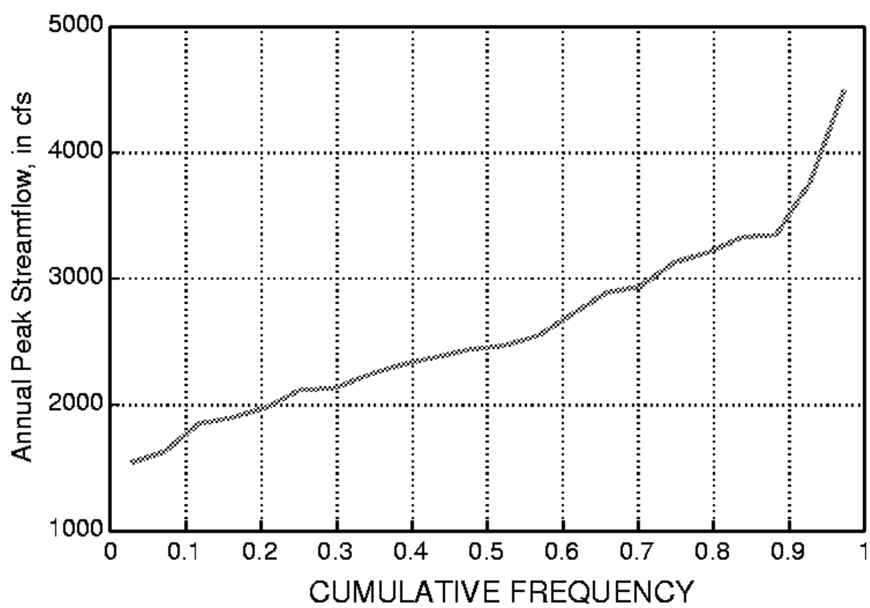
2.1 a)



b)

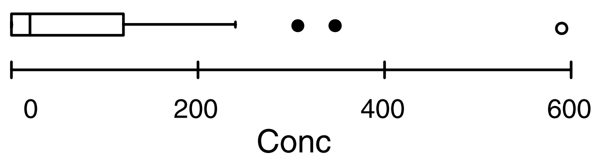


c)

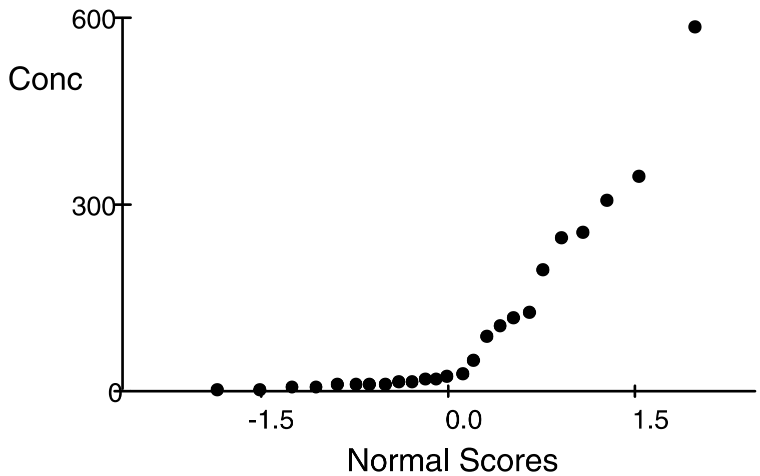


Either a cube root or logarithmic transform would increase symmetry.

2.2 a)

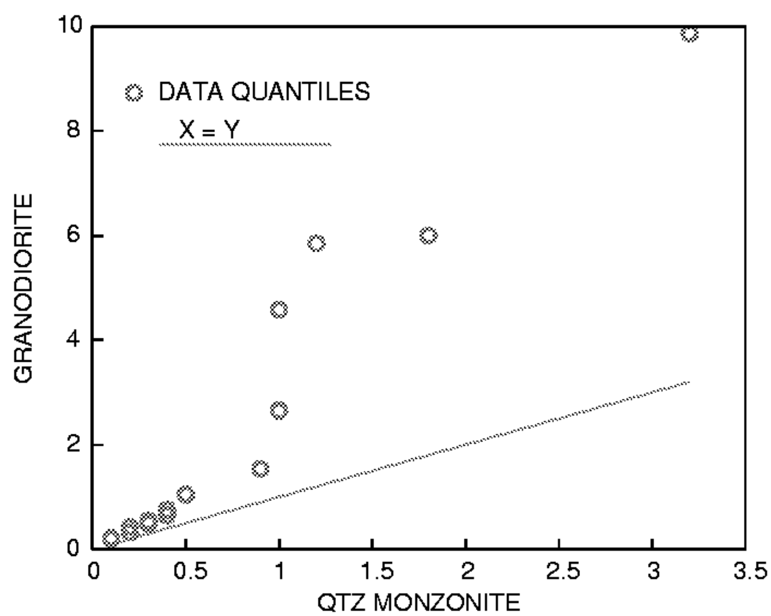


b)

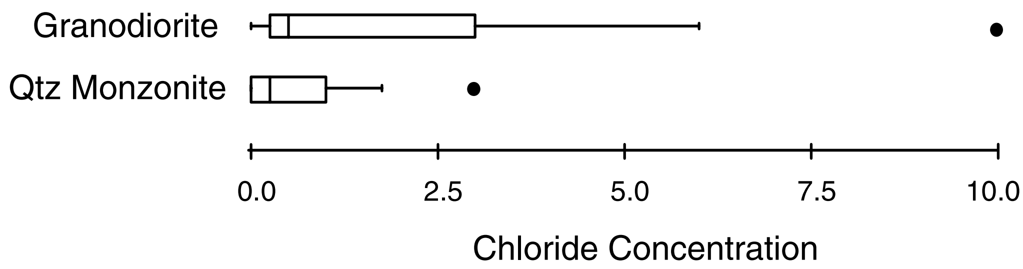


The data are strongly right-skewed. A log transformation makes these data more symmetric.

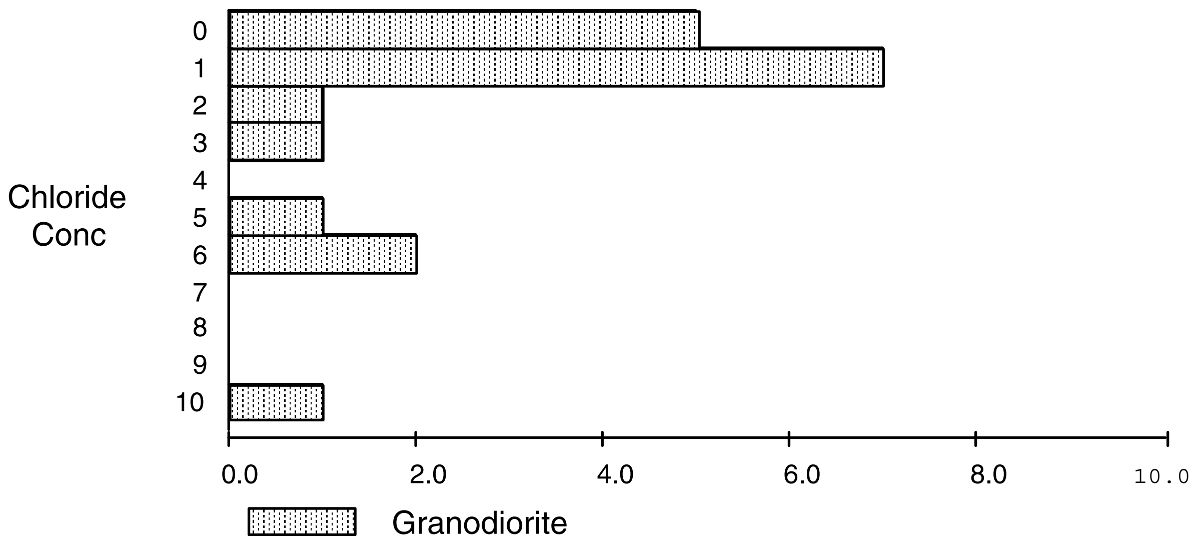
2.3 Q-Q plot.

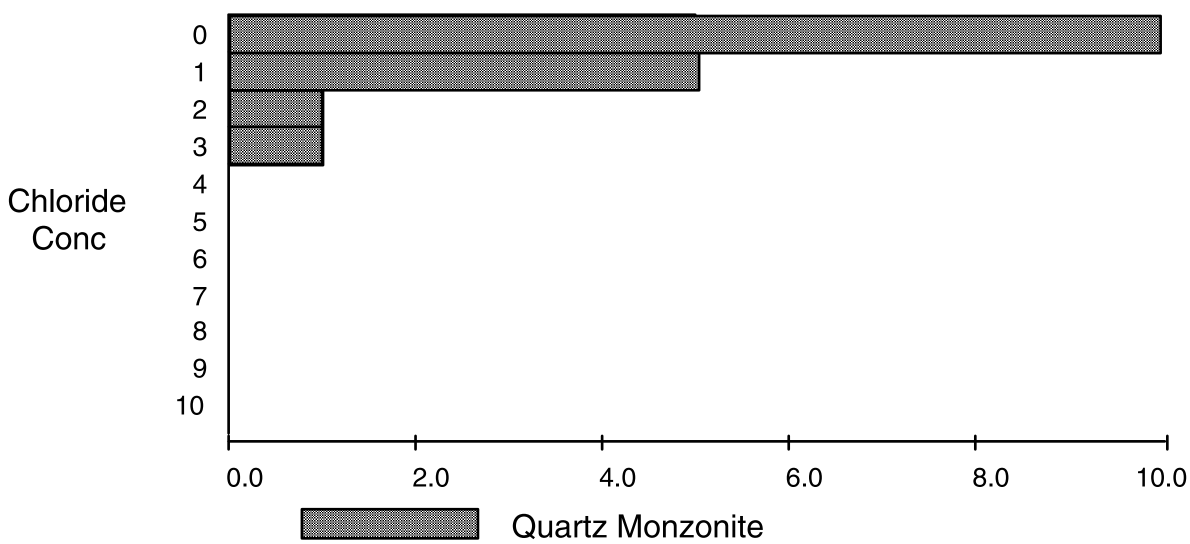


Boxplots:



Histograms:





The granodiorite shows higher chloride concentrations than does the quartz monzonite. This can be seen with any of the three graphs, but most easily with the boxplot. From the Q-Q plot, the relationship does not look linear.

2.4 There appears to be no effect of the waste treatment plant.

Chapter 3

3.1 nonparametric: $x' = 4$ ($\alpha/2 = .0154$). $R_l = 5$, $R_u = 14$.

$0.4 \le Cl_{0.5} \le 3.0$ at $\alpha = 0.031$.

This is as close to 0.05 as the table allows.

parametric: Using the natural logs of the data,

$$\exp\left(-0.045 - 2.11 \bullet \sqrt{1.63/18}\right) \le GM_x \le \exp\left(-0.045 + 2.11 \bullet \sqrt{1.63/18}\right).$$

$$0.51 \le GM_x \le 1.80.$$

Either of intervals is reasonable. The logs of the data still retain some skewness, so the nonparametric interval may be more realistic. The parametric interval might be preferred to obtain an alpha of 0.05. The choice would depend on whether the assumption of lognormality was believed.

3.2 symmetric: $0.706 - 2.12 \bullet \sqrt{0.639/17} \le \mu \le 0.706 + 2.12 \bullet \sqrt{0.639/17}$

$$0.30 \le \mu \le 1.12$$

Point estimates: mean $= 0.705$ (assuming normal distribution).

mean $= \exp(-0.849+0.5 \bullet 1.067)$

$= 0.73$ (assuming a lognormal distribution).

As the logs of the data are more symmetric than the data prior to transformation, the lognormal (2nd) estimate is preferred.

3.3 Parametric 95% prediction interval:

$0.19 - 2.20 \bullet \sqrt{0.0975 + (0.0975/12)}$ to $0.19 + 2.20 \bullet \sqrt{0.0975 + (0.0975/12)}$

or -0.53 to 0.91 gallons/min/foot. Includes 0.85, so same distribution.

Nonparametric 95% prediction interval:

$X_{[0.025 \bullet 13]}$ to $X_{[0.975 \bullet 13]}$ $X_{0.325}$ to $X_{12.675}$

The sample size is too small to produce such a wide (95%) nonparametric prediction interval. Therefore a parametric interval must be used. However, the negative lower end of the parametric prediction interval indicates that a symmetric interval is not appropriate. So an asymmetric interval resulting from taking logarithms should be used instead of the one above.

3.4 The data look relatively symmetric, so no logs taken.

mean: 683 ± 126, or 557->809 $\alpha = .05$.

median: $R_l=6$, $R_u=15$ 524->894 $\alpha = .041$.

3.5 The 90th percentile = 2445 cfs. A one-sided 95% confidence interval for the 90th percentile (an upper confidence limit to insure that the intake does not go dry) is found using the large-sample approximation of equation 3.17:

$$R_u = 365 \bullet 0.1 + z_{[0.95]} \bullet \sqrt{365 \bullet 0.1 \bullet (0.9)} + 0.5$$
$$= 36.5 + 1.645 \bullet 5.73 + 0.5 \quad = 46.4$$

The 46th ranked point is the upper CI, or 2700 cfs.

Chapter 4

4.1 For the before-1969 data, PPCC r=0.986. For the after-1969 data, PPCC r=0.971. Critical values of r are 0.948 and 0.929, respectively. Therefore normality cannot be rejected for either period at $\alpha = 0.05$.

4.2 For the arsenic data, PPCC r=0.844. The critical r* from Appendix table B3 is r*=0.959. Therefore reject normality. For log-transforms of the data, PPCC r=0.973. Normality of the transformed data is not rejected.

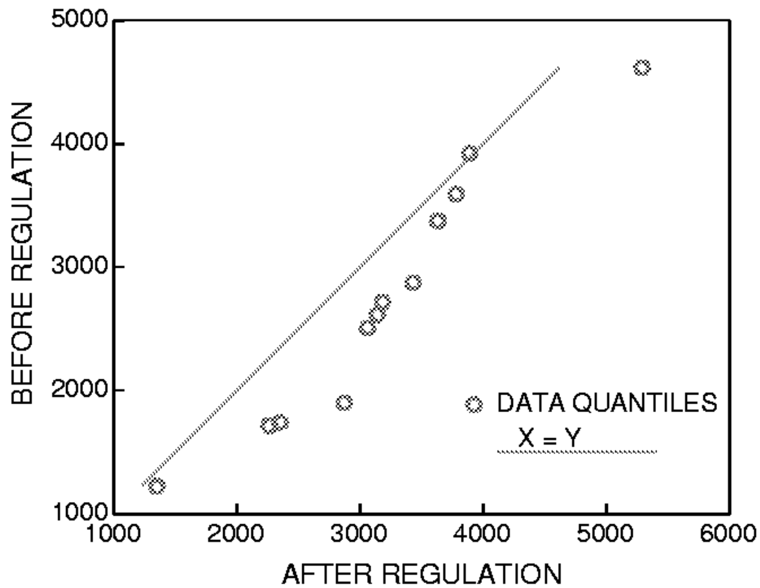
Chapter 5

5.1 The p-value remains the same.

5.2 Given that we wish to test for a change in concentration, but the direction of the change is not specified in the problem, this should be a two-sided test. If it had stated we were looking for an increase, or a decrease, the test would have been a one-sided test.

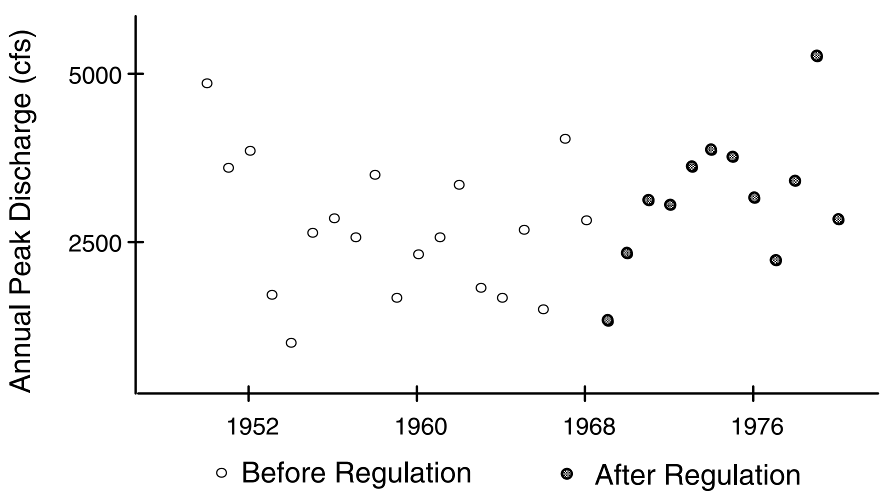
5.3 a. Quantiles are the 12 "after" data, and 12 quantiles computed from the 19 "before" data :

i	j	"after"	"before"
1	1.34	1350.00	1222.13
2	2.92	2260.00	1715.25
3	4.49	2350.00	1739.84
4	6.06	2870.00	1900.82
5	7.64	3060.00	2506.23
6	9.21	3140.00	2614.92
7	10.79	3180.00	2717.21
8	12.36	3430.00	2873.61
9	13.93	3630.00	3375.24
10	15.51	3780.00	3591.15
11	17.08	3890.00	3922.29
12	18.66	5290.00	4617.37



The relationship appears additive. The Hodges-Lehmann estimate (median of all possible after–before differences) = 480 cfs.

b. After regulation, the reservoir appears to be filling. Any test for change in flow should omit data during the transition period of 1969-1970. Plots of time series are always a good idea. They increase the investigator's understanding of the data. Low flows after regulation are not as low as those before. This produces the pattern seen in the Q-Q plot of the low quantiles being lower after regulation. while the upper quantiles appear the same, as shown by the drift closer to the x=y line for the higher values.



c. With 1969 and 70 included, W_{rs} = 273.5 p=0.22. The after flows are not significantly different. With 1969 and 70 excluded, W_{rs} = 243.5 p=0.06. The after flows are close to being significantly different -- more data after regulation is needed.

5.4 Exact test

X	Y	R(Y)	R(X)
1			1
	1.5	2	
2			3
	2.5	4	
3			5
	3.5	6	
4			7
	4.5	8	
	5.5	9	
	7.0	10	
	10.0	11	
	20.0	12	
	40.0	13	
	100.0	14	

$n = 4 \quad m = 10$ $\qquad W_{rs} = \Sigma R_x = 16$

From table B4, $\text{Prob}(W_{rs} \leq 16) = .027$. The two-sided exact p-value = 0.054

Large-sample approximation

The mean is $\mu_W = \dfrac{n \bullet (N+1)}{2} = \dfrac{4 \bullet 15}{2} = 30$

The standard deviation is given by $\sigma_W = \sqrt{\dfrac{n \bullet m \bullet (N+1)}{12}} = 7.0711$

$$Z_{rs} = \frac{16 - \mu_W + 1/2}{\sigma_W} = -1.909$$

Using linear interpolation between −1.9110 and −1.8957 in a table of the standard normal distribution gives the one-tail probability of 0.028. So the two-sided approximate p-value is 0.056.

t-test on the ranks

Replacing variable values by ranks gives

$\bar{x} = 4 \qquad S_x = 2.582 \qquad S^2_x = 6.667 \qquad n = 4$

$\bar{y} = 8.9 \qquad S_y = 3.928 \qquad S^2_y = 15.429 \qquad m = 10$

The pooled variance is :

$$S^2 = \frac{3S^2_x + 9S^2_y}{12} = 13.2386$$

$S = 3.639$

$$t = \frac{\bar{x} - \bar{y}}{S\sqrt{(1/n + 1/m)}} = -2.27610$$

Linear interpolation for a student's t with 12 degrees of freedom gives

$$.975 + \frac{(2.27610 - 2.1788)}{(2.6810 - 2.1788)} \bullet .015 = .97791 \qquad 1.0 - .97791 = .022$$

The two-sided rank transform p-value is .044.

Summary

Approach	**p-value**
Rank-Sum Exact	0.054
Rank-Sum Approx.	0.056
t test on ranks	0.044

To compute $\hat{\Delta}$, the (n•m)=40 differences ($X_i - Y_j = D_j$) are:

(Y_1)	0.5	1.5	2.5	**3.5**	4.5	6	9	19	39	99
(Y_2)	−0.5	0.5	1.5	2.5	3.5	5	8	18	38	98
(Y_3)	−1.5	−0.5	0.5	1.5	2.5	**4**	7	17	37	97
(Y_4)	−2.5	−1.5	−0.5	0.5	1.5	3	6	16	36	96

$\hat{D}$ = median of 40 D_j's $(D_{rank\ 20} + D_{rank\ 21})/2 = 3.75$

5.5 Yields with fracturing: $r_{crit} = .932$, accept normality
Yields without: $r_{crit} = .928$, reject normality

Because one of the groups is non-normal, the rank-sum test is performed. $W_{rs} = \Sigma R_{without} = 121.5$. The one-sided p-value from the large-sample approximation p= 0.032. Reject equality. The yields from fractured rocks are higher.

5.6 The test statistic changes very little ($W_{rs} = 123$), indicating that most information contained in the data below detection limit is extracted using ranks. Results are the same (one-sided p-value = 0.039. Reject equality). A t-test could not be used without improperly substituting some contrived values for less-thans which might alter the conclusions.

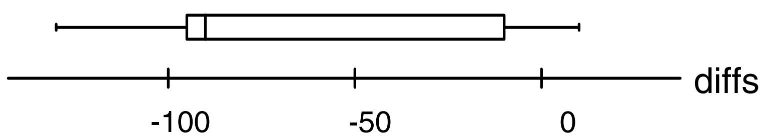
Chapter 6

6.1 The sign test is computed on all data after 683 cfs is subtracted. $S^+ = 11$. From table B5, reject if $S^+ \geq 14$ (one-sided test). So do not reject. p > 0.25.

6.2 c is not a matched pair.

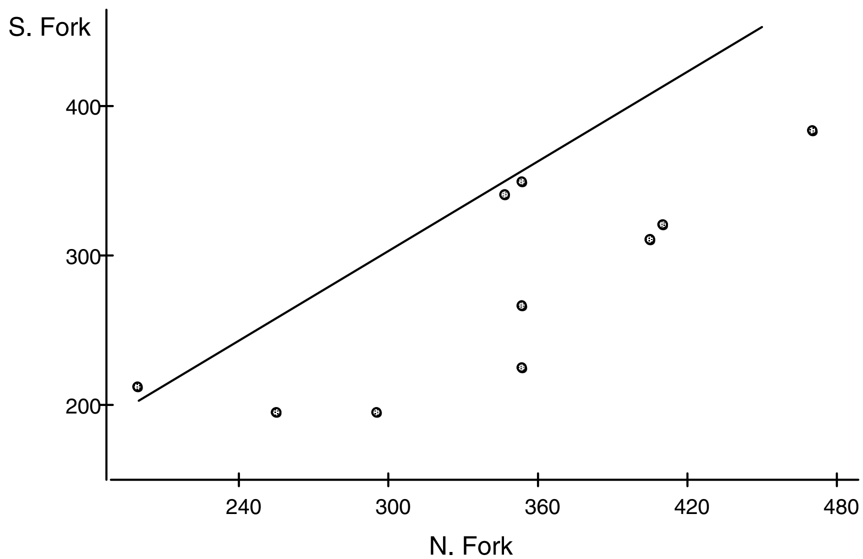
6.3 a. H_0: μ (South Fork) − μ (North Fork) = 0.
H_1: μ (South Fork) − μ (North Fork) ≠ 0.

b. A boxplot of the differences shows no outliers, but the median is low. Conductance data are usually not skewed, and the PPCC r=0.941, with normality not rejected. So a t-test on the differences is computed (parametric).



c. t = −4.24 p = 0.002 Reject H_0.

d. Along with the boxplot above, a scatterplot shows that the South Fork is higher only once:



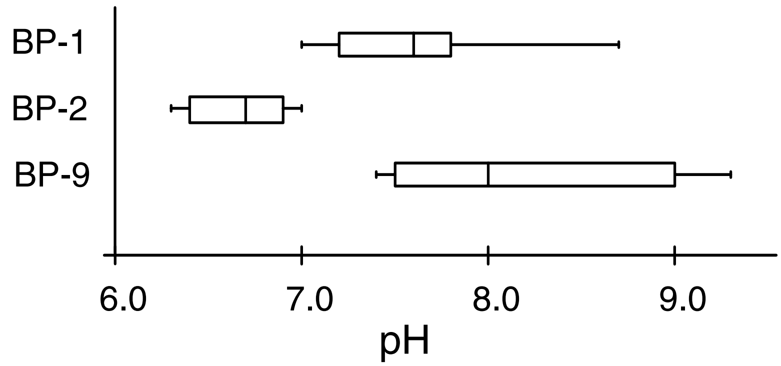
e. The mean difference is −64.7.

6.4 Because of the data below the reporting limit, the sign test is performed on the differences Sept–June. The one-sided p-value = 0.002. Sept atrazine concentrations are significantly larger than June concs before application.

6.5 For the t-test, t=1.07 with a one-sided p-value of 0.15. The t-test cannot reject equality of means because one large outlier in the data produces violations of the assumptions of normality and equal variance.

Chapter 7

7.1 As a log-transformed variable, pH often closely follows a normal distribution. See the following boxplots:



pH for three piezometer groups (from Robertson et al., 1984)

The PPCC for the three groups (0.955 for BP-1, 0.971 for BP-2, and 0.946 for BP-9) cannot disprove this assumption. Therefore ANOVA will be used to test the similarity of the three groups.

Anova Table:

Source	df	SS	MS	F	p-value
Piez Gp	2	7.07	3.54	9.57	0.002
Error	15	5.54	0. 37		
Total	17	12.61			

The groups are declared different. Statistics for each are:

GP	N	Mean	Std. Dev.	Pooled Std. Dev = 0.608
BP-1	6	7.65	0.596	
BP-2	6	6.68	0.279	
BP-9	6	8.20	0.822	

A Tukey's test on the data is then computed to determine which groups are different. The least significant range for Tukey's test is

$$\text{LSR} = q_{(0.95,\,2,\,15)} \bullet \sqrt{0.37/6} = 3.01 \bullet 0.248$$
$$= 0.75$$

Any group mean pH which differs by more than 0.75 is significantly different by the Tukey's multiple comparison test. Therefore two piezometer groups are contaminated, significantly higher than the uncontaminated BP-2 group:

BP-9 $\cong$ BP-1 $>$ BP-2

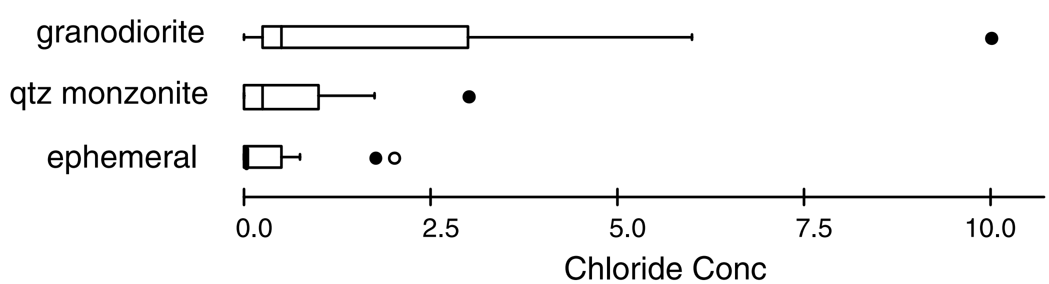
Since the sample sizes are small (n=6 for each group) one might prefer a Kruskal-Wallis test to protect against any hidden non-normality:

GP	N	MEDIAN	$\overline{R}_j$
BP-1	6	7.60	11.3
BP-2	6	6.75	3.6
BP-9	6	8.00	13.6

Overall Median = 9.5

K = 11.59 $\chi^2_{0.95,(2)} = 5.99$. Reject H_0, with p = 0.003.
ANOVA and Kruskal-Wallis tests give identical results.

7.2 Boxplots of the data indicate skewness. Therefore the Kruskal-Wallis test is computed:
K = 7.24 Corrected for ties, K = 7.31. p = 0.027
Reject that all groups have the same median chloride concentration.



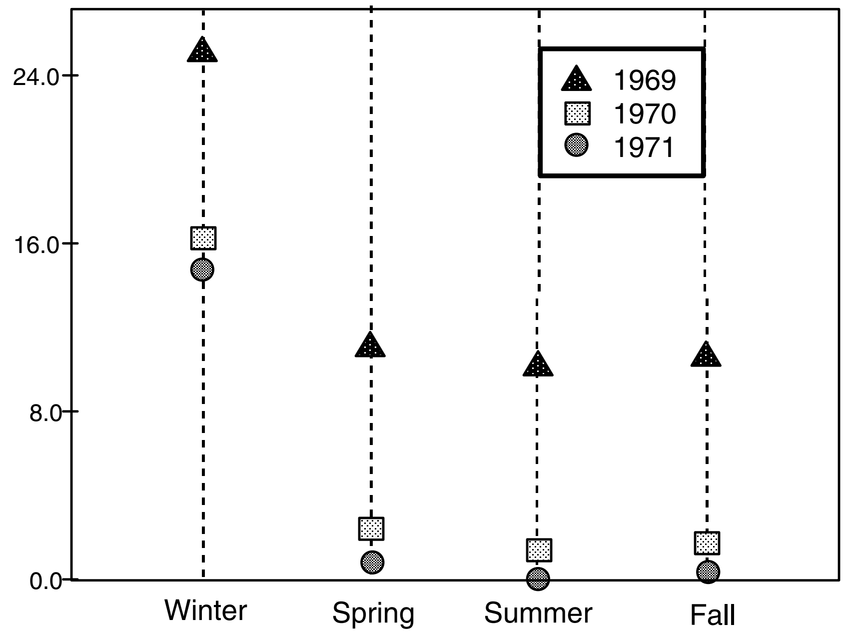
The medians are ranked as granodiorite > qtz monzonite > ephemeral. Individual K-W tests are computed for adjacent pairs at $\alpha = 0.05$:
granodiorite ≅ qtz monzonite (p = 0.086)
qtz monzonite ≅ ephemeral (p = 0.27). So:

granodiorite qtz monzonite ephemeral

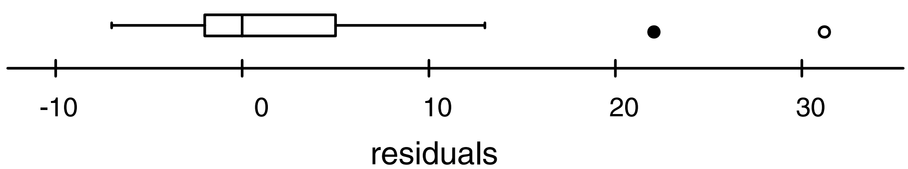
7.3 Median polish for the data of strata 1:

	Winter	Spring	Summer	Fall	Year median
1969	25.25	11.25	10.25	10.75	8.75
1970	16.5	2.5	1.5	2	0.00
1971	15	1	0	0.5	–1.50
Season median	14.25	0.25	–0.75	–0.25	**2.25**

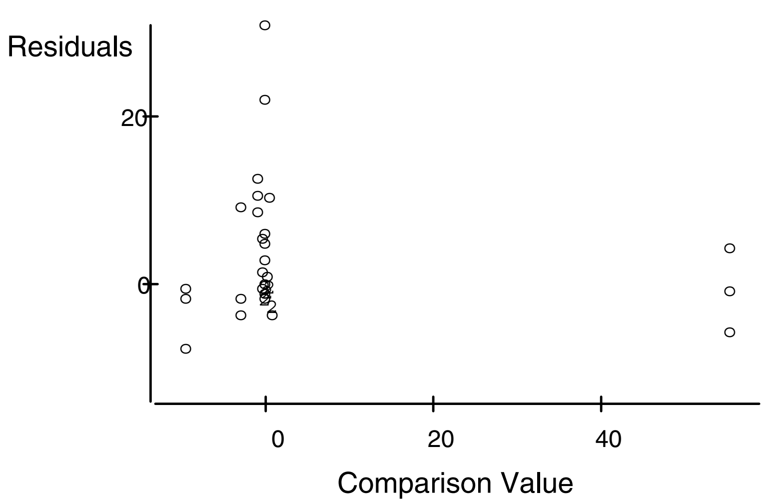
Corbicula densities were 14 units higher in winter than in other seasons, and 9 to 10 units higher in 1969 than 1970 or 1971. Those effects dominated all others. This is shown by a plot of the two-way polished medians:



The residuals are skewed, as shown in a boxplot:



However, a residuals plot of cell residuals versus the comparison value shows outliers, but an overall slope of zero, stating that no power transformation will improve the situation very much.



7.4 Due to the outliers noted above, ranks of the Corbicula data were used to test the effects of season and year on the number of organisms.

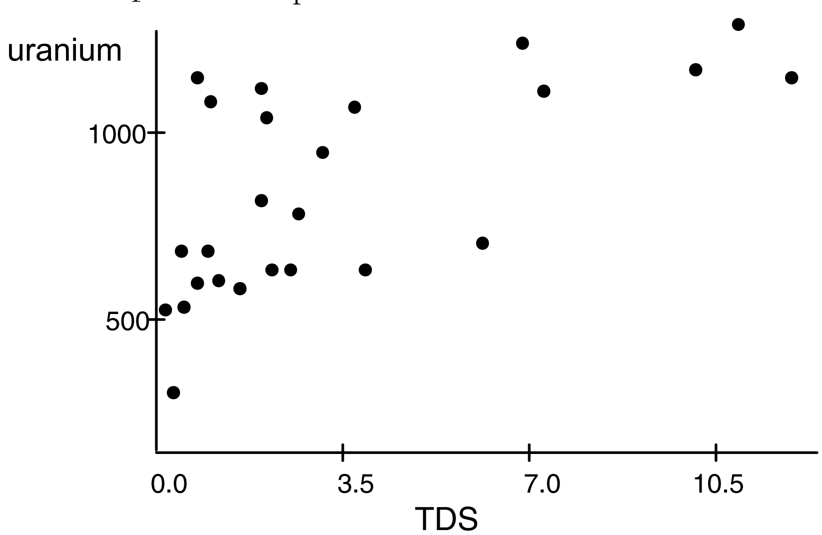
Source	df	SS	MS	F	p-value
Year	2	1064.7	532.33	13.78	0.000
Season	3	1300.7	433.57	11.23	0.000
Year*Season	6	560.8	93.46	2.42	0.057
Error	24	926.8	38.62		
Total	35	3853.0			

A two-way ANOVA on the ranks indicates that both season and year are significant influences on the density of Corbicula, and that there is no interaction. This is illustrated well by the plot of polished medians above.

7.5 Not answered.

Chapter 8

8.1 The plot of uranium versus total dissolved solids looks like it could be nonlinear near the 0 TDS boundary. So Spearman's rho was computed, and rho = 0.72 with t_r = 4.75 and p<0.001.



8.2 Pearson's r = 0.637 with t_r = 3.79 and p<0.001. Kendall's tau = 0.53 with p<0.001. The suggestion of nonlinearity would favor either rho or tau, though the nonlinearity is not serious in this case.

8.3 Not answered.

Chapter 9

9.1 A residuals plot for the untransformed variables shows strong curvature. A log-log regression gives an acceptable plot, with one outlier not influencing the line:

$$\log(\text{Yield}) = 6.74 + 1.39 \bullet \log(\text{Grain Size}) \qquad t= 8.14 \quad p < 0.001$$

9.2 The overall mean yield will be the average of estimates of mean yield for the four wells from the regression equation. Applying the 1/2 s^2 correction factor to obtain the mean yield rather than the median, the estimated mean yields are:
46.104 120.830 316.669 556.380 with overall mean = 260 gal/day/ft^2.

9.3 Here are some possible transformations, including the log. Can logQ be improved on?

explanatory variable	R^2
Q	40.8%
$Q^{0.5}$	51.1%
$\log Q$	57.3%
$Q^{-0.25}$	57.4%
$Q^{-0.5}$	55.4%
Q^{-1}	47.9%
$1/(1+0.00001Q)$	41.8%
$1/(1+0.0001Q)$	47.6%
$1/(1+0.001Q)$	58.5%
$1/(1+0.01Q)$	52.4%
$1/(1+0.1Q)$	48.5%

There are perhaps two other good candidate explanatory variables on this list, $Q^{-0.25}$, and $1/(1+0.001Q)$. Neither improve significantly over logQ, based on R^2 or on residuals plots. A residuals plot and probability plot of residuals for the hyperbolic transformation having b=0.001 are below.

When b=0.00001 or smaller, the model is virtually identical to the linear model TDS = b_0+b_1Q [a power transformation with θ =1]. When b=0.1 or larger, the model is virtually identical to the inverse model TDS = $b_0+b_1(1/Q)$ [a power transformation with θ =−1]. Values of b in between these provide functions similar to moving down the ladder of powers from θ =1 to θ =−1. The advantage of using the hyperbolic function is its interpretability as a mixing of ground and surface waters (Johnson et al., 1969).

9.4 If the objective is to predict LOAD, then that (or its transform) should be the dependent variable. The regression statistics (especially PRESS) will then tell how well the predictions will do. If ln(C) is used as the dependent variable, the standard error s = 0.3394, exactly the same as in the equation for ln(LOAD), but R^2=17.3 % rather than 67.9 % for ln(LOAD). The t statistic on β_1 is −4.43, also significant but not as much as when y = ln(LOAD). In other words, the error of the ln(C) values is exactly the same magnitude as the errors of ln(LOAD). The percent variation explained drops from 67.9 % to 17.3 %, the difference being the strong effect of Q on variation in LOAD. Note the changes in regression coefficients. The previous model was ln(LOAD) = 0.789 + 0.761 ln(Q). This one is ln(C) = −.194 − .239 ln(Q). The intercept decreased by an amount equal to ln(2.7) (the log of the unit conversion coefficient) and the slope decreased by exactly 1 because Q is removed from both sides. The standard errors of the coefficients are both unchanged.

If LOAD were computed by using the regression for ln(C) and then multiplying that result by 2.7 Q, exactly the same estimates would result as when using the equation for ln(LOAD). This is true regardless of which estimation method is employed (median, MLE, or Smearing), and will always be true for log-log regression estimation. The moral of the story is: if your boss thinks that you shouldn't use ln(LOAD) as the dependent variable and you can't convince him or her otherwise, go ahead and predict ln(C), and from that ln(LOAD), and you will still get the results you got doing it the simple way.

Chapter 10

10.1

X	Y	Slopes				
1	10	30	10	15	13	9.2
2	40	−10	7.5	7.33	4	
3	30	25	16	8.67		
4	55	7	0.5			
5	62	−6				
6	56					

Ranked slopes: −10, −6, 0.5, 4, 7, 7.33, 7.55, 8.67, 9.2, 10, 13, 15, 16, 25, 30

a) Median slope = 8.67 = Theil slope estimator $\hat{b}_1$
Median X = 3.5
Median Y = 47.5
S = P −M = 13 − 2 = 11

b) $$\tau = \frac{S}{n(n-1)/2} = \frac{11}{6\bullet 5/2} = 0.73$$

c) intercept $\hat{b}_0 = Y_{med} - \hat{b}_1 \bullet X_{med} = 47.5 - 8.67\bullet 3.5 = 17.17$
Y = 17.17 + 8.67• X is the Kendall-Theil equation
(Y = 10.07 + 9.17• X is the OLS equation for the same data)

d) from table B8, for S=11 and n=6, two-sided p-value = 2•0.028 = 0.056.

10.2

X	Y	Slopes				
1	10	30	10	15	47.5	9.2
2	40	−10	7.5	53.33	4	
3	30	25	85	8.67		
4	55	145	0.5			
5	200	−144				
6	56					

Ranked slopes −144, −10, 0.5, 4, 7.5, 8.67, 9.2, 10, 15, 25, 30, 47.5, 53.33, 85, 145

a) Median slope = 10 = Theil slope estimator $\hat{b}_1$
Median X = 3.5
Median Y = 47.5

b) S and τ are unchanged

c) $\hat{b}_0 = 47.5 - 10 \bullet 3.5 = 12.5$
$Y = 12.5 + 10 \bullet X$ the Kendall-Theil equation is similar to ex. 10.1.
($Y = -8.33 + 21 \bullet X$ the OLS slope has changed a lot from ex. 10.1)

d) the p-value is unchanged.

e) for a 95% confidence interval, the closest entry in table B8 to $\alpha/2=0.025$ is $p=0.028$ for $X_u=11$. From eqs. 10.3 and 10.4,

$$R_u = \frac{(15 + 11)}{2} = 13 \text{ for } N=15 \text{ and } X_u=11.$$

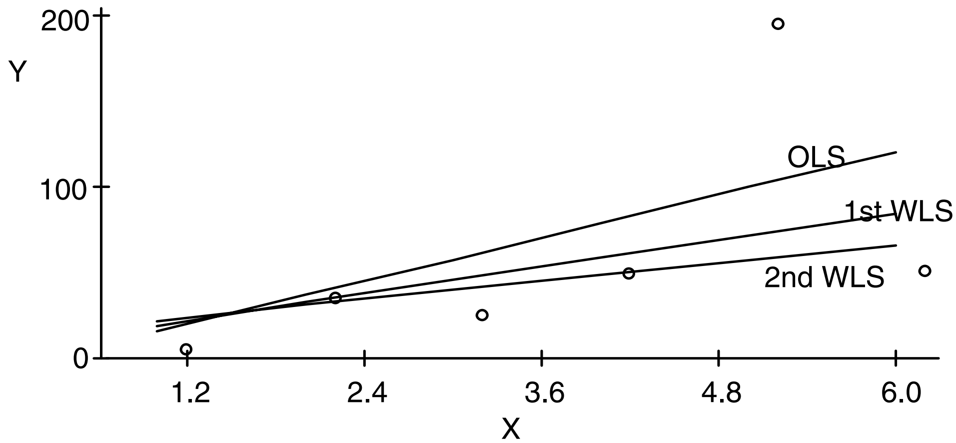
The rank R_l of the pairwise slope corresponding to the lower confidence limit is

$$R_l = \frac{(15 - 11)}{2} + 1 = 3.$$

So an $\alpha = 0.054$ confidence limit for $\hat{\beta}_1$ is the interval between the 3rd and 13th ranked pairwise slope (the 3rd slope in from either end), or

$$0.5 \leq \hat{\beta}_1 \leq 53.3 .$$

10.3 The unweighted OLS regression equation is
$Y = -8.3 + 21.0 \bullet X$ $t = 1.41$ $p = 0.23$



The residuals are then divided by 6•(MAD), where the MAD is the median of the absolute values of the residuals. Bisquare weights are computed for each data point:

pt #	1	2	3	4	5	6
weight	0.999	0.996	0.935	0.954	0.179	0.631

A first weighted least squares is then computed:

$Y = 3.1 + 13.1\ X$ $\quad$ $t = 1.49$ $\quad$ $p = 0.21$

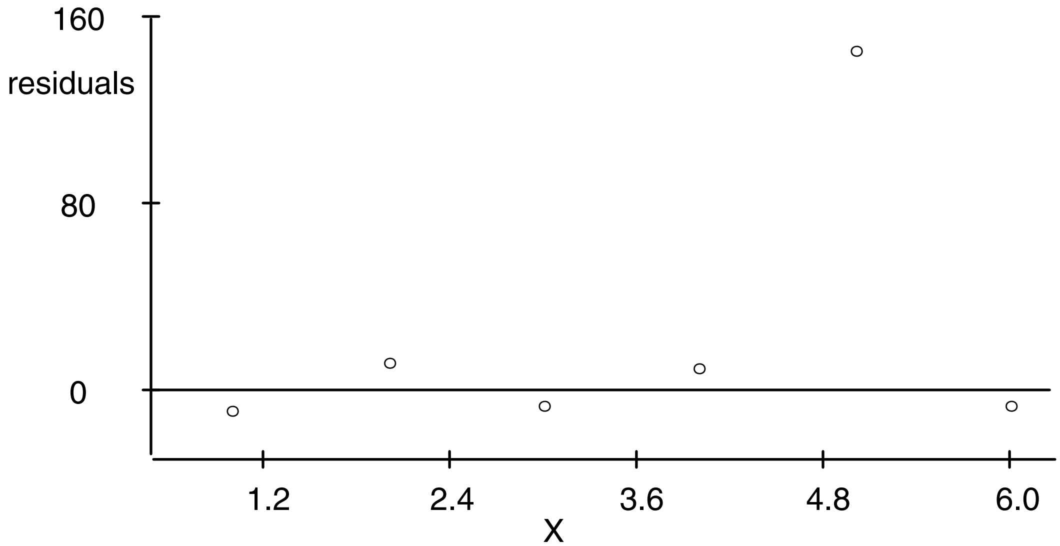
Bisquare weights are again computed for each data point, using residuals from the first WLS:

pt #	1	2	3	4	5	6
weight	0.984	0.952	0.938	1.000	0.000	0.746

A second WLS is then computed:

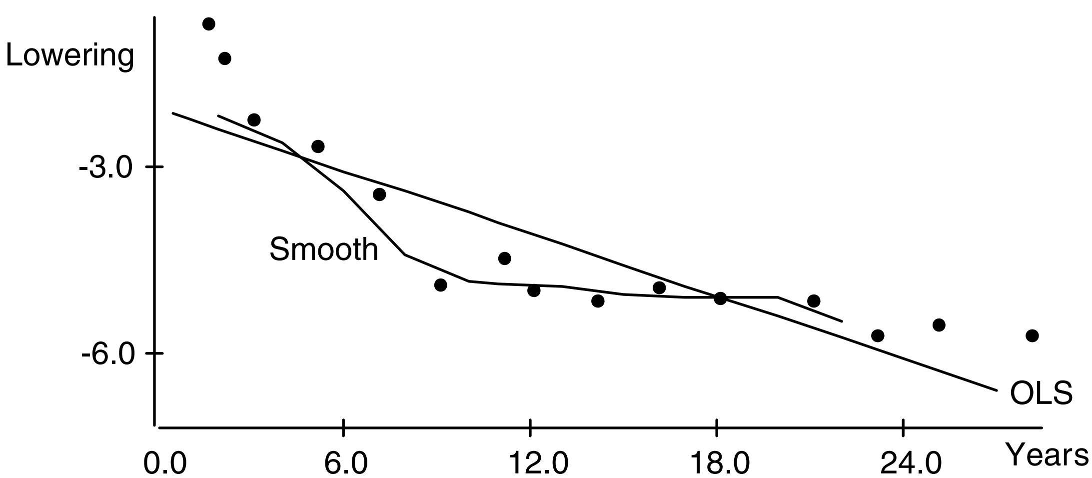
$Y = 10.4 + 8.80\ X$ $\quad$ $t = 2.73$ $\quad$ $p = 0.07$

Though the slope has diminished from the OLS line, the significance has greatly increased due to the lower weight of the outlier. Note the similarity between this WLS and the Kendall's robust line. A residuals plot shows that the WLS line fits most of the data much better than with OLS. The outlier's influence on the slope has diminished, and its residual remains large.



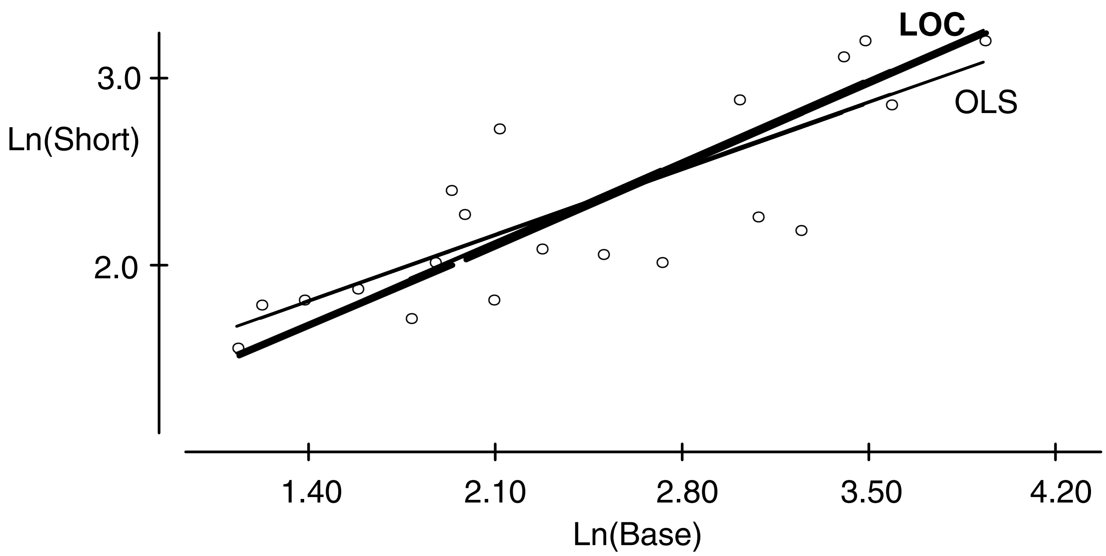
Some object: "Isn't this WLS line the same as throwing away the outlier -- it has a weight of zero?" The difference is that the outlier was determined to be downweighted to zero by the data itself, not an arbitrary decision by the data analyst. Weighted least squares also allows outliers to have partial weights, not simply a zero or one weight as with discarding the outlier. So WLS is far less arbitrary and far more consistent in its assignment of weights to all data points than is throwing away outliers.

10.4 Lowering = –2.07 – 0.167 •Years $\qquad r^2 = 0.76$



OLS does not follow the data as well as the smooth because the data are nonlinear.

10.5 Plotting the 20 years of joint data shows that curvature and heteroscedasticity exist, and transformation is required before regression. Thus the natural logs of both are taken. A linear relation results, as shown in the following plot.



Regression between the 20 year joint record at the two stations is:

Ln(Short) = 1.095 + 0.507•Ln(Base) t = 6.00 $p < 0.001$ $R^2 = 0.67$

Using this equation and the 30 additional years of record at Base, 30 years of simulated flows at Short are generated. Now the LOC is used to generate estimates of the "Short" 30-year record. Summary statistics for the 20 years of joint Ln(Base) and Ln(Short) records are as follows:

	n	Mean	Stdev	Median	P25	P75
Ln(Short)	20	2.319	0.524	2.160	1.862	2.850
Ln(Base)	20	2.414	0.844	2.190	1.802	3.200

From equation 10.10,

$$Y_i = \overline{Y} + \mathrm{sign}[r] \bullet \frac{s_y}{s_x} \bullet (X_i - \overline{X}), \quad \text{or}$$

$$\mathrm{Ln(Short)} = 2.319 + (.524/.844) \bullet (\mathrm{Ln(Base)} - 2.414)$$

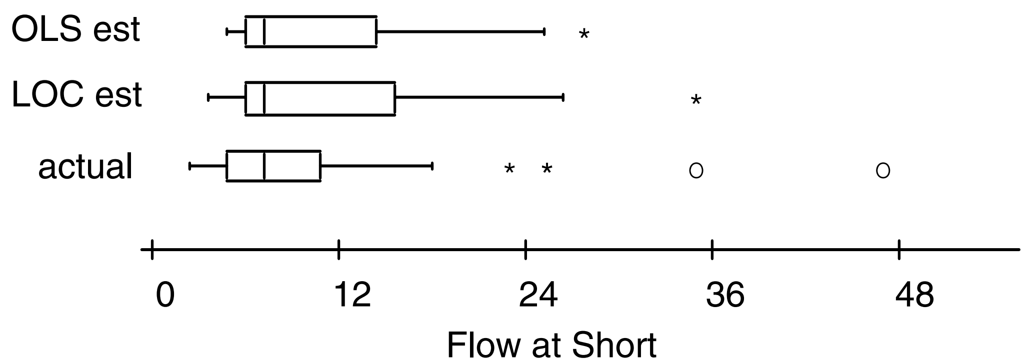
$$= 0.820 + .621 \bullet \mathrm{Ln(Base)}$$

Note how the slope and intercept for LOC differ from the regression coefficients. Summary statistics for the estimated flows at "Short" by the two methods are compared to the true 30-year record from Appendix C13 in the following table.

	n	Mean	Stdev	Median	P25	P75
OLS est.	30	2.2087	0.4975	2.0228	1.7731	2.6249
LOC est.	30	2.184	0.609	1.956	1.651	2.694
true values	30	2.079	0.613	1.930	1.630	2.290

The standard deviation for the regression estimate is too small, as expected.

Boxplots are presented below for three groups: the 30-year estimates using regression and LOC combined with the 20-year record at Short, and the actual 50-year record. LOC comes closer to correctly estimating the lowest and highest flows. The regression estimates are too low for high flows, and too high for low flows. They "regress" toward the mean more than the actual data because the standard deviation of the estimates is too small, as $R^2 < 1$.



10.6 Not answered.

Chapter 11

11.1 The full multiple regression model contains strong multi-collinearity. The VIFs among the four percentage variables are huge:

LOGTN = − 1.3 + 0.596 LOGDA + 0.346 LOGIMP + 0.0314 MMJTEMP
− 0.0494 MSRAIN + 0.040 PRES + 0.035 PNON + 0.037 PCOMM +
0.024 PIND

n = 42   s = 0.61 R^2 = 0.59

Parameter	Estimate	Std.Err(β)	t-ratio	p	VIF
Intercept β_0	−1.28	24.60	−0.05	0.959	
Slopes β_k					
LOGDA	0.596	0.121	4.94	0.000	1.8
LOGIMP	0.346	0.228	1.52	0.138	3.8
MMJTEMP	0.031	0.019	1.65	0.107	10.1
MSRAIN	−0.049	0.021	−2.32	0.026	9.1
PRES	0.040	0.245	0.16	0.873	9227.2
PNON	0.035	0.246	0.14	0.888	3062.2
PCOMM	0.037	0.245	0.15	0.882	8311.4
PIND	0.024	0.246	0.10	0.922	2026.2

Table 11.9 Regression statistics and VIF's for Exercise 11.1

To determine why the multi-collinearity is so strong, the correlation matrix is computed.

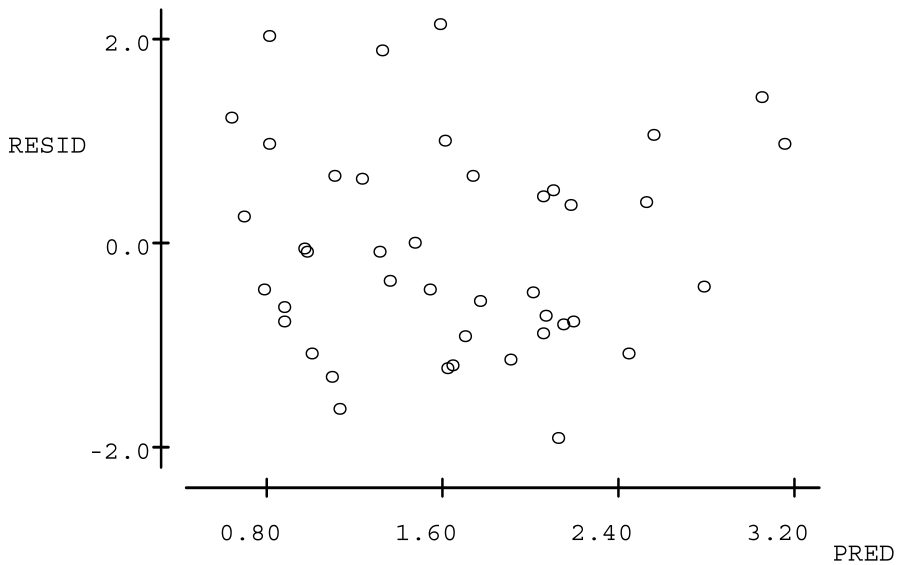
	LOGTN	LOGDA	LOG IMP	MMJ TEMP	MS RAIN	PRES	PNON	PCOMM
LOGDA	0.565							
LOGIMP	0.058	-0.382						
MMJTEMP	-0.205	-0.188	0.094					
MSRAIN	-0.259	-0.083	0.018	0.915				
PRES	0.294	0.210	-0.246	0.040	0.003			
PNON	-0.042	0.319	-0.639	0.066	0.065	-0.321		
PCOMM	-0.218	-0.441	0.589	-0.027	0.039	-0.747	-0.206	
PIND	-0.131	0.060	0.114	-0.111	-0.164	-0.226	-0.124	-0.180

Surprisingly, the percentage terms do not have large pair-wise correlation coefficients. Instead, they are strongly related in that the four of them add to 100%, except for rounding error. This is why the VIFs are so large. Therefore at least one of them should be dropped. The variable with the smallest partial F (PIND) could be chosen. This brings the VIF down from over 9000 to 10, still large. In order to save much time the Cp and PRESS statistics can be computed for all possible models. The results below show that the best 5-variable model, containing LOGDA, LOGIMP, MMJTEMP, MSRAIN, and PIND, is the best in terms of prediction errors (PRESS) and model bias/standard error (Cp). VIFs are below 10 ($R^2 < 0.9$) and so are acceptable.

X: Variable is in the model

# of Vars	R-sq	PRESS	C-p	Max VIF	LOGDA	LOGIMP	MMJTEMP	MSRAIN	PRES	PNON	PCOMM	PIND
1	32.0	22.9	17.0	----	X							
1	8.6	30.0	35.8	----					X			
2	40.7	21.2	11.9	1.2	X	X						
2	37.5	22.2	14.5	1.1	X					X		
3	46.4	20.2	9.3	1.2	X	X			X			
3	45.2	19.8	10.3	1.2	X	X						X
4	51.2	18.9	7.4	1.2	X	X		X				X
4	50.9	19.8	7.7	1.2	X	X		X	X			
* 5	57.0	17.8	4.7	6.8	X	X	X	X				X
5	56.2	18.0	5.4	7.5	X		X	X	X		X	
6	59.1	18.2	5.1	7.1	X	X	X	X	X			X
6	58.6	18.9	5.4	7.1	X	X	X	X		X		X
7	59.1	19.4	7.0	10.0	X	X	X	X	X	X	X	
7	59.1	19.4	7.0	13.2	X	X	X	X	X		X	X
8	59.1	20.1	9.0	9227	X	X	X	X	X	X	X	X

A residuals plot from the minimum PRESS and Cp model shows no hint of curvature or increasing variance. Therefore this model is preferred.



Residuals plot for the regression of Exercise 11.1

11.2 First compare those models which have equal numbers of parameters and eliminate the ones with higher SSE.

Compare 4 to 7 , eliminate 4
Compare 3 to 6 , eliminate 3
Compare 2 to 5 , eliminate 2
Compare 6 to 8 , eliminate 6

Now, for the remaining models (1, 5, 7, 8, 9. 10) perform F tests between pairs of nested models. The order in which to proceed is arbitrary.

Compare	F	df_{num}	df_{denom}	F_{crit}	conclusion
Models 1 to 5	11.18	1	123	3.9	reject H_0,eliminate model 1
Models 5 to 7	0.28	2	121	3.1	do not reject eliminate model 7
Models 5 to 8	1.39	1	122	3.9	do not reject eliminate model 8
Models 5 to 9	0.77	3	120	2.68	do not reject eliminate model 9
Models 5 to 10	0.88	5	118	2.29	do not reject eliminate model 10

Model 5 is the preferred model.

Another possible approach is to use either PRESS or Mallows Cp.

Model	p	s^2	Cp
1	3	0.5636	14.29
2	4	0.5350	8.37
3	5	0.5343	9.17
4	6	0.5359	10.51
5	4	0.5183	4.41
6	5	0.5207	5.98
7	6	0.5245	7.84
8	5	0.5166	5.00
9	7	0.5212	8.06
10	9	0.5208	9.95

The results are interpreted as: the transport curve is quadratic with a shift in intercept for the winter months. Only two seasons (not three) can be distinguished. The slope of the curve does not change with season.

11.3 Not answered.

11.4 Not answered.

Chapter 12

12.1 Regression

load = 25,250 − 12.6 year $\quad r^2 = 10.6\%$

(t) (1.53) (−1.50) two-sided p value = 0.134

Multiple regression

load = 28,152 − 14.4 year + 0.696 q $\quad r^2 = 88.3\%$

(t) (4.69) (−4.60) (10.91) two-sided p value ≅ 0.0001

Mann-Kendall

load = 11,800 − 5.8 year two-sided p value = 0.415

Mann-Kendall on Residuals

Regression model is load = −110 + 0.681 q $r^2 = 74.5\%$

(t) (−1.24) (7.44)

Kendall fit: residual = 28,250 − 14.4 year two-sided p value = 0.0001

therefore load = −110 + 0.681q + residual

= −110 + 0.681q + 28,250 − 14.4 year

= 28,140 − 14.4 year + 0.681q

12.2 Winter: P = 16, M = 34, S = -18
1 tie of 3, 2 ties of 2
Var[S] = 159.33
Z = −1.347
p = 0.18 very little evidence of downtrend in winter lead

Spring: P = 27, M = 38, S = −11
3 ties of 2, 1 tie of 5
Var[S] = 249
Z = 0.633
p = 0.53 no evidence of downtrend in spring lead

Summer: P= 16, M = 33, S = −17
1 tie of 4
Var[S] = 156.33
Z = −1.28
p = 0.20 very little evidence of downtrend in summer lead

Fall: P = 11, M = 37, S = −26
1 tie of 4, 1 tie of 2
Var[S] = 155.33
Z = 2.005
p = 0.045 fairly strong evidence of downtrend in fall lead

Seasonal Kendall: S = −72
VAR[S] = 720
Z = −2.646
p (2-sided)= 0.008

Thus, even though the evidence from no individual season was highly conclusive, the data from all seasons taken together provides highly conclusive evidence of a down-trend in lead.

12.3 Maumee River Trends in Total Phosphorus

12.3.1 Parametric analysis first: LOAD vs TIME
Simple linear regression: LOAD = 444 – 0.221 TIME
t = -0.42 p = 0.673
s = 20.59 R-sq = 0.1% R-sq(adj) = 0.0%

A boxplot of the residuals shows them to be terribly skewed. A transformation is required. Try logarithms. Then the regression equation is:

ln(LOAD) = 117 – 0.0592 TIME

t = –1.32 p = 0.189

s = 1.770 R-sq = 1.3% R-sq(adj) = 0.6%

There is a fairly normal distribution of residuals, so a test based on regression seems legitimate. Very weak evidence of trend -- (two-sided) p-value of 0.189. But are there strong flow and/or seasonal effects? A plot of the residuals versus log of streamflow (LQ) shows a strong dependence on flow. Removing this should greatly enhance the power to detect any trend which is present.

```
          -                                                 *   *
      2.4+                                                     *   *
          -
 RESID    -                                              **    *
          -                                              *
          -                                          2*   2
      1.2+                                          26 ***
          -                                    * *2*
          -                              * * 22 2 ***
          -                              * ** * 2*
          -                      **   *2* 2 *
      0.0+                         ***3***2
          -                   * *   *2* **    2
          -                 *  33*2 *2*
          -           * * ** 2   2     *
          -             2 43*      *
     -1.2+       **2**4**  *
          -     *2  2 **      *
          -       *
            --------+---------+---------+---------+---------+--------
lnQ
               -1.2       0.0       1.2       2.4       3.6
```

Boxplots of residuals by month also show a strong seasonal cycle, high in the winter & spring, low in summer. The best model we could find includes time, ln(Q), ln(Q)^2, and sine and cosine of $2\pi T$:

LLOAD = 83.3 – 0.0425 TIME + 1.08 lnQ + 0.0679 ln(Q)^2 – 0.0519 SIN + 0.141 COS

Predictor	Coef	Stdev	t-ratio	p
Constant	83.32	22.06	3.78	0.000
TIME	-0.04250	0.01115	-3.81	0.000
lnQ	1.08175	0.04947	21.87	0.000
ln(Q)^2	0.06789	0.01868	3.63	0.000
SIN	-0.05190	0.06252	-0.83	0.408
COS	0.14058	0.05441	2.58	0.011

s = 0.4398 R-sq = 94.1% R-sq(adj) = 93.9%

This is interpreted as a strong evidence of downtrend, with a p-value <.001 The slope (in log units) = –0.0425 per year. All coefficients are significant at $\alpha = 0.05$ except for sine(2πT). The sine must either be left in, or both it and the cosine taken out. To test whether together they are significant, an F test is performed. The model without these terms, with the standard error s = 0.449, is:

LLOAD = 85.1 – 0.0434 TIME + 1.06 LQ + 0.0748 LQSQ

Predictor	Coef	Stdev	t-ratio	p
Constant	85.09	22.51	3.78	0.000
TIME	-0.04340	0.01138	-3.81	0.000
lnQ	1.05921	0.04518	23.44	0.000
$\ln(Q)^2$	0.07481	0.01865	4.01	0.000
s = 0.4490	R-sq = 93.7%		R-sq(adj) = 93.6%	

So the F test to compare these two models is:

$$F = \frac{(26.01 - 24.57) / 2.0}{0.193} = 3.73$$

Comparing to an F distribution with 2 and 127 degrees of freedom, the two-sided p-value is 0.027. Therefore reject the simpler model in favor of including the seasonal sine and cosine terms.

To predict estimates of load for the two times and two flow conditions above, natural logs of these values are input to the regression equation. The third column below reports the predicted logs of Load from the regression equation.

lnQ	Time	Predicted lnL	Bias-Corrected L
2.4	1972.5	2.3356	11.3852
0.0	1972.5	-0.6516	0.574136
2.4	1986.5	1.7406	6.27963
0.0	1986.5	-1.2467	0.316640

These predicitons must be transformed and corrected for bias. Using the Ferguson (MLE) bias correction, $0.5 \bullet s^2 = 0.5 \bullet (0.4398)^2 = 0.097$. So the bias correction equals exp(0.097), or about 10%. The four predicted total phosphorus loads are given above in the fourth column.

Therefore the percent change at high flow over the 14-year time period is:

$(6.2763 - 11.3852) / 11.3852 = -0.448732$

The change in percent per year is

$-0.448732 \bullet 100 / 14 = -3.205$. That is a –3.2% change in total P per year.

The same analysis at lower flow over the 14 years is:

(0.31664 – 0.574136) / 0.574136 = –0.4485, the same amount as at high flow.

Re-expressing the slope estimate in original units as a percent change, the average change equals –4.2% per year:

100 • [exp (–0.0425) – 1.0] = –4.16096

12.3.2 The nonparametric approach

The seasonal-Kendall test on the original observations, using 12 seasons (months): Tau = –0.06 with a p-value of 0.3835.

log(Load) = 0.505 – 0.046 •time ,

where time = 0 at the begining of the first year of the record (typically a water year), and time is in units of years.

Residuals from a regression of log(Load) versus logQ and $\log Q^2$ removes the effect of flow:

$$\log(\text{Load}) = -0.745 + 1.06 \log Q + 0.0758 \log Q^2$$

The S-K test on the regression residuals: tau = –0.25 with p = 0.0002

and log(Load) = 0.312 – 0.048 •time

So, if flow is not first removed, the significant trend would be missed. Both the Seasonal Kendall on the residuals and multiple regression give a highly significant p-value. The S-K slope is 4.8% rather than 4.16% because of the effect of the low residuals during 1972-1977.

<u>Residuals plot from regression with logQ and $\log Q^2$</u>

```
         -
 RESID   -
         -    *              *
         -            *           *
      1.0+    *
         -         *
         -     *2    *  *  *          *   *
         -        2* **  32      *  **   *  * **  **
         -      *     * * *2*2  2    * ***   *  *  *     *
      0.0+       ** 3  2    *  2 * **  *   ** ***2  ** *  *  *   *
         -     * *      *     2***** ** *** *2     **   *  *  *
         -       *            *   * * *** *2  ***           *  *  *
         -                  *                     2            *   *
         -    **   *   *                         *  * **
     -1.0+
         -           *
         -                         *
         -
         -
          ----+---------+---------+---------+---------+---------+--
TIME
          1972.5    1975.0    1977.5    1980.0    1982.5    1985.0
```

12.4 Not answered.

12.5 Nat answered.

Chapter 13

13.1 Not answered

13.2 Because there is only one reporting limit, Kendall's tau can easily be computed for this data: tau = −0.40 with p = 0.023. There is a significant decrease in TPT concentrations with depth.

13.3 Estimates of the four descriptive statistics for each of 5 multiple-threshold methods (see Helsel and Cohn, 1988) are:

Method	MEAN	ST.DEV.	MEDIAN	IQR
ZE (substitute zero)	12.36	75.48	0.00	1.10
HA (substitute 1/2 dl)	13.91	75.28	1.10	3.30
DL (substitute the dl)	15.45	75.19	1.30	4.10
MR (prob plot regression)	**12.57**	**75.44**	0.29	1.54
MM (lognormal MLE)	8.30	61.52	**0.34**	**1.62**

Because of the outlier at 560 μg/L the data have more skewness than a lognormal distribution, and methods which assume a lognormal distribution for all the data (MM) would not be expected to estimate moment statistics well. It is not surprising therefore that the MLE method produces moment estimates dissimilar to the other methods. We generally select the MR moment estimates and the MM quantile estimates (those printed in bold), due to the results of Helsel and Cohn (1988).

Chapter 14

14.1 a) Contingency table

Expected values E_{ij}

Trend in Cl^- (1974-81, α=0.1)

Δ road salt appl.	Down	No trend	Up	Totals
Down	5.44	23.84	16.71	46
No change	9.82	43.02	30.15	83
Up	13.73	60.13	42.14	116
Totals	29	127	89	**245**

Table of $\frac{(0\text{-}E)^2}{E}$

Trend in Cl^- (1974-81, α=0.1)

Δ road salt appl.	Down	No trend	Up	Totals
Down	0.04	2.79	3.56	
No change	1.78	0.02	0.88	
Up	1.01	1.39	3.93	

$\chi^2 = 15.39$ df = 4 p = 0.004

The results indicate that the category of chloride trends is dependent on the category of salt applications, with a p-value of 0.004. Where increases in road salt occurred, there are more up trends and fewer down trends than would be expected from the marginal distributions of up trends and down trends. Where decreases in road salt occurred, there are fewer up trends than would be expected.

b) Kendall's tau

P = no. pluses = 5(44+25+51+55) + 32(25+55) + 14(51+55) + 44(55) = 7339

M = no. minuses = 32(14+10) + 9(14+44+10+51) + 44(10) + 25(10+51) = 3804

S = 7339 - 3804 = 3535.

$$\tau_b = \frac{3535}{\dfrac{\sqrt{(245^2 - (46^2 + 83^2 + 116^2))\,(245^2 - (29^2 + 127^2 + 89^2))}}{2}}$$

$$= 3535 / 18164 = 0.19$$

To test for significance,

$$\sigma_S \cong \sqrt{\frac{1}{9} * \left(1 - \left(.19^3 + .34^3 + .47^3\right)\right) * \left(1 - \left(.12^3 + .52^3 + .36^3\right)\right)}$$

$$= \sqrt{\frac{(0.85 * 0.81 * 245^3)}{9}} = 1061$$

and so Z_S = 3534/ 1061 = 3.33 and two-sided p= 0.0008.

The two variables are significantly and positively related.

c) Kendall's tau is more appropriate because

1. It includes the information that the variables are ordinal into the test. The p-value for Kendall's tau is lower than that for the contingency table, reflecting this additional information.
2. It provides a measure of the direction of association τ_b. Since τ is positive, the trends in Cl^- increase with increasing trends in road salt application.

14.2 Based on the tables below, there is a significant association between location of the well and the probability of detecting volatiles. The more protected downdip wells indeed have less chance of being contaminated than do the outcrop wells.

Expected values E_{ij}

Location	Non-detects	Detect VOC	Totals
Downdip	91.92	23.08	115
Outcrop	143.08	35.92	179
Totals	235	59	**294**

Table of $\frac{(0\text{-}E)^2}{E}$

Location	Non-detects	Detect VOC
Downdip	2.16	8.59
Outcrop	1.39	5.52

$\chi^2 =$ 17.647 df = 1 p = <0.0001

14.3 To test for association, Kendall's tau-b is used because both time and concentration variables are ordinal. Computations are shown in the boxes below.

Year	TBT Concentration ≤ 200	TBT Concentration > 200	A_i	a_i
1988	2	7	9	0.18
1989	9	13	22	0.43
1990	10	10	20	0.39
C_j	21	30	**51**	
c_j	0.41	0.59		**1.0**

The number of pluses P = 2(13+10) + 9(10) = 136
The number of minuses M = 7(9+10) + 13(10) = 263
So S = 136 – 263 = –127.

To compute the denominator of τ_b,

$$SS_a = 9^2 + 22^2 + 20^2 = 965.$$

$$SS_c = 21^2 + 30^2 = 1341.$$

and

$$\tau_b = \frac{-127}{\frac{\sqrt{(51^2 - 965)(51^2 - 1341)}}{2}} = \frac{-127}{717.9} = -0.18.$$

From equation 14.9 the approximate value of σ_S is

$$\sigma_S \cong \sqrt{\frac{1}{9} \bullet \left(1\text{-}(0.18^3+0.43^3+0.39^3)\right) \bullet \left(1\text{-}(0.41^3+0.59^3)\right) \bullet 51^3}$$

$$\cong \sqrt{\frac{(0.86) \bullet (0.73) \bullet 51^3}{9}} \quad = \sqrt{9253} \quad = 96.2$$

$$Z_S \cong \frac{-127+1}{96.2} = -1.31$$

and from a table of the normal distribution the one-sided p-value is $p = 0.095$. Therefore H_O: $\tau_b = 0$ is not rejected at α=0.05, but is for $\alpha = 0.10$. Thus there is weak evidence of a downtrend in TBT concentrations based on a split at 200 ng/L. Stronger evidence could be obtained by collecting data for subsequent years, or by obtaining better resolution of the data (the original data reported concentration values rather than a split at 200 ng/L).

Chapter 15

15.1 Logistic regression for the full model with four explanatory variables gives:

Variable Name	Parameter Estimate	Standard Error	Wald's t	p-value
Constant	−13.20539	3.55770	−3.71	0.0002
Thick	0.51527	0.15093	3.41	0.0004
Yields	0.42909	0.27484	1.56	0.0607
GW Qual	0.03035	0.32460	0.09	0.4642
Hazard	1.08952	0.29860	3.65	0.0002

with a likelihood ratio $lr_O = 49.70$ and $p<0.000$ as compared to the intercept-only model. However, two of the variables (Yields and GW Qual) have insignificant t-statistics. In the following model they are dropped, and lr_O recomputed:

Variable Name	Parameter Estimate	Standard Error	Wald's t	p-value
Constant	−10.89039	2.43434	−4.47	<0.0001
Thick	0.46358	0.13575	3.41	0.0004
Hazard	1.07401	0.28301	3.80	0.0001

with a likelihood ratio $lr_O = 52.54$ and $p<0.000$ as compared to the intercept-only model. The partial likelihood ratio to test whether the first model is significantly better than the simpler second model is:

$$lr = lr_{O(simple)} - lr_{O(complex)} = 52.54 - 49.70 = 2.84$$

which for a chi-square distribution with 2 degrees of freedom gives:

$p = 0.242.$

Therefore the two additional variables (Yields and GW Qual) do not appreciably add to the explanatory power of the model.

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