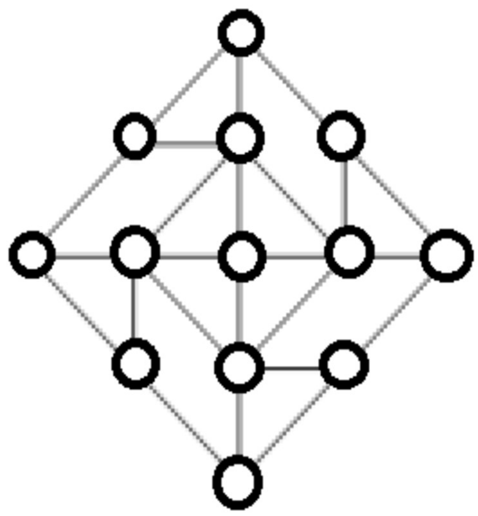
1. Constraint Satisfaction Problems (4 parts, 50 points total).
Consider the problem of coloring each node of this 13-node graph with one of three colors (**R**, **G**, **B**):



a) (5 points) CSP Graph Representation. Specify what the edges (links or neighborhood function) and vertices (nodes) in the above CSP mean.

Ans: In above CSP, edges mean binary constraints and nodes mean variables.

b) (20 points) CSP Methods. Suppose Node 1 (top) is colored **Blue**. Define **all** of the following and **choose one** to illustrate with an example using the above graph.

i) *Most constrained variable / Minimum remaining values (MRV) heuristic* for variable selection
ii) *Least constraining value* for value ordering
iii) *Forward checking* for speeding up constraint checking

i) *Most constrained variable / Minimum remaining values (MRV)* heuristic for variable selection

Ans: The variable which is most likely to cause failure is assigned first. If the variable has higher chances of failure and we are supposed to assign variable then, it's better to assign early and find out failure earlier. For ties, the degree heuristic is used to select the node with the most constraints on remaining nodes

ii) *Least constraining value* for value ordering

Ans: For a given variable, choose its value as the one which leaves fewest choices for neighboring variables and maximum flexibility for future variables.

iii) *Forward checking* for speeding up constraint checking

Ans: Forward checking detects the inconsistency which will lead to failure earlier compare to simple backtracking. It keeps track of remaining possible values for unassigned nodes. If does not have any possible values, then it terminates the algorithm.

I have used forward checking illustration. PFA pdf for solution.

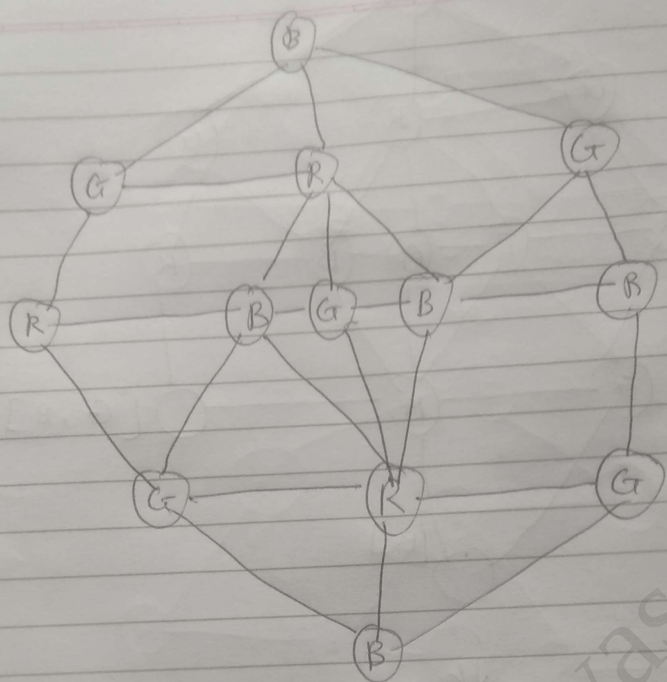
c) (10 points) AC-3. Explain in your own words how to use a table to store coloring constraints and show how it is updated using the AC-3 algorithm for the above graph. Give enough details to distinguish AC-3 from forward checking.

Ans: 1) AC-3 algorithm propagates constraints to neighboring node, when value for current node is selected.
2) AC-3 algorithm keep revising domain, i.e. removing inconsistent values with respect to neighbors until domain is empty. This avoids going down the path which does not have legal value.
3) Forward checking on the other hand, does not terminate search until no legal value is left for the node.

4) A table with n columns (one for each node) can store the coloring constraints by letting each cell in the initial row be the value set (R,G,B). Then, each row after would choose a value for the selected node and then remove that value from the value set of any neighboring nodes.

d) (15 points) 3-Coloring. Show that the graph **is** 3-colorable by finding a 3-coloring consistent with part (b). (Deciding 3-colorability, i.e., whether a graph is 3-colorable, is actually NP-complete.) You need not use any of the above heuristics, but they should help.

Ans: Graph is 3-colorable using forward checking. PFA pdf for complete solution.



3 - Colorable graph.