CS 221 Assignment #7 CAR (Written Part)

Thomas Wentao Jiang

TOTAL POINTS

18 / 25

QUESTION 1

1. Bayesian Network Basics 7 pts

1.1 **2 / 2**

- ✓ **- 0 pts** Correct
- **- 1 pts** Only shows what it is proportional to but do not normalize. Correct answer should be \eta.
- **- 1 pts** Calculation error, and the correct answer should be \eta.
- **- 1 pts** Did not simplify with \eta and \epsilon.
- **- 1 pts** Incorrect inference.
- **- 0.1 pts** Answer untagged
- **- 2 pts** Blank answer/ Incorrect.

1.2 **2 / 2**

- ✓ **- 0 pts** Correct
- **- 1 pts** Partially correct
- **- 2 pts** Incorrect or missing

1.3 **3 / 3**

- ✓ **+ 1 pts** (i) correct
- ✓ **+ 1 pts** (ii) correct
- ✓ **+ 1 pts** (iii) correct
- **+ 0 pts** Incorrect/missing
- **+ 0 pts** Point adjustment
- **- 0.5 pts** Need to Show More Work

QUESTION 2

5. Which car is it? 17 pts

2.1 **5 / 5**

- ✓ **- 0 pts** Correct
- **- 1 pts** Minor calculation error
- **- 2 pts** Major calculation erro
- **- 2 pts** Product over possible assignments instead of sum
- **- 3 pts** Assumes both assignments are equivalent
- **- 3 pts** Doesn't consider both assignments
- **- 5 pts** Incorrect

2.2 **4 / 4**

- ✓ **- 0 pts** Correct
- **- 4 pts** No Attempt
- **- 4 pts** Incorrect
- **- 2 pts** Does not allude to same prior probability

2.3 **2 / 2**

- ✓ **- 0 pts** Correct
- **- 1 pts** correct treewidth, incorrect/missing explanation
- **- 2 pts** incorrect / missing answer

2.4 (Extra Credit) **0 / 6**

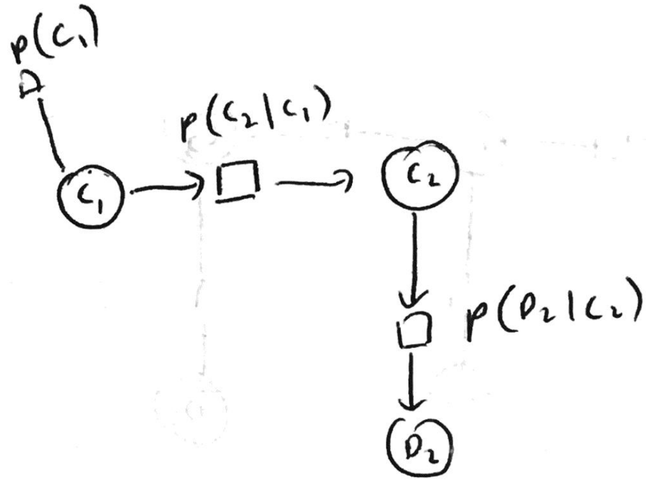
- **- 0 pts** Correct
- **- 3 pts** Partially correct
- **- 5 pts** Worthy attempt.
- ✓ **- 6 pts** Not found / Totally incorrect

QUESTION 3

3 Extra Credit (Typed) 0 / 1

- **+ 1 pts** Typeset
- ✓ **+ 0 pts** Not typeset

CS221 Thomas Jiang Car.

1a) 

$$\therefore P(C_2 = c_2,\ D_2 = d_2) = \sum_{c_1} p(c_1)\, p(c_2|c_1)\, p(d_2|c_2)$$

$$\therefore P(C_2 = 1 \mid D_2 = 0) = \frac{\sum_{c_1} p(c_1)\, p(c_2 = 1|c_1)\, p(d_2 = 0|c_2 = 1)}{P(D_2 = 0)}$$

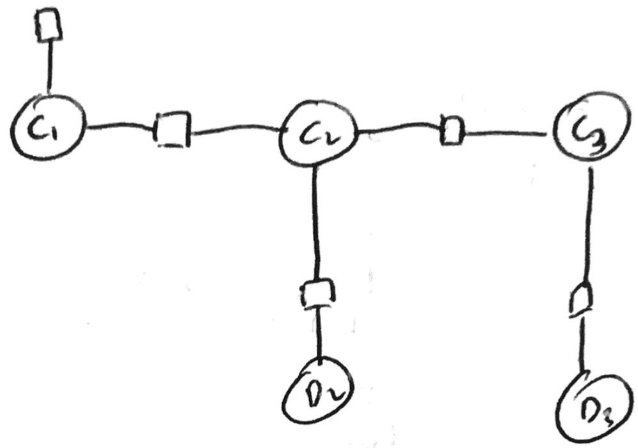
$$= \frac{\cancel{0.5}(\epsilon\mu + (1-\epsilon)\mu)}{\cancel{0.5}[\epsilon\mu + (1-\epsilon)\mu] + \cancel{0.5}[1 - \epsilon - \mu + \epsilon\mu + \epsilon - \epsilon\mu]}$$

$$= \frac{\mu}{\mu + 1 - \mu} = \underline{\underline{\mu}}.$$

1.1 2 / 2

- ✓ **- 0 pts** Correct
- **- 1 pts** Only shows what it is proportional to but do not normalize. Correct answer should be \eta.
- **- 1 pts** Calculation error, and the correct answer should be \eta.
- **- 1 pts** Did not simplify with \eta and \epsilon.
- **- 1 pts** Incorrect inference.
- **- 0.1 pts** Answer untagged
- **- 2 pts** Blank answer/ Incorrect.

1b)



$$p(C_2, D_2, D_3) = \sum_{c_1, c_3} p(c_1)\, p(c_2 | c_1)\, p(d_2 | c_2)\, p(c_3 | c_2)\, p(d_3 | c_3)$$

let $D_2 = 0$
$D_3 = 1$

$$P(C_2 = 1 | D_2, D_3) = \frac{P(C_2 = 1, D_2, D_3)}{P(D_2, D_3)}$$

$$= \frac{P(C_2 = 1, D_2, D_3)}{\sum_{C_2} P(C_2, D_2, D_3)}$$

$$= \frac{0.5\Big[\overset{c_1=0 \;\; c_3=0}{\varepsilon\varepsilon\mu\mu} + \overset{c_1=0 \;\; c_3=1}{\varepsilon(1-\varepsilon)\mu(1-\mu)} + (1-\varepsilon)\varepsilon\mu\mu + (1-\varepsilon)^2\mu(1-\mu)}{\sum_{C_2} P(C_2, D_2=0, D_3=1)}$$

$$= \frac{0.5\left[\cancel{\epsilon^2\mu^2} + \epsilon\mu - \cancel{\epsilon^2\mu} - \cancel{\epsilon\mu^2} + \cancel{\epsilon^2\mu^2} + \cancel{\epsilon\mu^2} - \cancel{\epsilon^2\mu^2} + \mu\left[1 - 2\epsilon \cancel{+\epsilon^2} - \mu + 2\epsilon\mu - \cancel{\epsilon^2\mu}\right]\right]}{\sum_{C_2} P(C_2, D_2, D_3)}$$

$$= \frac{0.5\left[\mu - \mu^2 - \epsilon\mu + 2\epsilon\mu^2\right]}{\sum_{C_2} P(C_2, D_2 = 0, D_3 = 1)}$$

$$= \frac{\eta - \eta^2 - \epsilon\eta + 2\epsilon\eta^2}{\epsilon + 2\eta - 2\eta^2 - 4\epsilon\eta + 4\epsilon\eta^2}$$ from Wolfram alpha

(I realised I have been writing μ instead of η)

1c) i) $P(C_2 = 1 \mid D_2 = 0) = \eta = 0.2$

$P(C_2 = 1 \mid D_2 = 0, D_3 = 1) = 0.4157$ (4sf)

ii) We know $P(C_2 = 1 \mid D_2 = 0, D_3 = 1) > P(C_2 = 1 \mid D_2 = 0)$.

This is as expected since there is a low error rate of η, which means more likely that car is at position 1 at $t = 3$, which in turn increases chances that car at position 1 at time 2 since $\epsilon < \eta$. $\therefore$ more likely to have car at position 1 at time 2 given $D_3 = 1$.

1.2 2 / 2

- ✓ **- 0 pts** Correct
- **- 1 pts** Partially correct
- **- 2 pts** Incorrect or missing

$$= \frac{0.5\left[\epsilon^2\mu^2 + \epsilon\mu - \cancel{\epsilon^2\mu} - \cancel{\epsilon\mu^2} + \cancel{\epsilon^2\mu^2} + \cancel{\epsilon\mu^2} - \cancel{\epsilon^2\mu^2} + \mu\left[1 - 2\epsilon + \cancel{\epsilon^2} - \mu + 2\epsilon\mu - \cancel{\epsilon^2\mu}\right]\right]}{\sum_{C_2} P(C_2, D_2, D_3)}$$

$$= \frac{0.5\left[\mu - \mu^2 - \epsilon\mu + 2\epsilon\mu^2\right]}{\sum_{C_2} P(C_2, D_2 = 0, D_3 = 1)}$$

$$= \frac{\eta - \eta^2 - \epsilon\eta + 2\epsilon\eta^2}{\epsilon + 2\eta - 2\eta^2 - 4\epsilon\eta + 4\epsilon\eta^2}$$ from Wolfram alpha

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1c iii) We want $P(C_2 = 1 \mid D_2 = 0) = P(C_2 = 1 \mid D_2 = 0, D_3 = 1)$

$$\therefore \eta = \frac{\eta - \eta^2 - \epsilon\eta + 2\epsilon\eta^2}{\epsilon + 2\eta - 2\eta^2 - 4\epsilon\eta + 4\epsilon\eta^2}$$

$$\therefore \epsilon + 2\eta - 2\eta^2 - 4\epsilon\eta + 4\epsilon\eta^2 = 1 - \eta - \epsilon + 2\epsilon\eta$$

$$2\epsilon + 3\eta - 2\eta^2 - 6\epsilon\eta + 4\epsilon\eta^2 = 1$$

$$\epsilon(2 - 6\eta + 4\eta^2) = 1 - 3\eta + 2\eta^2$$

$$2\epsilon = \frac{1 - 3\eta + 2\eta^2}{1 - 3\eta + 2\eta^2} = 1$$

$$\therefore \epsilon = \frac{1}{2}$$

This makes sense as it ~~then~~ no longer depends with respect to $p(d_2 = 0 \mid c_2 = 1)$ since they are held constant.

We instead look for car positions ~~with~~ conditioned on each other.

$\therefore$ car must have equal chance of moving or staying.

$\therefore$ Adding extra info for 3rd sensor doesn't change probability of C_2 position.

1.3 **3 / 3**

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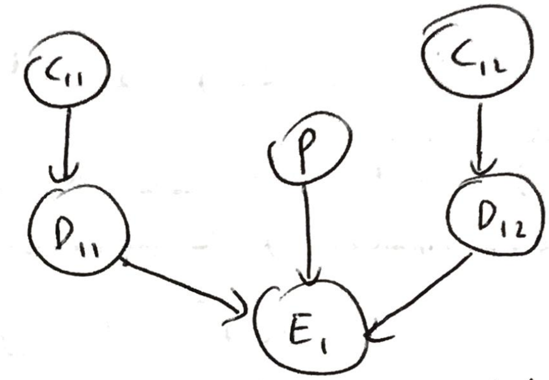
5a)



which are distance readings

P is a permutation of E with uniform distribution.

There are two permutations $\{D_{11}, D_{12}\}$ and $\{D_{12}, D_{11}\}$

$$P(C_{11}, C_{12} \mid E_1 = e_1)$$

$$\cancel{P(C_{11} = c_{11}, C_{12} = c_{12} \mid E_1 = e_1) + P(C_{11} = c_{12}, C_{12} = c_{11})}$$

$$= P(C_{11}, C_{12} \mid D_{11} = d_{11}, D_{12} = d_{12}) + P(C_{11}, C_{12} \mid D_{11} = d_{12}, D_{12} = d_{11})$$

$$= p(c_{11})\,p(d_{11} \mid c_{11})\,p(c_{12})\,p(d_{12} \mid c_{12})$$
$$+ \, p(c_{11})\,p(d_{12} \mid c_{11})\,p(c_{12})\,p(d_{11} \mid c_{12})$$

$$= p(c_{11})\,p_N(e_{11}; \|a_1 - c_{11}\|, \sigma^2)\,p(c_{12})\,p_N(e_{12}; \|a_1 - c_{12}\|, \sigma^2)$$
$$+ \, p(c_{11})\,p_N(e_{12}; \|a_1 - c_{11}\|, \sigma^2)\,p(c_{12})\,p_N(e_{11}; \|a_1 - c_{12}\|, \sigma^2)$$

2.1 5 / 5

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5b) We know that $p(c_{ii})$ is all the same for any i.
We also realise that there are $K!$ permutations for K cars as alligned by the random variable.

Thus there are at least $K!$ allignments for K cars that obtain max $P(C_{11} = c_{11} \dots C_{1n} = c_{1n})$ since we know every $p(c_{ii})$ is the same.

5c) The treewidth is the maximum arity of the factor graph created through variable elimination with best variable ordering.

For our example this would be $= K$ since we start at node E_{1t} and find K factors for all the noisy distances.

If we continue down, eliminating nodes, we find max arity is still K which is our tree width.

2.2 4 / 4

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- **- 4 pts** No Attempt
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