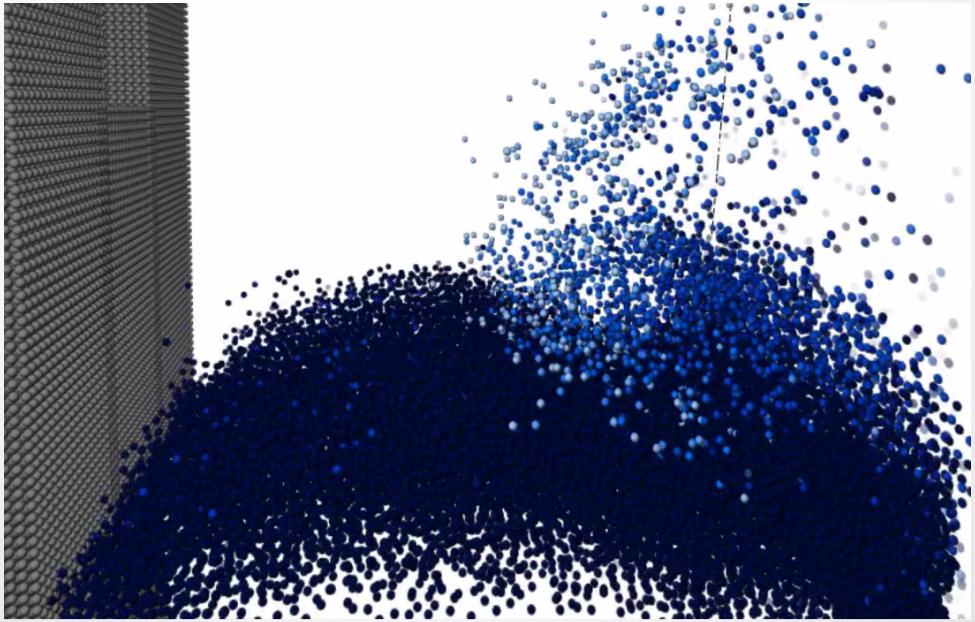
Smoothed Particle Hydrodynamics



SPH Fluids

Each particle is treated as a discrete element of fluid.

React to other particles in a finite radius.

Reaction amount weighted by kernel function

Simulation Algorithm

Each Timestep:

1. Update density and pressure for all particles
2. Update acceleration for all particles
3. Integrate acceleration to find new position of all particles

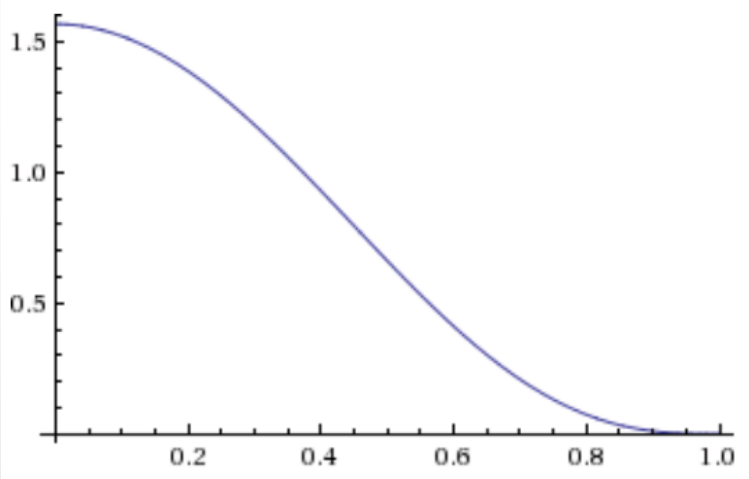
Computing Density and Pressure

Density: $$p_i = \sum_j m_j W_{ij}$$

Pressure: $$P = K(p - p_0)$$

Poly6 Smoothing Kernel

$$W_{ij} = \frac{315}{64\pi h^9}(h^2 - r^2)^3$$



Computing Acceleration

Acceleration due to pressure, density and mass of other particles

$$a_i = -\sum_j \frac{m_j}{m_i} \frac{P_i + P_j}{2 p_i p_j} \bigtriangledown W_{ij} \hat{r_{ij}}$$

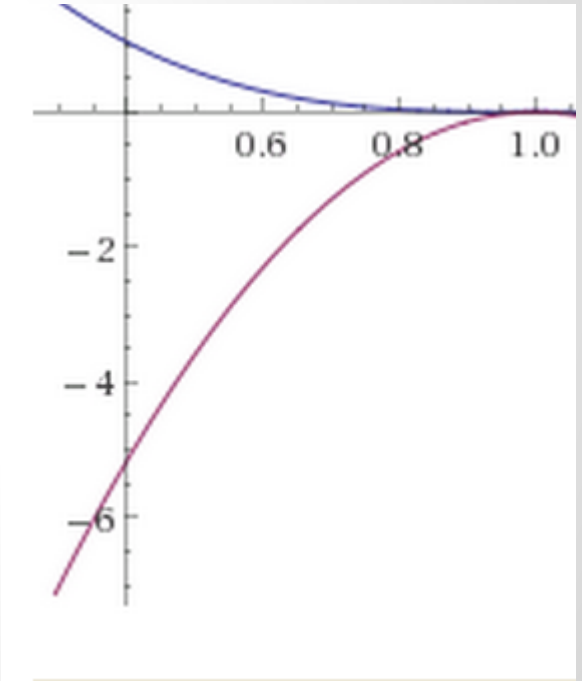
Uses the “Spikey Kernel” function

Spikey Kernel Function

$$W_{ij} = -\frac{45}{\pi h^6}(h - r)^2$$

Blue = function

Red = derivative



Viscosity

Damp motion by adding viscosity to acceleration

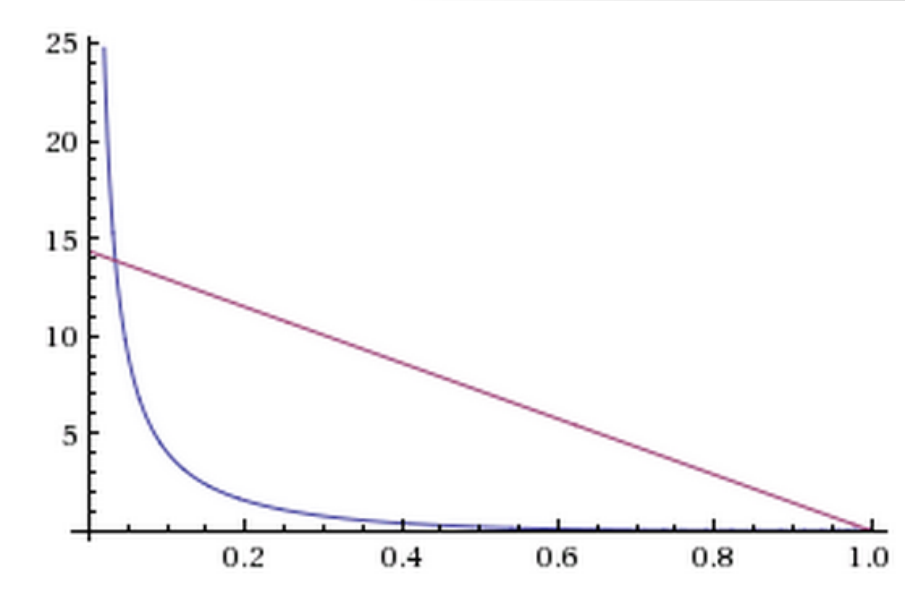
$$av_i = \epsilon \sum_j \frac{m_j}{m_i} \frac{1}{p_j} (v_j - v_i) \bigtriangledown^2 W_{ij} \hat{r}_{ij}$$

Viscosity Laplacian Kernel

$$W_{ij} = -\frac{r^3}{2h^3} + \frac{r^2}{h^2} + \frac{h}{2r} - 1$$

Blue = function

Red = laplacian



Calculating Timestep

Don’t want a particle to move more than one smoothing length in a single timestep

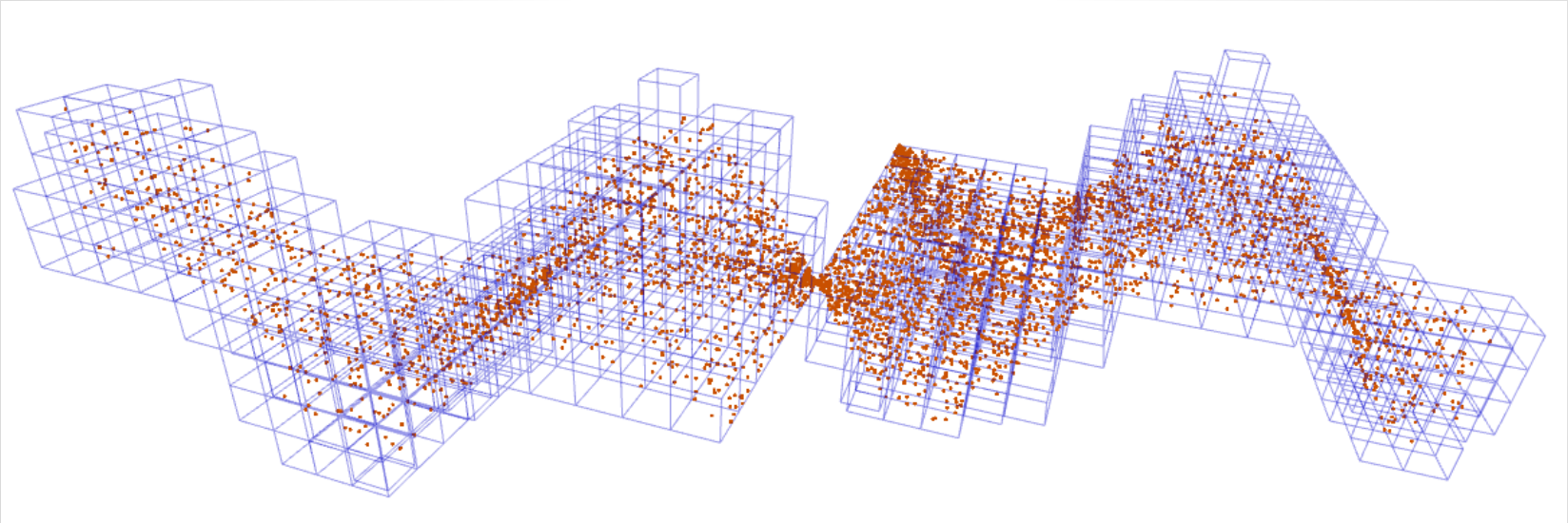
$$\triangle t = max\left(\frac{Ch}{max(1, v_{max})}, \sqrt{\frac{h}{a_{max}}}, \frac{Ch}{c_{max}}\right)$$

C = Courant Safety Factor

c = speed of sound

Nearest Neighbour Search

Dynamic Spatial grid

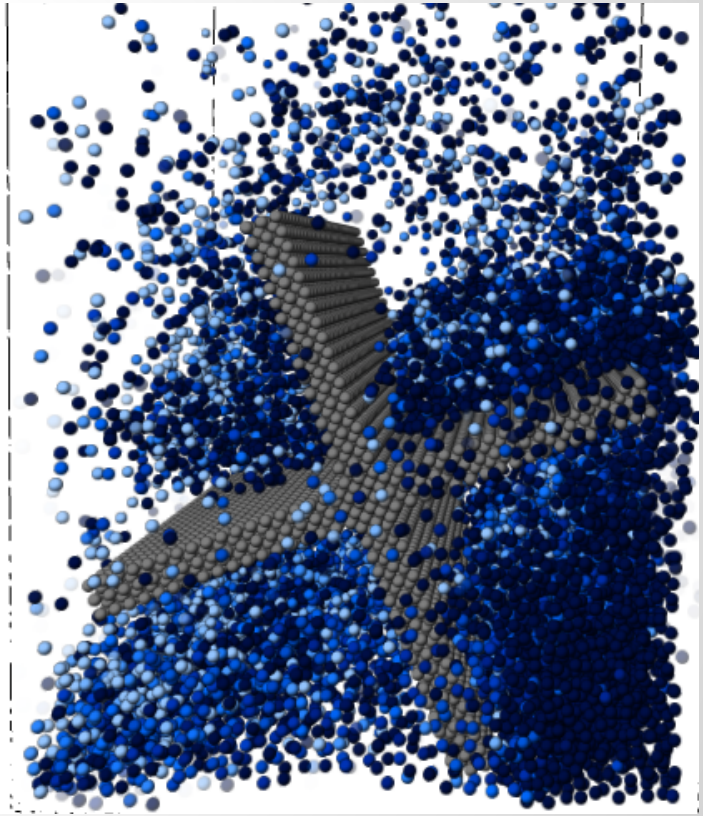


hash grid cells by indices

$$hash = 541i + 79j + 31k$$

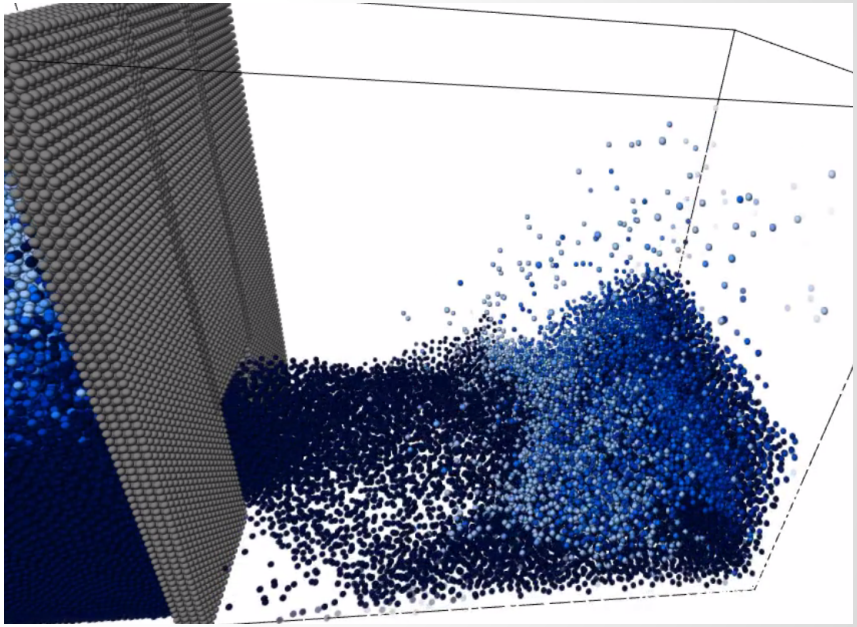
Object-Fluid Interaction

Treat object as a collection of fluid particles controlled by an external source



Dam Break Scenario

Tested on datasets of
50000,
100 000,
200 000
particles



Timing Results

Minutes per simulation second

Number of Particles	minutes/simulation second
48958	6.4
98958	17.8
210504	60.0

Table 1. Simulation Times

Each test ran for
1000 frames at
120 frames/sec

Timing Proportions

Dataset	Neighbour Search	Simulation	Graphics update	draw
48958	86.5%	9.2%	1.3%	3%
98958	80.5%	15.3%	1.2%	3%
210504	77%	20%	1%	2%

Table 2. Proportion of Simulation Times

Nearest neighbour search is the bottleneck

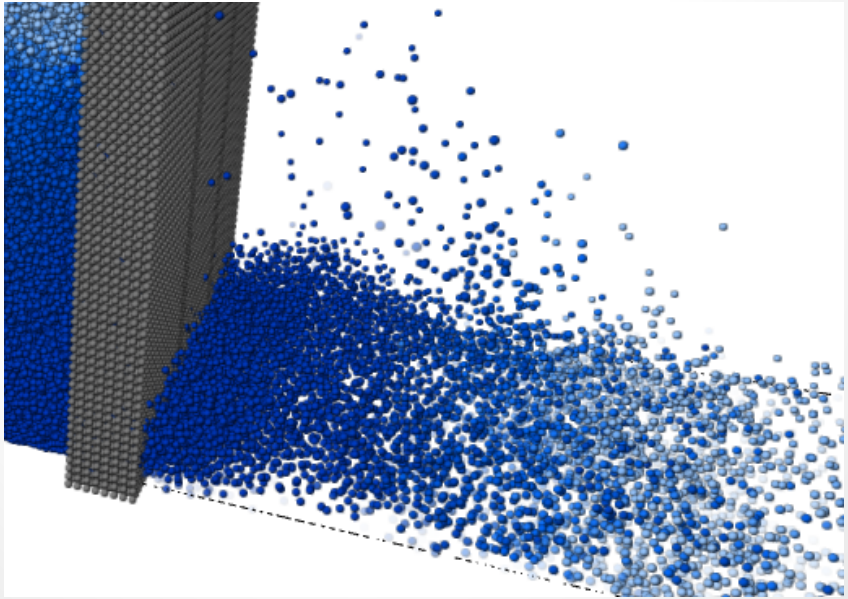
Conclusions

Implementation not suitable for incompressible fluids such as water.

Decrease simulation times by optimizing nearest neighbour search.

Conclusions

Fluids can leak through objects when pressure is very high



Videos