课程作业

杨佳乐

2021 年 10 月 24 日

题目 1. 5.9

解答.

$$\hat{f}(x_i) = N(N^T N + \lambda\Omega_N)^{-1} N^T y = S_\lambda y$$

$$S_\lambda = N(N^T N + \lambda\Omega_N)^{-1} N^T$$

当 N 可逆:

$$S_\lambda = ((N^T)^{-1}(N^T N + \lambda\Omega_N)N^{-1})^{-1}$$

$$= ((N^T)^{-1} N^T N N^{-1} + \lambda(N^T)^{-1}\Omega_N N^{-1})^{-1}$$

$$= (I + \lambda(N^T)^{-1}\Omega_N N^{-1})^{-1}$$

所以:

$$K = (N^T)^{-1}\Omega_N N^{-1}$$

K 不依赖于 λ

题目 2. 5.15

解答. (a)

$$< K(\bullet, x_i), f) >_{H_k} = \sum_{i=1}^{\infty} \frac{c_i}{r_i} < K(\bullet, x_i), \phi_i(\bullet) > = \sum_{i=1}^{\infty} \frac{c_i}{r_i} [r_i \phi_i(x_i)] = \sum_{i=1}^{\infty} c_i \phi_i(x_i) = f(x_i)$$

(b)

$$< K(\bullet, x_i), K(\bullet, x_j) >_{H_k} = \sum_{k=1}^{\infty} \frac{1}{r_k} < K(\bullet, x_i), K(\bullet, x_j) >$$

$$= \sum_{k=1}^{\infty} \frac{1}{r_k} [r_k \phi_k(x_i) r_k \phi_k(x_j)] = \sum_{k=1}^{\infty} r_k \phi_k(x_i) r_k \phi_k(x_j) = K(x_i, x_j)$$

(c)

$$J(g) = < g(x), g(x) >_{H_k} = \sum_{i=1}^{N} \sum_{i=1}^{N} < K(x, x_i), K(x, x_i) >_{H_k} \partial i \partial j = \sum_{i=1}^{N} \sum_{i=1}^{N} K(x_i, x_j) \partial i \partial j$$

(d) ρ 在 H_k 中正交，所以 $< K(\bullet, x_i), \rho(x) > = 0$

$$\tilde{g}(x_i) = < K(\bullet, \tilde{g}(x_i)) >_{H_k} = < K(\bullet, x_i), g(x_i) >_{H_k} + < K(\bullet, x_i), \rho(x_i) >_{H_k}$$

$$g(x_i)$$

$$J(\tilde{g}) = < \tilde{g}(x), \tilde{g}(x) >_{H_k} = < g(x), g(x) >_{H_k} + 2 < g(x), \rho(x) >_{H_k} + < \rho(x), \rho(x) >_{H_k}$$

$$= J(g) + 2 \sum_{i=1}^{N} \partial_i < K(x, x_i), \rho(x) >_{H_k} + J(\rho)$$

$$= J(g) + J(\rho)$$

所以最后可以推出：

$$\sum_{i=1}^{N} L(y_i, \tilde{g}(x_i)) + \lambda J(\tilde{g})$$

$$= \sum_{i=1}^{N} L(y_i, g(x_i)) + \lambda (J(g) + J(\rho)) \geq \sum_{i=1}^{N} L(y_i, g(x_i)) + \lambda J(\rho)$$

题目 3. 5.16

解答. (a)

$$K(x,y)=\sum_{m=1}^{M}h_m(x)h_m(y)=\sum_{i=1}^{\infty}\gamma_i\phi_i(x)\phi_i(y)$$

$$\Rightarrow \sum_{m=1}^{M}(\int h_m(x)\phi_k(x)dx)h_m(y)=\sum_{m=1}^{M}\gamma_i(\int \phi_i(x)\phi_k(x)dx)\phi_i(y)$$

因为 $\phi_i(x)$ 有如下性质:

$$\int \phi_i(x)\phi_k(x)dx=<\phi_i(x),\phi_k(x)>=\begin{cases}1 & i=k\\ 0 & i\neq k\end{cases}$$

所以:

$$\sum_{m=1}^{M}(\int h_m(x)\phi_k(x)dx)h_m(y)=\gamma_k\phi_k(y)$$

$\cdots(b)$

ESL 书中公式 (5.63) 对 β 求偏导, 并使其等于 0，可得:

$$-H^T(y-H\hat{(\beta)})+\lambda\hat{\beta}=0$$

$$\hat{\beta}=(H^TH+\lambda I)^{-1}H^Ty$$

$$\Rightarrow \hat{f}=H\hat{\beta}=H(H^TH+\lambda I)^{-1}H^Ty=(I+\lambda K^{-1})^{-1}y$$

$$=KK^{-1}(I+\lambda K^{-1})^{-1}y=K(K+\lambda I)^{-1}y$$

(c)

$$\hat{f}=H\hat{\beta}=K\hat{\alpha}=K(\alpha_1,\alpha_2,\cdots,\alpha_N)^T$$

$$\hat{f}(x)=h(x)^T\hat{\beta}=\sum_{i=1}^{N}K(x,x_i)\hat{\alpha}_i$$