KoRA - A Framework for Compositional Fine-Tuning

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Abstract

Low-Rank Adaptation (LoRA) is a standard for parameter-efficient fine-tuning (PEFT), but its layer-wise isolation limits compositional feature learning and out-of-distribution generalization. This proposal introduces KoRA (Kolmogorov-inspired Rank Adapters), a novel PEFT method that enables inter-adapter communication via lightweight, learnable connections. The core idea is to facilitate a directed flow of information, allowing the model to learn a unified, compositional adaptation strategy. Preliminary results on CIFAR-100 and Tiny ImageNet suggest that KoRA significantly outperforms LoRA in cross-domain transfer. The goal of this research is to extensively validate and refine the KoRA architecture to demonstrate its superiority for building robust, generalizable foundation models.

1 Introduction

The advent of large-scale foundation models, such as the Vision Transformer (ViT)[2], has marked a paradigm shift in computer vision. However, full fine-tuning has become computationally prohibitive. To address this, Parameter-Efficient Fine-Tuning (PEFT) methods[4] have gained prominence, with Low-Rank Adaptation (LoRA)[5] being one of the most popular techniques.

However, the very design that makes LoRA efficient also imposes a critical architectural limitation: **layer-wise isolation**. Each LoRA adapter operates independently, with no mechanism for coordination. We argue that this is a primary factor constraining the generalization capabilities of LoRA-tuned models, particularly when faced with out-of-distribution data.

This research proposes to move beyond isolated adaptation and introduce a model of structured compositionality. We present **KoRA (Kolmogorov-inspired Rank Adapters)**, a novel PEFT method that reframes adapters as nodes in a directed computational graph, facilitating a sequential flow of information from early to late layers. The goal of this work is to develop and validate this idea, showing that a compositional approach leads to more robust and generalizable models.

2 Methodology

2.1 Revisiting Low-Rank Adaptation (LoRA)

A pre-trained model can be described as a function $f(x; W_0)$. LoRA approximates the update matrix ΔW by factorizing it into two smaller matrices: $A \in \mathbb{R}^{r \times d_{in}}$ and $B \in \mathbb{R}^{d_{out} \times r}$. The forward pass is modified as:

$$h = W_0 x + \Delta W x = W_0 x + \alpha(BA)x \tag{1}$$

where α is a scaling scalar. The critical limitation is that the total adaptation is the sum of isolated perturbations, $\{\Delta W^{(1)}, \ldots, \Delta W^{(L)}\}$, preventing the model from learning dependencies between layer adaptations.

2.2 KoRA: A Compositional Adaptation Strategy

KoRA reframes adapters into nodes within a directed computational graph. This approach is motivated by the Kolmogorov-Arnold representation theorem[12, 8], which suggests any complex function can be decomposed into a composition of simpler functions.

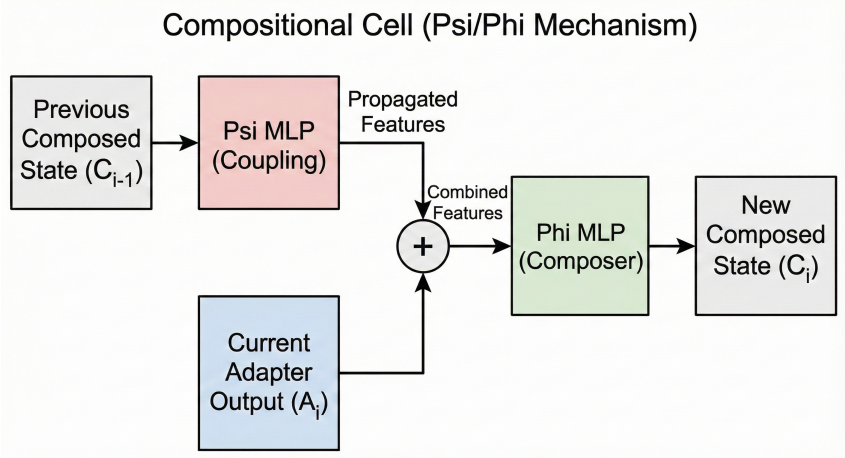

Figure 1: Compositional Cell

The KoRA architecture extends LoRA with a recursive flow:
1. Inner Adaptation and Projection. We compute the LoRA output, $o^{(i)} = (B^{(i)}A^{(i)})x^{(i)}$, and project it into a common composition space of dimension d_{comp}:

$$p^{(i)} = P^{(i)}_{\text{proj}}(o^{(i)}) \tag{2}$$

2. Sequential Compositional Flow. The projected vectors are sequentially integrated. The composed feature vector $c^{(i)}$ is updated recursively:

$$c^{(i)} = \begin{cases} \Phi^{(1)}(p^{(1)}) & \text{if } i = 1 \\ \Phi^{(i)}(p^{(i)} + \Psi^{(i-1)}(c^{(i-1)})) & \text{if } i > 1 \end{cases} \tag{3}$$

Here, Ψ is a lightweight *coupling network* propagating the state, and Φ is a *composer network* integrating new information.
3. Final Aggregation. The final composed vector, $c^{(L)}$, is projected back to the model's hidden dimension and added to the [CLS] token representation:

$$h_{\text{final}} = h_{\text{CLS}} + \alpha \cdot \text{Proj}_{\text{final}}(c^{(L)}) \tag{4}$$

3 Experimental Setup

This section outlines the initial experiments conducted to provide a proof-of-concept for the KoRA architecture.

3.1 Implementation Details

The classification experiments were conducted on a Kaggle GPU (NVIDIA P100). The depth estimation experiments utilized a more powerful A100 GPU, accessed via the Lightning AI platform.

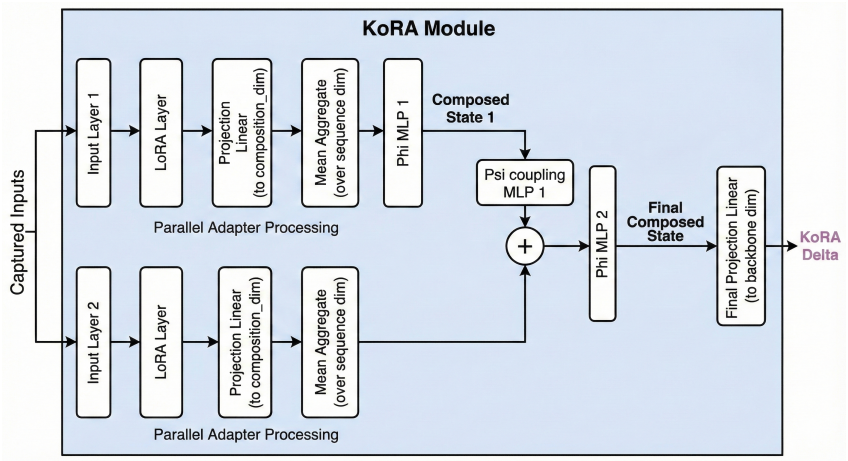

Figure 2: KoRA Module Dataflow

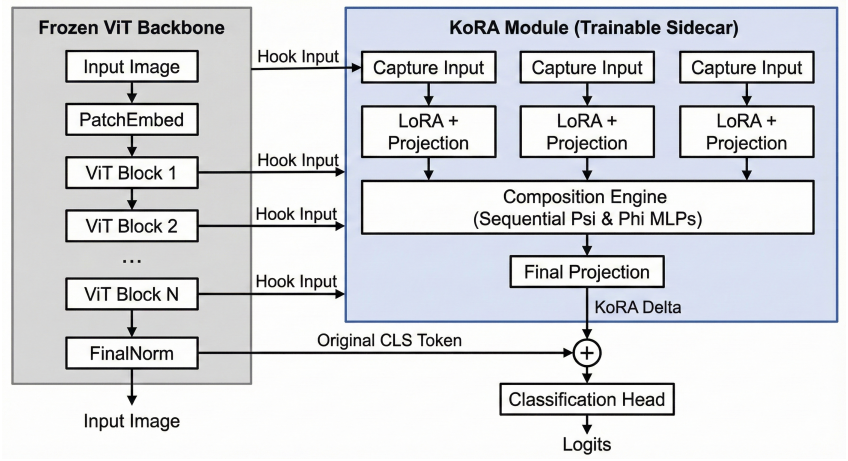

Figure 3: System-Wide Integration

3.2 Task Specialization on CIFAR-100

Models were fine-tuned for 5 epochs on the CIFAR-100[7] training set. Table 1 compares the performance of PEFT methods.

Table 1: Comparison of ViT Fine-Tuning Methods on CIFAR-100 (Task Specialization).

Tuning Method	Params Tuned (%)	CKA Sim.	Accuracy (%)	F1 Score
PEFT Methods				
LoRA ($r = 8$)	1.45	0.73	**92.48**	**0.924**
Adapter Fusion[10]	1.45	0.71	92.22	0.922
KoRA ($d_{\text{comp}} = 4$)	1.80	**0.76**	83.96	0.840
KoRA ($d_{\text{comp}} = 8$)	2.18	**0.76**	84.19	0.842

3.3 Task Generalization on Tiny ImageNet

Models pre-trained on CIFAR-100 were then fine-tuned for one epoch on 1% of the Tiny ImageNet[1] training data. Table 2 shows the results.

Table 2: Task Generalization: Classification Performance on Tiny ImageNet.

Tuning Method	Accuracy (%)	F1 Score
LoRA ($r = 8$)	71.04	0.8307
Adapter Fusion	46.67	0.6364
KoRA ($d_{\text{comp}} = 4$)	97.37	0.9867
KoRA ($d_{\text{comp}} = 8$)	**98.24**	**0.9911**

3.4 Representation Analysis

Preliminary Centered Kernel Alignment (CKA) analysis[6] suggests that KoRA learns more structured, cross-layer dependencies than LoRA. Early visualizations show higher similarity scores between layers in the KoRA model, indicating a more stable feature hierarchy.

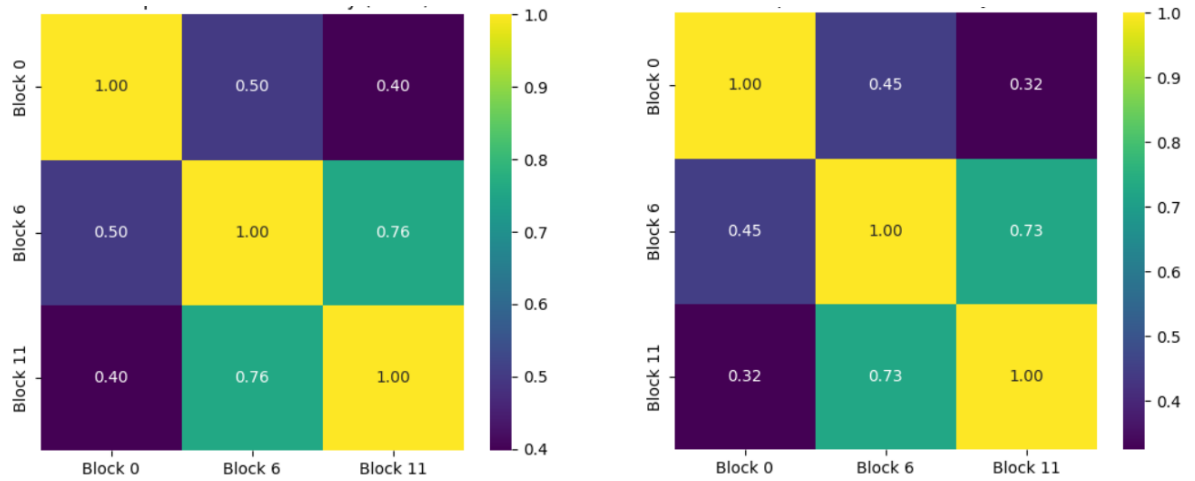

Figure 4: CKA analysis: KoRA (Left) vs LoRA (Right)

3.5 Depth Estimation on NYU Depth V2

The models were also fine-tuned for one epoch on 1% of the NYU Depth V2[13] training data.

Table 3: Transfer learning on NYU Depth V2.
Note: For these metrics, lower is better.

Method	Metric	Score
LoRA	RMSE	**0.2800**
	AbsRel	**0.5629**
KoRA	RMSE	0.3327
	AbsRel	0.6271

These initial results show LoRA performing better on this specific dense prediction task, highlighting an important area for future investigation of domain generalisation[9].

4 Expected Goals and Future Work

The central goal of this research is to rigorously validate and refine the KoRA architecture, with the aim of establishing compositional adaptation as a superior strategy for out-of-domain generalization[3]. Our preliminary findings, which show a trade-off between strong in-domain specialization and exceptional cross-domain transfer, motivate a detailed exploration of this new approach.

The future work is structured around three primary objectives:

1. **Architectural Refinement and Exploration:** The current implementation of KoRA uses simple MLPs[11] for the composer (Φ) and coupling (Ψ) networks. A key goal is to explore more sophisticated architectures. This includes investigating attention-based mechanisms for inter-adapter communication, where adapters could dynamically weigh information from previous layers. We will also experiment with different basis functions for the initial projection to better capture salient features for composition.

2. **Extensive Empirical Validation:** To prove the hypothesis that KoRA learns more fundamental features, we must expand our benchmarking significantly. The expected goal is to demonstrate strong performance across a diverse set of downstream tasks. This will involve:

    - Extending evaluation to object detection and semantic segmentation to test the learned feature hierarchy on tasks requiring spatial reasoning.
    - Conducting rigorous ablation studies on the dimensionality of the composition space (d_{comp}) and the complexity of the Φ and Ψ networks to understand the trade-offs between parameter count, specialization, and generalization.
    - Investigating the performance on the depth estimation task further. We hypothesize that the current KoRA architecture may be too biased towards classification and will explore modifications to better suit dense prediction tasks.

3. **In-Depth Theoretical and Representation Analysis:** We aim to deepen our understanding of *why* KoRA generalizes well. The expected goal is to provide concrete evidence that KoRA learns a more robust and transferable "adaptation algorithm." This will be achieved by:

    - Conducting a comprehensive CKA analysis across all layers of the transformer to map the flow of information and visualize the compositional structures that emerge.
    - Exploring the formal connection to Kolmogorov complexity by analyzing the compressibility of the learned KoRA parameters versus LoRA parameters.

By pursuing these objectives, we aim to move beyond promising preliminary results and establish KoRA as a robust, next-generation PEFT method that addresses the critical need for better generalization in fine-tuned foundation models.

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