CBSE BOARD | SHARMA TUTION CLASSES | Let's Rule it

CLASS - XII th ~~INVERS~~ MATRICES [PREVIOUS YEAR 2015-19]

(1) Write the element a_{23} of a 3×3 matrix $A = (a_{ij})$ whose element a_{ij} are given $a_{ij} = \frac{|i-j|}{2}$. [CBSE-2015]

Sol

$$a_{23} = \frac{|2-3|}{2} = \frac{1}{2}$$

(2) If $A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ -2 & -1 & 0 \end{bmatrix}$ find $A^2 - 5A + 4I$. and hence find a matrix X such that [CBSE 2015]

$A^2 - 5A + 4I + X = O$ or

If $A = \begin{bmatrix} 1 & -2 & 3 \\ 0 & -1 & 4 \\ -2 & 2 & 1 \end{bmatrix}$, find A^{-1}

Sol. Given $A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ -2 & -1 & 0 \end{bmatrix}$, getting $A^2 = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$

$$= \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix}$$

Now,

$$A^2 - 5A + 4I = \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix} - 5 \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ -2 & -1 & 0 \end{bmatrix} + 4 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix} + \begin{bmatrix} -10 & 0 & -5 \\ -10 & -5 & -15 \\ -5 & 5 & 0 \end{bmatrix} + \begin{bmatrix} 4 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & -1 & -3 \\ -1 & -3 & -10 \\ -5 & 4 & 2 \end{bmatrix}$$

$$X = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & 10 \\ 5 & -4 & -2 \end{bmatrix}$$

OR

$$A^{-1} = \begin{bmatrix} 1 & 0 & -2 \\ -2 & -1 & 2 \\ 3 & 4 & 1 \end{bmatrix}$$

So, $|A'| = 1(-9) - 2(-5)$

$= -9 + 10 = 1 \neq 0$

$$\text{Adj } A' = \begin{bmatrix} -9 & -8 & -2 \\ 8 & 7 & 2 \\ -5 & -4 & -1 \end{bmatrix}$$

(3) Use elementary column operation $C_2 \to C_2 + 2C_1$ in the following matrix eqn. [CBSE-2016]

$$\begin{bmatrix} 2 & 1 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}$$

Sol Given,

$$\begin{bmatrix} 2 & 1 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}$$

We know that, for elementary column operation

$A = AI$ (Column)
$A = IA$ (Row)

Applying $C_2 \rightarrow C_2 + 2C_1$

$$\begin{bmatrix} 2 & 1+4 \\ 2 & 0+4 \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0+2 \\ -1 & 1-2 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 5 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix}$$

(4) Write the no. of possible matrices of order 2×2 with each entry 1, 2, or 3 [CBSE - 2016]

Sol. Each element of 2×2 matrices can be filled in 3 ways, either 1, 2 or 3.

Let A be the 2×2 matrix such that,

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix},$$ There are 4 element in the matrix

So, if 1 element can be filled in 3 way, then

no. of ways $= 3^1$ (for 1 element)

Similarly for 4 elements

no. of ways $= 3^4$

$= 81$ Ans.

(5) If for any 2×2 square matrix A, $A(\text{adj}A) = \begin{bmatrix} 8 & 0 \\ 0 & 8 \end{bmatrix}$, then write the value of $|A|$

[CBSE 2017]

Sol Given, $A(\text{adj}A) = \begin{bmatrix} 8 & 0 \\ 0 & 8 \end{bmatrix}$

we know that $A(\text{adj}A) = |A| I$ — (1)

$A(\text{adj}A) = 8\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$A(\text{adj}A) = 8I$ $\left[I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right]$ — (2)

Comparing (1) and (2)

$|A| = 8$

(6) If A is a skew-symmetric matrix of order 3, then prove that det A = 0.

[CBSE - 2017]

Sol. We know that, for skew symmetric matrix

$A = -A^T$

Let $A = \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}$ be a skew symmetric matrix,

Now, $\det A = -a(0+bc) + b(ac-0)$

$= -abc + abc$

$|A| = 0$ Hp.

(7) If the matrix $A = \begin{bmatrix} 0 & a & -3 \\ 2 & 0 & -1 \\ b & 1 & 0 \end{bmatrix}$ is a skew symmetric, find the value of a and b. [RBSE 2018]

Sol. given $A = \begin{bmatrix} 0 & a & -3 \\ 2 & 0 & -1 \\ b & 1 & 0 \end{bmatrix}$ a skew sym.

$$A = -A^T$$

$$-A^T = \begin{bmatrix} 0 & -2 & -b \\ -a & 0 & -1 \\ 3 & 1 & 0 \end{bmatrix}$$

So, $a_{12} = -2$, $a_{31} = 3$

(8) Give $A = \begin{bmatrix} 2 & -3 \\ -4 & 7 \end{bmatrix}$ compute A^{-1} and show that $2A^{-1} = 9I - A$.

Sol Given $A = \begin{bmatrix} 2 & -3 \\ -4 & 7 \end{bmatrix}$

we know that $A^{-1} = \frac{1}{|A|} adj(A)$

$|A| = 14 - 12 = 2$, $adj(A) = \begin{bmatrix} A_{11} & A_{22} \\ A_{21} & A_{22} \end{bmatrix}^T$

$A_{11} = 7$, $A_{12} = 4$, $A_{21} = 3$, $A_{22} = 2$

$$\text{adj}(A) = \begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|}\,\text{adj}\,A = \frac{1}{2}\begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}$$

$$LHS = 2A^{-1} = \begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}$$

$$RHS = 9I - A = \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix} - \begin{bmatrix} 2 & -3 \\ -4 & 7 \end{bmatrix} = \begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}$$

H.P.

(9) If A is a square matrix of order 3 with $|A| = 4$, then write the value of $|-2A|$ [CBSE 2019]

Sol. Since, order of matrix, $n = 3$

$$|-2A| = (-2)^n |A|$$

$$|-2A| = (-2)^3 \times 4$$

$$= -32$$

(10) If $A = \begin{bmatrix} 0 & 2 \\ 3 & -4 \end{bmatrix}$, $KA = \begin{bmatrix} 0 & 3a \\ 2b & 24 \end{bmatrix}$, find K, a, b. [CBSE 2019]

Sol. Now, Imply $KA = \begin{bmatrix} 0 & 2K \\ 3K & -4K \end{bmatrix}$, comparing both $\begin{bmatrix} 0 & 2K \\ 3K & -4K \end{bmatrix} = \begin{bmatrix} 0 & 3a \\ 2b & 24 \end{bmatrix}$

$-4K = 24$, $K = 6$; $2K = 3a$, $2 \times 6 = 3a$, $a = 4$; $2b = 3K$, $b = \frac{3 \times 6}{2}$, $b = 9$