Tribhuvan University

Institute of Science and Technology

2074

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Bachelor Level / First Year/ First Semester/ Science
Computer Science and Information Technology (MTH. 112)
(Mathematics I)

Full Marks: 80
Pass Marks: 32
Time: 3 hours.

Candidates are required to give their answers in their own words as for as practicable.
The figures in the margin indicate full marks.

Attempt <u>any three</u> questions: (3×10=30)

1. (a) A function is defined by $f(x)=\begin{cases} x+2 & if \quad x<0 \\ 1-x & if \quad x>0 \end{cases}$, calculate f(-1), f(3), and sketch the graph. (5)

   (b) Prove that the $\lim_{x\to 0}\frac{|x|}{x}$ does not exist. (5)

2. (a) Find the derivative of $f(x)=\sqrt{x}$ and to state the domain of f'. (3+2)

   (b) Estimate the area between the curve $y^2 = x$ and the lines $x = 0$ and $x = 2$. (5)

3. (a) Find the Maclaurin series for e^x and prove that it represents e^x for all x. (4)

   (b) Define initial value problem. Solve that initial value problem of $y' + 5y = 1, y(0) = 2$. (4)

   (c) Find the volume of a sphere of radius r. (2)

4. (a) For what values of x does the series $\sum_{n=1}^{\infty}\frac{(x-3)^n}{x}$ converge? (5)

   (b) Calculate $\iint_R f(x,y)dA$ for $f(x,y)=100-6x^2y$ and R: $0\le x\le 2, -1\le y\le 1$. (5)

Attempt <u>any ten</u> questions: (10×5=50)

5. If $f(x)=\sqrt{x}$ and $g(x)=\sqrt{3-x}$, find gof and gog. (5)

6. Use continuity to evaluate the limit, $\lim_{x\to 4}\frac{5+\sqrt{x}}{\sqrt{5+x}}$. (5)

7. Verify Mean value theorem of $f(x) = x^3 - 3x + 3$ for [-1, 2]. (5)

8. Sketch the curve $y = x^3 + x$. (5)

9. Determine whether the integral $\int_1^{\infty}\frac{1}{x}dx$ is convergent or divergent. (5)

10. Find the length of the arc of the semicubical parabola $y^2 = x^3$ between the points (1, 1) and (4, 8). (5)

11. Find the solution of $y'' + 6y' + 9 = 0$, $y(0) = 2$, $y'(0) = 1$. (5)

12. Test the convergence of the series $\sum_{n=1}^{\infty} \frac{n^n}{n!}$. (5)

13. Define cross product of two vectors. If $a = i + 3j + 4k$ and $b = 2i + 7j - 5k$, find the vector $\mathbf{a} \times \mathbf{b}$ and $\mathbf{b} \times \mathbf{a}$. (1+2+2)

14. Define limit of a function. Find $\lim_{x \to \infty} \left(x - \sqrt{x}\right)$. (1+4)

15. Find the extreme values of $f(x, y) = y^2 - x^2$. (5)