4.2 Exercises

Answers to selected odd-numbered problems begin on page ANS-9.

4.2.1 Inverse Transforms

In Problems 1–30, use Theorem 4.2.1 to find the given inverse transform.

1. $\mathscr{L}^{-1}\left\{\dfrac{1}{s^3}\right\}$

2. $\mathscr{L}^{-1}\left\{\dfrac{1}{s^4}\right\}$

3. $\mathscr{L}^{-1}\left\{\dfrac{1}{s^2} - \dfrac{48}{s^5}\right\}$

4. $\mathscr{L}^{-1}\left\{\left(\dfrac{2}{s} - \dfrac{1}{s^3}\right)^2\right\}$

5. $\mathscr{L}^{-1}\left\{\dfrac{(s+1)^3}{s^4}\right\}$

6. $\mathscr{L}^{-1}\left\{\dfrac{(s+2)^2}{s^3}\right\}$

7. $\mathscr{L}^{-1}\left\{\dfrac{1}{s^2} - \dfrac{1}{s} + \dfrac{1}{s-2}\right\}$

8. $\mathscr{L}^{-1}\left\{\dfrac{4}{s} + \dfrac{6}{s^5} - \dfrac{1}{s+8}\right\}$

9. $\mathscr{L}^{-1}\left\{\dfrac{1}{4s+1}\right\}$

10. $\mathscr{L}^{-1}\left\{\dfrac{1}{5s-2}\right\}$

11. $\mathscr{L}^{-1}\left\{\dfrac{5}{s^2+49}\right\}$

12. $\mathscr{L}^{-1}\left\{\dfrac{10s}{s^2+16}\right\}$

13. $\mathscr{L}^{-1}\left\{\dfrac{4s}{4s^2+1}\right\}$

14. $\mathscr{L}^{-1}\left\{\dfrac{1}{4s^2+1}\right\}$

15. $\mathscr{L}^{-1}\left\{\dfrac{2s-6}{s^2+9}\right\}$

16. $\mathscr{L}^{-1}\left\{\dfrac{s+1}{s^2+2}\right\}$

17. $\mathscr{L}^{-1}\left\{\dfrac{1}{s^2+3s}\right\}$

18. $\mathscr{L}^{-1}\left\{\dfrac{s+1}{s^2-4s}\right\}$

19. $\mathscr{L}^{-1}\left\{\dfrac{s}{s^2+2s-3}\right\}$

20. $\mathscr{L}^{-1}\left\{\dfrac{1}{s^2+s-20}\right\}$

21. $\mathscr{L}^{-1}\left\{\dfrac{0.9s}{(s-0.1)(s+0.2)}\right\}$

22. $\mathscr{L}^{-1}\left\{\dfrac{s-3}{(s-\sqrt{3})(s+\sqrt{3})}\right\}$

23. $\mathscr{L}^{-1}\left\{\dfrac{s}{(s-2)(s-3)(s-6)}\right\}$

24. $\mathscr{L}^{-1}\left\{\dfrac{s^2+1}{s(s-1)(s+1)(s-2)}\right\}$

25. $\mathscr{L}^{-1}\left\{\dfrac{1}{s^3+5s}\right\}$

26. $\mathscr{L}^{-1}\left\{\dfrac{s}{(s+2)(s^2+4)}\right\}$

27. $\mathscr{L}^{-1}\left\{\dfrac{2s-4}{(s^2+s)(s^2+1)}\right\}$

28. $\mathscr{L}^{-1}\left\{\dfrac{1}{s^4-9}\right\}$

29. $\mathscr{L}^{-1}\left\{\dfrac{1}{(s^2+1)(s^2+4)}\right\}$

30. $\mathscr{L}^{-1}\left\{\dfrac{6s+3}{s^4+5s^2+4}\right\}$

4.2.2 Transforms of Derivatives

In Problems 31–40, use the Laplace transform to solve the given initial-value problem.

31. $\dfrac{dy}{dt} - y = 1, \quad y(0) = 0$

32. $2\dfrac{dy}{dt} + y = 0, \quad y(0) = -3$

33. $y' + 6y = e^{4t}, \quad y(0) = 2$

34. $y' - y = 2\cos 5t, \quad y(0) = 0$

35. $y'' + 5y' + 4y = 0, \quad y(0) = 1,\ y'(0) = 0$

36. $y'' - 4y' = 6e^{3t} - 3e^{-t}, \quad y(0) = 1,\ y'(0) = -1$

37. $y'' + y = \sqrt{2}\sin\sqrt{2}t, \quad y(0) = 10,\ y'(0) = 0$

38. $y'' + 9y = e^t, \quad y(0) = 0,\ y'(0) = 0$

39. $2y''' + 3y'' - 3y' - 2y = e^{-t}, \quad y(0) = 0,\ y'(0) = 0,\ y''(0) = 1$

40. $y''' + 2y'' - y' - 2y = \sin 3t, \quad y(0) = 0,\ y'(0) = 0,\ y''(0) = 1$

The inverse forms of the results in Problem 50 in Exercises 4.1 are

$$\mathscr{L}^{-1}\left\{\frac{s-a}{(s-a)^2+b^2}\right\} = e^{at}\cos bt$$

and

$$\mathscr{L}^{-1}\left\{\frac{b}{(s-a)^2+b^2}\right\} = e^{at}\sin bt.$$

In Problems 41 and 42, use the Laplace transform and these inverses to solve the given initial-value problem.

41. $y' + y = e^{-3t}\cos 2t, \quad y(0) = 0$

42. $y'' - 2y' + 5y = 0, \quad y(0) = 1,\ y'(0) = 3$

≡ Discussion Problems

43. **(a)** With a slight change in notation the transform in (6) is the same as

$$\mathscr{L}\{f'(t)\} = s\,\mathscr{L}\{f(t)\} - f(0).$$

With $f(t) = te^{at}$, discuss how this result in conjunction with part (c) of Theorem 4.1.1 can be used to evaluate $\mathscr{L}\{te^{at}\}$.

(b) Proceed as in part (a), but this time discuss how to use (7) with $f(t) = t\sin kt$ in conjunction with parts (d) and (e) of Theorem 4.1.1 to evaluate $\mathscr{L}\{t\sin kt\}$.

44. Make up two functions f_1 and f_2 that have the same Laplace transform. Do not think profound thoughts.

45. Reread *Remark (iii)* on page 220. Find the zero-input and the zero-state response for the IVP in Problem 36.

46. Suppose $f(t)$ is a function for which $f'(t)$ is piecewise continuous and of exponential order c. Use results in this section and Section 4.1 to justify

$$f(0) = \lim_{s\to\infty} sF(s),$$

where $F(s) = \mathscr{L}\{f(t)\}$. Verify this result with $f(t) = \cos kt$.