a certain first-order initial value problem $dy/dx = f(x, y)$, $y(0) = 1$. Equivalent results were obtained using three different commercial numerical solvers, yet the graph is hardly a plausible solution curve. (Why?) There are several avenues of recourse when a numerical solver has difficulties; three of the more obvious are decrease the step size, use another numerical method, or try a different numerical solver.

2.6 Exercises

Answers to selected odd-numbered problems begin on page ANS-3.

In Problems 1 and 2, use Euler's method to obtain a four-decimal approximation of the indicated value. Carry out the recursion of (3) by hand, first using $h = 0.1$ and then using $h = 0.05$.

1. $y' = 2x - 3y + 1,\quad y(1) = 5;\ y(1.2)$
2. $y' = x + y^2,\quad y(0) = 0;\ y(0.2)$

In Problems 3 and 4, use Euler's method to obtain a four-decimal approximation of the indicated value. First use $h = 0.1$ and then use $h = 0.05$. Find an explicit solution for each initial-value problem and then construct tables similar to Tables 2.6.3 and 2.6.4.

3. $y' = y,\quad y(0) = 1;\ y(1.0)$
4. $y' = 2xy,\quad y(1) = 1;\ y(1.5)$

In Problems 5–10, use a numerical solver and Euler's method to obtain a four-decimal approximation of the indicated value. First use $h = 0.1$ and then use $h = 0.05$.

5. $y' = e^{-y},\quad y(0) = 0;\ y(0.5)$
6. $y' = x^2 + y^2,\quad y(0) = 1;\ y(0.5)$
7. $y' = (x - y)^2,\quad y(0) = 0.5;\ y(0.5)$
8. $y' = xy + \sqrt{y},\quad y(0) = 1;\ y(0.5)$
9. $y' = xy^2 - \dfrac{y}{x},\quad y(1) = 1;\ y(1.5)$
10. $y' = y - y^2,\quad y(0) = 0.5;\ y(0.5)$

In Problems 11 and 12, use a numerical solver to obtain a numerical solution curve for the given initial-value problem. First use Euler's method and then the RK4 method. Use $h = 0.25$ in each case. Superimpose both solution curves on the same coordinate axes. If possible, use a different color for each curve. Repeat, using $h = 0.1$ and $h = 0.05$.

11. $y' = 2(\cos x)y,\quad y(0) = 1$
12. $y' = y(10 - 2y),\quad y(0) = 1$

Discussion Problem

13. Use a numerical solver and Euler's method to approximate $y(1.0)$, where $y(x)$ is the solution to $y' = 2xy^2$, $y(0) = 1$. First use $h = 0.1$ and then $h = 0.05$. Repeat using the RK4 method. Discuss what might cause the approximations of $y(1.0)$ to differ so greatly.

2.7 Linear Models

Introduction In this section we solve some of the linear first-order models that were introduced in Section 1.3.

Growth and Decay The initial-value problem

$$\frac{dx}{dt} = kx,\quad x(t_0) = x_0, \tag{1}$$

where k is the constant of proportionality, serves as a model for diverse phenomena involving either **growth** or **decay**. We have seen in Section 1.3 that in biology, over short periods of time, the rate of growth of certain populations (bacteria, small animals) is observed to be proportional to the population present at time t. If a population at some arbitrary initial time t_0 is known, then the solution of (1) can be used to predict the population in the future—that is, at times $t > t_0$. The constant of proportionality k in (1) can be determined from the solution of the initial-value problem using a subsequent measurement of x at some time $t_1 > t_0$. In physics