Question 1

(a) (5 points) Show that $f(x) = x^2 + 4x$ is $O(x^2)$.

- $x^2 + 4x \le c\,|\,x^2\,|$
- $x^2 + 4x \le x^2 + 4x^2 \quad \forall x > 1$
- $x^2 + 4x \le 5x^2 \quad \forall x > 1$
- $x_0 = 1 \; and \; c = 5$

Therefore, $f(x) = O(x^2)$.

(b) (5 points) Show that $f(x) = x^2$ is NOT $O(\sqrt{x})$.

If $x^2 \le c\sqrt{x}$;

$x^{3/2} \le c$;

As $x \to \infty$, $x^{3/2} \to \infty$, but c is a constant.

Therefore, $f(x) = x^2$ is NOT $O(\sqrt{x})$.

Note:

$x^2 \le x^2 \quad \forall x > 1$

$x^2 \le x^2 \quad \forall x > 1$

$x_0 = 1 \; and \; c = 1$

(c) (5 points) Show that $f(x) = x$ is $\Omega(\log x)$.

$x \Rightarrow c \log x$

$2^x \Rightarrow cx$

$x_0 = 1$, $c = 2$

Therefore, $f(x) = x$ is $\Omega(\log x)$.

(d) (10 points) Show that $f(x) = (2x^2 - 3)/((3x^4 + x^3 - 2x^2 - 1)$ is $\Theta(x^{-2})$.

$f(x) = (2x^2 - 3)/((3x^4 + x^3 - 2x^2 - 1)$

$2x^2 - 3$ is nearly equal to $2x^2$;

$3x^4 + x^3 - 2x^2 - 1$ is nearly equal to $3x^4$;

$f(x) \approx 2/3x^2$;

Big-O:

$f(x) <= cg(x), x > x_0$

$2/3x^2 <= c x^{-2}$

Therefore, c = 1and $x_0 = 1$, $O(x^{-2})$;

Big-Ω:

$f(x) => cg(x), x > x_0$

$2/3x^2 => c x^{-2}$

Therefore, c = 1/3 and $x_0 = 1$, $\Omega(x^{-2})$;

Thus, that $f(x) = (2x^2 - 3)/((3x^4 + x^3 - 2x^2 - 1)$ is $\Theta(x^{-2})$.

Question 2 (15 points). *Please rank the following functions based on their $O(\cdot)$ complexity of running time. The function that has the least complexity should be ranked 1. Please explain your answer to get full credit.*

$f_1(x) = x \log_2 x$
$f_2(x) = 3^x$
$f_3(x) = \sqrt{x}$
$f_4(x) = x!$
$f_5(x) = 2^x$

Rank:

$f_3(x)$ -> $f_1(x)$ -> $f_5(x)$ -> $f_2(x)$ -> $f_4(x)$

Explain my answer(Big-O):

$f_3(x) = \sqrt{x} <= c\sqrt{x} \quad \forall x > 1 \Rightarrow c = 2; x_0 = 1$

Big-O Complexity is $O(\sqrt{x})$, so it is slower than x.

$f_1(x) = x\log_2 x <= cx^2 \quad \forall x > 1 \Rightarrow c = 1; x_0 = 1$

Big-O Complexity is $O(x\log_2 x)$, so it is faster than x.

$f_5(x)$ and $f_2(x)$ belong to n^x type.

$f_5(x) = 2^x => c\, x^2 \; \forall x > 4 \Rightarrow c = 1; x_0 = 4$

Therefore, Big- Ω for 2^x is x^2.

Big-O Complexity is $O(n^x)$, so 3^x is faster than 2^x

$f_4(x) = x! => c3^x \; \forall x > 4 \Rightarrow c = 1; x_0 = 7$

Therefore, Big- Ω for $x!$ is 3^x.

And then,

Big-O

$f_4(x) = x! <= cx! \ \forall x > 1 \Rightarrow c = 2; x_0 = 1$

Result of Rank:

$f_3(x) \ -> f_1(x) -> f_5(x) -> f_2(x) -> f_4(x)$