Question 1 (15 points). *Suppose you are given a directed graph* $G = (V, E)$ *with* $V = \{1, 2, 3, 4, 5, 7, 8\}$ *and the depth first intervals* $([pre, post])$ *of each vertex are as follows.* $\{1 : [4, 5], 2 : [7, 8], 3 : [12, 13], 4 : [14, 15], 5 : [3, 6], 6 : [2, 9], 7 : [1, 10], 8 : [11, 16]\}$.

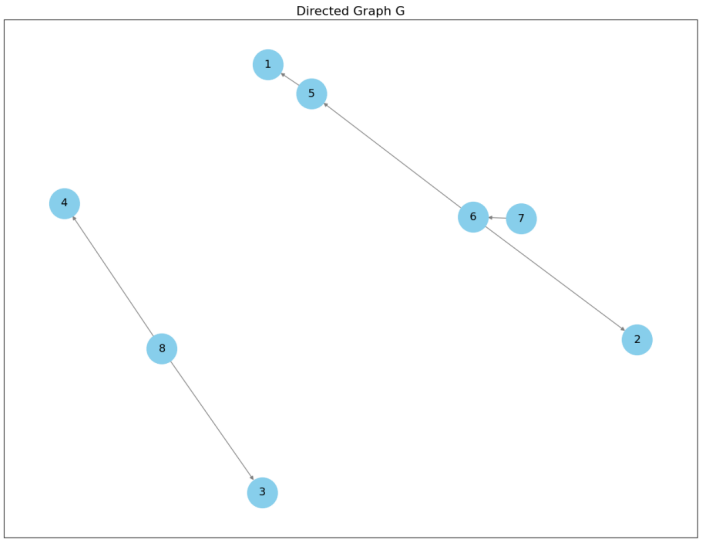
(a) (7 points) Draw this directed graph using networkx package in Python and include the image.

(b) (3 points) What are the descendent and ancestor vertices of vertex 6?

(c) (2 points) How many connected components does the graph have?

(d) (3 points) Identify three pairs of vertices that form a cross edge (i.e., one is neither a descendent nor an ancestor of the other).

a.



b.

Descendent Vertices Of Vertex 6 :1/2/5

Ancestor Vertices Of Vertex 6 :7

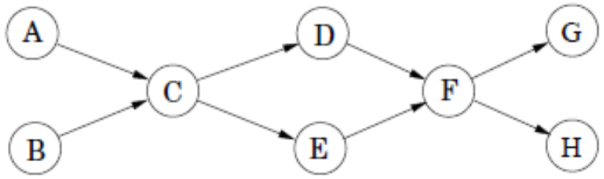
c.

CC is 2

d.

4/3; 1/2; 2/5

Question 2 (20 points). *Run the DFS-based topological ordering algorithm on the following graph. Whenever you have a choice of vertices to explore, always pick the one that appears first in alphabetical order.*



(a) (12 points) Indicate the pre- and post- numbers of the nodes.

(b) (4 points) What are the sources and sinks of the graph?

(c) (2 points) Write down one topological ordering found by the algorithm.

(d) (2 points) How many topological orderings does this graph have?

a.

Pre-order: {'A': 1, 'C': 2, 'D': 3, 'F': 4, 'G': 5, 'H': 7, 'E': 11, 'B': 15}

Post-order: {'G': 6, 'H': 8, 'F': 9, 'D': 10, 'E': 12, 'C': 13, 'A': 14, 'B': 16}

b.

Sources Of The Graph :A/B

Sinks Of The Graph: G/H

c.

BACEDFHG

d.

The form is {A,B}, C,{D/E}, {F}, {G,H}. The total of topological orderings is 8.

Question 3 (15 points). *Given an example of a graph with n vertices for which the queue of Breadth-first Search (BFS) will have $n-1$ vertices at one time, whereas the height of the recursion tree of Depth-First Search (DFS) is at most one. Both searches are started from the same vertex.*

BFS explores neighbors level by level while DFS follows a path as deep as possible before backtracking.

BFS start from node A, then the queue contains: {C, D, E, F, G, H} (size = n-1).

DFS also from node A, immediately visit all neighbors (size = 1)..

Question 4 (15 points). *You are given a binary tree $T = (V, E)$ that is not skewed and $|V| \geq 2$. Please describe in English a linear (in terms of $|V|$ and $|E|$) time algorithm to find the maximum sum of a path between any two leaves in T. Please explain why you are algorithm running time in linear.*

1.

The binary tree includes root, left tree and right tree. Due to the tree that is not skewed and |V | ≥ 2, computing the potential max path sum through this node:

maxSum = leftSum + rightSum + node.value

2.The reason for running time in linear:

Each node is visited only once ⟶ DFS traversal takes O(n).

Each edge is processed once ⟶ Every edge is used to compute the path sum, contributing to O(n).

the total runtime is O(|V|+|E|) that is O(n), which is optimal for tree traversal.

Question 5 (35 points). *For each vertex u in an undirected graph, let twodegree[u] be the sum of the degrees of u's neighbors. You are given an undirected graph $G = (V, E)$ in adjacency-list format.*

(a) (15 points) Please describe in English an efficient algorithm to compute the entire array of twodegree[·] values in time linear in $|V|$ and $|E|$.

(b) (15 points) Please write the pseudocode of your algorithm in (a).

(c) (5 points) Please explain why the running time of your pseudocode in (b) is linear in $|V|$ and $|E|$.

a.

Description:

1.Computing the degree of Vertex

- Traversal the adjacency-list. So store vertex and degrees of u's neighbor into array that is named by "degree".

2.Using two loop to traversal adjacency-list and sum the degrees of u's neighbor in the "degree".

Time Complexity:

Computing degrees traversals the vertex and edge: O(O(|V| + |E|)

Calculating twodegree --Each edge is visited twice ⟶ O(|E|) and traversal every vertex: O(O(|V| + |E|)

Overall complexity: O(|V| + |E|) (linear time)

b.

```
def c_two_degree(graph):
  degree = {}
  two_degree = {}
  for v, neighbors in graph.items():
    degree[v] = len(neighbors)

  for v, neighbors in graph.items():
    sum = 0
```

```
    for u in neighbors:
        sum += degree[u]
    two_degree[v] = sum

  return two_degree
```

c.

Step 1:

degree = {u: len(neighbors) for u, neighbors in graph.items()}

Computes the degree of each vertex in O(|V| + |E|) time.

Step 2:

For each vertex u, it sums the degrees of its neighbors: O(|V|).

Since every edge is counted twice in an undirected graph, it takes O(|E|).

Total Time Complexity:

O(|V| + |E|) — linear in the size of the graph.