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Program Structure and Algorithms (INFO 6205)
Quiz #2 – SAMPLE SOLUTIONS – 30 points

Student NAME:

Student ID:

Question 1 (15 points). *Please use the Master method to derive a $O(\cdot)$ or $\Theta(\cdot)$ complexity for the following recurrences. Please clearly write down the values of $a, b, d, f(n)$ and the case numbers to receive full credit.*

(a) *(5 points)* $T(n) = 3T(n/4) + \Theta(n \log n)$.

$a = 3, b = 4, d = 1, f(n) = n \log n, \log_4 3 < d$, *so Case 3.*

Thus, $T(n) = \Theta(n \log n)$.

(b) *(5 points)* $T(n) = 5T(n/2) + n^2 \log n$.

$a = 5, b = 2, d = 2, f(n) = n^2 \log n, \log_2 5 > d$, *so Case 1.*

Thus, $T(n) = \Theta(n^{\log_2 5})$.

(b) *(5 points)* $T(n) = 4T(n/2) + n^2 \log n$.

$a = 4, b = 2, d = 2, f(n) = n^2 \log n, \log_2 4 = d$, *so Case 2.*

Thus, $T(n) = \Theta(n^2 \log^2 n)$.

Question 2 (15 points). *Consider the following recurrence that we will solve by using recurrence trees.* $T(n) = 2T(n/2) + n$.

(a) *(3 points) How many levels are there in the recursion tree if the size of subproblems at the last level is 1?*

Size of subproblems at any level $i = n/2^i$. *Since at the leaf level, the size of subproblems = 1, we have the number of levels as* $n/2^i = 1 \Rightarrow i = \log_2 n$.

(b) *(3 points) How many leaf nodes are in the tree?*

At any level i, *there are* 2^i *nodes, so when* $i = \log_2 n$, *there are* $2^{\log_2 n} = n^{\log_2 2} = n$ *leaf nodes.*

(c) *(4 points) What is the cost per leaf node and total cost at the last level?*

Cost per leaf node is $\Theta(1) = c$. *The total cost for* n *leaf nodes is therefore,* $\Theta(n) = cn$.

(d) *(5 points) What is the total cost of the tree (i.e., the sum of costs at all levels)?*

Cost across all levels is, $T(n) = \sum_{i=0}^{\log_2 n} (2/2)^i \cdot cn + \Theta(n) \Rightarrow n \log n + \Theta(n)$.

Total cost is, $\Theta(n \log n)$.