

**Assignment 1** (100 points). You are given a directed graph  $G = (V, E)$  with positive edge lengths. Please develop an efficient algorithm by using **Dijkstra's algorithm** to return the length of the shortest cycle in the graph (if the graph is acyclic, it should say so) and implement it. Your algorithm implementation should take time at most  $O(|V|^3)$ .

Your input will be a graph described in a file, e.g., "testcase-1.txt". The format will be as follows.  
source vertex : list of destination vertices

```
v1 : dest1 wt11 dest2 wt12 ...  
v2 : dest3 wt23 dest7 wt27 ...  
...  
vN : destN-1 wtNN-1 destN-2 wtNN-2...
```

1. [https://github.com/OnetoTommy/Program-Structure-Algorithms-/tree/main/Course/Course\\_10/PA\\_1](https://github.com/OnetoTommy/Program-Structure-Algorithms-/tree/main/Course/Course_10/PA_1)

## 2. Describe Algorithm (Dijkstra's Algorithm)

### 1. Graph Representation:

The graph is stored as an adjacency list using a dictionary where keys are nodes, and values are lists of tuples representing neighbors and edge weights.

### 2. Function `dijkstra_shortest_cycle(start)`:

- This function uses **Dijkstra's Algorithm** to find the shortest cycle starting and ending at start.
- It initializes a `shortest_cycle` dictionary where all nodes have infinite distance except the start node (set to 0).
- A **priority queue (min-heap)** is used to always process the node with the smallest distance.
- For each node, its neighbors are checked, and distances are updated if a shorter path is found.
- **If a neighbor is the start node and it's not a direct back edge**, a cycle is detected, and its length is returned.

### 3. Function `finding_shortest_cycle()`:

- This function calls `dijkstra_shortest_cycle(start)` for every node in the graph.
- It keeps track of the shortest cycle found.
- If no cycle exists, it returns 0.

### 4. Reading Input Graph from File:

- The input file contains graph edges and each row represents a source vertex followed by pairs of (neighbor, weight).
- This is parsed into the adjacency list representation.

### 5. Final Output:

The algorithm prints the length of the shortest cycle found like "The length of the shortest cycle is: 8"

## 3. Testcase(s) on which you validated your program.

Case-1: Acyclic

0 : 1 1

1 : 2 2

2 : 3 3

#### **Case-2: Single Cycle**

**0 : 1 1 2 4**

**1 : 3 2**

**2 : 3 1**

**3 : 0 3**

#### **Case-3: Double Cycle**

**0 : 1 1**

**1 : 2 2**

**2 : 3 3**

**3 : 1 4 0 5**

#### **Case-4: Empty**

Link: [https://github.com/OnetoTommy/Program-Structure-Algorithms-/tree/main/Course/Course\\_10/PA\\_1](https://github.com/OnetoTommy/Program-Structure-Algorithms-/tree/main/Course/Course_10/PA_1)