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Program Structure and Algorithms (INFO 6205)
Midterm Exam – 120 points

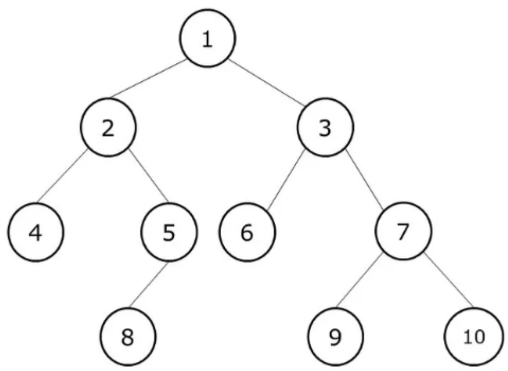
Student NAME:

Student ID:

Question 1 (30 points). *Please provide short and concise answers and reasons for the following.*

(a) *[3 points] Express the function* $2n^5 - n^3 + 6n^2 + 10n + 5$ *in* Θ*-notation.*

(b) *[6 points] Please list the order in which data will be printed for the following traversals on the tree below.*



Preorder: ______________________________

Inorder: ______________________________

Postorder: ______________________________

(c) *[9 points] Consider the following sequence of inputs,* $7, 5, 20, 4, 6, 19, 25, 30, 15, 3$.

(i) Draw a binary search tree (BST).

(ii) For the BST you drew in (i), redraw the BST after deleting node 5.

(iii) Redraw the BST from (ii), after deleting node 20.

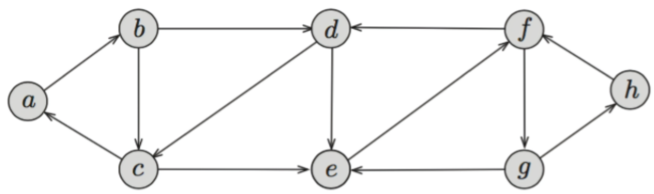
(d) *[6 points] Please state whether the following statements are* **True** *or* **False**. *You do not need to explain your answer.*

(i) If DFS traversal from vertex u *of a graph contains at least one back edge, then any other DFS traversal from some other vertex* $v \neq u$ *of the same graph will also contain at least one back edge.*

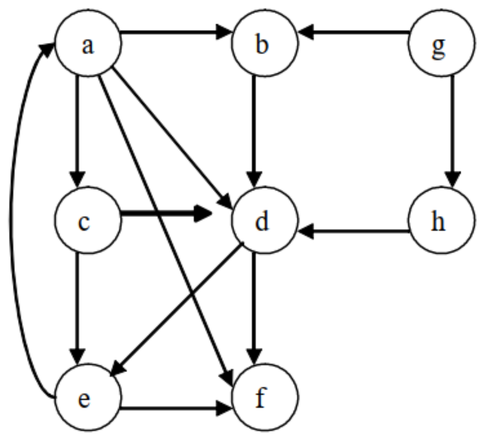
(ii) Any DFS traversal from any vertex of an undirected graph will always contain exactly the same number of connected components.

(iii) Every DAG has exactly one topological ordering.

(e) *[6 points] In the graph below, how many strongly connected components (SCC) are present? Identify the SCCs.*



Question 2 (20 points)**.** *Consider the directed graph G below. Break all ties lexicographically (i.e, according to alphabetical order).*



(a) *[8 points] Please execute DFS on G and list the pre and post numbers for each vertex.*

	a	b	c	d	e	f	g	h
pre								
post								

(b) *[6 points] Please list the different types of edges separated by commas.*

Tree edge(s): ______________________________

Cross edge(s): ______________________________

Forward edge(s): ______________________________

Back edge(s): ______________________________

(c) *[6 points] List the strongly connected components (SCCs) of G, in the order that they are found using the algorithm given in class.*

Question 3 (20 points). *Consider the following three divide-and-conquer algorithms.*

- ***Algorithm A*** *when run on an input of size n solves four recursive subproblems of size $n/2$ in addition to $n^2 \log n + \log n$ overhead work.*
- ***Algorithm B*** *when run on an input of size n solves two recursive subproblems of size $n/4$ in addition to $\log n$ overhead work.*
- ***Algorithm C*** *when run on an input of size n solves three recursive subproblems of size $n/9$ in addition to $n^{0.51}/\log n$ overhead work.*

(a) *[9 points] Please express the recursion function of each algorithm as $T(n) = aT(n/b) + f(n)$. Please clearly indicate what are a, b and $f(n)$ to receive full credit.*

(b) *[9 points] What are the asymptotic runtimes of these three algorithms?*

(c) *[2 points] Please rank the algorithms from the fastest (1) to the slowest (3).*

Question 4 (25 points). *In Excel spreadsheets we can perform mathematical operations based on values in other cells. Consider the spreadsheet below where the computations in each cell are given.*

	A	B
1	**20** =B1*2	**10**
2	**30** =A1+B1	**25** =A1+5
3	**50** =SUM(A1:A2)	**0.833** =B2/A2

(a) *[8 points] Suppose you want to represent the computations using a graph. Please draw the graph for the above example.*

(b) *[8 points] Please describe* **in English** *a linear time algorithm to accurately compute values of different cells in any spreadsheet if you use a similar graph as in (a).*

(c) *[5 points] Please draw a graph to show how calculations will happen accurately for the example in the figure above using your algorithm in (b).*

(d) *[4 points] Please explain why the running time of your algorithm in (b) is linear.*

Question 5 (25 points). *You are given an array* $A[1:n]$ *integers such that* $A[1] \geq A[2]$ *and* $A[n-1] \leq A[n]$. *An element* $A[x]$ *is called the local minimum if both* $A[x-1] \geq A[x]$ *and* $A[x] \leq A[x+1]$.

For example, the array $A = [9, 7, 7, 2, 1, 3, 7, 5, 4, 7, 3, 3, 4, 8, 6, 9]$ *has six local minima.*

- $A[2], A[5], A[9], A[11], A[12]$ *and* $A[15]$

(a) *(10 points) Please describe an efficient divide-and-conquer algorithm* **in English** *and write pseudocode for find_local_minima(A, start, end) to find* **any local minima** *given an input array A. Your solution* **MUST** *use recursion to receive any credit.*

(b) *(3 points) Please write the* **recurrence relation** *($T(n)$) of your pseudocode in (a). That is,* $T(n) =$ *???.*

(c) *(3 points) Please solve your recurrence in (e) using the Master method. Please clearly write the asymptotic running time of your algorithm in* $O(\cdot)$ *or* $\Theta(\cdot)$.

(d) *(9 points) For the array* $A = [9, 7, 7, 2, 1, 3, 7, 5, 4, 7, 3, 3, 4, 8, 6, 9]$, *please explain and demonstrate every recursion call with the start and end indices of the array until you find a local minima. Please use the start index for A as 1. Please mention value of your middle index and the element in A corresponding to it in each recursion.*