IC-03 Addition of Vectors

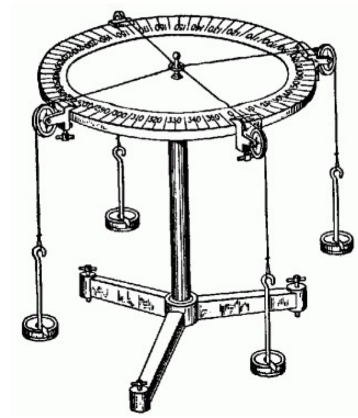
Rev 5-14-2023

3.1 OBJECTIVES

The objectives of this lab are to use graphical, analytic and experimental methods to Resolve a force vector into its rectangular components, and 2. To find the resultant of a number of forces acting on a body.

TAKE HOME LAB KIT LAB: This Manual was made for in-class lab. With LAB KIT, do only the Graphical and Analytical parts. Do these on paper, take nice pictures, and upload with your report

3.2 EQUIPMENT



Graphical and Analytic Part:

1. Protractor
2. Sheets of plain paper
3. Pencil
4. Ruler

Experimental Part:

1. Force table
2. Four weight holders
3. Four pulleys
4. Slotted weights
5. Strings for suspending the masses
6. A ring
7. A metal pin

3.3 THEORY OF VECTOR ADDITION

When a number of forces passing through the same point act on an object, they may be replaced by a single force which is called the resultant or the sum. The resultant therefore is a single force which is similar in effect to the effect produced by the several forces acting on the body. It is therefore a single force that replaces those forces. We can fine this resultant force by representing each force as a vector, and then adding the vectors together. There are several ways in which vectors may be added.

3.3.1 Graphical Methods

A. Parallelogram Method

Vectors are represented graphically by arrows. The length of a vector arrow (drawn to scale on graph paper) is proportional to the magnitude of the vector, and the arrow points in the direction of the vector. The length scale is arbitrary and usually selected for convenience so that the vector graph fits nicely on the graph paper. See Fig 1a, where **R** = **A** + **B**. The magnitude ***R*** of the resultant vector is proportional to the length of the diagonal arrow and the direction of the resultant vector is that of the diagonal arrow **R**. The direction of **R** may be specified as being at an angle θ relative to **A**.

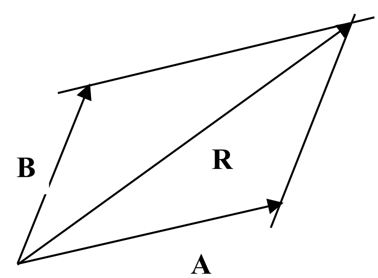
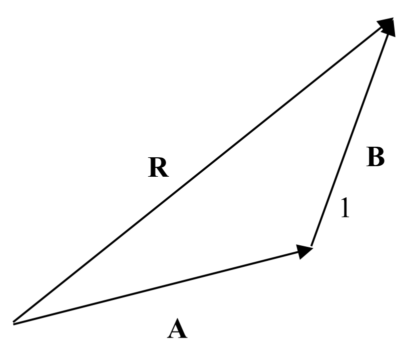


Figure 1a

B. Triangle Method

An equivalent method of finding **R** is to place the vectors to be added "head to tail" (head of **A** to tail of **B**, Fig. 1 b). Vector arrows may be



moved as long as they remain pointed in the same direction. The length and direction of the resultant is measured from the graph.

Figure 1b

C. *Polygon Method*

If more than two vectors are added, the head-to-tail method forms a polygon (Fig. 2). For four vectors, the resultant **R = A + B + C + D** is the vector arrow from the tail of the **A** arrow to the head of the vector **D**. The length (magnitude) and the angle of orientation of R can be measured from the diagram.

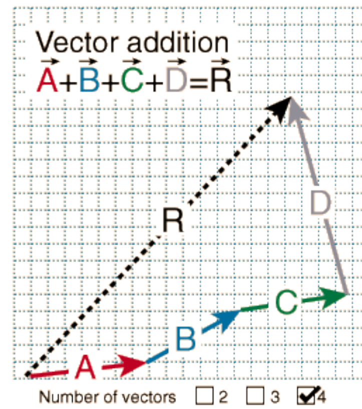


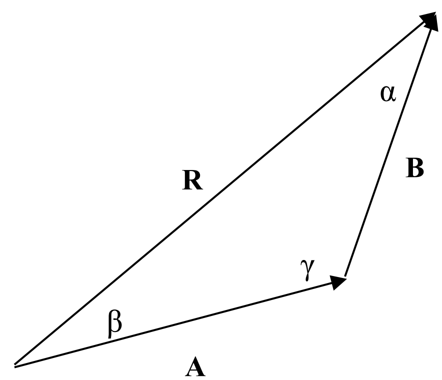
Figure 2

3.3.2 Analytical Methods

A. *Triangle Method*

The magnitude of **R** in Fig. 3 can also be computed by using trigonometry.

The Law of Sines and the Law of Cosines are especially useful for this:

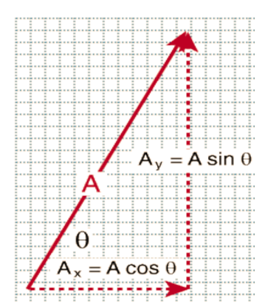


Law of Sines: $A/\sin\alpha = B / \sin\beta = C / \sin\gamma$. (3-1)

Law of Cosines: $C^2 = A^2 + B^2 - 2AB\cos\gamma$ (3-2)

Figure 3

B. *Method Of Components*



A vector **A** can be written as a sum of two vectors $\mathbf{A_x}$ and $\mathbf{A_y}$ along the x and y axis respectively, as shown**.** We call them the components of vector **A** and are given by**:**

$\mathbf{A_x} = A\cos\theta\, \boldsymbol{i}$ (3-3)

$\mathbf{A_y} = A\sin\theta\, \boldsymbol{j}$ (3-4)

where θ is the angle between the vector **A** and the x axis.

To find the resultant vector **R** of a system of vectors **A**, **B**, **C**, etc… we follow these steps:

a) Find the x and y components for each vector using the above equations. i.e. find $\mathbf{A_x}, \mathbf{B_x}, \mathbf{C_x}$ ... and $\mathbf{A_y}, \mathbf{B_y}, \mathbf{C_y}$ ....

Remember they can be positive or negative depending on their direction.

b) Add up these components to get:

$$\mathbf{R_x} = \mathbf{A_x} + \mathbf{B_x} + \mathbf{C_x} + \ldots \qquad (3\text{-}5)$$

$$\mathbf{R_y} = \mathbf{A_y} + \mathbf{B_y} + \mathbf{C_y} + \ldots \qquad (3\text{-}6)$$

c) Now, the magnitude of **R** is : $[\mathbf{R_x}^2 + \mathbf{R_y}^2]^{½}$

The direction of **R** is : $\theta = \tan^{-1} [\mathbf{R_y} / \mathbf{R_x}]$ where θ is the angle between **R** and x axis.

If $\theta > 0$ then **R** is either in the 1st or 3rd quadrant

If $\theta < 0$ then **R** is either in the 2nd or 4th quadrant.

3.3.3 Experimental Method

This method is described in a later section.

3.4 PROCEDURE

For Graphical method, be sure to:

1) Make ARROWS, NOT Lines.
2) LABEL the vectors, and the angles
3) MEASURE the lengths to get the forces
4) MEASURE the angles to get the angles.
5) The only calculation you can do is to convert N or gram to cm, and vice versa. DO NOT use trigonometry in the Graphical part. DO NOT use cm in the analytical part.

For the Experimental part:

We will use a Force Table, in which we balance "forces", and not masses. We will be creating each force by hanging a known mass, and the force will be given by F = m g. Here mass is in kg, and g is 9.8 m/s^2. The masses that we will be using are in grams, and should be divided by 1000 to get kg. However, since the mass will come in each term of the equations, we may cancel out the factor of 1000, and leave the masses in grams. Similarly, the term 'g' comes in each term, and can also be cancelled out. Therefore, using the "force" as a mass in grams leads to no errors, if we realize that, e.g. 300 grams means the "force due to gravity acting on 300 grams", and not the 300-gram mass itself. The concept is important because force is a vector, while mass is a scalar. Hence a mass of 300 gram cannot balance two masses of 200 gram each, but the force due to a 300-gram mass can balance the forces due to two 200-gram masses if they are placed at the appropriate angles. With this in mind, we will deal with masses in grams, understanding that they represent forces due to these masses. The term 'g' in the following refers to gram, not acceleration due to gravity.

VECTOR RESOLUTION: Consider a force vector = 300 g at the 40º angle. Resolve this vector into its x- and y-components by the following methods:

A) ***Graphical***: Make the X- and Y-axes. Use a scale of 30 g = 1.0 cm, and draw an arrow of appropriate length at 40º. Drop perpendiculars from the tip of the vector to the X- and Y-axes. Measure the lengths of these lines and hence find the magnitudes of $\mathbf{F_x}$ and $\mathbf{F_y}$. (do not calculate using trigonometry) Record the results.

B) ***Analytical***: Compute the magnitudes of $\mathbf{F_x}$ and $\mathbf{F_y}$ by using the Component Method (equations 3-3 and 3-4). Record the results.

C) ***Experimental***: The 'force' of 300 gram is equal to that due to a suspended mass of 300 grams. The Equilibrant Vector is the force vector of equal magnitude, but opposite in direction to the Resultant (i.e. **E** = **-R**). The Equilibrant is therefore equal to a suspended mass of 300 grams at the 40+180 =

220º angle. On the force table, clamp a pulley at 220º and hang a mass of 300-gram. Now attach two pulleys, one at 0º and the other at 90º (these will become the X- and Y-components), and add weights to them until they balance the equilibrant force. Equilibrium is seen when the ring in the center of the force table is centered. The two forces are the X- and Y- components (i.e. $\mathbf{F}_x$ and $\mathbf{F}_y$) of the original force. Record them in the Data Table.

VECTOR ADDITION: Find the sum of two 'forces': 100 g at 30º and 200 g at 120º, by:

A) ***Graphical Method***: Make the X- and Y-axes. Use a scale of 25 g = 1.0 cm. Draw arrows of appropriate lengths at 30º and 120º, both starting from the origin. Add them using the Parallelogram method. Measure the length and angle of the resultant. Convert length to magnitude of the resultant vector using the scale used to draw the vectors. Record the results in the Data Table.

B) ***Analytical Method***: Compute the sum of the two vectors by using the Component Method (equations 3-3 and 3-4) as well as the Triangle Method (equations 3-1 and 3-2). Record the results in the Data Table.

C) ***Experimental method***: Clamp two pulleys at the 30º and 120º positions and suspend masses of 100 g and 200 g on them respectively. Now find the Equilibrant by clamping a pulley at the angle found by the component method plus 180°. Attach enough mass to this third pulley to balance the system. If needed, adjust the position (angle) of the pulley. The magnitude and position of the equilibrant is thus found. The resultant of the two forces has the same magnitude, and opposite direction to the equilibrant. Note this in the Data Table.

VECTOR ADDITION: In addition to the two forces of procedure 2, add a third force = 150 g at 230º. Find the vector sum by the Graphical, Component and Experimental methods, and record in the Data Table. Use a scale of 25 g = 1.0 cm for the graphical method.

3.5 CALCULATIONS

Show your calculations for the analytic method and the method of components.

3.6 PRECAUTIONS

1. Don't forget to include the mass of the mass hangars.
2. The strings should be tied to the ring such that the direction of the strings pull it directly away from the pin.
3. Make sure that the pulleys are pointing radially away from the pin.
4. The strings from pulley to ring should be horizontal. Adjust the height of the pulleys.
5. When the system is in balance, the Pin should be in the center of the ring.
6. The weights should not be swinging like a pendulum.
7. Draw the vectors as ARROWS with a sharp pencil. Do not make them as LINES.
8. Label the vectors and angles
9. Measure all angles counter-clockwise from the positive X-axis.
10. MEASURE the length and the angle of the resultant in the Graphical Method. DO NOT calculate using trigonometry.
11. In Analytic Method, DO NOT convert 'g' into 'cm'.

3.7 ADDITIONAL INFORMATION:

http://www.physicsclassroom.com/class/vectors/u3l1b.cfm

http://phet.colorado.edu/sims/vector-addition/vector-addition_en.html

https://www.youtube.com/watch?v=ZYbX8cL5LNE

3.8 TOPICS TO THINK ABOUT

1. What is/are the purpose of the pulleys in the force table?
2. Would the results be affected if the strings do not point to the center of the pin?
3. Should the mass of the weight hangers be included in the calculations?
4. Should the strings be parallel to the force table surface? What if the top of the pulley is higher or lower than the ring?
5. Would it matter if the ring is made of plastic or steel?
6. Should the Force Table surface be horizontal? What would be the effect on the results if it is tilted in any direction?
7. Does it matter if the weights are hanging close to or far below the pulleys?
8. Is the mass of the threads important in this experiment?
9. How would friction in the pulleys effect the results?
10. How can the results if the experiment be improved?
11. Which of the methods used in vector addition seem to be the easiest for you?

Name: ______________________________ Date: _____________

1-A: VECTOR RESOLUTION: Resolve the Force vector **F** = 300 g at the 40º angle into its X- and Y-components, by using the *Graphical Method*. Use Scale: 30 g = 1.0 cm.

Result:

F_x = ______ cm = ______ g

F_y = ______ cm = ______ g

1-B: VECTOR RESOLUTION: Resolve the Force vector **F** = 300 g at 40º angle into its X- and Y-components, by using the *Component Method (Analytical)*.

2-A: VECTOR ADDITION: Find the sum of two forces: **R** = **A** + **B**, where **A**=100 g at 30º and **B**=200 g at 120º, by using *Graphical Method*: Scale: 25 g = 1.0 cm.

Use *Parallelogram method*.

Results:

Measured

Length = ____ cm

Magnitude= ____ g

Measured

Direction = _____°

2-B: VECTOR ADDITION: Find the sum of two forces: **R** = **A** + **B**, where **A**=100 g at 30º and **B**=200 g at 120º, by using the *Component Method (Analytical)*. Use equation 3.3 and 3.4

2-C: VECTOR ADDITION: Find the sum of two forces: **R** = **A** + **B**, where **A**=100 N at 30º and **B**=200 N at 120º, by using the ***Triangle Method (Analytical)*** (eqn. 3-1 and 3-2).

3-A: VECTOR ADDITION: Find the sum of three forces **R** = **A** + **B** + **C**, where **A**=100 g at 30º, **B**=200 g at 120º, and **C**=150 g at 230º by ***Graphical Method***: Use a scale of 25 g = 1.0 cm.

Use ***Polygon Method*** (Tip-To-Tail rule).

Results:

Measured

Length = _____ cm

Magnitude= _____ g

Measured
Direction = ______°

3-B: VECTOR ADDITION: Find the sum of three forces **R** = **A** + **B** + **C**, where **A**=100 g at 30º, **B**=200 g at 120º, and **C**=150 g at 230º by the ***Component Method (Analytical).***

3.10 RESULTS

	FORCE/S	R E S U L T S			
		GRAPHICAL METHOD	ANALYTICAL (TRIANGLE METHOD)	ANALYTICAL (COMPONENT METHOD)	EXPERIMENTAL METHOD (in-class only)
VECTOR RESOLUTION	F= 300 g, θ = 40º	Fx = Fy =		Fx = Fy =	Fx = Fy =
VECTOR ADDITION (TWO FORCES)	F_1 = 100 g, θ = 30º F_2 = 200 g, θ = 120º	Mag = Dir =	Mag = Dir =	Mag = Dir =	*<u>EQUILIBRANT</u>:* Mag = Dir = *<u>RESULTANT</u>:* Mag = Dir =
VECTOR ADDITION (THREE FORCES)	F_1 = 100 g, θ = 30º F_2 = 200 g, θ = 120º F_3 = 150 g, θ = 230º	Mag = Dir =		Mag = Dir =	*<u>EQUILIBRANT</u>:* Mag = Dir = *<u>RESULTANT</u>:* Mag = Dir =

3.11 REPORT SUBMISSION

Hand in or Upload pages 5 – 9 of this manual as your report, i.e. the drawings of the vectors, the detailed calculations for the analytic and component methods, and the Table of Results. Add a page for Sources of error (graphical and experimental) and Discussion / Conclusion.

		Points in report In-Class Lab	Points in report Online Lab
1.	Each part 1A, 1B, 2A, 2B, 2C, 3A, 3B is 5 points	7*5 = 35	7*5 = 35
2.	Table of Results (parts 1, 2, 3 only)	5	5
3.	Experimental Results (For In-Class Version) Attach a picture of the ring in center for reach case	5 3x5=15	-
4.	Sources of Error	5	5
5.	Discussion of the Results / Conclusions	10	10
	Total	75	55

Points taken off for:

Vectors drawn as Lines instead of arrows

Wrong lengths of vectors

Wrong angles of vectors

Calculated instead of measured values in graphical methods

Calculation error in component / analytical part

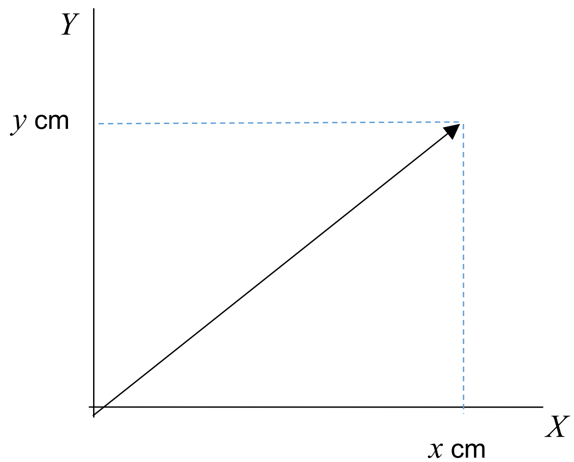
Too many / few significant figures in the answers

Missing / wrong units where needed

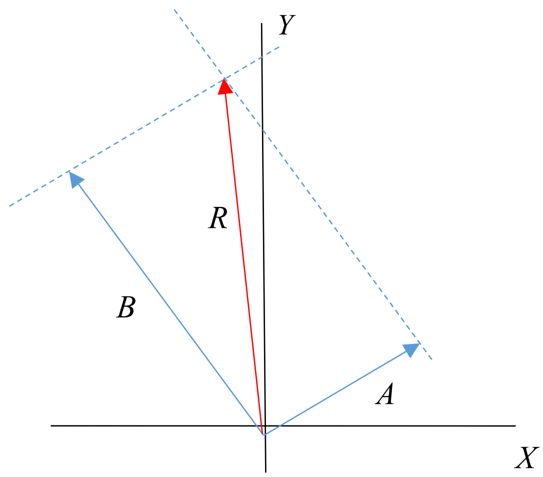
3.12 SAMPLE DATA

The drawings of the vectors in the Graphical Method should look like this:

1-A: VECTOR RESOLUTION



2-A: VECTOR ADDITION (2 VECTORS)



3-A: VECTOR ADDITION (THREE VECTORS)