3 Applications of Differentiation





3.6 A Summary of Curve Sketching

Objective

- Analyze and sketch the graph of a function.

Analyzing the Graph of a Function

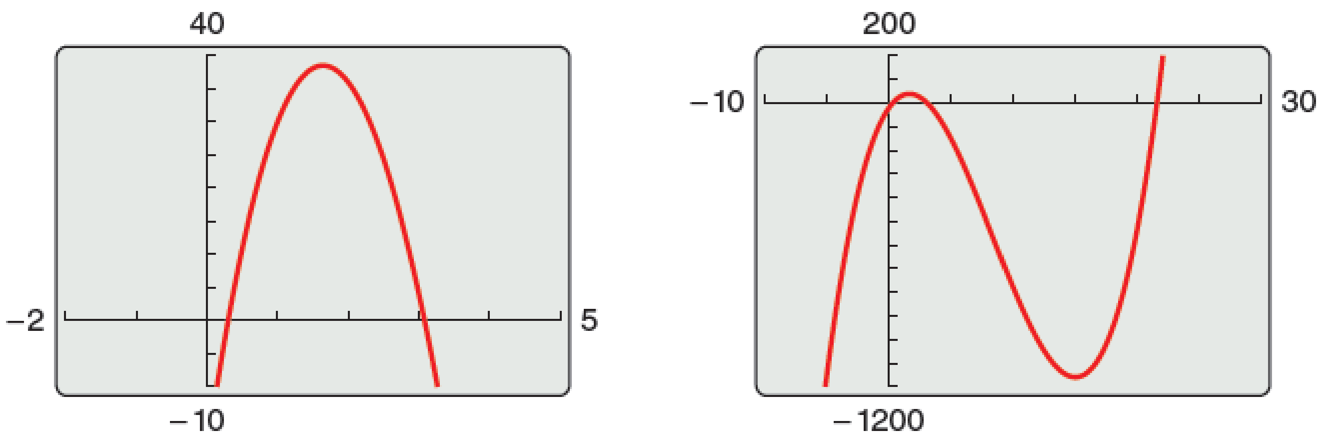
Analyzing the Graph of a Function

When you are sketching the graph of a function, either by hand or with a graphing utility, remember that normally you cannot show the *entire* graph.

The decision as to which part of the graph you choose to show is often crucial.

Analyzing the Graph of a Function

For instance, which of the viewing windows in Figure 3.44 better represents the graph of $f(x) = x^3 - 25x^2 + 74x - 20$?



Different viewing windows for the graph of $f(x) = x^3 - 25x^2 + 74x - 20$

Figure 3.44

Analyzing the Graph of a Function

By seeing both views, it is clear that the second viewing window gives a more complete representation of the graph.

But would a third viewing window reveal other interesting portions of the graph?

To answer this, you need to use calculus to interpret the first and second derivatives.

Analyzing the Graph of a Function

To determine a good viewing window for a function, use these guidelines to analyze its graph.

GUIDELINES FOR ANALYZING THE GRAPH OF A FUNCTION

1. Determine the domain and range of the function.
2. Determine the intercepts, asymptotes, and symmetry of the graph.
3. Locate the x-values for which $f'(x)$ and $f''(x)$ either are zero or do not exist. Use the results to determine relative extrema and points of inflection.

Example 1 – *Sketching the Graph of a Rational Function*

Analyze and sketch the graph of $f(x) = \dfrac{2(x^2 - 9)}{x^2 - 4}$.

Solution:

Domain: All real numbers except $x = \pm 2$

Range: $(-\infty, 2) \cup \left[\frac{9}{2}, \infty\right)$

x-intercepts: $(-3, 0), (3, 0)$

y-intercept: $\left(0, \frac{9}{2}\right)$

Vertical asymptotes: $x = -2, x = 2$

Horizontal asymptote: $y = 2$

Example 1 – *Solution*

cont'd

Symmetry: With respect to y-axis

First derivative: $f'(x) = \dfrac{20x}{(x^2 - 4)^2}$

Second derivative: $f''(x) = \dfrac{-20(3x^2 + 4)}{(x^2 - 4)^3}$

Critical number: $x = 0$

Possible points of inflection: None

Test intervals: $(-\infty, -2), (-2, 0), (0, 2), (2, \infty)$

Example 1 – *Solution*

cont'd

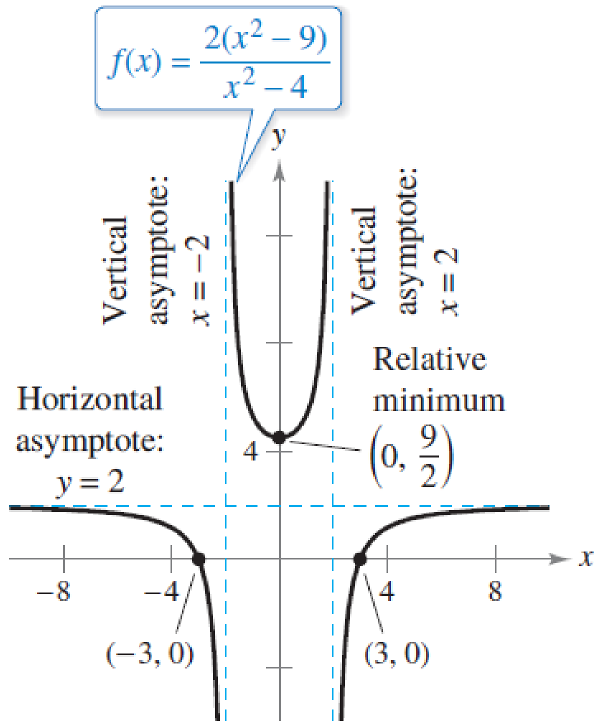
The table shows how the test intervals are used to determine several characteristics of the graph.

	$f(x)$	$f'(x)$	$f''(x)$	Characteristic of Graph
$-\infty < x < -2$		$-$	$-$	Decreasing, concave downward
$x = -2$	Undef.	Undef.	Undef.	Vertical asymptote
$-2 < x < 0$		$-$	$+$	Decreasing, concave upward
$x = 0$	$\frac{9}{2}$	0	$+$	Relative minimum
$0 < x < 2$		$+$	$+$	Increasing, concave upward
$x = 2$	Undef.	Undef.	Undef.	Vertical asymptote
$2 < x < \infty$		$+$	$-$	Increasing, concave downward

Example 1 – *Solution*

cont'd

The graph of *f* is shown in Figure 3.45.



Using calculus, you can be certain that you have determined all characteristics of the graph of *f*.

Figure 3.45

Analyzing the Graph of a Function

The graph of a rational function (having no common factors and whose denominator is of degree 1 or greater) has a **slant asymptote** when the degree of the numerator exceeds the degree of the denominator by exactly 1.

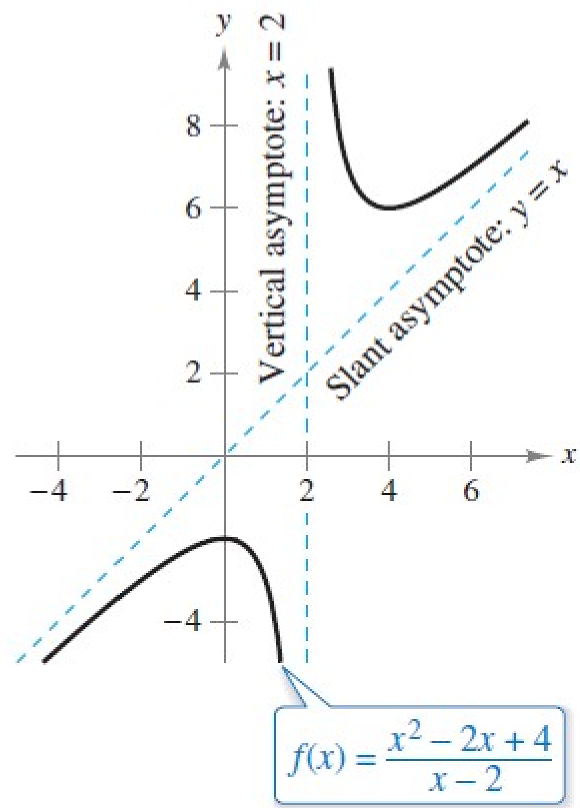
To find the slant asymptote, use long division to rewrite the rational function as the sum of a first-degree polynomial (the slant asymptote) and another rational function.

$$f(x) = \frac{x^2 - 2x + 4}{x - 2} \qquad \text{Write original equation.}$$

$$= x + \frac{4}{x - 2} \qquad \text{Rewrite using long division.}$$

Analyzing the Graph of a Function

In Figure 3.48, note that the graph of *f* approaches the slant asymptote *y* = *x* as *x* approaches $-\infty$ or ∞.



A slant asymptote

Figure 3.48