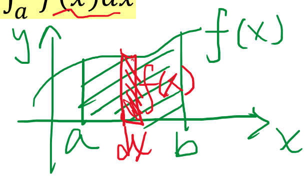
Sec.4.3

p.273: The Definite Integral as the Area of a Region (see Fig. 4.22): $Area = \int_a^b f(x)dx$



p.274: Examples 3

p.275 - 276: Properties of Definite Integrals; Examples 4 - 6; Theorem 4.8

p.277-278:
Evaluating a Definite Integral Using a Geometric Formula In Exercises 23–32, sketch the region whose area is given by the definite integral. Then use a geometric formula to evaluate the integral (a > 0, r > 0).

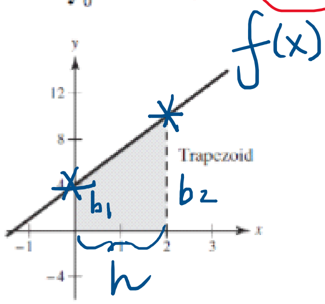
27. $\int_0^2 (3x + 4)\, dx$ $\qquad f(x) = 3x + 4$

$b_1 = 4, \quad b_2 = 10, \quad h = 2$

Trapezoid

$$A = \frac{b_1 + b_2}{2}h = \left(\frac{4 + 10}{2}\right)2 = 14$$

$$A = \int_0^2 (3x + 4)\, dx = 14$$



31. $\int_{-7}^{7} \sqrt{49 - x^2}\, dx$

$f(x) = \sqrt{49 - x^2} \rightarrow y = \sqrt{49 - x^2} \quad y \geq 0$

$y^2 = 49 - x^2 \rightarrow x^2 + y^2 = 49 \quad C(0,0)$

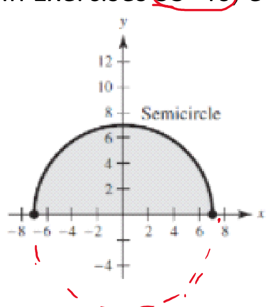
$r = 7$

Using Properties of Definite Integrals In Exercises 33–40, evaluate the definite integral

Semicircle

$$A = \frac{1}{2}\pi r^2 = \frac{1}{2}\pi(7)^2 = \frac{49\pi}{2}$$

$$A = \int_{-7}^{7} \sqrt{49 - x^2}\, dx = \frac{49\pi}{2}$$



$A_O = \pi r^2$

$= \pi \cdot 7^2 = 49\pi$

$A_\triangle = \frac{49}{2}\pi$

using the values below.

$\int_2^6 x^3\, dx = 320, \quad \int_2^6 x\, dx = 16, \quad \int_2^6 dx = 4$

34. $\int_2^2 x\, dx = 0$

36. $\int_2^6 -3x\, dx = -3\int_2^6 x\,dx = -3(16) = -48$

38. $\int_2^6 \left(6x - \frac{1}{8}x^3\right) dx = \int_2^6 6x\,dx - \int_2^6 \frac{1}{8}x^3 dx = 6\int_2^6 x\,dx - \frac{1}{8}\int_2^6 x^3 dx = 6(16) - \frac{1}{8}(320)$

$= 56$

38. $\int_2^6 \left(6x - \frac{1}{8}x^3\right) dx = \int_2^6 6x\,dx - \int_2^6 \frac{1}{8}x^3\,dx = 6\int_2 x\,dx - 8\int_2 x$ [illegible] $= 6(16) - 8($[illegible]$)$

$= 56$

40. $\int_2^6 (21 - 5x - x^3)\,dx = \int_2^6 21\,dx - \int_2^6 5x\,dx - \int_2^6 x^3\,dx$

$= 21\int_2^6 dx - 5\int_2^6 x\,dx - \int_2^6 x^3\,dx = 21(4) - 5(16) - 320 = -316$

42. **Using Properties of Definite Integrals** Given

$$\int_0^3 f(x)\,dx = 4 \quad \text{and} \quad \int_3^6 f(x)\,dx = -1$$

evaluate

(a) $\int_0^6 f(x)\,dx. = \int_0^3 f(x)\,dx + \int_3^6 f(x)\,dx = 4 + (-1) = 3$

(b) $\int_6^3 f(x)\,dx. = -\int_3^6 f(x)\,dx = -(-1) = 1$

(c) $\int_3^3 f(x)\,dx. = 0$

(d) $\int_3^6 -5f(x)\,dx. = -5\int_3^6 f(x)\,dx = -5(-1) = 5$