Sec.2.4

p.133-134: Theorem 2.10; p.135: Examples 2 & 3

THEOREM 2.10 ~~The Chain Rule~~

If $y = f(u)$ is a differentiable function of u and $u = g(x)$ is a differentiable function of x, then $y = f(g(x))$ is a differentiable function of x and

$$1\quad \frac{dy}{dx} = \frac{dy}{du}\cdot\frac{du}{dx} \quad u'$$

p.135: Theorem 2.11; p.136 - 137: Examples 4 – 9

THEOREM 2.11 The General Power Rule

If $y = [u(x)]^n$, where u is a differentiable function of x and n is a rational number, then 2

$$\frac{d}{dx}[u^n] = nu^{n-1}u'.$$

p.140: In Exercises 9–34, find ~~the derivative~~ of the function.

10. $y = 5(2 - x^3)^4$ $\quad u = 2 - x^3 \rightarrow u' = -3x^2$

$$y = 5u^4 \rightarrow y' = 5\cdot 4u^3\cdot(-3x^2) = \boxed{-60x^2(2-x^3)^3}$$

14. $g(x) = \sqrt{4 - 3x^2} = (4-3x^2)^{1/2}$ $\quad u = 4-3x^2 \rightarrow u' = -6x$

$$g = u^{1/2} \rightarrow g'(x) = \tfrac{1}{2}u^{-1/2}\cdot(-6x) = -3x(4-3x^2)^{-1/2}$$

$$= \boxed{-\frac{3x}{\sqrt{4-3x^2}}}$$

20. $y = -\dfrac{3}{(t-2)^4} = -3(t-2)^{-4}$ $\quad u = t-2 \rightarrow u' = 1$

$$y' = -3(-4)(t-2)^{-5}(1) = 12(t-2)^{-5} = \boxed{\frac{12}{(t-2)^5}}$$

28. $y = \dfrac{x}{\sqrt{x^4+4}} = \underset{f}{x}\cdot\underset{g}{(x^4+4)^{-1/2}}$ $\quad u = x^4+4 \rightarrow u' = 4x^3$

$$y' = 1\cdot(x^4+4)^{-1/2} + x\cdot\left[-\tfrac{1}{2}(x^4+4)^{-3/2}\cdot(4x^3)\right] \qquad x\cdot\left(-\tfrac{1}{\cancel{2}}\right)\left(\cancel{4}^{2}x^3\right)$$

$$= (x^4+4)^{-1/2} - 2x^4(x^4+4)^{-3/2} \qquad u^{-1/2} - 2x^4u^{-3/2}$$

$$= (x^4+4)^{-3/2}\left[(x^4+4) - 2x^4\right] = \boxed{\frac{4-x^4}{(x^4+4)^{3/2}}} = u^{-3/2}(u - 2x^4)$$

$x^4 + 4 - 2x^4$

$y^5 + 2y^3 = y^3(y^2+2)$

p.138 - 139: Trigonometric Functions and the Chain Rule; Examples 10 - 13

p.140: In Exercises 35–54, find the derivative of the trigonometric function.

36. $y = \sin \pi x \rightarrow y' = (\cos\pi x)\pi = \pi\cos\pi x$

38. $h(x) = \sec 6x \rightarrow h'(x) = (\sec 6x\tan 6x)(6) = 6\sec 6x\tan 6x$

40. $y = \csc(1-2x)^2$ $\quad \csc,\ 2,\ (1-2x)$

$$y' = \left[-\csc(1-2x)^2\cot(1-2x)^2\right]\left[2(1-2x)^1\right](-2) = 4(1-2x)\csc(1-2x)^2\cot(1-2x)^2$$

$\sin 2\theta = 2\sin\theta\cos\theta$

$y' = [-\csc(1-2x)\cot(1-2x)][2(1-2x)](-2)$

$\sin 2\theta = 2\sin\theta\cos\theta$

46. $g(t) = 5\cos^2 \pi t = 5(\cos \pi t)^2$ 2, cos, πt

$g'(t) = 5\cdot[2(\cos\pi t)^1][-\sin\pi t](\pi) = -5\pi[2\cos\pi t\sin\pi t]$
$= -5\pi\sin 2\pi t$

p.140: In Exercises 61–68, find the slope of the graph of the function at the given point.

62. $y = \sqrt[5]{3x^3 + 4x}$, $(2, 2)$

$y = (3x^3+4x)^{1/5}$

1) $y' = \frac{1}{5}(3x^3+4x)^{-4/5}(9x^2+4)$
$= \dfrac{9x^2+4}{5(3x^3+4x)^{4/5}}$

2) @ $(2, 2)$

$m = y'(2) = \dfrac{9\cdot 2^2+4}{5(3\cdot 2^3+4\cdot 2)^{4/5}}$
$= \dfrac{\cancel{40}\ 8}{\cancel{5}\cdot(32)^{4/5}} = \dfrac{8}{16} = \dfrac{1}{2}$

$\sqrt[5]{32}$

$(32^{1/5})^4 = 2^4$
$= 16$

p.141: In Exercises 69–76, (a) find an equation of the tangent line to the graph of the function at the given point.

76. $y = 2\tan^3 x$, $\left(\frac{\pi}{4}, 2\right)$

$y = 2(\tan x)^3$

1) $y' = 2\cdot[3(\tan x)^2](\sec^2 x)$
$= 6\tan^2 x\sec^2 x$

2) @ $\left(\frac{\pi}{4}, 2\right)$

$m = y'\left(\frac{\pi}{4}\right)$
$= 6\times\tan^2\frac{\pi}{4}\times\sec^2\frac{\pi}{4}$
$= 6\times 1^2\times(\sqrt{2})^2 = 12$

$\sec\frac{\pi}{4} = \frac{1}{\cos\frac{\pi}{4}}$
$= \frac{1}{\sqrt{2}/2} = \sqrt{2}$

3) T.L.

$m = 12$
$\left(\frac{\pi}{4}, 2\right)$ x_1, y_1

$y - 2 = 12\left(x - \frac{\pi}{4}\right)$
$y - 2 = 12x - 3\pi$
$\boxed{y = 12x - 3\pi + 2}$

p.141: Find the second derivative of the function. f''

84. $f(x) = \dfrac{8}{(x-2)^2} = 8(x-2)^{-2}$

$f'(x) = 8(-2)(x-2)^{-3}(1)$
$= -16(x-2)^{-3}$

$f''(x) = -16(-3)(x-2)^{-4}(1)$
$= \boxed{\dfrac{48}{(x-2)^4}}$

90. $f(x) = \dfrac{1}{\sqrt{x+4}}$, $\left(0, \frac{1}{2}\right)$

$f(x) = (x+4)^{-1/2}$
$f'(x) = -\frac{1}{2}(x+4)^{-3/2}(1)$
$= -\frac{1}{2}(x+4)^{-3/2}$

$f''(x) = -\frac{1}{2}\left(-\frac{3}{2}\right)(x+4)^{-5/2}(1)$
$= \frac{3}{4}(x+4)^{-5/2} = \dfrac{3}{4(x+4)^{5/2}}$

$4^{5/2} = (4^{1/2})^5 = 2^5 = 32$

@ $\left(0, \frac{1}{2}\right)$ x

$f''(0) = \dfrac{3}{4(0+4)^{5/2}}$
$= \dfrac{3}{4\cdot 4^{5/2}} = \dfrac{3}{4\cdot 32} = \dfrac{3}{128}$

p.139: Summary of Differentiation Rules