Table of Contents

U-substitiution. ... 2

Recognizing the $f\left(g\left(x\right)\right)g'\left(x\right)$ form. ... 3

1/22/2024 ... 4

Change of Variables. ... 4

Integration of Trigonometric Functions. ... 4

1 ... 4

#10 Diagnostic Test. ... 5

1/23/2024 ... 5

Basic Integrals and Derivatives ... 6

5.5 Derivative of exponential functions. ... 7

5.5#4 ... 7

5.5#5 ... 7

Integration of $\int a^u du$ ... 8

5.5#9 ... 8

#23 Exam Review ... 9

Exam Review #24 ... 10

5.7 Inverse Trigonometric Functions: Differentiation. ... 10

5.8#4 ... 11

1/30/2024 10:39 AM ... 11

Exam Review #23 ... 11

Exam I Review#24 ... 12

5.8 Inverse Trigonometric Functions: Integration ... 12

$d\ dxarcsinu$ ... 12

Find derivative of y = arcsinln$x$2 ... 13

Find the Derivative of arcsine (u) ... 13

Thursday, February 1, 2024 ... 14

5.8 Inverse Trig Functions: Integration. ... 14

5.8#1 ... 14

5.8#3 ... 15

Derivation of the derivative of cosecant u. ... 16

5.8#5 16
Hyperbolic Functions. 16
Monday, February 5, 2024 17
Derivative of sinh x 17
5.9#4 17
7.1#3 17
Tuesday, February 6, 2024 18
5.5#6 18
5.5#7 19
5.5#12 19
5.7#9 19
5.9 Hyperbolic Functions. 20
Wednesday, February 7, 2024 21
7.2#4 21
7.3 Shell Method. 22

U-substitiution.

$$\int \underbrace{f}_{\substack{\text{Outside}\\ \text{function}}}\left(\underbrace{g}_{\text{inside function}}(x)\right)\underbrace{g'(x)}_{\substack{\text{Derivative of}\\ \text{inside function}}}\,dx = F\left(g(x)\right)+C$$

Recognizing the $f\left(g\left(x\right)\right)g'\left(x\right)$ form.

$$\int x\left(x^2+1\right)^2 dx$$
$$u = x^2+1$$
$$du = 2xdx$$
$$\frac{dy}{2} = \frac{2xdx}{2} \Rightarrow xdx = \frac{du}{2}$$
$$\int x\left(x^2+1\right)^2 dx \Rightarrow \int \frac{1}{2}\cdot\left(u\right)^2 du$$
$$= \frac{1}{2}\int u^2 du$$
$$= \frac{1}{2}\cdot\frac{u^3}{3}+C$$
$$\frac{u^3}{6}+C$$

$$\int \sqrt{2x-1}dx$$
$$u = 2x-1 \Rightarrow du = 2dx$$
$$\int \sqrt{u}\cdot\frac{du}{2}$$
$$\frac{1}{2}\int u^{1/2}du$$
$$\frac{1}{2}\cdot\frac{2}{3}u^{3/2}+C$$
$$\therefore \int \sqrt{2x-1}dx = \frac{1}{3}\cdot\left(2x-1\right)^{3/2}+C$$

1/22/2024

Change of Variables.

$$\int_0^1 x\left(x^2+1\right)^3 dx$$

Let $u=x^2+1$

$$du = 2xdx$$

$$\frac{1}{2}du = xdx$$

$$\boxed{\begin{array}{ll} x=0 & x=1 \\ u=1 & u=2 \end{array}}$$

$$\int_0^1 x\left(x^2+1\right)^3 dx \Rightarrow \frac{1}{2}\int_1^2 u^3 du$$

$$=\frac{1}{2}\cdot\frac{u^4}{4}\Bigg]_1^2 = \frac{15}{8}$$

Integration of Trigonometric Functions.

$$\int \sec x dx$$

$$\int \frac{\sec x\left(\sec x+\tan x\right)}{\left(\sec x+\tan x\right)}dx$$

$$\int \frac{\sec^2 x+\sec x\tan x}{\sec x+\tan x}dx$$

$$\ln\left|\sec x+\tan x\right|+C$$

1

$$\int\left(1+\cot^2 x\right)dx$$

$$\int \csc^2 x dx = \cot x + C$$

#2

$$\int \frac{\left(\ln x\right)^3}{x}dx \qquad u=\ln x \qquad du=\frac{1}{x}dx$$

$$\int u^3 du$$

$$\frac{1}{4}u^4+C$$

$$\frac{1}{4}\left(\ln x\right)^4+C$$

#10 Diagnostic Test.

$$\int \frac{\sqrt[7]{3-\ln z^6}}{z}dz$$

Logarithmic Properties

$\log_b b = 1$

$\log_b a^c = c\log_b a$

$a^{\log_a b} = b$

$\log a - \log b = \log \frac{a}{b}$

$\log a + \log b = \log ab$

$$\int \frac{\sqrt[7]{3-\ln z^6}}{z}dz$$

$$\int \frac{\sqrt[7]{3-6\ln z}}{z}dz$$

$$\boxed{\begin{aligned} u &= 3-6\ln z \\ du &= -6\cdot\frac{1}{z}dz \end{aligned}}$$

$$\int \sqrt[7]{3-6\ln z}.\frac{-6dz}{-6z}$$

$$-\frac{1}{6}\int u^{1/7}du = -\frac{1}{6}\cdot\frac{u^{\frac{1}{7}+1}}{\frac{1}{7}+1}+C$$

$$-\frac{7}{48}(3-6\ln z)^{\frac{8}{7}}+C$$

1/23/2024

Differentiate implicitly the equation $x = \ln(x+y+1)$ to find the slope of the tangent line at any given point (x,y)

$$\frac{d}{dx}(x) = \frac{d}{dx}\left(\ln(x+y+1)\right)$$
$$1 = \frac{1}{x+y+1} \cdot \frac{d}{dx}(x+y+1)$$
$$1 = \frac{1}{x+y+1} \cdot 1 + y' + 0$$
$$1 + y' = x + y + 1$$
$$y' = x + y$$
$$\text{Random Slope of } \left(-1, e^{-1}\right)$$

$$y' = -1 + \frac{1}{e}$$

Basic Integrals and Derivatives

$\frac{d}{dx}(\sin u) = \cos u du$	$\int \cos u du = \sin u + C$
$\frac{d}{dx}(\cos u) = -\sin u du$	$\int \sin u du = -\cos u + C$
$\frac{d}{dx}(\tan u) = \sec^2 u du$	$\int \sec^2 u du = \tan u + C$
$\frac{d}{dx}(\sec u) = \sec u \tan u du$	$\int \sec u \tan u du = \sec u + C$
$\frac{d}{dc}(\cot u) = -\csc^2 u du$	$\int \csc^2 du = -\cot u + C$
$\frac{d}{dc}(\csc u) = -\csc u \cot u du$	$\int \csc u \cot u du = -\csc u + C$

5.5 Derivative of exponential functions.

$$\frac{d}{dx}\left(e^u\right)=e^u du$$

$$a^{\log_a b}=b$$

Remember that if $a^u = x$ then $\log_a x = u$

Logarithm is another word for exponent!!

$$\frac{d}{dx}\left(2^x\right)$$

$$\therefore \frac{d}{dx}\left(2^x\right)=\frac{d}{dx}\left[e^{\ln 2^x}\right]$$

Remember that $\ln(x)=\log_e(x)$

$$\therefore \frac{d}{dx}\left(2^x\right)=\frac{d}{dx}\left[e^{\ln 2^x}\right]=e^{\ln 2^x}\frac{d}{dx}\left(\ln 2^x\right)$$

$$=e^{\ln 2^x}\frac{d}{dx}\left[x\cdot\ln 2\right]$$

$$=3^{\log_e 2^x}\ln 2$$

$$=2^x\ln 2$$

$$\therefore \boxed{\frac{d}{dx}\left(a^u\right)=a^u\left(\ln a\right)}$$

5.5#4

Find the derivative of y=5^{-2x}

$$\frac{d}{dx}\left[a^u\right]=a^u\left(\ln a\right)du$$

$$\frac{d}{dx}\left[5^{-2x}\right]=5^{-2x}\left(\ln 5\right)\left(-2\right)$$

5.5#5

Find the derivative of the function $f(x)=x5^x$

$$f(x)=x5^x$$

$$f'(x)=\frac{d}{dx}(x)\cdot 5^x+x\cdot\frac{d}{dx}\left(5^x\right)$$

$$=1\cdot 5^x+x\cdot 5^x\cdot\ln 5\cdot\frac{d}{dx}(x)$$

$$=5^x+x\cdot 5^x\ln 5$$

$$=5^x\left(1+x\ln 5\right)$$

Integration of $\int a^u du$

$$\int a^{\omega} d\omega$$

$$\boxed{\text{Remmember } a^u = e^{\ln a^u}}$$

$$\int e^{\omega} d\omega$$

$$\int e^{\ln a^u} du$$

$$e^{\omega} + C$$

$$\omega = \ln a^u$$

$$\omega = u \ln a$$

$$\frac{d\omega}{du} = \ln a$$

$$d\omega = \ln a du$$

$$\frac{d\omega}{\ln a} = du$$

$$\therefore \int e^{\omega} d\omega$$

$$\int e^{\ln a^u} du \Rightarrow \int e^{\omega} \cdot \frac{d\omega}{\ln a}$$

$$= \frac{1}{\ln a} \int e^{\omega} d\omega$$

$$= \frac{1}{\ln a} e^{\omega} + C$$

$$\therefore \boxed{\int a^u du = \frac{1}{\ln a} a^u + C}$$

5.5#9

Find the integral of $\int 7^{-x} dx$

$$\int 7^{-x}\,dx$$

$$a = 7 \qquad u = -x$$

$$\frac{du}{dx} = -1 \qquad du = -dx$$

$$-\int 7^{u}\,du = -\int e^{\ln 7^{u}}\,du$$

$$= -\int e^{w}\,dw$$

$$w = \ln 7^{u} \qquad w = u(\ln 7) \qquad \frac{dw}{du} = \ln 7 \qquad dw = \ln 7\,du$$

$$= -\frac{1}{\ln 7}\int e^{\ln 7^{u}}\,du$$

$$-\frac{1}{\ln 7}\cdot e^{\ln 7^{u}} + C$$

$$-\frac{1}{\ln 7}\cdot 7^{u} + C$$

#23 Exam Review

$$\int_{1}^{2} 6x^{2}4x^{3}\,dx$$

$$u = x^{3} \qquad du = 3x^{2}2x$$

$$2du = 6x^{2}2x$$

$$x = 1, u = 1 \qquad x = 2, u = 8$$

$$\int_{1}^{0} 4^{u}\,du = 2\cdot\frac{1}{\ln 4}\cdot 4^{u}\Big]_{1}^{8}$$

$$= 2\left[\frac{1}{\ln 4}\right]\left(4^{8} - 4\right)$$

$$= \frac{2}{\ln 4}(65536 - 4)$$

$$= \frac{2}{\ln 4}(65532)$$

$$= \frac{2}{\ln 2^{2}}(65532)$$

$$= \frac{1}{\ln 2}(65532)$$

Exam Review #24

$$\int_1^2 \frac{6\ln x}{x}\,dx$$

$$u = \ln x \quad du = \frac{1}{x}dx$$

$$\int_0^{\ln 2} 6^u\,du$$

$$\frac{1}{\ln 6}6^u\Big]_0^{\ln 2}$$

$$\frac{1}{\ln 6}\left[6^{\ln 2} - 6^0\right]$$

$$\frac{1}{\ln 6}\left[6^{\ln 2} - 1\right]$$

$$\frac{6^{\ln 2} - 1}{\ln 6}$$

5.7 Inverse Trigonometric Functions: Differentiation.

Key Concepts.

- For a function to have a derivative, it needs to be one-to-one.
- Trigonometric functions need to have their domain limited for them to be one-to-one.

If given a function such as y= arcsin u and asked to find the derivative, we proceed as follows.

$$y = \arcsin u$$

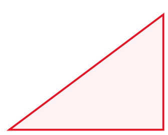
$$\sin y = u$$

$$\frac{d}{dx}(\sin y) = \frac{d}{dx}(u)$$

$$\cos y y' = du$$

$$y' = \frac{du}{\cos y}$$

If given $y = \arcsin e^{x^2}$ what is y'=?



$r = e^{x^2}$

$q =$

$$\sin y = e^{x^2} = \frac{e^{x^2}}{1}$$

$$\frac{d}{dx}(\sin y) = \frac{d}{dx}\left[e^{x^2}\right]$$

$$\cos y \cdot y' = e^{x^2} \cdot 2x$$

$$\cos y \cdot y' = 2xe^{x^2}$$

$$y' = \frac{2xe^{x^2}}{\cos y}$$

$$y' = \frac{2xe^{x^2}}{\sqrt{1 - e^{2x^2}}}$$

Since

$$\boxed{\frac{d}{dx}(\arcsin u) = \frac{u'}{\sqrt{1-u^2}}}$$

Then we can say that

$$\boxed{\int \frac{du}{\sqrt{1-u^2}} = \arcsin u + C}$$

5.8#4

Find the integral

$$\int \frac{6x+7}{\sqrt{1-x^2}}dx$$

We split it into two in order to fit the arcsin

$$\int \frac{6xdx}{\sqrt{1-x^2}} + \int \frac{7}{\sqrt{1-x^2}}dx$$

For $\int \frac{6xdx}{\sqrt{1-x^2}}$

$$u = 1-x^2 \qquad du = -2x$$

$$-3\int \frac{du}{\sqrt{u}} = -3\int u^{-\frac{1}{2}}du$$

For $\int \frac{7}{\sqrt{1-x^2}}dx$

$$= 7\int \frac{1}{\sqrt{1-x^2}}dx$$

Combining the two yields

$$C - 6\sqrt{1-x^2} + 7\sin^{-1} x$$

1/30/2024 10:39 AM

Exam Review #23

$$\int_1^2 6x^2 4^{x^3}\,dx$$

$$u = x^3 \qquad du = 3x^2 dx$$

$$x = 1, u = 1 \qquad dx = \frac{du}{3x^2}$$

$$x = 2, u = 8$$

$$2\int_1^8 4^u\,du$$

$$2\frac{1}{\ln 4}4^u\Big]_1^8$$

$$\frac{2}{\ln 2^2}\left[4^u\right]_1^8$$

$$\frac{1}{\ln 2}\left[4^8 - 4\right]$$

Exam I Review#24

$\int_1^2 \frac{6^{\ln x}}{x}dx$	$u=\ln x$	$du=\frac{1}{x}dx=\frac{dx}{x}$

$\int_1^2 6^{\ln x}\frac{dx}{x}$

$\int 6^u\,du=\frac{1}{\ln 6}\cdot 6^u=\frac{1}{\ln y}\left[6^{\ln x}\right]_1^2$

$=\frac{1}{\ln y}\left[6^{\ln 2}-6^{\ln 1}\right]$

5.8 Inverse Trigonometric Functions: Integration

It is important to note that.

$\frac{d}{dx}[\arcsin u]=-\frac{d}{dx}[\arccos u]$	
$\frac{d}{dx}[\arctan u]=-\frac{d}{dx}[\operatorname{arccot} u]$	
$\frac{d}{dx}[\operatorname{arcsec} u]=-\frac{d}{dx}[\operatorname{arccsc} u]$	

- In order to master a formular, learning how to derive it a sure way.
- The composition of a function and it's inverses simply gives a variable as shown below

$(f\circ g)(x)=x$

For instance

$y=x^2-4$	$[-\infty,0]$
$x=y^2-4$	$[-4,\infty]$
$\pm\sqrt{x+4}=y$	

$g(x)=f^{-1}(x)=\left(-\sqrt{x+4}\right)$

$f(g(x))=(g(x))^2-4$

$=\left(-\sqrt{x+4}\right)^2-4$

$=x+4-4=x$

$\frac{d}{dx}[\arcsin u]$

$\sin y=\sin(\arcsin u)$	$y'=\frac{du}{\cos y}$
$\sin y=u$	$\sin y=u$
$\frac{d}{dx}[\sin y]=\frac{d}{dx}u$	$\sin y=\frac{u}{1}$
$\cos y\cdot y'=du$	

Find derivative of $y = \arcsin \ln(x^2)$

$$y' =$$

$$\sin y = \sin\left[\arcsin \ln\left(x^2\right)\right]$$

$$\sin y = \ln x^2$$

$$\frac{d}{dx}[\sin y] = \frac{d}{dx}\left[\ln x^2\right]$$

$$\cos y \cdot y' = \frac{2x}{x^2} = \frac{2}{x}$$

$$y' = \frac{2}{x \cos y}$$

$$but \cos y = \frac{\sqrt{1-\left(\ln x^2\right)^2}}{1}$$

$$y' = \frac{2}{x\sqrt{1-\left(\ln x^2\right)^2}}$$

Find the Derivative of arcsine (u)

$$\operatorname{arccsc} u$$

$$\csc y = \csc\left(\operatorname{arccsc} u\right)$$

$$\frac{d}{dx}[\csc y] = \frac{d}{dx}(u)$$

$$-\cot y \csc y \cdot y' = u'$$

$$y' = -\frac{u'}{\cot y \csc y}$$

Triange with;

$$opp = 1, hyp = u, adj = \sqrt{u^2 - 1}$$

$$= \frac{-u'}{\frac{\sqrt{u^2-1}}{1}\frac{u}{1}}$$

$$= -\frac{u'}{u\sqrt{u^2-1}}$$

Thursday, February 1, 2024

5.8 Inverse Trig Functions: Integration.

$\frac{d}{dx}\left(\arctan\frac{u}{a}\right)$	$= \frac{du}{a\left(\frac{\sqrt{a^2+u^2}}{a}\right)^2}$
$\tan y = \tan\left(\arctan\frac{u}{a}\right)$	$= \frac{adu}{a^2+u^2}$
$\frac{d}{dx}[\tan y] = \frac{d}{dx}\left[\frac{u}{a}\right]$	$\frac{d}{dx}\left(\arctan\frac{u}{a}\right) = \frac{adu}{a^2+u^2}$
Triangle: Opp=u,adj=a, hyp=$\sqrt{a^2+u^2}$	$\Rightarrow \int\frac{adu}{a^2+u^2} = \arctan\frac{u}{a} + C$
$\sec^2 y \cdot y' = \frac{1}{a}\cdot du$	$a\int\frac{du}{a^2+u^2} = \arctan\frac{u}{a} + C$
$y' = \frac{\frac{1}{a}du}{\sec^2 y} = \frac{du}{a\sec^2 y}$	$\int\frac{du}{a^2+u^2} = \frac{1}{a}\cdot\arctan\frac{u}{a} + C$

5.8#1

Find the indefinite integral.

$$\int \frac{16}{1+64x^2}\,dx$$

$$16\int \frac{dx}{1+64x^2} \quad \boxed{\begin{array}{l} a=1, u=8x \\ du=8dx \\ dx=\frac{8}{du} \end{array}} \quad \boxed{\frac{1}{a}\arctan\frac{u}{a}+C}$$

$$=\frac{16}{8}\cdot\frac{1}{1}\arctan\frac{8x}{1}+C$$

$$=2\arctan 8x+C$$

Proof

$$\frac{d}{dx}\left[16\arctan 8x+C\right]$$

$$y=\arctan 8x+C$$

$$\tan y=$$

Triangle:Opp=8x,adj=1

hyp=$\sqrt{64x^2+1}$

$$y=\arctan 8x+C$$

$$\tan y=8x$$

$$\frac{d}{dx}\left[\tan y\right]=\frac{d}{dx}\left[8x\right]$$

$$\sec^2 y\cdot y'=8$$

$$y'=(2)\frac{8}{\sec^2 y}=(2)\frac{8}{\left(\frac{\sqrt{64x^2+1}}{1}\right)^2}$$

$$=\frac{16}{64x^2+1}$$

5.8#3

Find the indefinite integral.

$$\int \frac{e^{2x}}{16+e^{4x}}\,dx$$

$a=4$	$u=e^{2x}$
$\frac{du}{dx}=2e^{2x}$	$du=2e^{2x}dx$

$$\frac{1}{2}\int \frac{du}{a^2+u^2}=\frac{1}{2}\cdot\frac{1}{a}\cdot\arctan\frac{u}{a}+C$$

$$=\frac{1}{2}\cdot\frac{1}{4}\arctan\frac{1}{4}e^{2x}+C$$

Derivation of the derivative of cosecant u.

$$y = \operatorname{arccsc}\frac{u}{a}$$

$$\csc y = \frac{u}{a}$$

Triangle opp=a,hyp=u
adj=$\sqrt{u^2 - a^2}$

$$\frac{d}{dx}(\csc y) = \frac{d}{dx}\left(\frac{u}{a}\right)$$

$$-\csc y \cot y \cdot y' = \frac{1}{a} \cdot du$$

$$y' = \frac{du}{a(-\csc y \cot y)}$$

$$= \frac{du}{-a \cdot \frac{u}{a} \cdot \frac{\sqrt{u^2 - a^2}}{a}}$$

$$= -\frac{du}{\frac{\sqrt{u^2 - a^2}}{a}}$$

$$\frac{d}{dx}\left(\operatorname{arccsc}\frac{u}{a}\right) = -\frac{adu}{u\sqrt{u^2 - a^2}}$$

$$\int \frac{-adu}{u\sqrt{u^2 - a^2}} = \operatorname{arccsc}\frac{u}{a}$$

$$-\frac{1}{a}\int \frac{-adu}{u\sqrt{u^2 - a^2}} = -\frac{1}{a}\operatorname{arccsc}\frac{u}{a}$$

$$\int \frac{du}{u\sqrt{u^2 - a^2}} = -\frac{1}{a}\operatorname{arcsec}\frac{|u|}{a} + C$$

5.8#5

$$\int_1^3 \frac{1}{x\sqrt{16x^2 - 4}}\,dx$$

$a = 2$	
$u = 4x$	$du = 4dx$

$$\frac{4}{4}\int_1^3 \frac{4}{x4\sqrt{16x^2 - 4}}\,dx = \frac{1}{2}\operatorname{arcsec}\frac{|4x|}{2}\Bigg]_1^3$$

$$= \frac{1}{2}\left[\operatorname{arcsec}\frac{4(3)}{2} - \operatorname{arcsec}\frac{4(1)}{2}\right]$$

$$= \frac{1}{2}\left(\operatorname{arcsec} 6 - \sec^{-1} 2\right)$$

$$= \frac{1}{2}\left(\operatorname{arcsec}(6) - \frac{\pi}{2}\right)$$

Hyperbolic Functions.

Definitions of the Hyperbolic Functions.

$$\sinh x = \frac{e^x - e^{-x}}{2} \qquad \operatorname{csch} x = \frac{1}{\sinh x}, \qquad x \neq 0$$

$$\cosh x = \frac{e^x + e^{-x}}{2} \qquad \operatorname{sech} x = \frac{1}{\cosh x}$$

$$\tanh x = \frac{\sinh x}{\cosh x} \qquad \coth x = \frac{1}{\tanh x}, x \neq 0$$

Monday, February 5, 2024

Derivative of sinh x

$$y = \sinh x$$
$$y' = ?$$
$$y = \frac{e^x - e^{-x}}{2}$$
$$y = \frac{1}{2}\left(e^x - e^{-x}\right)$$
$$y' = \frac{1}{2}\left[e^x(1) - e^{-x}(-1)\right]$$
$$y' = \frac{1}{2}\left[e^x + e^{-x}\right]$$
$$\therefore \int \cosh x = \sinh x + C$$

Proof of $\cosh^2 - \cos^2 x = 1$

$$\sinh^2 x - \cosh^2 x = -1$$
$$\sinh x = \frac{e^x - e^{-x}}{2}$$
$$\cosh x = \frac{e^x + e^{-x}}{2}$$
$$\frac{1}{4}\left(e^{2x} - 2e^x e^{-x} + e^{-2x}\right) - \frac{1}{4}\left(e^{2x} + 2e^x e^{-x} + e^{-2x}\right) \stackrel{?}{=} 1$$
$$e^{2x} - 2e^x e^{-x} + e^{-2x} - \left(e^{2x} + 2e^x e^{-x} + e^{-2x}\right) = 4$$
$$e^{2x} - 2 + e^{-2x} - e^{2x} - 2 - e^{-2x} = 4$$
$$-4 = 4$$

Which was an error but also able to prove that

$$\cosh^2 x - \sinh^2 x = 1$$

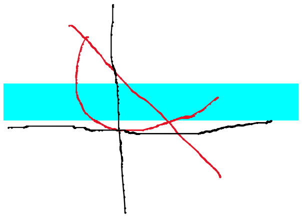
5.9#4

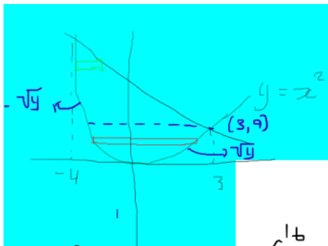
Find the derivative of $\sinh 6x$

$$y' = \cosh 6x \cdot 6$$
$$y' = 6\cosh 6x$$

7.1#3

Consider the following. $Y=x^2$, $y=12-x$





$\int_0^9 (\sqrt{y} - (-\sqrt{y}))\,dy + \int_9^{16} [(12-y) - (-\sqrt{y})]\,dy$

Find limits $x^2 = 12 - x$ $(x+4)(x-3) = 0$ $x = -4, 3$ $\int_{-4}^{3} 12 - x - (x^2)\,dx$ $12x - \frac{1}{2}x^2 - \frac{1}{3}x^3 \Big]_{-4}^{3}$ $12(3) - \frac{1}{2}(3)^2 - \frac{1}{3}(3)^3 - \left[12(-4) - \frac{1}{2}(-4)^2 - \frac{1}{3}(-4)^3\right]$ $36 - \frac{9}{2} - 9 - \left(-48 - 8 + \frac{64}{3}\right)$	$\int_0^9 2\sqrt{y}\,dy + \int_9^{16} (12 - y + \sqrt{y})\,dy$ $2\int_0^9 y^{1/2}\,dy$ $2 \cdot \frac{2}{3} y^{3/2} \Big]_0^9 + 12 - \frac{1}{2}y^2 + \frac{2}{3}y^{3/2} \Big]_9^{16}$ $12(16) - \frac{1}{2}(16)^2 + \frac{2}{3}(4^2)^{3/2} - \left[12(9) - \frac{1}{2}(81) + \frac{2}{3}(3^2)^{3/2}\right]$ $12(16) - \frac{1}{2} \cdot 16 \cdot 16 + \frac{2}{3} \cdot 64 - 12(9) + \frac{81}{2} - 18$ $12(16-9) + \frac{81}{2} - 146$	

Tuesday, February 6, 2024

5.5#6

$h(x) = \log_3 \frac{x\sqrt{x-6}}{3}$ $= \log_3 x\sqrt{x-6}$ $= \log_3 x\sqrt{x-6} - 1$ $= \log_3 x + \log_3 \sqrt{x-6} - 1$ $= \log_3 x + \log_3 (x-6)^{\frac{1}{2}} - 1$ $h(x) = \log_3 x + \frac{1}{2}\log_3 (x-6) - 1$ $= \frac{\ln x}{\ln 3} + \frac{1}{2}\left[\frac{\ln(x-6)}{\ln 3}\right] - 1$	$h'(x) = \frac{1}{\ln 3} \cdot \frac{1}{x} + \frac{1}{\ln 9} \cdot \frac{1}{x-6}$ $h'(x) = \frac{1}{\ln 3^3} + \frac{1}{\ln 9^{(x-6)}}$	

5.5#7

$\ln y = \ln x^{\frac{3}{x}}$ $\ln y = \frac{3}{x}\ln x$ $\frac{1}{y}\cdot y' = 3\cdot\frac{d}{dx}\left[\frac{1}{x}\cdot\ln x\right]$ $\frac{y'}{y} = 3\left[-\frac{1}{x^2}\cdot\ln x + \frac{1}{x}\cdot\frac{1}{x}\right]$	$\frac{y'}{Y} = 3\left[-\frac{1}{x^2}\ln x + \frac{1}{x^2}\right]$ $y' = y\left\{\frac{3}{x^2}[1-\ln x]\right\}$ $y' = x^{3/x}\left\{\frac{3}{x^2}[1-\ln x]\right\}$	

5.5#12

$\int_{-2}^{2} 36^{\frac{1}{2}x}\,dx$ $\int a^u du = \frac{1}{\ln a}a^u + C$ $u = \frac{1}{2}x \qquad \frac{du}{dx} = \frac{1}{2}$ $du = \frac{1}{2}dx$ if x=-2, u=-1 x=2, u=1 $\int_{-1}^{1} 36^u 2du$ $2\int_{-1}^{1} 36^u du$ $2\left[\frac{1}{\ln 36}36^u\right]_{-1}^{1}$ $\frac{2}{\ln 36}\left[36^1 - 36^{-1}\right]$	$\frac{1}{\ln 6}\left(36 - \frac{1}{36}\right)$ $\frac{1}{\ln 6}\left(\frac{36^2 - 1}{36}\right)$ $\frac{36^2 - 1}{36\ln 6}$ $\boxed{\frac{36^2 - 1}{\ln 6^{36}}}$	

5.7#9

$\int_1^2 \frac{2x-3}{\sqrt{4x-x^2}}$ $\int \frac{2x-3}{\sqrt{4x-x^2}}dx$ $\int \frac{2x}{\sqrt{4x-x^2}}dx - 3\int\frac{dx}{\sqrt{4x-x^2}}$ $-\int \frac{(-1)2x}{\sqrt{4x-x^2}}dx - 3\int\frac{dx}{\sqrt{4x-x^2}}$ $-\int \frac{-2x+4-4}{\sqrt{4x-x^2}}dx - 3\int\frac{dx}{\sqrt{4x-x^2}}$ $-\int \frac{4-2x}{\sqrt{4x-x^2}}dx + 4\int\frac{dx}{\sqrt{4x-x^2}} - 3\int\frac{dx}{\sqrt{4x-x^2}}$	$\int\frac{dx}{\sqrt{4x-x^2}} - \int\frac{4-2x}{\sqrt{4-x^2}}dx$ $\int\frac{dx}{\sqrt{2^2-(x-2)^2}} - \int\frac{4-2x}{\sqrt{4-x^2}}dx$ $= \arcsin\frac{(x-2)}{a}\Big]_1^2 - \int\frac{du}{u}$ $= \arcsin\frac{(x-2)}{a}\Big]_1^2 - 2\sqrt{u}\Big]_1^2$ $= \arcsin\frac{(x-2)}{a}\Big]_1^2 - 2\sqrt{u}\Big]_1^2$	$= \sin^{-1}(0) - \arcsin\left(-\frac{1}{2}\right) - 2\left[2-\sqrt{3}\right]$ $= 0 - \left(-\frac{\pi}{6}\right) - 4 + 2\sqrt{3}$ $= \frac{\pi}{6} - 4 + 2\sqrt{3}$

5.9 Hyperbolic Functions.

Proof of $\cosh^2 x - \sinh^2 x = 1$

$$\left(\frac{e^x + e^{-x}}{2}\right)^2 - \left(\frac{e^x - e^{-x}}{2}\right)^2 = 1$$

$$\frac{e^{2x} + 2\cdot e^x e^{-x} + e^{-2x}}{4} - \frac{e^{2x} - 2e^x e^{-x} + e^{-2x}}{4} = 1$$

$$\frac{e^{2x} + 2 + e^{-2x} - e^{2x} + 2 - e^{-2x}}{4} = 1$$

$$\frac{4}{4} = 1$$

$$1 = 1$$

Proof of $\frac{d}{dx}(\sinh u) = \cosh u$

$$\frac{d}{dx}(\sinh u) = \frac{d}{dx}\left[\frac{e^u - e^{-u}}{2}\right]$$

$$\frac{1}{2}\cdot\frac{d}{dx}\left[e^u - e^{-u}\right]$$

$$= \frac{1}{2}\left[e^u du - e^{-u}(-1)du\right]$$

$$= \cosh u$$

Proof of sinhx=ln$\left(x + \sqrt{x^2+1}\right)$

$$y = \sinh x = \frac{e^x - e^{-x}}{2}$$

$$x = \sinh y = \frac{e^y - e^{-y}}{2}$$

$$2x = e^y - \frac{1}{e^y}$$

$$2x = \frac{e^{2y} - 1}{e^y}$$

$$2xe^y = e^{2y} - 1$$

$$0 = e^{2y} - 2xe^y - 1$$

$$e^y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(1) \pm \sqrt{(-2x)^2 - 4(1)(-1)}}{2(1)}$$

$$= e^y = \frac{2x \pm \sqrt{4x^2 + 4}}{2}$$

$$= \frac{2x \pm 2\sqrt{x^2+1}}{2}$$

$$\frac{2\left(x \pm \sqrt{x^2+1}\right)}{2} = e^y$$

$$e^y = x + \sqrt{x^2+1}$$

$$\ln e^y = \ln\left(x + \sqrt{x^2+1}\right)$$

$$y = \ln\left(x + \sqrt{x^2+1}\right)$$

derivative of $\left[\sinh^{-1}u\right]=\dfrac{u'}{\sqrt{u^2+1}}$ $\dfrac{d}{dx}\left[\sinh^{-1}u\right]=\dfrac{d}{dx}\left[\ln\left(u+\sqrt{u^2+1}\right)\right]$ $=\dfrac{1du+\frac{1}{}}{}$ $\boxed{\text{NOT FINISHED}}$		

Wednesday, February 7, 2024

7.2#4

Find the volume of the solids generated by revolving the region bounded by the graphs of the equations about the given lines.

$y=\sqrt{x}$ $y=0$ $x=3$ $V=\pi\int_0^3\left(\sqrt{x}-0\right)^2dx$ $=\pi\int_0^3 xdx$ $=\pi\left[\dfrac{x^2}{2}\right]_0^3=\dfrac{9\pi}{2}$		

$\pi\int_0^{\sqrt{3}}(9-y)^2-(9-3)^2\,dx$ $\pi\int_0^{\sqrt{3}}81-18y+y^2-9dx$ $=\dfrac{144\pi\sqrt{3}}{5}$

7.3 Shell Method.

$\int_0^2$ The volume of the shell is given by $r-\frac{w}{2}=$ radius of hole $r+\frac{w}{2}=$ radius of outside cylinder $V=\pi r^2 h$ $V_0=\pi\left(r+\frac{w}{2}\right)^2 h$ $V_H=\pi\left(r-\frac{w}{2}\right)^2 h$	Volume of Shell V_0-V_H $\pi\left(r+\frac{w}{2}\right)^2 h-\pi\left(r-\frac{w}{2}\right)^2 h$ $\pi h\left[r^2+rw+\frac{w^2}{4}-\left(r^2-rw+\frac{w^2}{4}\right)\right]$ $\pi h\left[r^2+rw+\frac{w^2}{4}-r^2+rw-\frac{w^2}{4}\right]$ $=\pi h 2rw$ $\therefore V_{\text{SHELL}}=2\pi\int_0^2 \overbrace{r(x)}^{\text{Radius}}\overbrace{h(x)}^{\text{Height}}\overbrace{dy}^{\text{Thickness}}$	For our example we have $2\pi\int_0^2 y\left(4-y^2\right)dy$ $2\pi\int_0^2\left(4y-y^3\right)dy$ $2\pi\left[4\cdot\frac{1}{2}y^2-\frac{1}{4}y^4\right]_0^2$ $2\pi\left[2\cdot 2^2-\frac{1}{4}\cdot 2^4-\left(2\cdot 0^2-\frac{1}{4}\cdot 0\right)\right]$ $2\pi[8-4]$ $=\boxed{8\pi}$