Sec.4.1

p.248 - 249:
Definition of Antiderivative; Representation of Antiderivative;
Differential equation, General solution (GS); Indefinite integration

DE

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C$$

$n \neq -1$

p.250-252: Basic Integration Rules; Examples 2 - 6

p.253: Initial condition (IC), particular solution (PS); Example 8

p.255-256: Find the indefinite integral and check the result by differentiation

16. $\int (13 - x)\, dx = \int 13 dx - \int x^1 dx = 13x + C_1 - \left(\frac{x^{1+1}}{1+1} + C_2\right) = \boxed{13x - \frac{x^2}{2} + C}$

$C_1 - C_2$

26. $\int \frac{x^4 - 3x^2 + 5}{x^4} dx = \int \left(\frac{x^4}{x^4} - \frac{3x^2}{x^4} + \frac{5}{x^4}\right) dx$

$= \int (1 - 3x^{-2} + 5x^{-4}) dx = x - 3 \cdot \frac{x^{-2+1}}{-2+1} + 5 \cdot \frac{x^{-4+1}}{-4+1} + C$

$= x + 3x^{-1} - \frac{5x^{-3}}{3} + C = \boxed{x + \frac{3}{x} - \frac{5}{3x^3} + C}$

20. $\int \left(\sqrt{x} + \frac{1}{2\sqrt{x}}\right) dx$

$= \int \left(x^{1/2} + \frac{1}{2} x^{-\frac{1}{2}}\right) dx$

$= \frac{x^{1/2+1}}{\frac{1}{2}+1} + \frac{1}{2} \cdot \frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + C = \frac{x^{3/2}}{\frac{3}{2}} + \cancel{\frac{1}{2}} \cdot \frac{x^{1/2}}{\cancel{\frac{1}{2}}} + C = \boxed{\frac{2}{3} x^{3/2} + x^{\frac{1}{2}} + C}$

30. $\int (\sin x - 6 \cos x)\, dx$

$= \boxed{-\cos x - 6 \sin x + C}$

36. $\int (4x^1 - \csc^2 x)\, dx$

$= 4 \cdot \frac{x^{1+1}}{1+1} - (-\cot x) + C = 4 \cdot \frac{x^2}{2} + \cot x + C = \boxed{2x^2 + \cot x + C}$

P.S. DE. I.C.

Find the particular solution of the differential equation that satisfies the initial condition(s).

D.E. I.C.

40. $f'(s) = 10s - 12s^3,\ f(3) = 2$ $\rightarrow f(s) = ?$ $\quad f'(s) = \frac{df}{ds} \rightarrow \int df = \int f'(s)\, ds$

f $\quad 10 \cdot \frac{s^2}{2} - 12 \cdot \frac{s^4}{4}$

$f(s) = \int (10s - 12s^3)\, ds = 5s^2 - 3s^4 + C$ (GS), $\leftarrow f(3) = 2$ (IC)

$5(3^2) - 3(3^4) + C = 2 \rightarrow 45 - 243 + C = 2 \rightarrow C = 200$

$\boxed{f(s) = 5s^2 - 3s^4 + 200}$ (PS)

$f(x)^2$?

DE. I.C

42. $f''(x) = 3x^2,\ f'(-1) = -2,\ f(2) = 3$

1st

$f'(x) = \int 3x^2\, dx = x^3 + C_1$ (GS), $\leftarrow f'(-1) = -2$ (IC) $\quad 3 \cdot \frac{x^3}{3}$

$(-1)^3 + C_1 = -2 \rightarrow -1 + C_1 = -2 \rightarrow C_1 = -1 \rightarrow f'(x) = x^3 - 1$ (PS)

2 nd.

$f(x) = \int (x^3 - 1)\, dx = \frac{1}{4}x^4 - x + C_2$ (GS), $\leftarrow f(2) = 3$ (IC)

$\frac{1}{4}(2)^4 - 2 + C_2 = 3 \rightarrow 4 - 2 + C_2 = 3 \rightarrow C_2 = 1 \rightarrow \boxed{f(x) = \frac{1}{4}x^4 - x + 1}$ (PS)

DE I.C

44. $f''(x) = \sin x,\ f'(0) = 1,\ f(0) = 6$

$\frac{1}{4}(2)^4 - 2 + C_2 = 3 \;\rightarrow\; 4 - 2 + C_2 = 3 \;\rightarrow\; C_2 = 1 \;\rightarrow\; \boxed{f(x) = \frac{1}{4}x^4 - x + 1}$ (PS)

DE   I.C

44. $f''(x) = \sin x$, $f'(0) = 1$, $f(0) = 6$

1st

$f'(x) = \int \sin x \, dx = -\cos x + C_1 \quad (GS), \quad f'(0) = 1 \;(IC)$

2nd

$-\cos 0 + C_1 = 1 \;\rightarrow\; -1 + C_1 = 1 \;\rightarrow\; C_1 = 2 \;\rightarrow\; f'(x) = -\cos x + 2 \quad (PS)$

$f(x) = \int(-\cos x + 2)\, dx = -\sin x + 2x + C_2 \quad (GS), \quad f(0) = 6 \;(IC)$

$-\sin 0 + 2(0) + C_2 = 6 \;\rightarrow\; C_2 = 6 \;\rightarrow\; \boxed{f(x) = -\sin x + 2x + 6} \quad (PS)$

56. Population Growth The rate of growth dP/dt of a population of bacteria is proportional to the square root of t, where P is the population size and t is the time in days $(0 \leq t \leq 10)$. That is,

$$\frac{dP}{dt} = k\sqrt{t}.$$

$P' = k\sqrt{t}$ $P(0) = 500$, $P(1) = 600 \rightarrow P(7)$?

$P(t)'$

The initial size of the population is 500. After 1 day, the population has grown to 600. Estimate the population after 7 days.

$$\frac{t^{\frac{1}{2}+1}}{\frac{1}{2}+1} = \frac{t^{3/2}}{3/2}$$

$P(t) = \int k\sqrt{t}\, dt = k\int t^{\frac{1}{2}}\, dt = k \cdot \frac{t^{\frac{3}{2}}}{\frac{3}{2}} + C = \frac{2}{3}kt^{\frac{3}{2}} + C \quad (GS)$

IC: $P(0) = 500$: $\frac{2}{3}k \cdot 0^{\frac{3}{2}} + C = 500 \;\rightarrow\; C = 500$ $2k = 300$

$P(1) = 600$: $\frac{2}{3}k \cdot 1^{\frac{3}{2}} + 500 = 600 \;\rightarrow\; \frac{2}{3}k = 100 \;\rightarrow\; k = 150 \rightarrow$

$7 \; \boxed{y^x} \; \frac{3}{2}$

$P(t) = \frac{2}{\cancel{3}} \cdot \cancel{150}^{50} \cdot t^{\frac{3}{2}} + 500 = 100t^{\frac{3}{2}} + 500 \;\rightarrow\; \boxed{P(t) = 100t^{\frac{3}{2}} + 500} \quad (PS)$

$P(7) = 100 \cdot 7^{\frac{3}{2}} + 500 \approx 2352.0 \; bacteria$