Sec.5.4

p.342 -343:
Definition of the Natural Exponential Function; Examples 1 - 2

Operations with Exponential Functions; Properties of the Natural Exponential Function

p.344-345: Derivatives of the Natural Exponential Function; Examples 3 - 5

p.348: Find the derivative of the function.

38. $y = 5e^{x^2+5}$ $\quad u = x^2+5 \rightarrow u' = 2x \qquad 1^2 - (e^x)^2$

$y' = 5e^{x^2+5}(2x) = \boxed{10xe^{x^2+5}}$

$u \rightarrow u' = e^x \qquad u \rightarrow u' = -e^x$

46. $y = \ln\left(\frac{1+e^x}{1-e^x}\right) = \ln(1+e^x) - \ln(1-e^x) \qquad LCD = (1+e^x)(1-e^x)$

$y' = \frac{e^x}{1+e^x} - \frac{-e^x}{1-e^x} = \frac{e^x(1-e^x)+e^x(1+e^x)}{(1+e^x)(1-e^x)}$

$= \frac{e^x - \cancel{e^{2x}} + e^x + \cancel{e^{2x}}}{1-e^{2x}} = \boxed{\frac{2e^x}{1-e^{2x}}}$

48. $y = \frac{e^x - e^{-x}}{2} = \frac{1}{2}[e^x - e^{-x}] \qquad e^{-x} \rightarrow u = -x \rightarrow u' = -1$

$y' = \frac{1}{2}[e^x - e^{-x}(-1)] = \frac{1}{2}\left[\frac{e^x}{1} + \frac{1}{e^x}\right] = \frac{1}{2}\left(\frac{e^{2x}+1}{e^x}\right)$

$= \boxed{\frac{e^{2x}+1}{2e^x}}$

Find an equation of the tangent line to the graph of the function at the given point. $\quad x_1 \; y_1$

62. $y = xe^x - e^x, \quad (1, 0)$

1) $y' = \cancel{1e^x} + xe^x - \cancel{e^x}$
$= xe^x$

2) @ $(1, 0)$
$m = y'(1) = 1 \cdot e^1$
$= e$

3) T.L. $m = e \quad (1, 0)$
$y - 0 = e(x - 1)$
$\boxed{y = ex - e}$

Use implicit differentiation to find $\frac{dy}{dx}$. $\quad y' \quad y(x)$

64. $e^{xy} + x^2 - y^2 = 10$

$e^{xy}(y + xy') + 2x - 2yy' = 0 \rightarrow ye^{xy} + xy'e^{xy} + 2x - 2yy' = 0$

$ye^{xy} + 2x = 2yy' - xy'e^{xy} \rightarrow ye^{xy} + 2x = y'(2y - x\,e^{xy}) \rightarrow \boxed{y' = \frac{ye^{xy} + 2x}{2y - x\,e^{xy}}}$

p.346: Integration Rules for Exponential Functions; Examples 7 – 10

p.350: Find the indefinite integral.

$u = \frac{1}{x^2} = x^{-2} \rightarrow du = -2x^{-3}dx = -\frac{2}{x^3}dx$

98. $\int \frac{e^{1/x^2}}{x^3}dx$

98. $\int \frac{e^{1/x^2}}{x^3}\,dx$ $\quad u = \frac{1}{x^2} - x \rightarrow du =$

$= -\frac{1}{2}\int e^{\frac{1}{x^2}}\left(-\frac{2}{x^3}\right)dx = -\frac{1}{2}\int e^u\,du = -\frac{1}{2}e^u + C = \boxed{-\frac{1}{2}e^{1/x^2} + C}$

100. $\int \frac{e^{2x}}{1+e^{2x}}\,dx$ $\quad u = 1 + e^{2x} \rightarrow du = e^{2x}\cdot 2\,dx = 2e^{2x}\,dx$

$= \frac{1}{2}\int \frac{1}{1+e^{2x}}(2e^{2x}\,dx) = \frac{1}{2}\int \frac{1}{u}\,du = \frac{1}{2}\ln|u| + C$

$= \frac{1}{2}\ln|1+e^{2x}| + C = \boxed{\frac{1}{2}\ln(1+e^{2x}) + C}$

106. $\int \frac{e^{-3x} + 2e^{2x} + 3}{e^x}\,dx = \int \left(\frac{e^{-3x}}{e^x} + \frac{2e^{2x}}{e^x} + \frac{3}{e^x}\right)dx$

$= \int (e^{-4x} + 2e^x + 3e^{-x})\,dx$ $\quad \boxed{-\frac{1}{4}e^{-4x} + 2e^x - 3e^{-x} + C}$

$= -\frac{1}{4}e^{-4x} + 2e^x + 3(-e^{-x}) + C =$

u $\quad du = -4dx$

$\int e^{-4x}\,dx$

$= -\frac{1}{4}\int e^{-4x}(-4dx)$

$= -\frac{1}{4}e^{-4x} + C_1$

$\int e^{-x}\,dx \rightarrow du = -dx$

$= -\int e^{-x}(-dx)$

$= -e^{-x} + C_3$

108. $\int e^{2x}\csc(e^{2x})\,dx$ $\quad u = e^{2x} \rightarrow du = 2e^{2x}\,dx$

$= \frac{1}{2}\int \csc(e^{2x})(2e^{2x}\,dx) = \frac{1}{2}\int \csc u\,du$

$= -\frac{1}{2}\ln|\csc u + \cot u| + C = \boxed{-\frac{1}{2}\ln|\csc(e^{2x}) + \cot(e^{2x})| + C}$

Evaluate the definite integral.

110. $\int_{-1}^{1} e^{1+4x}\,dx$ $\quad u = 1 + 4x \rightarrow du = 4dx$ $\quad x=-1:\ u = 1+4(-1) = -3$ $\quad x = 1:\ u = 1+4(1) = 5$

$= \frac{1}{4}\int_{-1}^{1} e^{1+4x}(4dx) = \frac{1}{4}\int_{-3}^{5} e^u\,du = \frac{1}{4}\left[e^u\right]_{-3}^{5}$

$= \frac{1}{4}\left[e^5 - e^{-3}\right] = \frac{1}{4}\left[\frac{e^5}{1} - \frac{1}{e^3}\right] = \frac{1}{4}\left[\frac{e^8 - 1}{e^3}\right] = \boxed{\frac{e^8 - 1}{4e^3}}$