Sec.2.5

p.144 - 148: Implicit and Explicit Functions
Guidelines for Implicit Differentiation 1-4; Example 2; Examples 4 – 8

1. Differentiate each side of the equation *with respect to x*.
2. Collect all terms involving dy/dx on the left side of the equation and move all other terms to the right side of the equation.
3. Factor dy/dx out of the left side of the equation.
4. Solve for dy/dx. $\longleftarrow y'$

p.149: In Exercises 5–20, find dy/dx by implicit differentiation.

9. $x^3 - xy + y^2 = 7$

$$3x^2 - (1\cdot y + xy') + 2yy' = 0$$
$$3x^2 - y - xy' + 2yy' = 0$$
$$2yy' - xy' = y - 3x^2 \to y'(2y - x) = y - 3x^2$$
$$\boxed{y' = \frac{y - 3x^2}{2y - x}}$$

15. $\sin x + 2\cos 2y = 1$

$$\cos x + 2(-\sin 2y)(2y') = 0$$
$$\cos x - 4y'\sin 2y = 0$$
$$\cos x = 4y'\sin 2y$$
$$\boxed{y' = \frac{\cos x}{4\sin 2y}}$$

$$\frac{5\cancel{3}}{\cancel{3}\cdot 4} = \frac{5}{4}$$
$$\frac{5+3}{3-4} = \frac{8}{-1} = -8$$

p.149: In Exercises 25–32, find dy/dx (y') by implicit differentiation. Then find the slope of the graph at the given point.

$$\frac{\cancel{2xy}Z}{\cancel{2xy}A} = \frac{Z}{A}$$

29. $(x + y)^3 = x^3 + y^3$, $(-1, 1)$ $\quad \frac{1}{3}$

(1) $\cancel{3}(x+y)^2(1+y') = \cancel{3}x^2 + \cancel{3}y^2y'$

$$(x^2 + 2xy + y^2)(1 + y') = x^2 + y^2y'$$
$$\cancel{x^2} + x^2y' + 2xy + 2xyy' + y^2 + \cancel{y^2y'} = \cancel{x^2} + \cancel{y^2y'}$$
$$x^2y' + 2xyy' = -2xy - y^2 \to y'(x^2 + 2xy) = -(2xy + y^2)$$

$$(x^2y')' = 2xy' + x^2y''$$

$$\boxed{y' = -\frac{2xy + y^2}{x^2 + 2xy}}$$

(2) @$(-1, 1)$: $m = y'(-1,1) = -\frac{2(-1)(1) + 1^2}{(-1)^2 + 2(-1)(1)} = -\frac{-1}{-1} = -1$

$$\frac{4\cancel{x}(4 - 2xy)}{x^{\cancel{2}}}$$

p.150: In Exercises 49–54, find d^2y/dx^2 (y'') implicitly in terms of x and y.

50. $x^2y - 4x = 5$

1st: $2xy + x^2y' - 4 = 0 \longleftrightarrow y' = \frac{4 - 2xy}{x^2}$

2nd: $2y + 2xy' + 2xy' + x^2y'' = 0$

$$2y + 4xy' + x^2y'' = 0$$
$$2y + 4\cancel{x}\cdot\frac{(4 - 2xy)}{x^{\cancel{2}}} + x^{\cancel{2}}y'' = 0$$

$x\cdot$: $2y + \frac{16 - 8xy}{x} + xy'' = 0 \quad \cdot x$

$$2xy + 16 - 8xy + x^3y'' = 0$$
$$x^3y'' + 16 - 6xy = 0 \to x^3y'' = 6xy - 16$$
$$\boxed{y'' = \frac{6xy - 16}{x^3}}$$

p.150: In Exercises 57 and 58, find equations for the tangent line and normal line to the circle at each given point. (The normal line at a point is perpendicular to the tangent line at the point.)

58. $x^2 + y^2 = 36$ $\quad (6, 0), (5, \sqrt{11})$

$$2x + 2yy' = 0 \to 2yy' = -2x \to y' = -\frac{2x}{2y} = -\frac{x}{y}$$

$l_1 \perp l_2$

given point. (The normal line at a point is perpendicular to the tangent line at the point.)

58. $x^2 + y^2 = 36$ $(6, 0), (5, \sqrt{11})$

$\ell_1 \perp \ell_2$

$m_1 = -\frac{1}{m_2}$

$2x + 2yy' = 0 \rightarrow 2yy' = -2x \rightarrow y' = -\frac{2x}{2y} = -\frac{x}{y}$

① @$(6,0)$: T.L. $m = -\frac{6}{0} = \text{undef.} \rightarrow$ V.T.L. $\boxed{x=6}$

N.L. $\rightarrow$ HNL. $\boxed{y=0}$

$y - y_1 = m(x - x_1)$

② @$(5, \sqrt{11})$: T.L. $m = y'(5, \sqrt{11}) = -\frac{5}{\sqrt{11}}$

$y - \sqrt{11} = -\frac{5}{\sqrt{11}}(x-5) \rightarrow \sqrt{11}(y - \sqrt{11}) = -5(x-5)$

$\sqrt{11}y - 11 = -5x + 25 \rightarrow \boxed{5x + \sqrt{11}y - 36 = 0}$

N.L. $m = \frac{\sqrt{11}}{5}$

$y - \sqrt{11} = \frac{\sqrt{11}}{5}(x-5) \rightarrow 5(y - \sqrt{11}) = \sqrt{11}(x-5) \rightarrow 5y - \cancel{5\sqrt{11}} = \sqrt{11}x - \cancel{5\sqrt{11}}$

$\boxed{\sqrt{11}x - 5y = 0}$

p.150: In Exercises 61 and 62, find the points at which the graph of the equation has a vertical or horizontal tangent line.

62. $4x^2 + y^2 - 8x + 4y + 4 = 0$

$\cancel{2}y'(y+2) = \cancel{2}(4-4x)$

$8x + 2yy' - 8 + 4y' = 0 \rightarrow 2yy' + 4y' = 8 - 8x \rightarrow y'(2y+4) = 8 - 8x$

$y' = \frac{8-8x}{2y+4} = \frac{\cancel{2}(4-4x)}{\cancel{2}(y+2)} = \frac{4-4x}{y+2}$

V.T.L. $y' = \text{undef.} \rightarrow y + 2 = 0 \rightarrow y = -2$

@$(0,-2)$ & $(2,-2)$

$4x^2 + (-2)^2 - 8x + 4(-2) + 4 = 0 \rightarrow 4x^2 + \cancel{4} - 8x - \cancel{8} + \cancel{4} = 0$

$4x^2 - 8x = 0 \rightarrow 4x(x-2) = 0 \rightarrow x = 0, 2$

H.T.L. $y' = 0 \rightarrow 4 - 4x = 0 \rightarrow 4 = 4x \rightarrow x = 1$

$-4x = -4$

@$(1,0)$ & $(1,-4)$

$4(1^2) + y^2 - 8(1) + 4y + 4 = 0 \rightarrow \cancel{4} + y^2 - \cancel{8} + 4y + \cancel{4} = 0$

$y^2 + 4y = 0 \rightarrow y(y+4) = 0 \rightarrow y = 0, -4$