Exam 1 Math 2414 Litong Name _____Test No_____

Use logarithmic differentiation to find the derivative of y with respect to the independent variable.

1) $y = (\cos x)^x$

Evaluate the integral.

2) $\int \frac{dx}{2\sqrt{x}(1+x)}$

3) $\int \frac{8-3x}{\sqrt{25-64x^2}}\,dx$

4) $\int \frac{dx}{(x+7)\sqrt{x^2+14x+48}}$

5) $\int \frac{dx}{\sqrt{-x^2-14x-40}}$

Evaluate exactly.

6) $\sec\left(\tan^{-1}\left(\frac{3}{4}\right)\right)$

Evaluate the expression.

7) $\sin(\tan^{-1} x)$

8) $\tan\left(\sec^{-1}\frac{\sqrt{x^2+9}}{x}\right)$

Find the limit.

9) $\lim_{x\to 1^-} \cos^{-1} x$

Find the derivative of y with respect to x.

10) $y = \tan^{-1}\sqrt{11x}$

11) $y = \sin^{-1}(e^{6t})$

Provide an appropriate response.

12) Which of the following items is undefined and why?

$\csc^{-1}\frac{1}{2}$ or $\cos^{-1}\frac{1}{2}$

Evaluate the integral.

13) $\int_2^3 \frac{x^5+1}{x^6+6x}\,dx$

14) $\int_0^{\pi/24} \frac{\sec^2 6x}{4+\tan 6x}\,dx$

15) $\int \frac{dx}{x(5+6\ln x)}$

16) $\int_{\pi/8}^{\pi/4} 2\cot(2\theta)\,d\theta$

17) $\int_0^{\pi/8} 12\tan 2x\,dx$

18) $\int \frac{8e^{(8\sin 2x)}}{\sec 2x}\,dx$

19) $\int \frac{25e^{\sqrt{5x}}}{2\sqrt{x}}\,dx$

20) $\int \frac{e^{1/x}}{5x^2}dx$

21) $\int_0^{\pi/16} (1+e^{\tan 4x})\sec^2 4x\,dx$

22) $\int_0^{\sqrt{\ln \pi}} 2x\,e^{x^2}\sin\left(e^{x^2}\right)dx$

23) $\int_1^2 6x2^{4x^3}\,dx$

24) $\int_1^2 \frac{6^{\ln x}}{x}\,dx$

25) $\int \frac{\log_{10} x}{x}\, dx$

Solve the initial value problem.

26) $\frac{dy}{dt} = e^t \sin(e^t - 1),\ y(\ln 1) = 0$

27) $\frac{d^2y}{dt^2} = 4 - e^{-t},\ y(1) = \frac{-1}{e},\ y'(0) = -7$

Evaluate the integral.

28) $\int \frac{dx}{x \ln 10}$

29) $\int_0^{20} \frac{\log_5(x+5)}{x+5} dx$

Provide an appropriate response.

30) How do you know that $f(x) = -5e^x$ is concave down over every interval of x–values?

31) Show that the equation for converting base 10 logarithms to base 9 logarithms is $\log_9 x = \frac{\ln 10}{\ln 9} \log_{10} x$.

Solve the problem.

32) The amount of alcohol in the bloodstream, A, declines at a rate proportional to the amount, that is, $\frac{dA}{dt} = -kA$. If $k = 0.6$ for a particular person, how long will it take for his alcohol concentration to decrease from 0.10% to 0.05%? Give your answer to the nearest tenth of an hour.

Find the slowest growing and the fastest growing functions as $x \to \infty$.

33) $y = 2x^2 + 10x$

$y = e^x$

$y = e^x/5$

$y = \log_3 x$

A value of sinh x or cosh x is given. Use the definitions and the identity $\cosh^2 x - \sinh^2 x = 1$ to find the value of the other indicated hyperbolic function.

34) $\sinh x = -\frac{4}{3}$, $\tanh x =$

Rewrite the expression in terms of exponentials and simplify the results.

35) $(\sinh x + \cosh x)^6$

Find the derivative of y.

36) $y = \ln(\operatorname{sech}(7x + 10))$

37) $y = \operatorname{csch} \frac{14x}{3}$

Find the derivative of y with respect to the appropriate variable.

38) $y = (8 - 8\theta) \tanh^{-1} \theta$

Determine whether the integration formula is correct.

39) $\int x \operatorname{csch}^{-1} x\, dx = \frac{x^2}{2} \operatorname{csch}^{-1} x + \frac{1}{2}\sqrt{1 + x^2} + C$

Evaluate the integral.

40) $\int \operatorname{sech}^2(7x - 1)\, dx$

41) $\int_0^{\ln 5} \cosh x\, dx$

Express the value of the inverse hyperbolic function in terms of natural logarithms.

42) $\sinh^{-1}\left(\frac{-3}{4}\right)$

1) $(\cos x)^x\ (\ln \cos x - x \tan x)$

2) $\tan^{-1}\sqrt{x} + C$

3) $\sin^{-1}\left(\frac{8}{5}x\right) + \frac{3}{64}\sqrt{25 - 64x^2} + C$

4) $\sec^{-1}(x + 7) + C$

5) $\sin^{-1}\left(\frac{x + 7}{3}\right) + C$

6) $\frac{5}{4}$

7) $\frac{x\sqrt{x^2 + 1}}{x^2 + 1}$

8) $\frac{3}{x}$

9) 0

10) $\frac{11}{2(1 + 11x)\sqrt{11x}}$

11) $\frac{6\ e^{6t}}{\sqrt{1 - e^{12t}}}$

12) $\csc^{-1}\frac{1}{2}$, There is no angle whose cosecant is $\frac{1}{2}$.

13) $\frac{1}{6}\ln\left|\frac{747}{76}\right|$

14) $\frac{1}{6}\ln\left|\frac{5}{4}\right|$

15) $\frac{1}{6}\ln|5 + 6\ln x| + C$

16) $\frac{\ln 2}{2}$

17) $3 \ln 2$

18) $\frac{1}{2}e^{(8 \sin 2x)} + C$

19) $5\sqrt{5}\ e^{\sqrt{5x}} + C$

20) $-\frac{e^{1/x}}{5} + C$

21) $\frac{e}{4}$

22) $1 + \cos 1$

23) $\frac{131{,}064}{\ln 4}$

24) $\frac{6^{\ln 2} - 1}{\ln 6}$

25) $\frac{(\ln x)^2}{2 \ln 10} + C$

26) $y = -\cos(e^t - 1) + 1$

27) $y = 2t^2 - e^{-t} - 8t + 6$

28) $\frac{\ln|x|}{\ln 10} + C$

29) $\frac{3 \ln 5}{2}$

30) $f''(x) = -5e^x$. This is always negative so f(x) is always concave down.

31) $\frac{\ln 10}{\ln 9}\log_{10} x = \frac{\ln 10}{\ln 9} \cdot \frac{\ln x}{\ln 10} = \frac{\ln x}{\ln 9} = \log_9 x$

32) 1.2 hr

33) Slowest: $y = \log_3 x$

Fastest: $y = e^x$ and $y = e^x/5$ grow at the same rate

34) $-\frac{4}{5}$

35) e^{6x}

36) $-7 \tanh(7x + 10)$

37) $-\frac{14}{3} \operatorname{csch} \frac{14x}{3} \coth \frac{14x}{3}$

38) $\frac{8}{1+\theta} - 8 \tanh^{-1} \theta$

39) Yes

40) $\frac{1}{7} \tanh(7x - 1) + C$

41) $\frac{12}{5}$

42) $\ln \frac{1}{2}$