

Sec.3.3

p.181: Definition of Increasing/Decreasing Functions; Theorem 3.5

p.182: Guidelines for Finding Intervals on Which a Function is Increasing/Decreasing; Example 1
Strictly monotonic function.

p.183: Theorem 3.6 - the First Derivative Test; p.184 – 185: Examples 2 – 4

p.187: In Exercises 19–40, (a) find the critical numbers of f , if any, (b) find the open intervals on which the function is increasing or decreasing, (c) apply the First Derivative Test to identify all relative extrema

34. $f(x) = |x + 3| - 1$

See 2.1

$$f'(-3) = \lim_{x \rightarrow -3} \frac{f(x) - f(-3)}{x - (-3)} = \lim_{x \rightarrow -3} \frac{[|x+3| - 1] - (-1)}{x+3} = \lim_{x \rightarrow -3} \frac{|x+3|}{x+3}$$

$|a| = \begin{cases} a & a \geq 0 \\ -a & a < 0 \end{cases}$

$$= \begin{cases} \frac{-(x+3)}{x+3} = -1, & \text{for } x < -3 \\ \frac{x+3}{x+3} = 1, & \text{for } x > -3 \end{cases}$$

$x \rightarrow -3^-$
 $x \rightarrow -3^+$

-3^- -3^+ -2

-4 -2

1) C.#.
2) I/D.

$$f'(-3) = DNE \rightarrow C.\# : x = -3$$

x	$f(x)$	$f'(x)$	Conclusion
$(-\infty, -3)$		$-$ ✓	Decreasing ✓
$x = -3$	-1 ✓		R. min @ $(-3, -1)$
$(-3, \infty)$		$+$ ✓	Increasing ✓

p.187: In Exercises 41–48, consider the function on the interval $(0, 2\pi)$. (a) Find the open intervals on which the function is increasing or decreasing. (b) Apply the First Derivative Test to identify all relative extrema.

42. $f(x) = \sin x \cos x + 5$

$$f'(x) = (\cos x)(\cos x) + \sin x(-\sin x) = \cos^2 x - \sin^2 x = \cos 2x$$

let $f'(x) = 0$: $\cos 2x = 0 \rightarrow 2x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2} \rightarrow$

$$x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$$

x	$f(x)$	$f'(x)$	Conclusion
$(0, \frac{\pi}{4})$		$+$	Increasing
$x = \frac{\pi}{4}$	$5\frac{1}{2}$		<u>R. Max @ $(\frac{\pi}{4}, 5\frac{1}{2})$</u>
$(\frac{\pi}{4}, \frac{3\pi}{4})$		$-$	Decreasing
$x = \frac{3\pi}{4}$	$4\frac{1}{2}$		<u>R. Min @ $(\frac{3\pi}{4}, 4\frac{1}{2})$</u>
$(\frac{3\pi}{4}, \frac{5\pi}{4})$		$+$	Increasing
$x = \frac{5\pi}{4}$	$5\frac{1}{2}$		<u>R. Max @ $(\frac{5\pi}{4}, 5\frac{1}{2})$</u>
$(\frac{5\pi}{4}, 7\pi)$		$-$	Decreasing

$0 < x < 2\pi$
 $0 < 2x < 4\pi$
 $n^2 x = \cos 2x$
 \rightarrow
 $C.\#s$
 $f'(\frac{\pi}{6}) = \omega(2 \times \frac{\pi}{6}) = \omega \frac{\pi}{3} = (+)$
 $f'(\frac{\pi}{2}) = \omega(2 \times \frac{\pi}{2}) = \omega \pi = (-)$
 $f'(\pi) = \omega(2 \cdot \pi) = (+)$
 $f'(\frac{3\pi}{2}) = \omega(2 \times \frac{3\pi}{2}) = \omega 3\pi = (-)$
 $f'(\frac{11\pi}{6}) = \omega(2 \times \frac{11\pi}{6}) = \omega \frac{11\pi}{3} = \omega \frac{5\pi}{3} = (+)$

$x = \frac{5\pi}{4}$	$5\frac{1}{2}$		R. Max @ $(\frac{5\pi}{4}, 5\frac{1}{2})$ ✓
$(\frac{5\pi}{4}, \frac{7\pi}{4})$		- ✓	<u>Decreasing</u>
$x = \frac{7\pi}{4}$	$4\frac{1}{2}$		R. Min @ $(\frac{7\pi}{4}, 4\frac{1}{2})$ ✓
$(\frac{7\pi}{4}, 2\pi)$		+ ✓	<u>Increasing</u>

$$f'(\frac{11\pi}{6}) = \omega(2 \times \frac{11\pi}{6}) = \omega \frac{11\pi}{3} = \omega \frac{5\pi}{3} = (+)$$

$$\frac{11\pi}{3} = \frac{6\pi}{3} + \frac{5\pi}{3} = 2\pi + \frac{5\pi}{3}$$

$$f(\frac{7\pi}{4}) = \sin \frac{7\pi}{4} \omega \frac{7\pi}{4} + 5 = \frac{\sqrt{2}}{2} \times \frac{\sqrt{2}}{2} + 5 = \frac{1}{2} + 5 = 5\frac{1}{2}$$

$$f(\frac{3\pi}{4}) = \sin \frac{3\pi}{4} \omega \frac{3\pi}{4} + 5 = \frac{\sqrt{2}}{2} (-\frac{\sqrt{2}}{2}) + 5 = -\frac{1}{2} + 5 = 4\frac{1}{2}$$

$$f(\frac{5\pi}{4}) = \sin \frac{5\pi}{4} \omega \frac{5\pi}{4} + 5 = (-\frac{\sqrt{2}}{2})(-\frac{\sqrt{2}}{2}) + 5 = \frac{1}{2} + 5 = 5\frac{1}{2}$$

$$f(\frac{7\pi}{4}) = \sin \frac{7\pi}{4} \omega \frac{7\pi}{4} + 5 = (-\frac{\sqrt{2}}{2})(\frac{\sqrt{2}}{2}) + 5 = -\frac{1}{2} + 5 = 4\frac{1}{2}$$