Sec.3.2

p.174 - 175: Theorem 3.3; Examples 1 & 2

p.178: In Exercises 3–6, explain why Rolle's Theorem does not apply to the function even though there exist a and b such that $f(a) = f(b)$.

$\cot x = \frac{1}{\tan x}$

4. $f(x) = \cot \frac{x}{2}, \ [\pi, 3\pi]$

$$f(2\pi) = \cot\frac{2\pi}{2} = \cot\pi = \frac{1}{\tan\pi} = \frac{1}{0} = DNE \ \rightarrow \ f(x) \text{ is not continuous at } x = 2\pi$$

6. $f(x) = \sqrt[2]{(2 - x^{2/3})^3}, \ [-1, 1]$

$$f(x) = \left(2 - x^{\frac{2}{3}}\right)^{\frac{3}{2}} \rightarrow f'(x) = \frac{\cancel{3}}{\cancel{2}}\left(2 - x^{\frac{2}{3}}\right)^{\frac{1}{2}}\left(-\frac{\cancel{2}}{\cancel{3}}x^{-\frac{1}{3}}\right) = -\frac{\sqrt{2 - x^{\frac{2}{3}}}}{\sqrt[3]{x}} \rightarrow f'(0) = DNE$$

$\rightarrow$ $f(x)$ *is not differentiable at* $x = 0$

p.178: In Exercises 11–24, determine whether Rolle's Theorem can be applied to f on the closed interval $[a, b]$. If Rolle's Theorem can be applied, find all values of c in the open interval (a, b) such that $f'(c) = 0$. If Rolle's Theorem cannot be applied, explain why not.

24. $f(x) = \sec x, \quad [\pi, 2\pi]$

$$f(\pi) = \sec\pi = \frac{1}{\cos\pi} = -1, \quad f(2\pi) = \sec 2\pi = \frac{1}{\cos 2\pi} = 1 \rightarrow f(\pi) \neq f(2\pi) \ No$$

-1 1

14. $f(x) = (x - 4)(x + 2)^2, \quad [-2, 4]$

$f(x) = (x - 4)(x + 2)^2$ *is continuous on* $[-2, 4]$ *and differenciable on* $(-2, 4)$

$f(-2) = f(4) = 0 \quad \rightarrow$ *Yes* $\longrightarrow$

$$f'(x) = 1 \cdot (x + 2)^2 + (x - 4) \cdot 2(x + 2) \cdot 1 = (x + 2)[(x + 2) + (2x - 8)]$$

$$= (x + 2)(3x - 6) = 3(x + 2)(x - 2)$$

let $f'(x) = 0$: $3(x + 2)(x - 2) = 0 \ \rightarrow \ x = -2, 2 \ \rightarrow \ c = 2$

LE

20. $f(x) = \cos x, \quad [\pi, 3\pi]$

$f(x) = \cos x$ *is continuous on* $[\pi, 3\pi]$ *and differenciable on* $(\pi, 3\pi)$, $f(\pi) = f(3\pi) = -1 \ \rightarrow$ *Yes* $\longrightarrow$

$f'(x) = -\sin x$; *let* $f'(x) = 0$: $-\sin x = 0 \rightarrow \sin x = 0 \ \rightarrow c = 2\pi$

p.176-177: Theorem 3.4; Example 3; Alternative Form of Mean Value Theorem

p.179: In Exercises 39–48, determine whether the Mean Value Theorem can be applied to f on the closed interval $[a, b]$. If the Mean Value Theorem can be applied, find all values of c in the open interval (a, b) such that

MTL. $$f'(c) = \frac{f(b) - f(a)}{b - a}$$ Msl.

$f(0) = 4$

$f(-1) = (-1)^3 + 2(-1) + 4 = -1 - 2 + 4 = 1$

41. $f(x) = x^3 + 2x + 4, \quad [-1, 0]$ a, b

$f(x) = x^3 + 2x + 4$ *is continuous on* $[-1, 0]$ *and differenciable on* $(-1, 0) \ \rightarrow$ *Yes* $\longrightarrow$

41. $f(x) = x^3 + 2x + 4, \quad [-1, 0]$

$f(x) = x^3 + 2x + 4$ *is continuous on* $[-1, 0]$ *and differenciable on* $(-1, 0)$ $\rightarrow$ *Yes* $\longrightarrow$

$$\frac{f(0) - f(-1)}{0 - (-1)} = \frac{4 - 1}{1} = 3; \quad f'(x) = 3x^2 + 2 \quad \rightarrow \quad 3x^2 + 2 = 3 \rightarrow$$

M

S.L. $\quad x^2 = \frac{1}{3} \quad \rightarrow \quad x = \pm\frac{1}{\sqrt{3}} = \pm\frac{\sqrt{3}}{3} \quad \rightarrow \quad c = -\frac{\sqrt{3}}{3}$