# Sec.2.3

p.122 - 123: The Product Rule – Theorem 2.7; Examples 1-3

$$\frac{d}{dx}[f(x)g(x)] = f'(x)g(x) + f(x)g'(x)$$

$(fg)' \neq \cancel{f'g'}$

$= f'g + fg'$

p.124 – 125: The Quotient Rule – Theorem 2.8; Examples 4 & 5

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x)g(x) - f(x)g'(x)}{g^2(x)}, \qquad g(x) \neq 0$$

$\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$

p.129: Find the derivative of the function.

**6.** $y = (3x - 4)(x^3 + 5)$

$y' = 3(x^3+5) + (3x-4)(3x^2) = 3x^3 + 15 + 9x^3 - 12x^2 = \boxed{12x^3 - 12x^2 + 15}$

**12.** $g(t) = \dfrac{3t^2 - 1}{2t + 5}$

$g'(t) = \dfrac{6t(2t+5) - (3t^2-1)(2)}{(2t+5)^2} = \dfrac{12t^2 + 30t - 6t^2 + 2}{(2t+5)^2} = \boxed{\dfrac{6t^2 + 30t + 2}{(2t+5)^2}}$

p.129: In Exercises 17–22, find $f'(x)$ and $f'(c)$.

**18.** $f(x) = (2x^2 - 3x)(9x + 4)$ $\qquad c = -1$

$f'(x) = (4x-3)(9x+4) + (2x^2-3x)(9) = 36x^2 + 16x - 27x - 12 + 18x^2 - 27x$

$= 54x^2 - 38x - 12 \qquad f'(-1) = 54(-1)^2 - 38(-1) - 12 = \boxed{80}$

**22.** $f(x) = \dfrac{\sin x}{x}$ $\qquad c = \dfrac{\pi}{6}$

$f'(x) = \dfrac{(\cos x)x - (\sin x)(1)}{x^2} = \dfrac{x\cos x - \sin x}{x^2}$

$f'\left(\frac{\pi}{6}\right) = \dfrac{\frac{\pi}{6}\cos\frac{\pi}{6} - \sin\frac{\pi}{6}}{\left(\frac{\pi}{6}\right)^2} = \dfrac{\frac{\pi}{6}\left(\frac{\sqrt{3}}{2}\right) - \frac{1}{2}}{\frac{\pi^2}{36}} = \dfrac{\frac{\pi\sqrt{3}}{12} - \frac{6}{12}}{\frac{\pi^2}{36}}$

$= \dfrac{\pi\sqrt{3} - 6}{\cancel{12}} \cdot \dfrac{\cancel{36}\,3}{\pi^2} = \boxed{\dfrac{3(\pi\sqrt{3} - 6)}{\pi^2}}$

$\dfrac{3\pi\sqrt{3} - 18}{\pi^2}$

p.126 – 127: Theorem 2.9; Examples 8 & 9

p.129: Find the derivative of the function.

**46.** $y = x + \cot x \rightarrow y' = 1 - \csc^2 x$

**48.** $h(x) = \dfrac{1}{x} - 12\sec x = x^{-1} - 12\sec x \rightarrow h'(x) = -x^{-2} - 12\sec x\tan x = -\dfrac{1}{x^2} - 12\sec x \tan x$

**56.** $h(\theta) = 5\theta\sec\theta + \theta\tan\theta$

$h'(\theta) = 5\sec\theta + 5\theta\sec\theta\tan\theta + \tan\theta + \theta\sec^2 x$

p.130: In Exercises 61–64, find the slope of the graph of the function at the given point.

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**62.** $f(x) = \tan x \cot x = 1$ $\quad (1, 1)$

$f'(x) = 0 \qquad m = f'(1) = 0$

$\sin 2x = 2\sin x \cos x$

$\cos 2x = \cos^2 x - \sin^2 x$

**64.** $f(x) = \underbrace{(\sin x)}_{f}\underbrace{(\sin x + \cos x)}_{g} \quad \left(\frac{\pi}{4}, 1\right)$

$f'(x) = \cos x(\sin x + \cos x) + \sin x(\cos x - \sin x)$

$= \cos x \sin x + \cos^2 x + \sin x \cos x - \sin^2 x$

$= 2\sin x \cos x + \cos^2 x - \sin^2 x = \sin 2x + \cos 2x$

$f'\left(\frac{\pi}{4}\right) = \sin\left(2 \times \frac{\pi}{4}\right) + \cos\left(2 \times \frac{\pi}{4}\right) = \sin\frac{\pi}{2} + \cos\frac{\pi}{2} = 1 + 0 = 1$ $m$

p.130: In Exercises 75–78, determine the point(s) at which the graph of the function has a horizontal tangent line. H.T.L. $m = 0$

**78.** $f(x) = \dfrac{x - 4}{x^2 - 7} \quad \dfrac{f}{g}$

$f'(x) = \dfrac{1(x^2-7) - (x-4)(2x)}{(x^2-7)^2} = \dfrac{-x^2 + 8x - 7}{(x^2-7)^2}$

H.T.L. $f'(x) = 0 \longrightarrow -x^2 + 8x - 7 \overset{\times(-1)}{=} 0$

$x^2 - 8x + 7 = 0$

$(x-7)(x-1) = 0$

$x = 7, \; x = 1$

$x = 1: \; f(1) = \dfrac{1-4}{1^2-7} = \dfrac{-3}{-6} = \dfrac{1}{2}$

$x = 7: \; f(7) = \dfrac{7-4}{7^2-7} = \dfrac{3}{42} = \dfrac{1}{14}$

$\boxed{\text{H.T.L. @ } \left(1, \tfrac{1}{2}\right) \; \& \; \left(7, \tfrac{1}{14}\right)}$

p.128: Higher-Order Derivatives; Notations; Example 10

| | |
|---|---|
| $s(t)$ | Position function |
| $v(t) = s'(t)$ | Velocity function |
| $a(t) = v'(t) = s''(t)$ | Acceleration function |

p.131: Find the second derivative of the function. $f''$

**96.** $f(x) = x^2 + 3x^{-3}$

$f'(x) = 2x + 3(-3)x^{-4} = 2x - 9x^{-4} \longrightarrow f''(x) = 2 - 9(-4)x^{-5} = 2 + 36x^{-5} = \boxed{2 + \dfrac{36}{x^5}}$