Sec.3.6

p.209 - 214: Guidelines for Analyzing the Graph of a Function; Examples 1-6

p.215: Analyze and sketch a graph of the function over the given interval. Label any intercepts, relative extrema, points of inflection, and asymptotes.

33. $y = 3x^4 + 4x^3$

D: $(-\infty, \infty)$

y-int: $x=0 \to (0, 0)$

x-int: $y=0 \to (0,0)$ & $(-\frac{4}{3}, 0)$

$y = 3x^4 + 4x^3$

$y' = 12x^3 + 12x^2 = 12x^2(x + 1) = 0$ when $x = 0, x = -1$. ← Critical #s

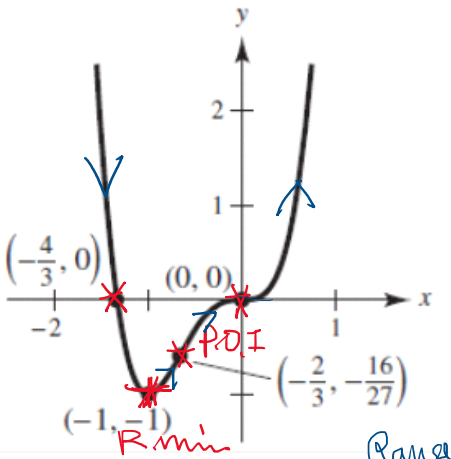
$y'' = 36x^2 + 24x = 12x(3x + 2) = 0$ when $x = 0, x = -\frac{2}{3}$. ← Possible P.O.I

$3x^4+4x^3=0$

$x^3(3x+4)=0$

$x=0 \quad x=-\frac{4}{3}$

x ↓	y	y'	y''	Conclusion
$-\infty < x < -1$		− ✓	+ ✓	Decreasing, concave up
$x = -1$	-1	0 ✓	+ ✓	Relative minimum @ $(-1,-1)$
$-1 < x < -\frac{2}{3}$		+ ✓	+ ✓	Increasing, concave up
$x = -\frac{2}{3}$	$-\frac{16}{27}$	+	0 ✓	Point of inflection @ $(-\frac{2}{3}, -\frac{16}{27})$
$-\frac{2}{3} < x < 0$		+ ✓	− ✓	Increasing, concave down
$x = 0$	0	0 ✓	0 ✓	Point of inflection @ $(0,0)$
$0 < x < \infty$		+ ✓	+ ✓	Increasing, concave up



Range $[-1, \infty)$

-2 $-\frac{3}{4}$ $-\frac{1}{2}$ 1

$-\infty$ −1 $-\frac{2}{3}$ 0 ∞

$y'(-2) = 12(-2)^3 + 12(-2)^2 = (-)$

$y''(-2) = 36(-2)^2 + 24(-2) = (+)$

$y''(-1) = 36(-1)^2 + 24(-1) = (+)$

R.min $y(-1) = 3(-1)^4 + 4(-1)^3$

$= 3(1) + 4(-1) = -1$

$y'(-\frac{3}{4}) = 12(-\frac{3}{4})^3 + 12(-\frac{3}{4})^2 = (+)$

$y''(-\frac{3}{4}) = 36(-\frac{3}{4})^2 + 24(-\frac{3}{4}) = (+)$

$y'(-\frac{1}{2}) = 12(-\frac{1}{2})^3 + 12(-\frac{1}{2})^2 = (+)$

$y''(-\frac{1}{2}) = 36(-\frac{1}{2})^2 + 24(-\frac{1}{2}) = (-)$

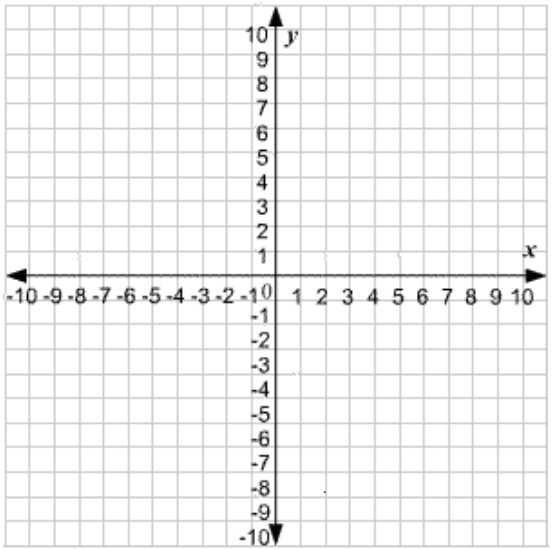
$y(-\frac{2}{3}) = 3(-\frac{2}{3})^4 + 4(-\frac{2}{3})^3 = -\frac{16}{27}$

$y'(1) = 12(1)^3 + 12(1)^2 = (+)$

$y''(1) = 36(1)^2 + 24(1) = (+)$

16. $y = \frac{x^2 + 1}{x^2 - 4}$

x	y	y'	y''	$Conclusion$
$(-\infty,-2)$		$+$	$+$	$Increasing, up$
$x=-2$	-----	-----	-----	$V.A.\ x=-2$
$(-2,0)$		$+$	$-$	$Increasing, down$
$x=0$	$-1/4$	0	$-$	$R.Max\ @(0,-1/4)$
$(0,2)$		$-$	$-$	$Decreasing, down$
$x=2$	-----	-----	-----	$V.A.\ x=2$
$(2,\infty)$		$-$	$+$	$Decreasing, up$



37. $f(x) = 2x - 4\sin x$ $\qquad 0 \le x \le 2\pi$

x	$f(x)$	$f'(x)$	$f''(x)$	$Conclusion$
$x=0$	0			$Left\ End\ @\ (0,0)$
$(0,\frac{\pi}{3})$		$-$	$+$	$Decreasing, up$
$x=\frac{\pi}{3}$	$\frac{2\pi}{3}-2\sqrt{3}$	0	$+$	$A.Min\ @\ (\frac{\pi}{3},\frac{2\pi}{3}-2\sqrt{3})$
$(\frac{\pi}{3},\pi)$		$+$	$+$	$Increasing, up$
$x=\pi$	2π		0	$POI\ @\ (\pi,2\pi)$
$(\pi,\frac{5\pi}{3})$		$+$	$-$	$Increasing, down$

$x = \frac{5\pi}{3}$	$\frac{10\pi}{3} + 2\sqrt{3}$	0	$-$	$A.Max\ @\ (\frac{5\pi}{3}, \frac{10\pi}{3} + 2\sqrt{3})$
$(\frac{5\pi}{3}, 2\pi)$		$-$	$-$	$Decreasing, down$
$x = 2\pi$	4π			$Right\ End\ @\ (2\pi, 4\pi)$