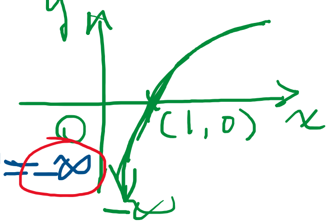
Sec.5.1

p.315-316:
Properties of the Natural Logarithmic Function; Logarithmic Properties; Example 1

$f(x) = \log_b x$

$\log_e = \ln$



p.317: The Number e: $e \approx 2.71828182846\ldots$

p.321: Find the limit.

$x \to 0^+$ $\ln x$

36. $\lim_{x\to 6^-} \ln(6-x) = \ln(6-6^-) = \ln(0^+) = -\infty$

38. $\lim_{x\to 5^+} \ln \dfrac{x}{\sqrt{x-4}} = \dfrac{5^+}{\sqrt{5^+-4}} = \dfrac{5}{\sqrt{1}} = 5$

p.318-319: Derivative of the Natural Logarithmic Function; Examples 3 - 6

p.320: Derivative Involving Absolute Value; Example 7

p.321-322: Find the derivative of the function.

42. $h(x) = \ln(2x^2+1)$ $\quad u = 2x^2+1 \to u' = 4x$ $\quad \dfrac{u'}{u}$

$h'(x) = \boxed{\dfrac{4x}{2x^2+1}}$

44. $y = \underset{f}{x^2} \underset{g}{\ln x}$

$y' = 2x\ln x + \dfrac{x^2}{1}\cdot\dfrac{1}{x} = \boxed{2x\ln x + x}$

54. $y = \ln(\ln x)$ $\quad u = \ln x \to u' = \dfrac{1}{x}$ $\quad$ LCD $= (x-1)(x+1)$

$y' = \dfrac{\frac{1}{x}}{\ln x} = \dfrac{1}{x}\cdot\dfrac{1}{\ln x} = \boxed{\dfrac{1}{x\ln x}}$

$u' = 1$ $\quad u' = 1$

56. $y = \ln\sqrt[3]{\dfrac{x-1}{x+1}} = \ln\left(\dfrac{x-1}{x+1}\right)^{1/3} = \dfrac{1}{3}\ln\dfrac{x-1}{x+1} = \dfrac{1}{3}\left[\ln\underset{u}{(x-1)} - \ln\underset{u}{(x+1)}\right]$

$y' = \dfrac{1}{3}\left[\dfrac{1}{x-1} - \dfrac{1}{x+1}\right] = \dfrac{1}{3}\left[\dfrac{1(x+1)}{(x+1)(x-1)} - \dfrac{1(x-1)}{(x+1)(x-1)}\right]$

$= \dfrac{1}{3}\left[\dfrac{(x+1)-(x-1)}{(x+1)(x-1)}\right] = \dfrac{\cancel{x}+1-\cancel{x}+1}{3(x^2-1)} = \boxed{\dfrac{2}{3(x^2-1)}}$

60. $y = \ln|\csc x|$ $\quad u = \csc x \to u' = -\csc x\cot x$

$y' = \dfrac{-\cancel{\csc x}\cot x}{\cancel{\csc x}} = \boxed{-\cot x}$

62. $y = \ln|\sec x + \tan x|$ $\quad u = \sec x + \tan x \to u' = \sec x\tan x + \sec^2 x$

$y' = \dfrac{\sec x\tan x + \sec^2 x}{\sec x + \tan x} = \dfrac{\sec x\cancel{(\tan x + \sec x)}}{\cancel{\sec x + \tan x}} = \boxed{\sec x}$

$y - y_1 = m(x - x_1)$

$$y' = \frac{\sec x \tan x + \sec x}{\sec x + \tan x} = \frac{\sec x \cancel{(\tan x + \sec x)}}{\cancel{\sec x + \tan x}} = \boxed{\sec x}$$

$y - y_1 = m(x - x_1)$

In Exercises 63–70, (a) find an equation of the tangent line to the graph of the function at the given point.

64. $y = \ln x^{2/3}, \quad (-1, 0)$

$y = \frac{2}{3}\ln x$

① $y' = \frac{2}{3} \cdot \frac{1}{x} = \frac{2}{3x}$

② @ $(-1, 0)$

$m = y'(-1) = \frac{2}{3(-1)} = -\frac{2}{3}$

③ T.L. $m = -\frac{2}{3}$ $\quad (-1, 0)$ with $x_1 = -1$, $y_1 = 0$

$y - 0 = -\frac{2}{3}(x + 1)$

$$\boxed{y = -\frac{2}{3}x - \frac{2}{3}}$$

$y = mx + b$

Use implicit differentiation to find $\frac{dy}{dx}$. y'

$\ln x^2 y = 2\ln x + \ln y$

80. $4xy + \ln x^2 y = 7$

$4xy + 2\ln x + \ln y = 7 \rightarrow 4y + 4xy' + \frac{2}{x} + \frac{y'}{y} = 0 \rightarrow$

$4xy' + \frac{y'}{y} = -4y - \frac{2}{x}$

$\left(4x + \frac{1}{y}\right)y' = -4y - \frac{2}{x} \rightarrow \frac{4xy + 1}{y}y' = -\frac{4xy + 2}{x} \rightarrow \boxed{y' = -\frac{y(4xy + 2)}{x(4xy + 1)}}$

$y' = -\frac{4xy + 2}{x} \cdot \frac{y}{4xy + 1}$