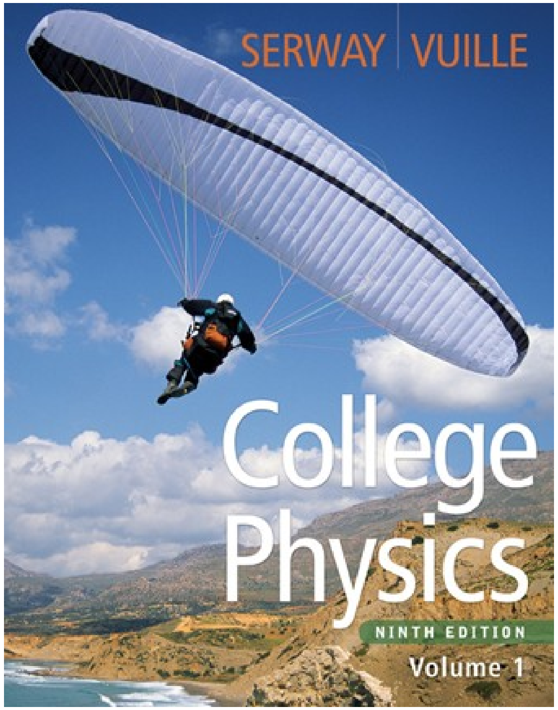

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Chapter Three

Two-Dimensional Motion

Topics Chapter 3

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Vectors and Motion

- In one-dimensional motion, vectors were used to a limited extent
- For more complex motion, manipulating vectors will be more important

Vector vs. Scalar Review

- All physical quantities encountered in this text will be either a scalar or a vector
- A **vector** quantity has both magnitude (size) and direction
- A **scalar** is completely specified by only a magnitude (size)

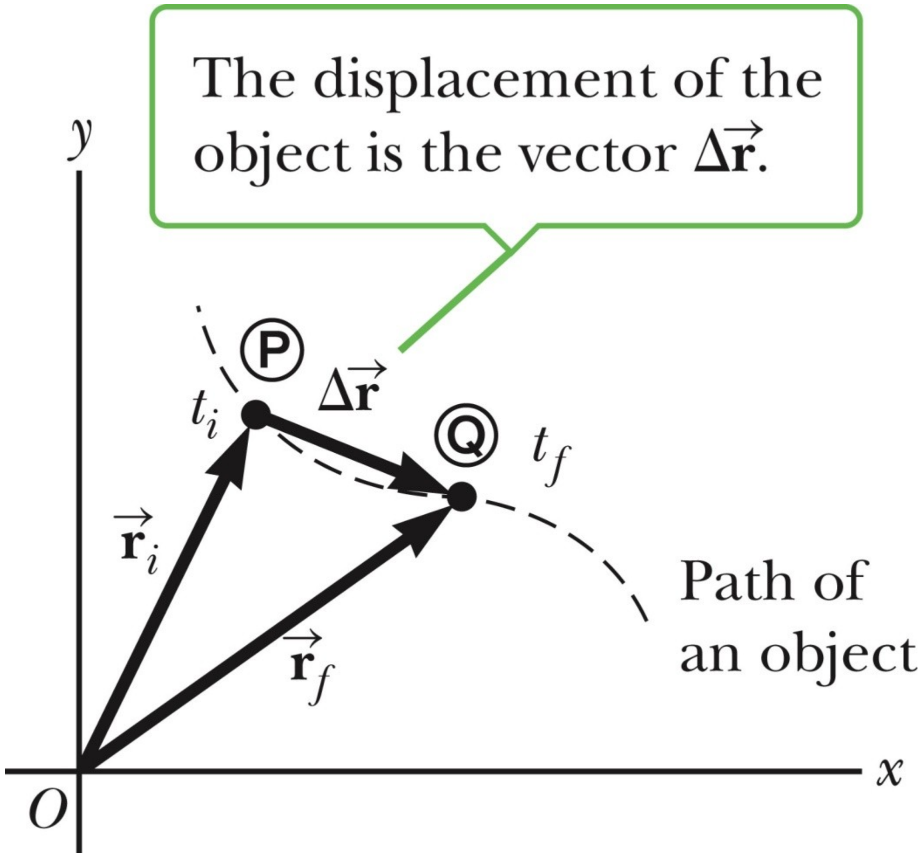
Displacement is the direction of a vector

Motion in Two Dimensions

- Using + or – signs is not always sufficient to fully describe motion in more than one dimension
  - Vectors can be used to more fully describe motion
- Still interested in displacement, velocity, and acceleration

Displacement

- The position of an object is described by its position vector, $\vec{\mathbf{r}}$
- The **displacement** of the object is defined as the ***change in its position***
  - $\Delta\vec{\mathbf{r}} = \vec{\mathbf{r}}_f - \vec{\mathbf{r}}_i$
  - SI unit: meter (m)



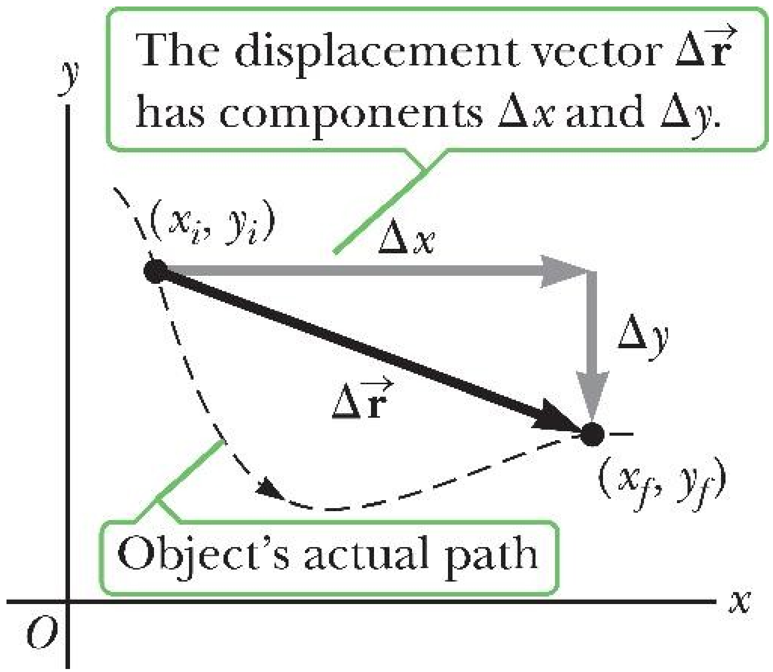
Velocity

- The average velocity is the ratio of the displacement to the time interval for the displacement

$$\vec{\mathbf{v}}_{av} \equiv \frac{\Delta\vec{\mathbf{r}}}{\Delta t}$$

- The instantaneous velocity is the limit of the average velocity as Δt approaches zero AKA the slope at a specific point on a time graph
  - The direction of the instantaneous velocity is along a line that is tangent to the path of the particle and in the direction of motion
- SI unit: meter per second (m/s)

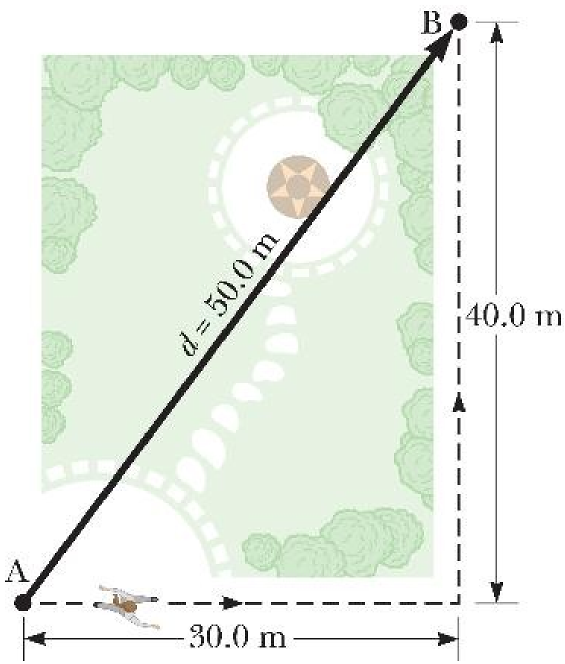
Displacement in Two Dimensions



$$\Delta x = x_f - x_i$$

$$\Delta y = y_f - y_i$$

Velocity in Two Dimensions (3 of 3)



Path length:

$$30.0\text{ m} + 40.0\text{ m} = 70.0\text{ m}$$

Average speed: $$\frac{70.0\text{ m}}{20.0\text{ s}} = 3.50\text{ m/s}$$

Average velocity: $$\frac{50.0\text{ m}}{20.0\text{ s}} = 2.50\text{ m/s}$$

Acceleration

- The average acceleration is defined as the rate at which the velocity changes

$$\vec{\mathbf{a}}_{av} = \frac{\Delta\vec{\mathbf{v}}}{\Delta t}$$

- The instantaneous acceleration is the limit of the average acceleration as Δt approaches zero.
- SI unit: meter per second squared (m/s^2)

Unit Summary (SI)

- Displacement
  - m
- Average velocity and instantaneous velocity
  - m/s
- Average acceleration and instantaneous acceleration
  - m/s^2

Ways an Object Might Accelerate

- The magnitude of the velocity (the speed) may change with time
- The direction of the velocity may change with time
  - Even though the magnitude is constant
- Both the magnitude and the direction may change with time

Example: Something spinning in a circle

Projectile Motion

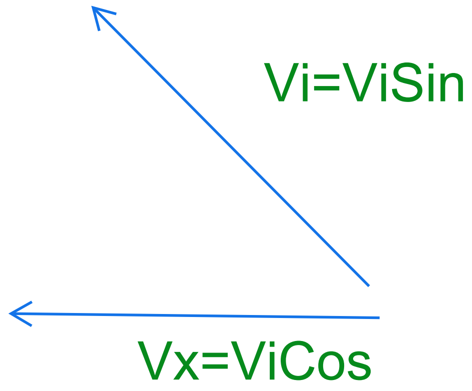
- An object may move in both the x and y directions simultaneously
  - It moves in two dimensions
- The form of two dimensional motion we will deal with is an important special case called **projectile motion**

Assumptions of Projectile Motion

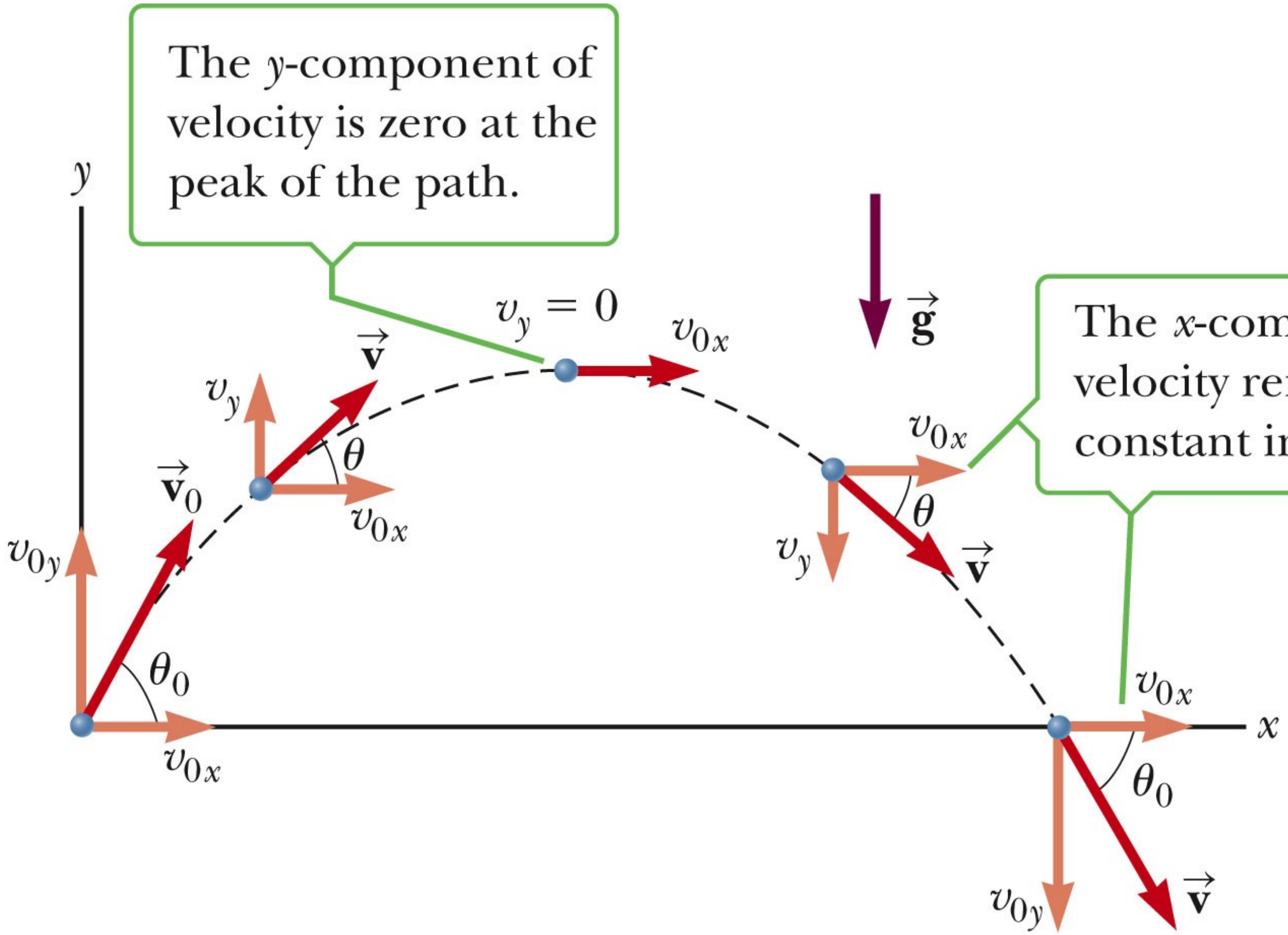
- We may ignore air friction
- We may ignore the rotation of the earth
- With these assumptions, an object in projectile motion will follow a parabolic path

Rules of Projectile Motion

- The x- and y-directions of motion are completely independent of each other
- The x-direction is uniform motion
  - $a_x = 0$
- The y-direction is free fall
  - $a_y = -g$
- The initial velocity can be broken down into its x- and y-components
  - $v_{Ox} = v_O \cos\theta_O \quad v_{Oy} = v_O \sin\theta_O$

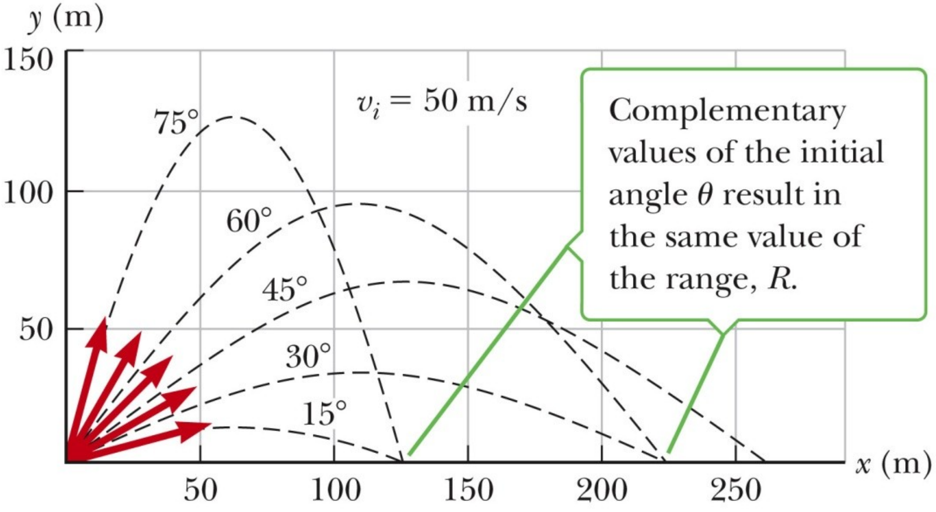


Projectile Motion



Projectile Motion at Various Initial Angles

- Complementary values of the initial angle result in the same range
  - The heights will be different
- The maximum range occurs at a projection angle of 45°



Two-Dimensional Motion

$$v_x = v_{0x} + a_x t$$

$$\Delta x = v_{0x} t + \frac{1}{2} a_x t^2$$

$$v_x^{\ 2} = v_{0x}^{\ 2} + 2a_x \Delta x$$

$$v_{0x} = v_0 \cos\theta$$

$$v_y = v_{0y} + a_y t$$

$$\Delta y = v_{0y} t + \frac{1}{2} a_y t^2$$

$$v_y^{\ 2} = v_{0y}^{\ 2} + 2a_y \Delta y$$

$$v_{0y} = v_0 \sin\theta$$

$$v = \sqrt{v_x^{\ 2} + v_y^{\ 2}}$$

$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right)$$

Some Details About the Rules

- x-direction
  - $a_x = 0$
  - $v_x = v_{o_x} = v_o \cos\theta_o = constant$
  - $x = v_{ox}t$
    - This is the only operative equation in the x-direction since there is uniform velocity in that direction

More Details About the Rules

- y-direction
  - $v_{o_y} = v_o \sin\theta_o$
  - Free fall problem
    - a = -g
  - Take the positive direction as upward
  - Uniformly accelerated motion, so the motion equations all hold

Two-Dimensional Motion

$$v_x = v_{0x} = v_0 \cos\theta_0 = \text{constant}$$

$$\Delta x = v_{0x}t = (v_0 \cos\theta_0)t$$

$$v_y = v_0 \sin\theta_0 - gt$$

$$\Delta y = (v_0 \sin\theta_0)t - \frac{1}{2}gt^2$$

$$v_y^{\,2} = (v_0 \sin\theta_0)^2 - 2g\Delta y$$

Velocity of the Projectile

- The velocity of the projectile at any point of its motion is the vector sum of its x and y components at that point

$$v = \sqrt{v_x^2 + v_y^2} \quad and \quad \theta = \tan^{-1} \frac{v_y}{v_x}$$

  - Remember to be careful about the angle's quadrant

Projectile Motion Summary

- Provided air resistance is negligible, the horizontal component of the velocity remains constant
  - Since $a_x = 0$
- The vertical component of the acceleration is equal to the free fall acceleration $-g$
  - The acceleration in the *y*-direction is not zero at the top of the projectile's trajectory

Projectile Motion Summary, cont

- The vertical component of the velocity v_y and the displacement in the y-direction are identical to those of a freely falling body
- Projectile motion can be described as a superposition of two independent motions in the *x*- and *y*-directions




Problem-Solving Strategy

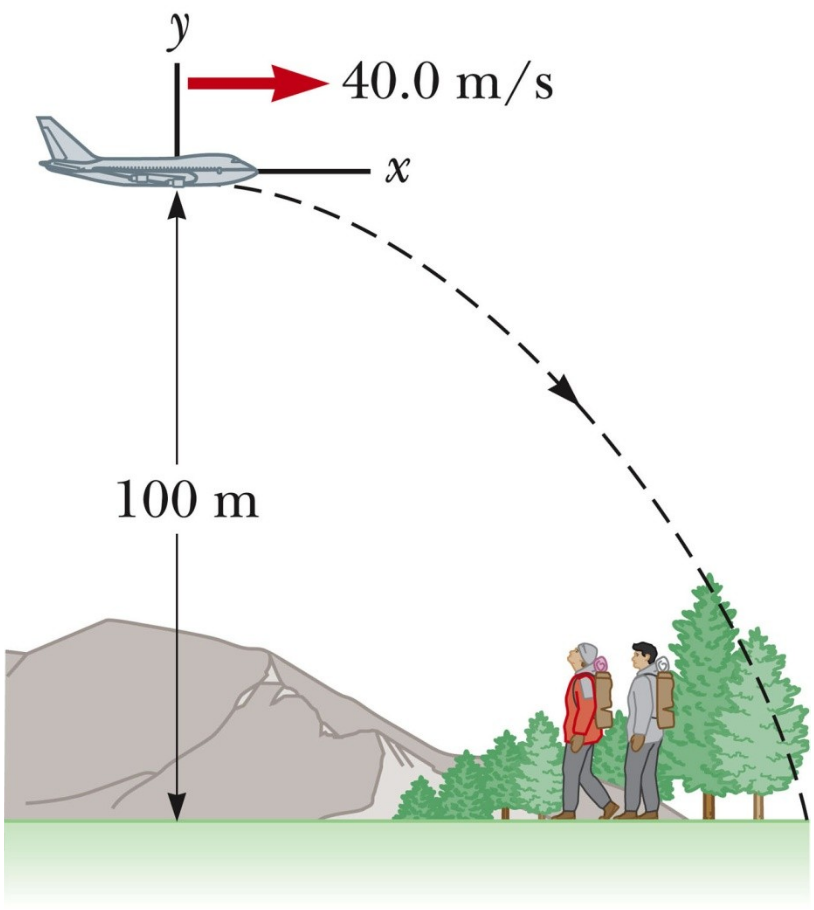
- **Select** a coordinate system and sketch the path of the projectile
  - Include initial and final positions, velocities, and accelerations
- **Resolve** the initial velocity into x- and y-components
- **Treat** the horizontal and vertical motions independently

Problem-Solving Strategy, cont

- **Follow** the techniques for solving problems with constant velocity to analyze the horizontal motion of the projectile
- **Follow** the techniques for solving problems with constant acceleration to analyze the vertical motion of the projectile

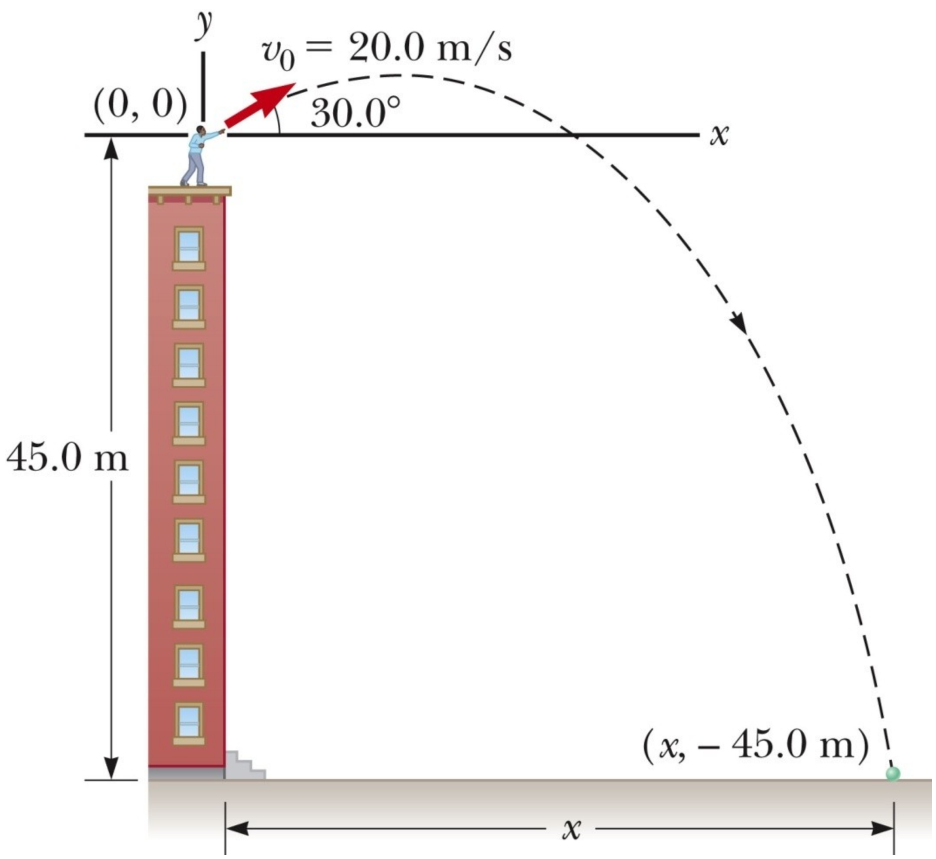
Some Variations of Projectile Motion

- An object may be fired horizontally
- The initial velocity is all in the x-direction
  - $v_o = v_x$ and $v_y = 0$
- All the general rules of projectile motion apply



Non-Symmetrical Projectile Motion

- Follow the general rules for projectile motion
- Break the y-direction into parts
  - up and down
  - symmetrical back to initial height and then the rest of the height



Special Equations

- The motion equations can be combined algebraically and solved for the range and maximum height

$$\Delta x = \frac{v_o^2 \sin 2\theta_o}{g}$$

$$\Delta y_{max} = \frac{v_o^2 \sin^2 \theta_o}{2g}$$

Relative Velocity

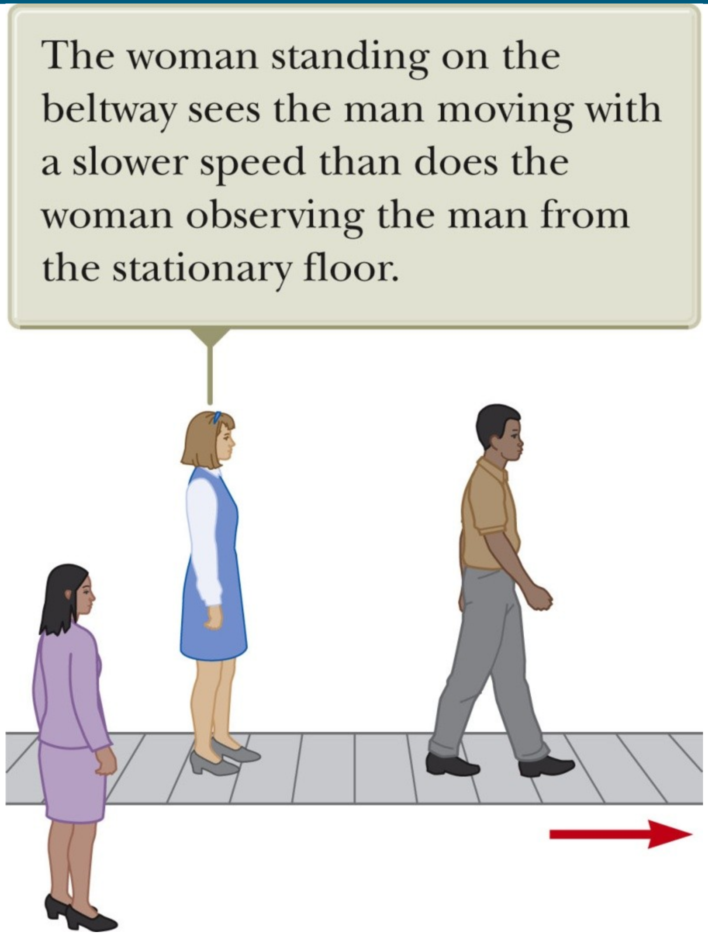
The man is walking on the moving beltway.

The woman on the beltway sees the man walking at his normal walking speed.

The stationary woman sees the man walking at a much higher speed.

- The combination of the speed of the beltway and the walking.

The difference is due to the relative velocity of their frames of reference.



Relative Velocity

- Relative velocity is about relating the measurements of two different observers
- It may be useful to use a moving frame of reference instead of a stationary one
- It is important to specify the frame of reference, since the motion may be different in different frames of reference
- There are no specific equations to learn to solve relative velocity problems

Relative Velocity Notation

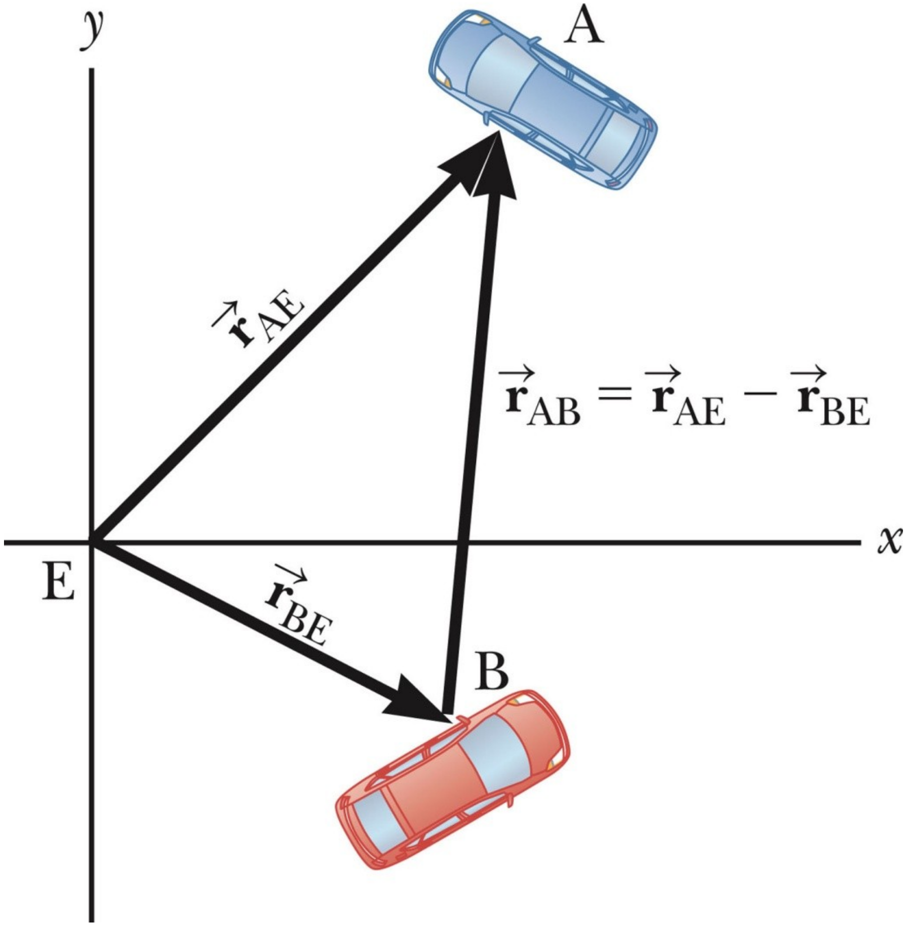
- The pattern of subscripts can be useful in solving relative velocity problems
- Assume the following notation:
  - E is an observer, stationary with respect to the earth
  - A and B are two moving cars

Relative Position Equations

- $\vec{\mathbf{r}}_{AE}$ is the position of car A as measured by E
- $\vec{\mathbf{r}}_{BE}$ is the position of car B as measured by E
- $\vec{\mathbf{r}}_{AB}$ is the position of car A as measured by car B
- $\vec{\mathbf{r}}_{AB} = \vec{\mathbf{r}}_{AE} - \vec{\mathbf{r}}_{BE}$

Relative Position

- The position of car A relative to car B is given by the vector subtraction equation



Relative Velocity Equations

- The rate of change of the displacements gives the relationship for the velocities

$$\vec{\mathbf{v}}_{AB} = \vec{\mathbf{v}}_{AE} - \vec{\mathbf{v}}_{BE}$$

Problem-Solving Strategy: Relative Velocity

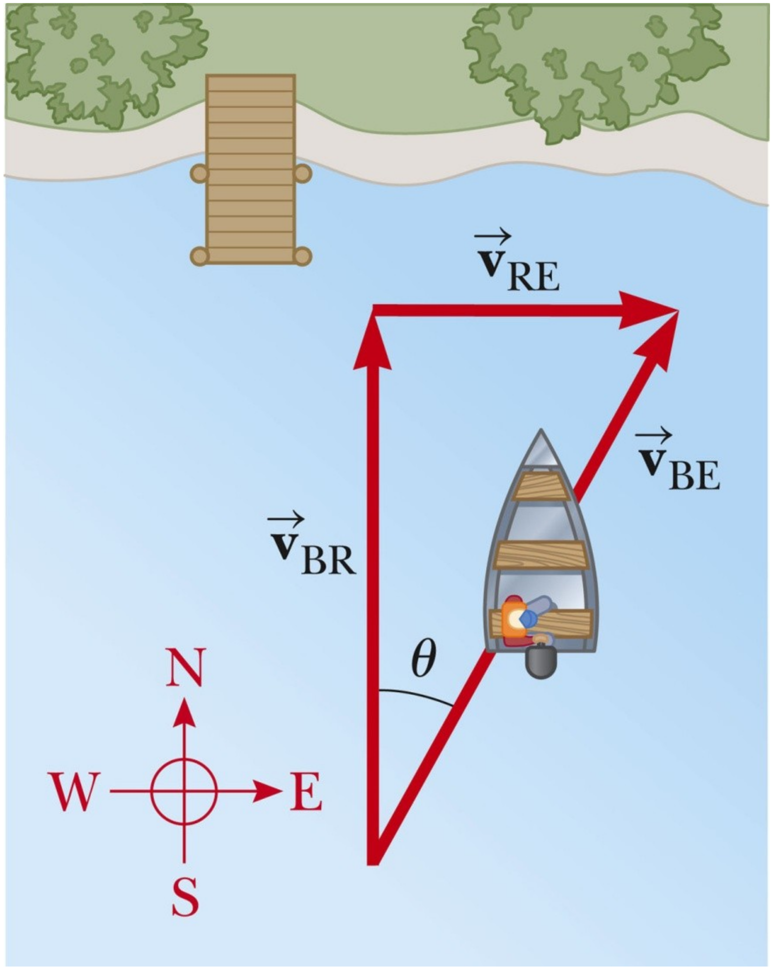
- **Label** all the objects with a descriptive letter
- **Look** for phrases such as “velocity of A relative to B”
  - Write the velocity variables with appropriate notation
  - If there is something not explicitly noted as being relative to something else, it is probably relative to the earth

Problem-Solving Strategy: Relative Velocity, cont

- **Take** the velocities and put them into an equation
  - Keep the subscripts in an order analogous to the standard equation
- **Solve** for the unknown(s)

Relative Velocity, Example

- Need velocities
  - Boat relative to river
  - River relative to the Earth
  - Boat with respect to the Earth (observer)
- Equation
  - $\vec{\mathbf{v}}_{BR} = \vec{\mathbf{v}}_{BE} - \vec{\mathbf{v}}_{RE}$



Assessing to Learn (1 of 2)

Consider the following situations:

a) a car slowing down at a stop sign Yes

b) a ball being swung in a circle at constant speed Yes

c) a vibrating string Yes, changing direction in the vibration

d) the Moon orbiting the Earth Yes

e) a skydiver falling at terminal speed No

f) an astronaut in an orbiting space station Yes

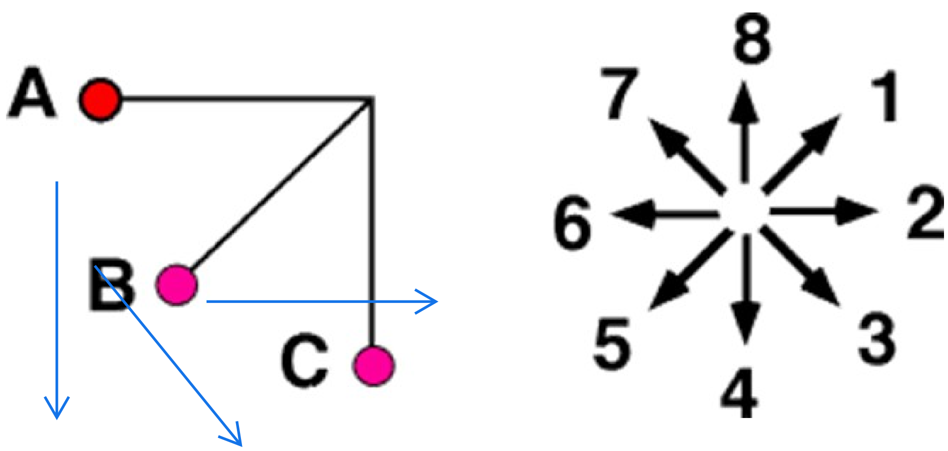
g) a ball rolling down a hill Yes, affected by gravity without being at terminal

h) a person driving down a straight section of highway at constant speed with her foot on the accelerator No

In how many of the situations is the object accelerating?

Assessing to Learn (2 of 2)

A pendulum is released from rest at position A and swings toward the vertical under the influence of gravity as depicted below. When at position B, which direction most nearly corresponds to the direction of the acceleration?



3 degree arrow is the change in velocity, and the resultant is in the 2 degree of which 3 - 2 = 1

Topic 3: Motion in Two Dimensions

TOPIC SUMMARY

Topic Summary (1 of 5)

- **Displacement, Velocity, and Acceleration in Two Dimensions**

$$\Delta \mathbf{r} \equiv \mathbf{r}_f - \mathbf{r}_i$$

$$\vec{\mathbf{v}}_{\text{av}} \equiv \frac{\Delta \mathbf{r}}{\Delta t} \qquad \vec{\mathbf{v}} \equiv \lim_{\Delta t \to 0} \frac{\Delta \mathbf{r}}{\Delta t}$$

$$\vec{\mathbf{a}}_{\text{av}} \equiv \frac{\Delta \mathbf{v}}{\Delta t} \qquad \vec{\mathbf{a}} \equiv \lim_{\Delta t \to 0} \frac{\Delta \mathbf{v}}{\Delta t}$$

Topic Summary (2 of 5)

- **Two-Dimensional Motion**

$$v_x = v_{0x} + a_x t \qquad v_y = v_{0y} + a_y t$$

$$\Delta x = v_{0x} t + \frac{1}{2} a_x t^2 \qquad \Delta y = v_{0y} t + \frac{1}{2} a_y t^2$$

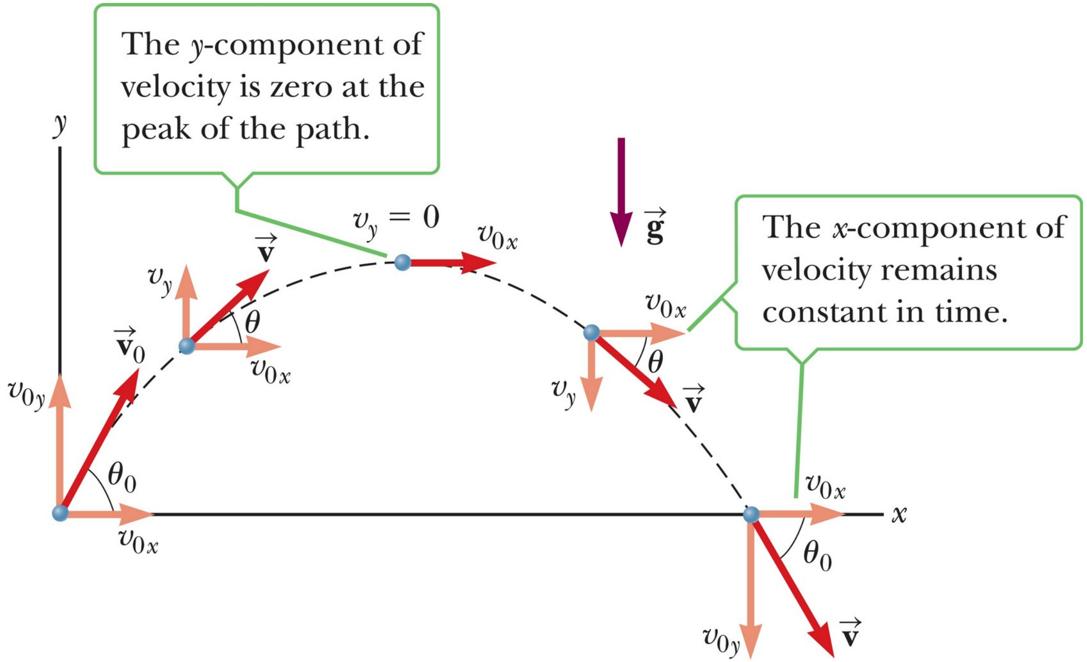
$$v_x^{\,2} = v_{0x}^{\,2} + 2 a_x \Delta x \qquad v_y^{\,2} = v_{0y}^{\,2} + 2 a_y \Delta y$$

$$v_{0x} = v_0 \cos\theta \qquad v_{0y} = v_0 \sin\theta$$

$$v = \sqrt{v_x^{\,2} + v_y^{\,2}} \qquad \theta = \tan^{-1}\left(\frac{v_y}{v_x}\right)$$

Topic Summary (3 of 5)

- **Two-Dimensional Motion**



- **Two-Dimensional Motion**

$$v_x = v_{0x} = v_0 \cos\theta_0 = \text{constant}$$

$$\Delta x = v_{0x} t = (v_0 \cos\theta_0) t$$

$$v_y = v_0 \sin\theta_0 - gt$$

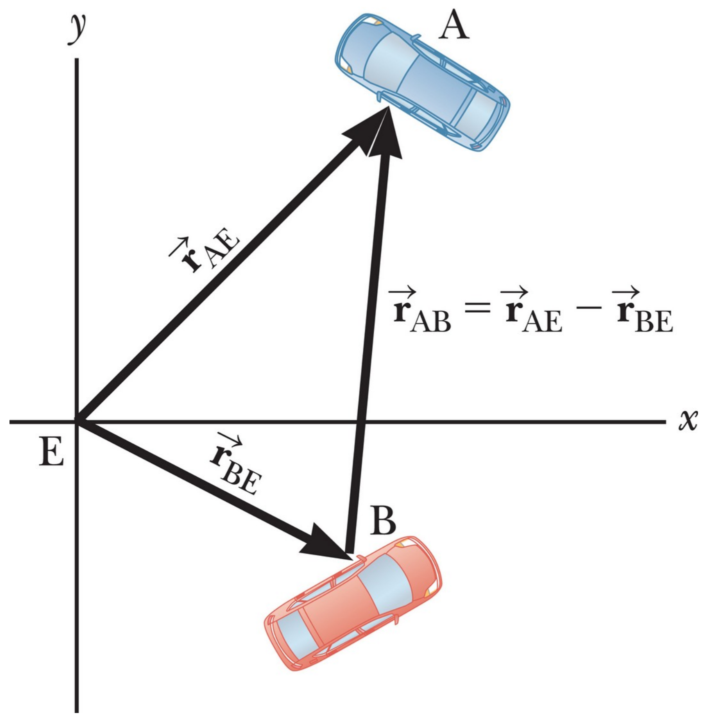
$$\Delta y = (v_0 \sin\theta_0) t - \frac{1}{2} g t^2$$

$$v_y^{\,2} = (v_0 \sin\theta_0)^2 - 2g\Delta y$$

Topic Summary (5 of 5)

- **Relative Velocity**

$$\mathbf{v}_{AB} = \mathbf{v}_{AE} - \mathbf{v}_{BE}$$



Reading Question 3-1 (1 of 2)

A book is moved once around the perimeter of a tabletop with the dimensions 1.0 m × 2.0 m. If the book ends up at its original position, what is its displacement and what is the distance it traveled?

1. displacement of 0 m, distance traveled 0 m
2. displacement of 0 m, distance traveled 6 m
3. displacement of 6 m, distance traveled 6 m
4. displacement of 6 m, distance traveled 0 m

Reading Question 3-1 (2 of 2)

A book is moved once around the perimeter of a tabletop with the dimensions 1.0 m × 2.0 m. If the book ends up at its original position, what is its displacement and what is the distance it traveled?

1. displacement of 0 m, distance traveled 0 m
2. **Answer: displacement of 0 m, distance traveled 6 m**
3. displacement of 6 m, distance traveled 6 m
4. displacement of 6 m, distance traveled 0 m

Reading Question 3-2 (1 of 2)

A vector lies in the *x*-*y* plane. For what orientations will both components be negative?

1. The vector lies between 180° and 270° from the *x*-axis.
2. The vector lies between 0° and 90° from the *x*-axis.
3. The vector lies between 90° and 180° from the *x*-axis.
4. None. Just like for vector magnitudes, components are always positive.

Reading Question 3-2 (2 of 2)

A vector lies in the *x*-*y* plane. For what orientations will both components be negative?

1. **Answer: The vector lies between 180° and 270° from the *x*-axis.**
2. The vector lies between 0° and 90° from the *x*-axis.
3. The vector lies between 90° and 180° from the *x*-axis.
4. None. Just like for vector magnitudes, components are always positive.

Reading Question 3-3 (1 of 2)

Which of the following quantities, if any, remain constant as a projectile moves through its parabolic trajectory?

1. speed
2. acceleration
3. the horizontal component of velocity
4. the vertical component of velocity

1. (i) only
2. (ii) and (iii)
3. (ii) and (iv)
4. all of them

Reading Question 3-3 (2 of 2)

Which of the following quantities, if any, remain constant as a projectile moves through its parabolic trajectory?

1. speed
2. acceleration
3. the horizontal component of velocity
4. the vertical component of velocity

1. (i) only
2. **Answer: (ii) and (iii)**
3. (ii) and (iv)
4. all of them

Reading Question 3-4 (1 of 2)

A sailor drops a wrench from the top of a sailboat's mast while the boat is moving steadily and rapidly in a straight line. Where will the wrench hit the deck?

1. at the base of the mast
2. behind the mast, along the direction opposite to the motion of the boat
3. in front of the mast, along the direction of the motion of the boat
4. impossible to tell

Reading Question 3-4 (2 of 2)

A sailor drops a wrench from the top of a sailboat's mast while the boat is moving steadily and rapidly in a straight line. Where will the wrench hit the deck?

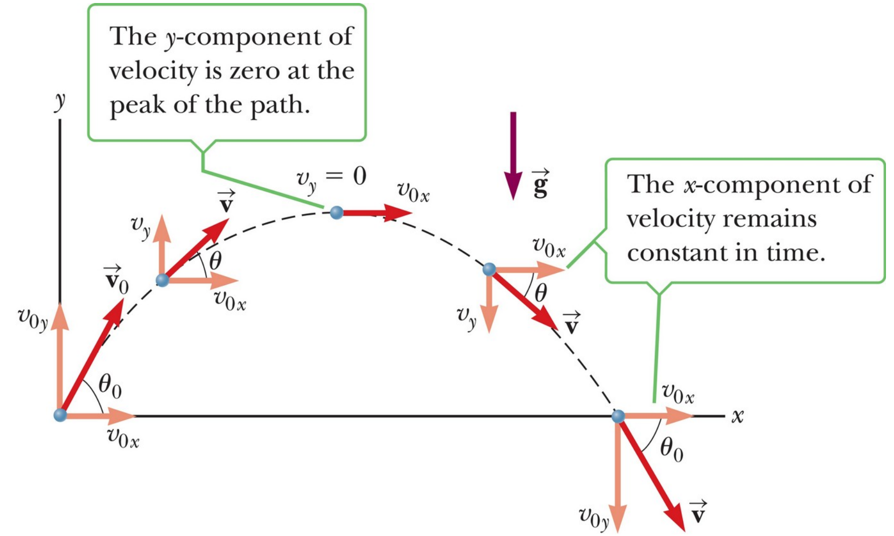
1. **Answer: at the base of the mast**
2. behind the mast, along the direction opposite to the motion of the boat
3. in front of the mast, along the direction of the motion of the boat
4. impossible to tell

Don't forget that the wrench has the same horizontal velocity as the boat, and so will move with the boat

Reading Question 3-5 (1 of 2)

As a projectile thrown upward at a non-vertical angle moves in a parabolic path, at what point along its path are the velocity and acceleration vectors for the projectile parallel to each other?

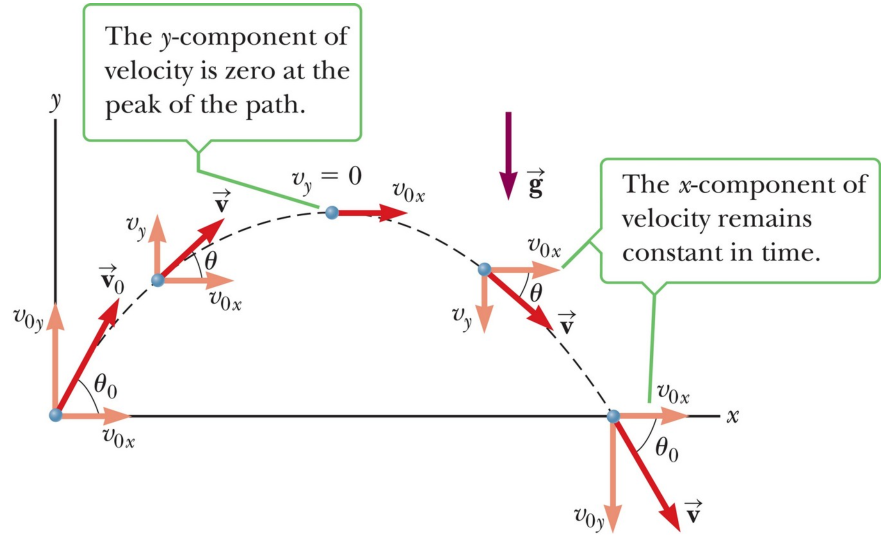
1. at the point just before the projectile lands
2. at the highest point
3. at the launch point
4. nowhere



Reading Question 3-5 (2 of 2)

As a projectile thrown upward at a non-vertical angle moves in a parabolic path, at what point along its path are the velocity and acceleration vectors for the projectile parallel to each other?

1. at the point just before the projectile lands
2. at the highest point
3. at the launch point
4. **Answer: nowhere**



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