

Topic 1

Units, Trigonometry, and Vectors

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Physics

Physics is a Fundamental Science

- •Concerned with the fundamental principles of the Universe
- •Foundation of other physical sciences
- •Has simplicity of fundamental concepts
- •Laws expressed as mathematical equations

Physics

Major branches of Physics:

- Classical Mechanics (motion, forces, developed before 1900)
- Thermodynamics (Heat, temperature, engines)
- Electromagnetism (electricity, magnetism)
- Vibrations and Waves (sound, electromagnetic waves)
- Optics (light)
- Relativity (motion at very high speeds)
- Quantum Mechanics (sub-microscopic objects)
- Astrophysics (Space, stars, galaxies)
- Particle and Nuclear Physics (Atoms and sub-atomic particles)
- •
- Modern Physics (developed after 1900)

Theories and Experiments

- The goal of physics is to develop theories based on experiments
- A physical theory, usually expressed mathematically, describes how a given system works
- The theory makes predictions about how a system should work
- Experiments check the theories' predictions
- Every theory is a work in progress

Fundamental Quantities and Their Dimension

- Mechanics uses three fundamental quantities
 - Length [L]
 - Mass [M]
 - Time [T]

Units

- To communicate the result of a measurement for a quantity, a unit must be defined
- Defining units allows everyone to relate to the same fundamental amount
- Fundamental Units: for mass, length and time.
- Derived Units: for speed, acceleration, force etc.

SI System of Measurement

Every region on Earth had developed their own systems of measurements of mass, length, area, time, etc.

This led to confusion as human interactions grew.

- SI Systéme International
 - Agreed to in 1960 by an international committee
 - Main system used in this text

Length

- Units
 - meter, m
- Originally defined as one ten millionth of distance from equator to north pole.
- Currently defined in terms of the distance traveled by light in a vacuum during a given time
 - Also establishes the value for the speed of light in a vacuum

Length

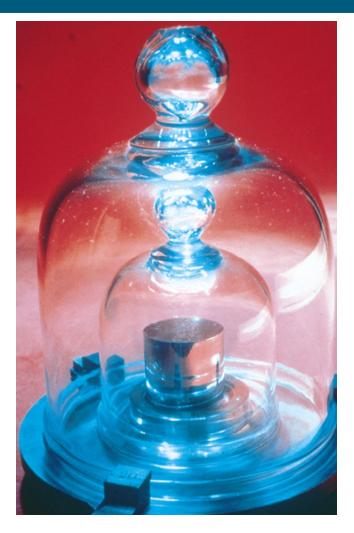
TABLE 1.1 Approximate Values of Some Measured Lengths

	Length (m)
Distance from the Earth to the most remote known quasar	2.7×10^{26}
Distance from the Earth to the most remote normal galaxies	3×10^{26}
Distance from the Earth to the nearest large galaxy (Andromeda)	2×10^{22}
Distance from the Sun to the nearest star (Proxima Centauri)	4×10^{16}
One light-year	9.46×10^{15}
Mean orbit radius of the Earth about the Sun	1.50×10^{11}
Mean distance from the Earth to the Moon	3.84×10^{8}
Distance from the equator to the North Pole	1.00×10^{7}
Mean radius of the Earth	6.37×10^{6}
Typical altitude (above the surface) of a satellite orbiting the Earth	2×10^5
Length of a football field	9.1×10^{1}
Length of a housefly	5×10^{-3}
Size of smallest dust particles	$\sim 10^{-4}$
Size of cells of most living organisms	$\sim 10^{-5}$
Diameter of a hydrogen atom	$\sim 10^{-10}$
Diameter of an atomic nucleus	$\sim 10^{-14}$
Diameter of a proton	$\sim 10^{-15}$

Mass

- Units
 - kilogram, kg
- Originally defined as the mass of a specific cylinder kept at the International Bureau of Weights and Measures.
- A more recent definition is based on the value of the Plank's Constant.

Standard Kilogram



Section 1.1

Mass

TABLE 1.2 Approximate Masses of Various Objects

2	Mass (kg)
Observable	
Universe	$\sim 10^{52}$
Milky Way	
galaxy	$\sim 10^{42}$
Sun	1.99×10^{30}
Earth	$5.98 imes 10^{24}$
Moon	7.36×10^{22}
Shark	$\sim 10^{3}$
Human	$\sim 10^{2}$
Frog	$\sim 10^{-1}$
Mosquito	$\sim 10^{-5}$
Bacterium	$\sim 1 \times 10^{-15}$
Hydrogen atom	1.67×10^{-27}
Electron	9.11×10^{-31}

Time

- Units
 - seconds, s
- Originally defined as 24*60*60 s in one mean solar day of the year 1900.

• Currently defined in terms of the period of oscillation of radiation from a cesium-133 atom.

Time

TABLE 1.3 Approximate Values of Some Time Intervals

	Time Interval (s)
Age of the Universe	4×10^{17}
Age of the Earth	1.3×10^{17}
Average age of a college student	6.3×10^{8}
One year	3.2×10^{7}
One day	8.6×10^{4}
One class period	3.0×10^{3}
Time interval between normal	
heartbeats	8×10^{-1}
Period of audible sound waves	$\sim 10^{-3}$
Period of typical radio waves	$\sim 10^{-6}$
Period of vibration of an atom	
in a solid	$\sim 10^{-13}$
Period of visible light waves	$\sim 10^{-15}$
Duration of a nuclear collision	$\sim 10^{-22}$
Time interval for light to cross	
a proton	$\sim 10^{-24}$

Scale of things

Scale of Size of the Universe

https://www.youtube.com/watch?v=uaGEjrADG
PA

Scale of time

https://www.bing.com/videos/search?q=scale+ of+universe&&view=detail&mid=D8B2D23F5BE EC47B039CD8B2D23F5BEEC47B039C&&FORM= VRDGAR

Other Systems of Measurements

- cgs Gaussian system
 - Named for the first letters of the units it uses for fundamental quantities

- US Customary
 - Everyday units
 - Often uses weight, in pounds, instead of mass as a fundamental quantity

Units in Various Systems

System	Length	Mass	Time
SI	meter	kilogram	second
cgs	centimeter	gram	second
US Customary	foot	slug	second

Prefixes

Prefixes correspond to powers of 10.

Each prefix has a specific name, and abbreviation.

Examples:

Kilometer =
$$1000 \text{ m}$$
 = 10^3 m

Millimeter =
$$1/1000 \text{ m} = 10^{-3} \text{ m}$$

$$kilo = 10^{3}$$

$$Mega = 10^6$$

Giga =
$$10^9$$

$$milli = 10^{-3}$$

$$micro = 10^{-6}$$

nano =
$$10^{-9}$$

Powers of ten

TABLE 1.4 Prefixes for Powers of Ten

Power	Prefix	Abbreviation	Power	Prefix	Abbreviation
10^{-24}	yocto	у	10^{3}	kilo	k
10^{-21}	zepto	Z	10 ⁶	mega	\mathbf{M}
10^{-18}	atto	a	109	giga	G
10^{-15}	femto	f	10^{12}	tera	\mathbf{T}
10^{-12}	pico	р	10^{15}	peta	P
10^{-9}	nano	n	10^{18}	exa	\mathbf{E}
10^{-6}	micro	μ	10^{21}	zetta	Z
10^{-3}	milli	m	10^{24}	yotta	Y
10^{-2}	centi	С		•	
10^{-1}	deci	d			

•Meter = m Kilo = k = thousand

Milli = m = 1 / 1000

•Kilometer = km = 1000 m

Millimeter = mm = 1/1000 m

•We read 4.5 km as: 4.5 kilo meters or as 4.5 thousand meters.

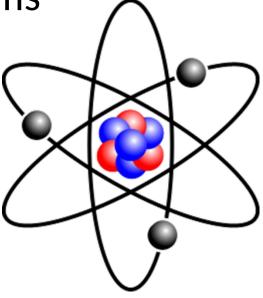
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Expressing Numbers

- Numbers with more than three digits are written in groups of three digits separated by spaces
 - Groups appear on both sides of the decimal point
- 10 000 instead of 10,000
- 3.141 592 65
- However, this is not convenient, so it is better to keep the 'thousands separator', i.e. "," after every three digits and write it as 10,000

Structure of Matter

- Matter is made up of molecules
 - The smallest division that is identifiable as a substance
- Molecules are made up of atoms
 - Correspond to elements

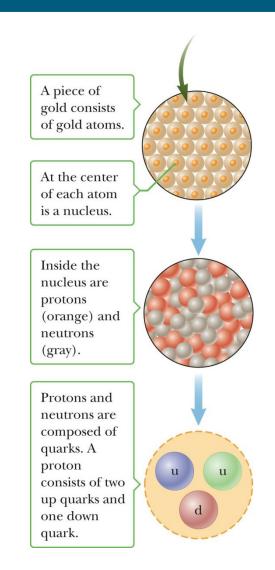


More structure of matter

- Atoms are made up of
 - Nucleus, very dense, contains
 - Protons, positively charged, "heavy"
 - Neutrons, no charge, about same mass as protons
 - Protons and neutrons are made up of quarks
 - Orbited by
 - Electrons, negatively charges, "light"
 - Fundamental particle, no structure

Structure of Matter





Dimensional Analysis

Technique to check correctness of an equation or to assist in deriving an equation.

Dimensions (length, mass, time, combinations) can be treated as algebraic quantities.

Any relationship (i.e. an equation) can be correct **only** if the dimensions (and units) on both sides of the equation are the same.

You can **only** add or subtract terms that have the same dimensions (and units), but you **can** multiply or divide terms that have different dimensions.

ay

Dimensional Analysis, cont.

- Cannot give numerical factors: this is its limitation
- Dimensions of some common quantities are listed in Table 1.5
- Allows a check for calculations which can show up in the units

Dimensions

• Table 1.5 Dimensions and Some Units of Area, Volume, Velocity, and Acceleration

System	Area (L ²)	Volume (L ³)	Velocity (L/T)	Acceleration (L/T²)
SI	m^2	m^3	m/s	m/s^2
cgs	cm^2	cm^3	cm/s	cm/s^2
U.S. customary	ft^2	ft^3	ft/s	ft/s^2

$$\lfloor v \rfloor = L/T$$

$$[x] = L$$
 $[t] = T$ $[v] = L/T$ $[a] = L/T^2$

$$[v] = \frac{L}{T} = \frac{L}{T^2} \cdot T = [a][t] \qquad v = at$$

$$[x] = L = L \cdot \frac{T}{T} = \frac{L}{T} \cdot T = [v][t] = [a][t]^2$$
 $x = \frac{1}{2}at^2$

Dimensional Analysis

What is the equation for area of a circle?

•
$$A=2\pi r$$

LHS = area, so must be Length² e.g. m²
RHS = 2 π (Length) e.g. m

Dimensions do not match. Equation is not dimensionally correct.

•
$$A = \pi r^2$$

RHS = 4π (Length²) = m² e.g. m²

This equation is dimensionally correct.

 An equation involving volume MUST include the Length three times (to get m³).

Dimensional Analysis

Can these equations be correct?

$$V_f = V_i \left(\frac{M_1}{M_1 + M_2} \right)$$

$$A = \frac{v_i^2 \sin^2 \theta_i}{2g}$$

$$V = v = \text{m/s}$$
 $M = \text{kg}$ $A = \text{m}$ $g = \text{m/s}^2$

- There is uncertainty in every measurement, this uncertainty carries over through the calculations
 - Need a technique to account for this uncertainty
- We will use rules for significant figures to approximate the uncertainty in results of calculations

There is uncertainty in every measurement – this uncertainty carries over through the calculations.

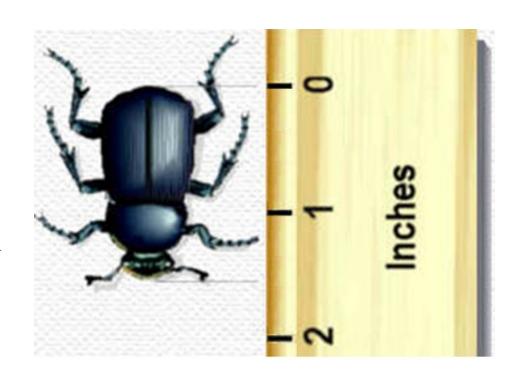
- May be due to the apparatus, the experimenter, and/or the number of measurements made
- Need a technique to account for this uncertainty

We will use rules for significant figures to approximate the uncertainty in results of calculations.

What is the size of the beetle?

- a. Between 0 and 2 in
- b. Between 1 and 2 in
- c. Between 1.5 and 1.6 in
- d. Between 1.54 and 1.56 in
- e. Between 1.546 and 1.547 in

We can estimate within the closest two marks



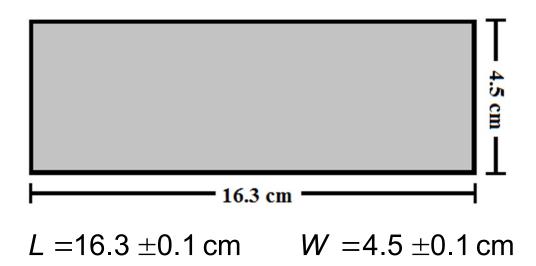
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We can estimate within the closest two marks

Better instrument yields more significant figures.

Uncertainty in Measurement and Significant Figures (1 of 6)



$$L \times W = (16.3 \text{ cm})(4.5 \text{ cm}) = 73.35 \text{ cm}^2 \rightarrow 73 \text{ cm}^2$$

Significant Figures

- A significant figure is a reliably known digit
- All non-zero digits are significant
- Zeros are not significant when they only locate the decimal point
 - Using scientific notion to indicate the number of significant figures removes ambiguity when the possibility of misinterpretation is present

- $0.03 \rightarrow 1$ significant figure
- $0.007 5 \rightarrow 2$ significant figure
- 1500 g \rightarrow ? significant figures
- $1.500 \times 10^3 \text{ g} \rightarrow 4 \text{ significant figures}$
- $1.50 \times 10^3 \text{ g} \rightarrow 3 \text{ significant figures}$
- $1.5 \times 10^2 \text{ g} \rightarrow 2 \text{ significant figures}$

Uncertainty in Measurement and Significant Figures (4 of 6)

 $3.00 \rightarrow 3$ significant figures

 $30.0 \rightarrow 3$ significant figures

300. → 3 significant figures

300 → 1 significant figure

Operations with Significant Figures

- When multiplying or dividing two or more quantities, the number of significant figures in the final result is the same as the number of significant figures in the least accurate of the factors being combined
 - Least accurate means having the lowest number of significant figures
- When adding or subtracting, round the result to the smallest number of decimal places of any term in the sum (or difference)

Uncertainty in Measurement and Significant Figures

Addition:

$$123 + 5.35 = 128.35$$

- → 123 has zero decimal places → sum =128
 - Subtraction:

$$1.002 - 0.998 = 0.004$$

Uncertainty in Measurement and Significant Figures (6 of 6)

Calculate

$$2.35 \times 5.89 \div 1.57$$

• Method 1:

$$2.35 \times 5.89 = 13.842 \rightarrow 13.8$$

$$13.8 \div 1.57 = 8.789 \ 8 \rightarrow |8.79|$$

Method 2:

$$5.89 \div 1.57 = 3.7516 \rightarrow 3.75$$

$$2.35 \times 3.75 = 8.8125 \rightarrow \boxed{8.81}$$

Method 3:

$$2.35 \div 1.57 = 1.496 \ 8 \rightarrow 1.50$$

$$1.50 \times 5.89 = 8.835 \rightarrow \boxed{8.84}$$

13.84

3.752

1.497

13.842

3.7516

1.4968

It is a good idea to keep extra significant figures until the end

Do all calculations with at least 4 but preferably 5 digits. Then round the final answer to three significant figures. Do not round to less than three significant figures, and not more than four.

Rounding

- Calculators will generally report many more digits than are significant
 - Be sure to properly round your results at the end
- Slight discrepancies may be introduced by both the rounding process and the algebraic order in which the steps are carried out
 - Minor discrepancies are to be expected and are not a problem in the problem-solving process
- In experimental work, more rigorous methods would be needed

Significant Figures

$$A = \pi r^2 = \pi (6.0 \text{ cm})^2 = 1.1 \times 10^2 \text{ cm}^2$$

calculator answer: 113.0973355



Conversions

- When units are not consistent, you may need to convert to appropriate ones
- See the inside of the front cover for an extensive list of conversion factors
- Units can be treated like algebraic quantities that can "cancel" each other
- Example:

$$15.0 in \times \frac{2.54 cm}{1 in} = 38.1 cm$$

Chain-link Units Conversions

Examples:

• 1.3 km
$$\times \frac{(1000 \text{ m})}{(1 \text{km})} = 1300 \text{ m} = 1.3 \times 10^3 \text{ m}$$

$$0.8 \text{ km} \times \frac{\text{(1000 m)}}{\text{(1 km)}} \times \frac{\text{(100 cm)}}{\text{(1 m)}} = 80000 \text{ cm}$$

$$= 8 \times 10^4 \text{ cm}$$

•
$$2845 \text{ mm} \times \frac{\text{(1 m)}}{\text{(1000 mm)}} \times \frac{\text{(3.281 ft)}}{\text{(1 m)}} = 9.334 \text{ ft}$$

Example: Convert 60 mile/hour to m/s.

$$60 \ \frac{mile}{hour} = 60 \frac{mile}{hour} \ x \ \frac{1609 \ meter}{1 \ mile} \ x \ \frac{1 \ hour}{3600 \ second} = 26.817 \ \frac{m}{s} = 26.8 \ \frac{m}{s}$$

Example: Convert 9.8 $\frac{m}{s^2}$ to $\frac{mile}{hour^2}$

$$9.8 \frac{m}{s^2} = 9.8 \frac{m}{s^2} x \frac{1 \, mile}{1609 \, m} x \frac{3600 \, s}{1 \, hour} x \frac{3600 \, s}{1 \, hour} = 78935 \frac{mile}{hour^2} = 78900 \frac{mile}{hour^2}$$

Estimates

- Can yield useful approximate answers
 - An exact answer may be difficult or impossible
 - Mathematical reasons
 - Limited information available
- Can serve as a partial check for exact calculations
- Eg: how many meters is the door?

Order of Magnitude

- Approximation based on a number of assumptions
 - May need to modify assumptions if more precise results are needed
- Order of magnitude is the power of 10 that applies

Coordinate Systems

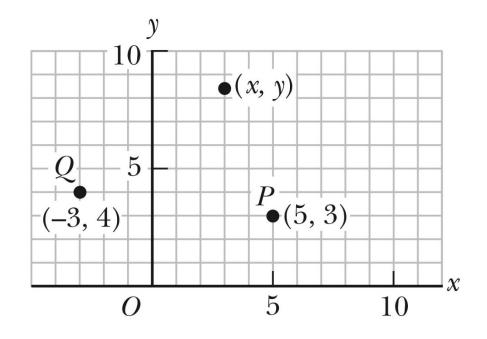
- Used to describe the position of a point in space
- Coordinate system consists of
 - A fixed reference point called the origin, O
 - Specified axes with scales and labels
 - Instructions on how to label a point relative to the origin and the axes

Types of Coordinate Systems

- Cartesian (rectangular)
- Plane polar

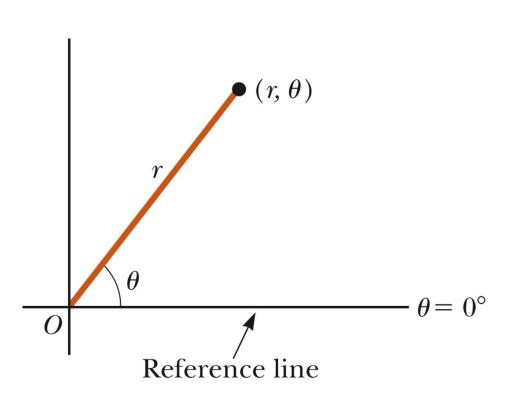
Cartesian coordinate system

- x- and y- axes
- Points are labeled (x,y)
- Positive x is usually selected to be to the right of the origin
- Positive y is usually selected to be to upward from the origin



Plane polar coordinate system

- Origin and reference line are noted
- Point is distance r from the origin in the direction of angle θ
- Positive angles are measured ccw from reference line
- Points are labeled (r,θ)
- The standard reference line is usually selected to be the positive x axis

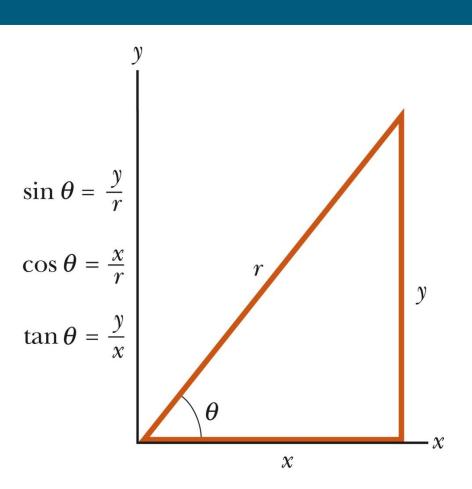


Trigonometry Review

$$\sin \theta = \frac{opposite \ side}{hypotenuse}$$

$$\cos \theta = \frac{adjacent \ side}{hypotenuse}$$

$$\tan \theta = \frac{opposite \ side}{adjacent \ side}$$



Trigonometry Review (2 of 2)

• Example:

$$\sin \theta = 0.866 \rightarrow \text{ What is } \theta ?$$

 $\sin^{-1}(0.866) = \theta = 60.0^{\circ}$

Example:

$$tan^{-1}(0.400) = \theta = 21.8^{\circ}$$

More Trigonometry

- Pythagorean Theorem
 - $r^2 = x^2 + y^2$
- To find an angle, you need the inverse trig function
 - For example, $\theta = \sin^{-1} 0.707 = 45^{\circ}$

Degrees vs. Radians

- Be sure your calculator is set for the appropriate angular units for the problem, i.e. degrees or radians.
- For example:
 - $tan^{-1} 0.5774 = 30.0^{\circ}$
 - $tan^{-1} 0.5774 = 0.5236 rad$

Rectangular ⇔ Polar

Rectangular to polar

$$r^{2} = x^{2} + y^{2}$$
$$\theta = tan^{-1} (y/x)$$

Polar to rectangular

$$x = r \cos \theta$$

$$y = r \sin \theta$$

Vector vs. Scalar Review

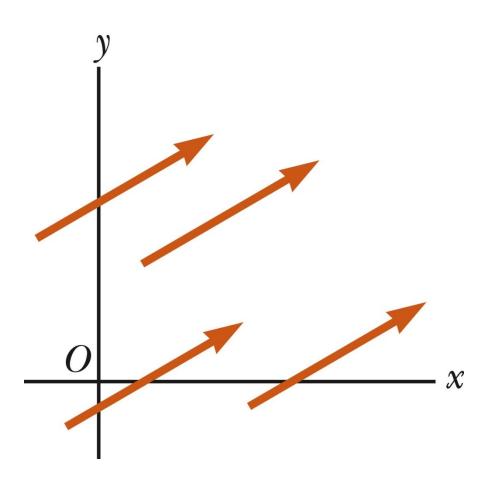
- All physical quantities encountered in this text will be either a scalar or a vector
- A vector quantity has both magnitude (size) and direction
- A scalar is completely specified by only a magnitude (size)

Vector Notation

- When handwritten, use an arrow: \vec{A}
- When printed, will be in bold print with an arrow: \(\overline{\Omega} \)
- When dealing with just the magnitude of a vector in print, an italic letter will be used: A
 - Italics will also be used to represent scalars

Properties of Vectors

- Equality of Two Vectors
 - Two vectors are equal if they have the same magnitude and the same direction
- Movement of vectors in a diagram
 - Any vector can be moved parallel to itself without being affected



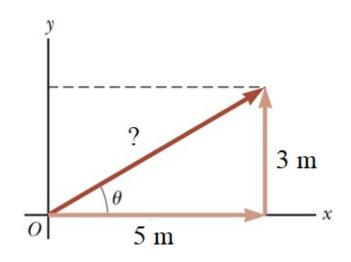
Adding Vectors

- When adding vectors, their directions must be taken into account
- Units must be the same
- Geometric Methods
 - Use scale drawings
- Algebraic Methods
- The resultant vector (sum) is denoted as ${f R}$

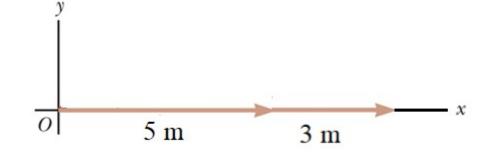
$$\vec{R} = \vec{A} + \vec{B}$$

Adding vectors

Is
$$R = 5 + 3 = 8m$$
?



Is
$$R = 5 + 3 = 8m$$
?

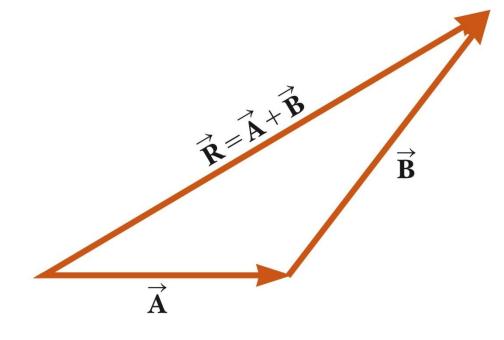


Adding Vectors Geometrically (Triangle or Polygon Method)

- Choose a scale
- Draw the first vector with the appropriate length and in the direction specified, with respect to a coordinate system
- Draw the next vector using the same scale with the appropriate length and in the direction specified, with respect to a coordinate system whose origin is the end of the first vector and parallel to the ordinate system used for the first vector

Graphically Adding Vectors, cont.

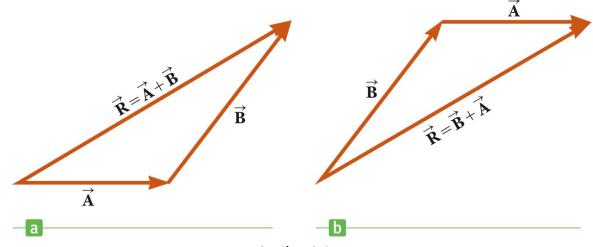
- Continue drawing the vectors "tip-to-tail"
- The resultant is drawn from the origin of the first vector to the end of the last vector
- Measure the length of the resultant and its angle
 - Use the scale factor to convert length to actual magnitude
- This method is called the triangle method



Notes about Vector Addition

- Vectors obey the Commutative Law of Addition
 - The order in which the vectors are added doesn't affect the result

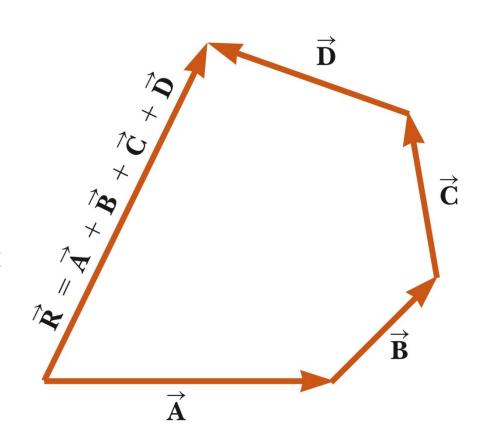
$$\vec{A} + \vec{B} = \vec{B} + \vec{A}$$



Section 3.1

Graphically Adding Vectors, cont.

- When you have many vectors, just keep repeating the "tip-to-tail" process until all are included
- The resultant is still drawn from the origin of the first vector to the end of the last vector



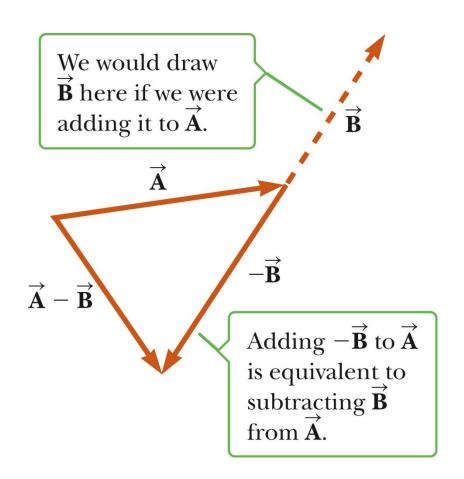
More Properties of Vectors

- Negative Vectors
 - The negative of the vector is defined as the vector that gives zero when added to the original vector
 - Two vectors are negative if they have the same magnitude but are 180° apart (opposite directions)

$$\vec{\mathbf{A}} + (-\vec{\mathbf{A}}) = 0$$

Vector Subtraction

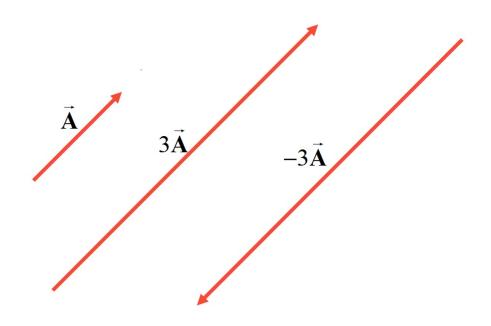
- Special case of vector addition
 - Add the negative of the subtracted vector
- $\vec{A} \vec{B} = \vec{A} + (-\vec{B})$
- Continue with standard vector addition procedure



Multiplying or Dividing a Vector by a Scalar

- The result of the multiplication or division is a vector
- The magnitude of the vector is multiplied or divided by the scalar
- If the scalar is positive, the direction of the result is the same as of the original vector
- If the scalar is negative, the direction of the result is opposite that of the original vector

Multiplying or Dividing a Vector by a Scalar

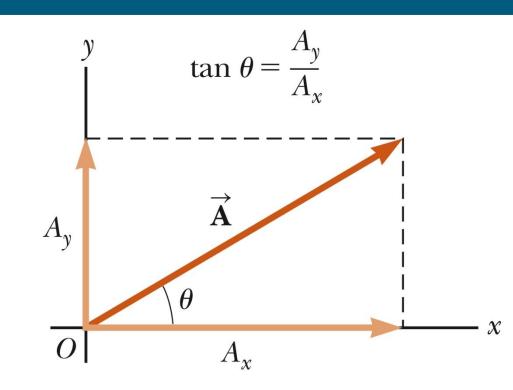




Components of a Vector

It is useful to use rectangular components to add vectors

These are the projections of the vector along the x- and y-axes



$$A_{r} = A \cos \theta$$

$$A_{v} = A \sin \theta$$

$$\vec{A} = \vec{A}_x + \vec{A}_y$$

More About Components of a Vector

- The previous equations are valid only if Θ is measured with respect to the x-axis
- The components can be positive or negative and will have the same units as the original vector

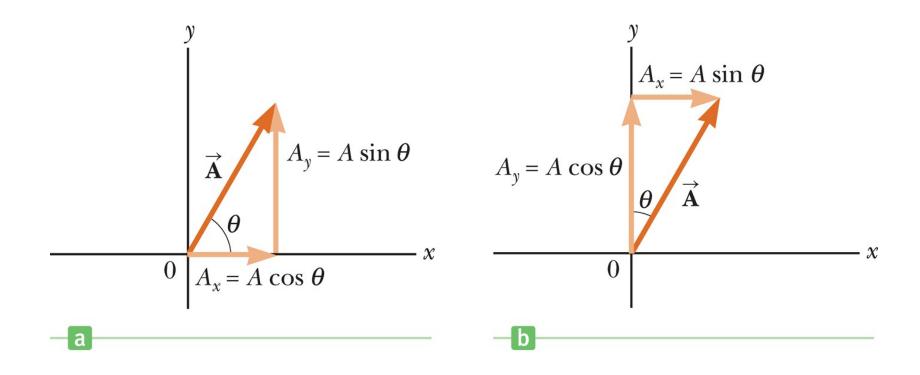
More About Components, cont.

 The components are the legs of the right triangle whose hypotenuse is A

$$A = \sqrt{A_x^2 + A_y^2}$$
 and $\theta = \tan^{-1} \left(\frac{A_y}{A_x}\right)$

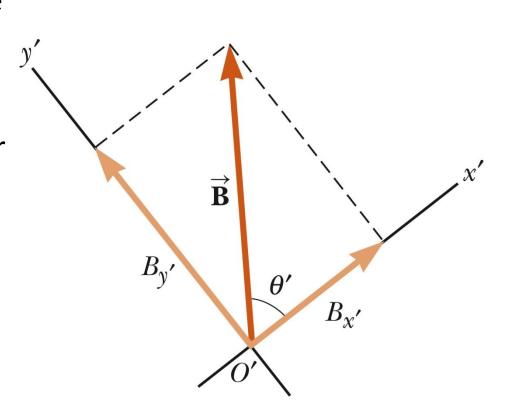
- May still have to find θ with respect to the positive x-axis
- The value will be correct only if the angle lies in the first or fourth quadrant
- In the second or third quadrant, add 180°

Components of a Vector (4 of 4)



Other Coordinate Systems

- It may be convenient to use a coordinate system other than horizontal and vertical
- Choose axes that are perpendicular to each other
- Adjust the components accordingly



Vector Addition using Components

$$R = A + B$$

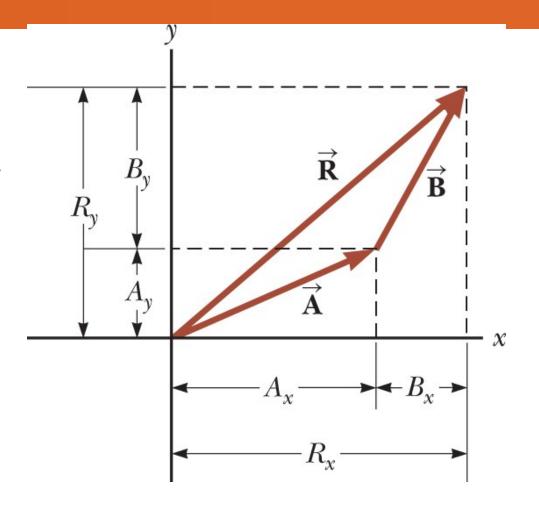
$$R_x + R_y = A_x + A_y + B_x + B_y$$

$$R_x = A_x + B_x$$

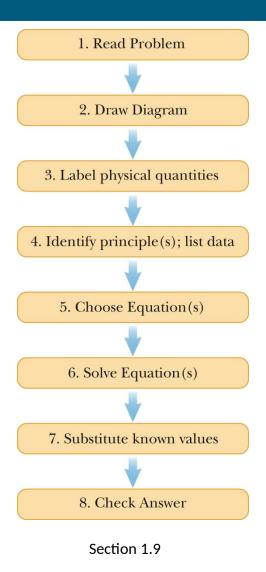
$$R_y = A_y + B_y$$

$$R = \sqrt{R_x^2 + R_y^2}$$

$$\tan \theta = \frac{R_y}{R_x} = \frac{A_y + B_y}{A_x + B_x}$$



Problem Solving Strategy



Problem Solving Strategy

- Problem
 - Read the problem
 - Read at least twice
 - Identify the nature of the problem
 - Draw a diagram
 - Some types of problems require very specific types of diagrams

Problem Solving cont.

- Problem, cont.
 - Label the physical quantities
 - Can label on the diagram
 - Use letters that remind you of the quantity
 - Many quantities have specific letters
 - Choose a coordinate system and label it

Strategy

- Identify principles and list data
 - Identify the principle involved
 - List the known(s) (given information)
 - Indicate the unknown(s) (what you are looking for)
 - May want to circle the unknowns

Problem Solving, cont.

- Strategy, cont.
 - Choose equation(s)
 - Based on the principle, choose an equation or set of equations to apply to the problem
- Solution
 - Solve for the unknown quantity
 - Substitute into the equation(s)
 - Substitute the data into the equation
 - Obtain a result
 - Include units

Problem Solving, final

- Check
 - Check the answer
 - Do the units match?
 - Are the units correct for the quantity being found?
 - Does the answer seem reasonable?
 - Check order of magnitude
 - Are signs appropriate and meaningful?

Problem Solving Summary

- Equations are the tools of physics
 - Understand what the equations mean and how to use them
- Carry through the algebra as far as possible
 - Substitute numbers at the end
- Be organized