Sec.3.4

p.191 - 193: Definition of Concavity; Theorem 3.7; Examples 1 & 2 f''

p.193 – 194: Definition of Point of Inflection; Theorem 3.8; Example 3; Figure3.29

P.O.I $f''=0$ or $f''=DNE$

p.195: Theorem 3.9 - The Second Derivative Test; Figure3.30; Example 4

Sec3.3: $f \uparrow I \to f' \oplus$; $f \downarrow D \to f' \ominus$

down, up

f upward $\to f' \uparrow I \to f'' \oplus$

f downward $\to f' \downarrow D \to f'' \ominus$

p.196: In Exercises 17–32, find the points of inflection and discuss the concavity of the graph of the function.

20. $f(x) = 4 - x - 3x^4$

$f'(x) = -1 - 12x^3,\ f''(x) = -36x^2 \ \to\ let\ f''(x) = 0{:}\ -36x^2 = 0 \ \to\ x = 0\ \ a\ possible\ POI$

x	$f''(x)$	Conclusion
$(-\infty, 0)$	$-$	Concave downward ✓
$x = 0$		Not a POI
$(0, \infty)$	$-$	Concave downward ✓

f'': $-\infty$, -1 $\ominus$, 0, 1 $\ominus$, ∞

p.196: In Exercises 33–44, find all relative extrema of the function. Use the Second Derivative Test where applicable.

36. $f(x) = -x^3 + 7x^2 - 15x$

$f'(x) = -3x^2 + 14x - 15, \qquad f''(x) = -6x + 14$

$let\ f'(x) = 0{:}\ \ -3x^2 + 14x - 15 = 0 \ \to\ 3x^2 - 14x + 15 = 0$

$3x \quad -5$
$x \quad -3$

$$(3x - 5)(x - 3) = 0 \ \to\ x = \frac{5}{3}, 3 \qquad C.\#'s$$

$3x-5=0 \to x = 5/3$
$x-3=0 \to x=3$

x	$f(x)$	$f''(x)$	Conclusion
$x = 3$	-9	$-$ ✓	$R.Max\ @(3, -9)$ ✓
$x = \frac{5}{3}$	$-\frac{275}{27}$	$+$ ✓	$R.Min\ @\ (\frac{5}{3}, -\frac{275}{27})$ ✓

$f''(\frac{5}{3}) = -6(\frac{5}{3}) + 14 = -10 + 14 = 4 > 0$ R min

$f(\frac{5}{3}) = -(\frac{5}{3})^3 + 7(\frac{5}{3})^2 - 15(\frac{5}{3}) = -\frac{275}{27}$

$f''(3) = -6(3) + 14 = -18 + 14 = -4 < 0$ R.max

$f(3) = -3^3 + 7(3^2) - 15(3) = -9$

$\sin 2x = 2\sin x \cos x$

44. $f(x) = 2\sin x + \cos 2x, \quad [0, 2\pi]$

$f'(x) = 2cosx - 2sin2x, \quad f''(x) = -2sinx - 4cos2x$

$let\ f'(x) = 0{:}\ \ 2cosx - 2sin2x = 0 \ \to 2cosx - 4sinxcosx = 0 \to$

$2cosx(1 - 2sinx) = 0 \ \to\ cosx = 0\ \ or\ \ 1 - 2sinx = 0 \ \leftrightarrow\ sinx = \frac{1}{2}$

$$x = \frac{\pi}{2}, \frac{3\pi}{2}\ \ or\ \ x = \frac{\pi}{6}, \frac{5\pi}{6} \ \to\ x = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{3\pi}{2} \qquad C.\#'s$$

$\pi/2$, $\pi/6$, $5\pi/6$, $3\pi/2$

$\cos\frac{\pi}{3}$

x	$f(x)$	$f''(x)$	Conclusion
$x = \frac{\pi}{6}$	$\frac{3}{2}$	$-$ ✓	$R.Max\ @\ (\frac{\pi}{6}, \frac{3}{2})$ ✓
$x = \frac{\pi}{2}$	1	$+$ ✓	$R.Min\ @\ (\frac{\pi}{2}, 1)$ ✓
$x = \frac{5\pi}{6}$	$\frac{3}{2}$	$-$ ✓	$R.Max\ @\ (\frac{5\pi}{6}, \frac{3}{2})$ ✓
$x = \frac{3\pi}{2}$	-3	$+$ ✓	$R.Min\ @\ (\frac{3\pi}{2}, -3)$ ✓

1) $f''(\frac{\pi}{6}) = -2\sin\frac{\pi}{6} - 4\cos(2\times\frac{\pi}{6})$

$= -2(\frac{1}{2}) - 4(\frac{1}{2}) = -1 - 2 = -3 < 0$ R.max

$f(\frac{\pi}{6}) = 2\sin\frac{\pi}{6} + \cos\frac{\pi}{3}$

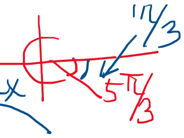
$= 2(\frac{1}{2}) + \frac{1}{2} = \frac{3}{2}$

2) $f''(\frac{\pi}{2}) = -2\sin\frac{\pi}{2} - 4\cos\pi = -2(1) - 4(-1) = -2 + 4 > 0$ R.min

$f(\frac{\pi}{2}) = 2\sin\frac{\pi}{2} + \cos\pi = 2(1) + (-1) = 2 - 1 = 1$

$\pi/3$, $5\pi/6$, Rmax

$$f\left(\frac{\pi}{2}\right) = 2\sin\frac{\pi}{2} + \cos\pi = 2(1) + (-1) = 2 - 1 = 1$$



3) $$f''\left(\frac{5\pi}{6}\right) = -2\sin\frac{5\pi}{6} - 4\cos\frac{5\pi}{3} = -2\left(\frac{1}{2}\right) - 4\left(\frac{1}{2}\right) = -1 - 2 < 0$$ Rmax

$$f\left(\frac{5\pi}{6}\right) = 2\sin\frac{5\pi}{6} + \cos\frac{5\pi}{3} = 2\left(\frac{1}{2}\right) + \frac{1}{2} = \frac{3}{2}$$

4) $$f''\left(\frac{3\pi}{2}\right) = -2\sin\frac{3\pi}{2} - 4\cos 3\pi = -2(-1) - 4(-1) = 2 + 4 > 0$$ Rmin

$$f\left(\frac{3\pi}{2}\right) = 2\sin\frac{3\pi}{2} + \cos 3\pi = 2(-1) + (-1) = -3$$