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Chapter Two

Motion in One Dimension

Topics Chapter 2

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Dynamics

- The branch of physics involving the motion of an object and the relationship between that motion and other physics concepts
- ***Kinematics*** is a part of dynamics
 - In kinematics, you are interested in the *description* of motion
 - *Not* concerned with the cause of the motion

Quantities in Motion

- Any motion involves three concepts
 - Displacement
 - Velocity
 - Acceleration
- These concepts can be used to study objects in motion

Position

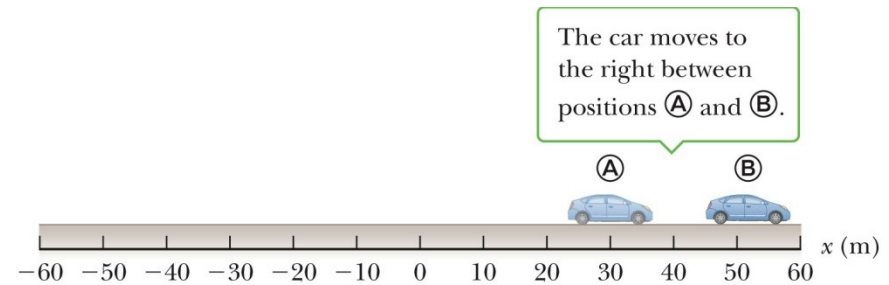
- Defined in terms of a **frame of reference**
 - A choice of coordinate axes
 - Defines a starting point for measuring the motion
 - Or any other quantity
 - One dimensional, so generally the x- or y-axis

Displacement

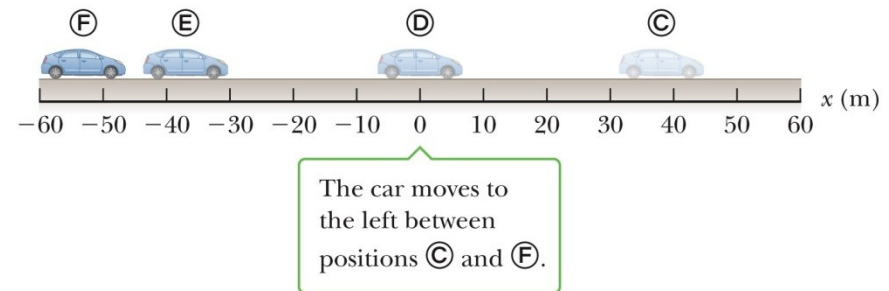
- Defined as the *change in position*
 - $\Delta x \equiv x_f - x_i$
 - f stands for final and i stands for initial
 - Units are meters (m) in SI

Displacement Examples

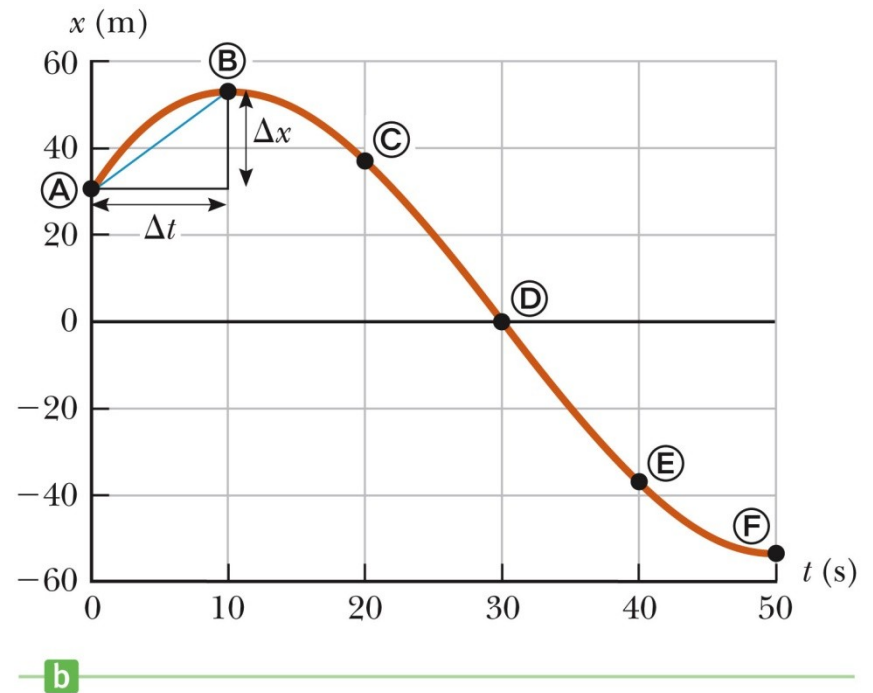
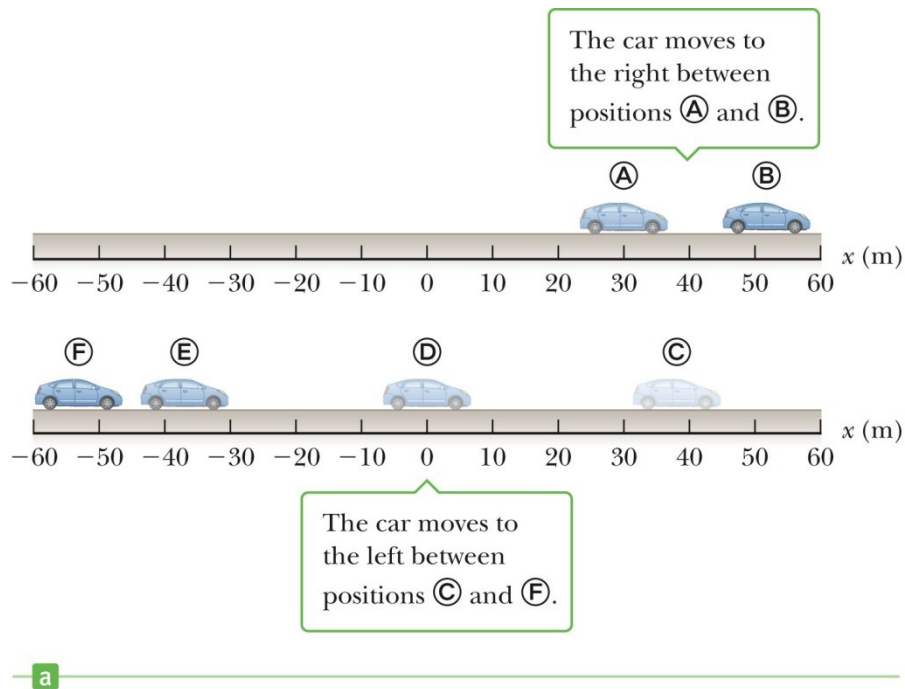
- From A to B
 - $x_i = 30 \text{ m}$
 - $x_f = 52 \text{ m}$
 - $\Delta x = 22 \text{ m}$
 - The displacement is positive, indicating the motion was in the positive x direction



- From C to F
 - $x_i = 38 \text{ m}$
 - $x_f = -53 \text{ m}$
 - $\Delta x = -91 \text{ m}$
 - The displacement is negative, indicating the motion was in the negative x direction



Displacement, Graphical



Vector and Scalar Quantities

- Vector quantities need both magnitude (size) and direction to completely describe them
 - Generally denoted by boldfaced type and an arrow over the letter
 - + or – sign is sufficient for this chapter
- Scalar quantities are completely described by magnitude only

Displacement Isn't Distance

- The displacement of an object is not the same as the distance it travels
 - Example: Throw a ball straight up and then catch it at the same point you released it
 - The distance is twice the height
 - The displacement is zero

Speed

- The **average speed** of an object is defined as the total distance traveled divided by the total time elapsed

$$\text{Average speed} = \frac{\text{path length}}{\text{elapsed time}}$$

$$v = \frac{d}{t}$$

- Speed is a scalar quantity

Speed, cont

- Average speed totally ignores any variations in the object's actual motion during the trip
- The path length and the total time are all that is important
 - Both will be positive, so speed will be positive
- SI units are m/s

Path Length vs. Distance

- Displacement depends only on the endpoints

$$\Delta s = \sqrt{(x_f - x_i)^2 + (y_f - y_i)^2}$$

- The displacement does not depend on what happens between the endpoints
 - Is the magnitude of the displacement
- Path length will depend on the actual route taken

Velocity

- It takes time for an object to undergo a displacement
- The **average velocity** is rate at which the displacement occurs

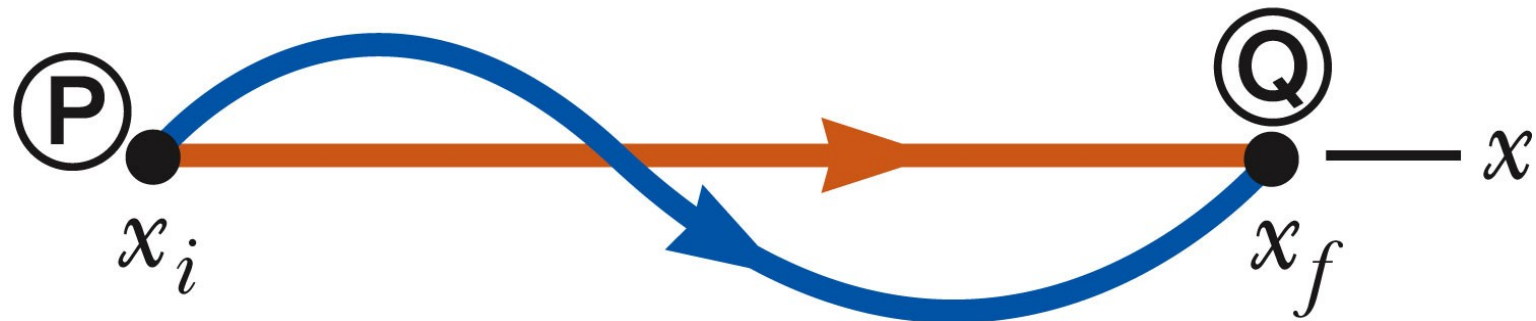
$$V_{average} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$$

- Velocity can be positive or negative
 - Δt is always positive
- Average speed is not the same as the average velocity

Velocity continued

- Direction will be the same as the direction of the displacement, + or - is sufficient in one-dimensional motion
- Units of velocity are m/s (SI)
 - Other units may be given in a problem, but generally will need to be converted to these
 - In other systems:
 - US Customary: ft/s
 - cgs: cm/s

Speed vs. Velocity

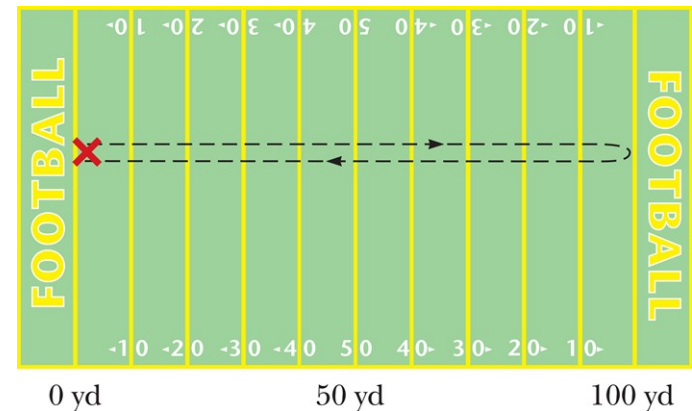


- Cars on both paths have the same average velocity since they had the same displacement in the same time interval
- The car on the blue path will have a greater average speed since the path length it traveled is larger

Think – Pair – Share 1

- The figure below shows the unusual path of a confused football player. After receiving a kickoff at his own goal, he runs downfield to within inches of a touchdown, then reverses direction and races back until he's tackled at the exact location where he first caught the ball. During this run, which took 25 s, what is the path length he travels?

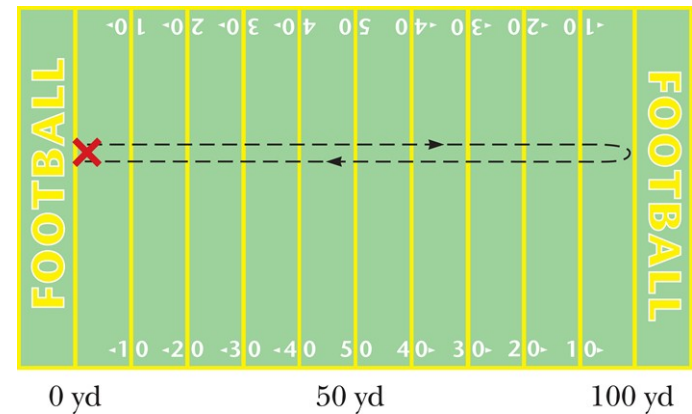
- 100 yd
- 200 yd
- 0.00 yd
- It is more than 100 yd, but you must know the exact path to say exactly



Think – Pair – Share 2

- What was the player's displacement?

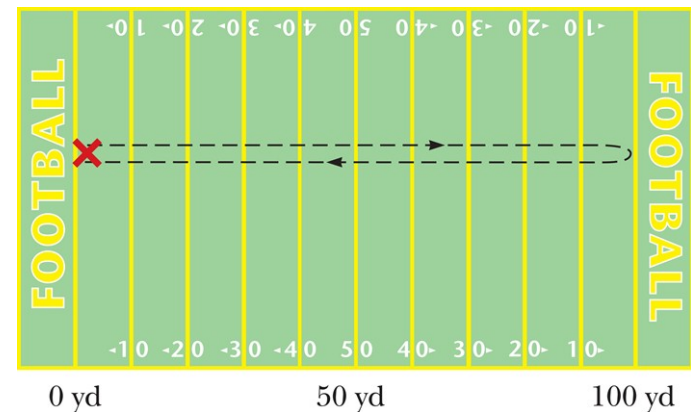
- 100 yd
- 200 yd
- 0.00 yd
- It is more than 100 yd, but you must know the exact path to say exactly.



Think – Pair – Share 3

- During the player's run, which took 25 s, what is his average velocity in the x-direction?

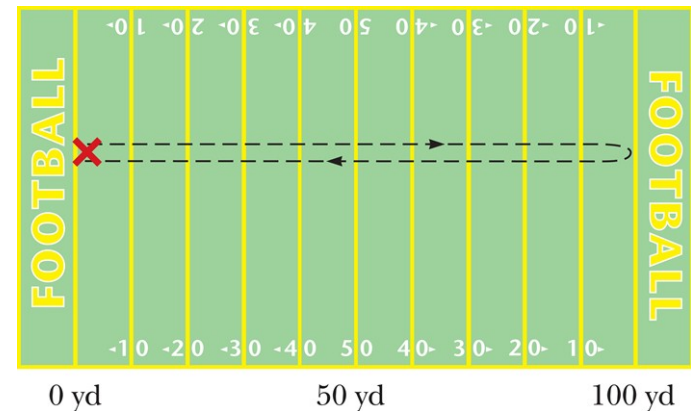
- 100 yd/s
- 25.0 yd/s
- 8.00 yd/s
- 0.00 yd/s



Think – Pair – Share 4

- During the player's run, which took 25 s, what is his average velocity in the x-direction?

- 100 yd/s
- 25.0 yd/s
- 8.00 yd/s
- 0.00 yd/s



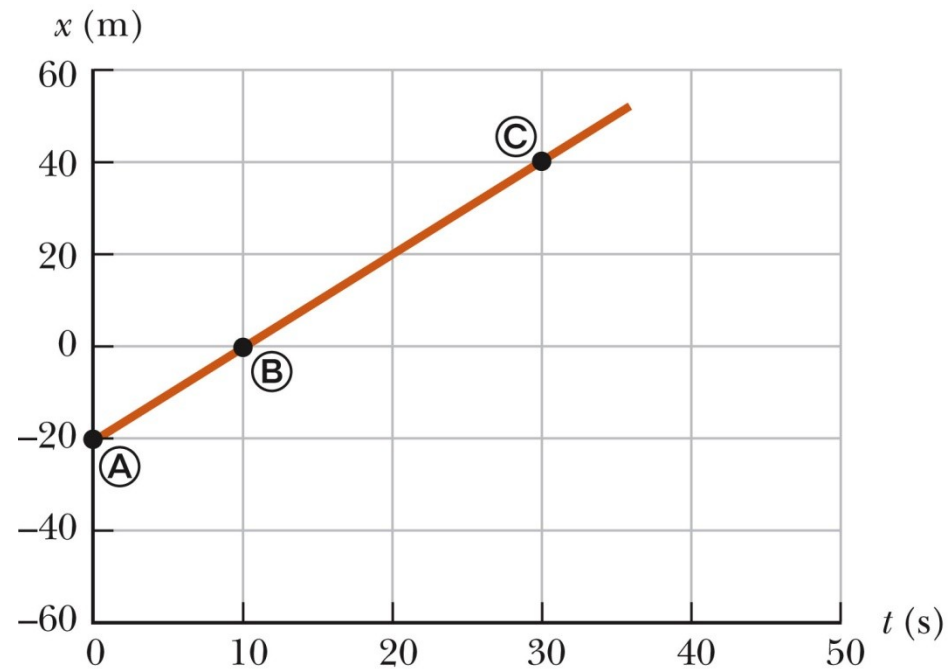
Graphical Interpretation of Velocity

- Velocity can be determined from a position-time graph
- Average velocity equals the slope of the line joining the initial and final points on the graph
- An object moving with a constant velocity will have a graph that is a straight line

Average Velocity, Constant

- The straight line indicates constant velocity
- The slope of the line is the value of the average velocity

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$$



Notes on Slopes

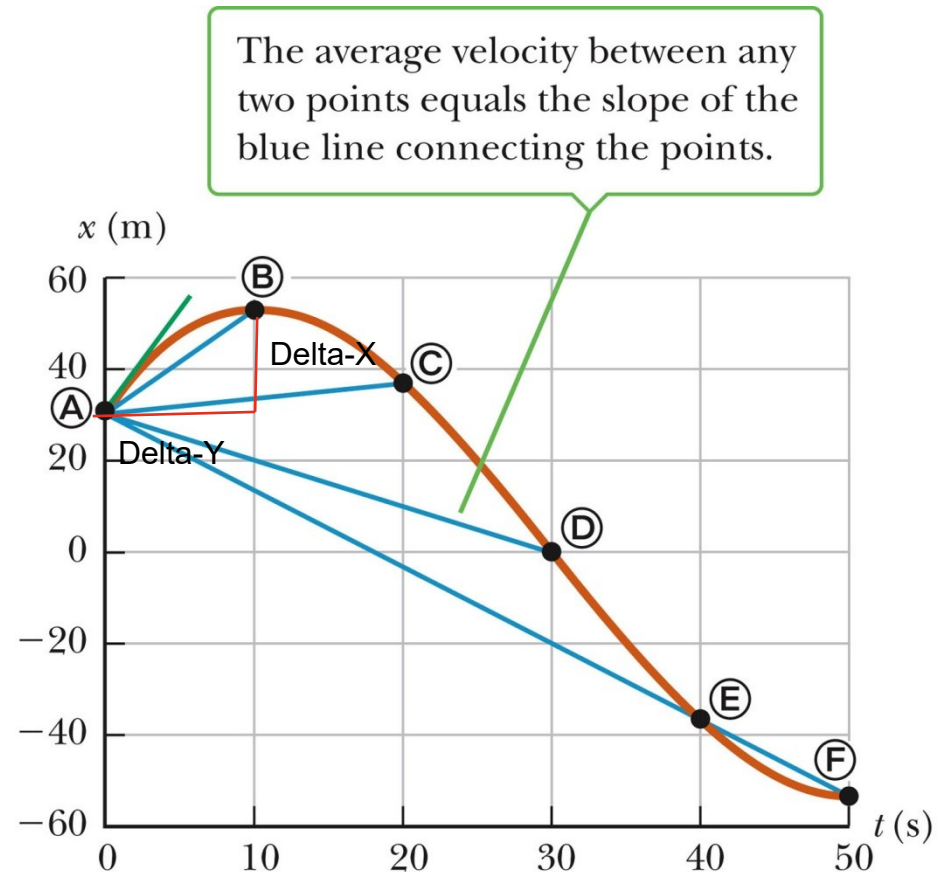
- The general equation for the slope of any line is

$$\text{slope} = \frac{\text{change in vertical axis}}{\text{change in horizontal axis}} \quad \text{AKA: Meters/Seconds}$$

- The meaning of a specific slope will depend on the physical data being graphed
- Slope carries units

Average Velocity, Non Constant

- The motion is non-constant velocity
- The average velocity is the slope of the straight line joining the initial and final points



Instantaneous Velocity (2 of 3)

- **Table 2.2** Positions of a Car at Specific Instants of Time

t (s)	x (m)
1.00	5.00
1.01	5.47
1.10	9.67
1.20	14.3
1.50	26.3
2.00	34.7
3.00	52.5

$t = 1.00$ s, $x = 5.00$ m

$t = 3.00$ s, $x = 52.5$ m

$$\frac{\Delta x}{\Delta t} = \frac{52.5 \text{ m} - 5.00 \text{ m}}{3.00 \text{ s} - 1.00 \text{ s}} = 23.8 \text{ m/s}$$

- **Table 2.3** Calculated Values of the Time Intervals, Displacements, and Average Velocities of the Car of Table 2.2

Time Interval (s)	Δt (s)	Δx (m)	\bar{v} (m/s)
1.00 to 3.00	2.00	47.5	23.8
1.00 to 2.00	1.00	29.7	29.7
1.00 to 1.50	0.50	21.3	42.6
1.00 to 1.20	0.20	9.30	46.5
1.00 to 1.10	0.10	4.67	46.7
1.00 to 1.01	0.01	0.470	47.0

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{0.470 \text{ m}}{0.0100 \text{ s}} = 47 \text{ m/s}$$

Instantaneous Velocity

- The limit of the average velocity as the time interval becomes infinitesimally short, or as the time interval approaches zero

$$v \equiv \lim_{\Delta t \rightarrow 0} \frac{\Delta x}{\Delta t}$$

- The instantaneous velocity indicates what is happening at every point of time
 - The magnitude of the instantaneous velocity is what you read on a car's speedometer

Instantaneous Velocity on a Graph

- The slope of the line tangent to the position vs. time graph is defined to be the
- instantaneous velocity at that time
 - The instantaneous speed is defined as the magnitude of the instantaneous velocity



Instantaneous Velocity (2 of 3)

Table 2.2 Positions of a Car at Specific Instants of Time

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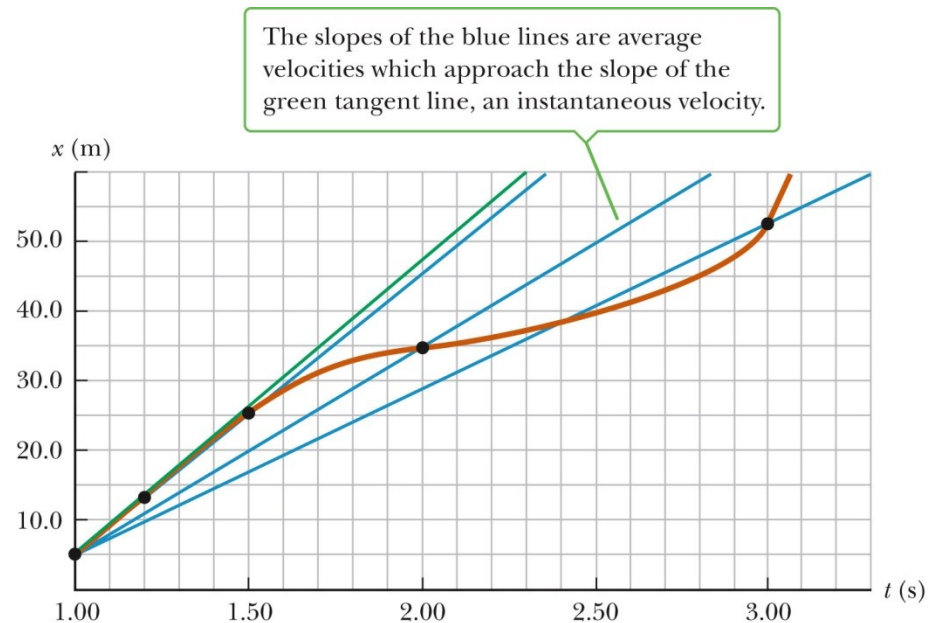
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$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{0.470 \text{ m}}{0.0100 \text{ s}} = 47 \text{ m/s}$$

Graphical Instantaneous Velocity

- Average velocities are the blue lines
- The green line (tangent) is the instantaneous velocity



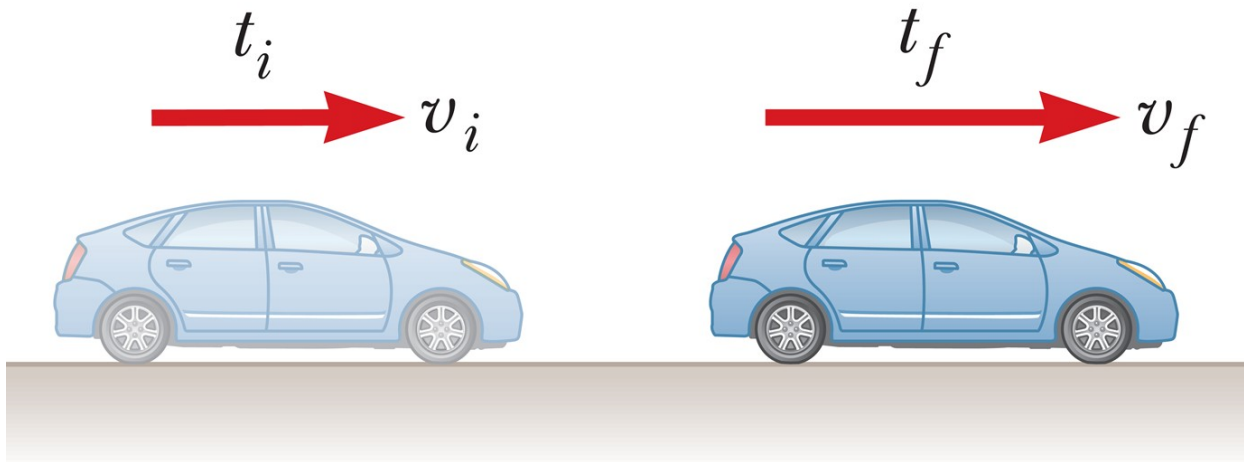
Acceleration

- Changing velocity means an acceleration is present
- Acceleration is the rate of change of the velocity

$$\vec{a} \equiv \frac{\Delta \mathbf{v}}{\Delta t} = \frac{v_f - v_i}{t_f - t_i}$$

- Units are m/s² (SI), cm/s² (cgs), and ft/s² (US Cust)

Average Acceleration (2 of 3)



$$\Delta v = v_f - v_i \qquad \Delta t = t_f - t_i$$

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{20 \text{ m/s} - 10 \text{ m/s}}{2 \text{ s}} = +5 \text{ m/s}^2$$

Average Acceleration

- Vector quantity
- When the object's velocity and acceleration are in the same direction (either positive or negative), then the speed of the object increases with time
- When the object's velocity and acceleration are in the opposite directions, the speed of the object decreases with time

Average Acceleration (3 of 3)

- velocity and acceleration in same direction → speed increases with time
- velocity and acceleration are in opposite directions → speed decreases with time.

$$v_i = -10 \text{ m/s} \quad v_f = -20 \text{ m/s} \quad \Delta t = 2 \text{ s}$$

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{-20 \text{ m/s} - (-10 \text{ m/s})}{2 \text{ s}} = -5 \text{ m/s}^2$$

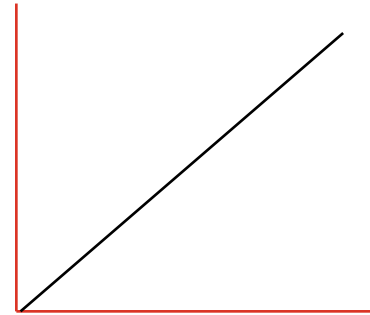
Negative Acceleration

- A negative acceleration does not necessarily mean the object is slowing down
- If the acceleration and velocity are both negative, the object is speeding up
- “Deceleration” means a decrease in speed, not a negative acceleration

Instantaneous and Uniform Acceleration

- The limit of the average acceleration as the time interval goes to zero

$$a \equiv \lim_{\Delta t \rightarrow 0} \frac{\Delta v}{\Delta t}$$



- When the instantaneous accelerations are always the same, the acceleration will be uniform

The instantaneous accelerations will all be equal to the average acceleration

Think – Pair – Share 5

- **True or False?** A car must always have an acceleration in the same direction as its velocity.



False

Think – Pair – Share 6

- **True or False?** It's possible for a slowing car to have a positive acceleration.

True

Think – Pair – Share 7

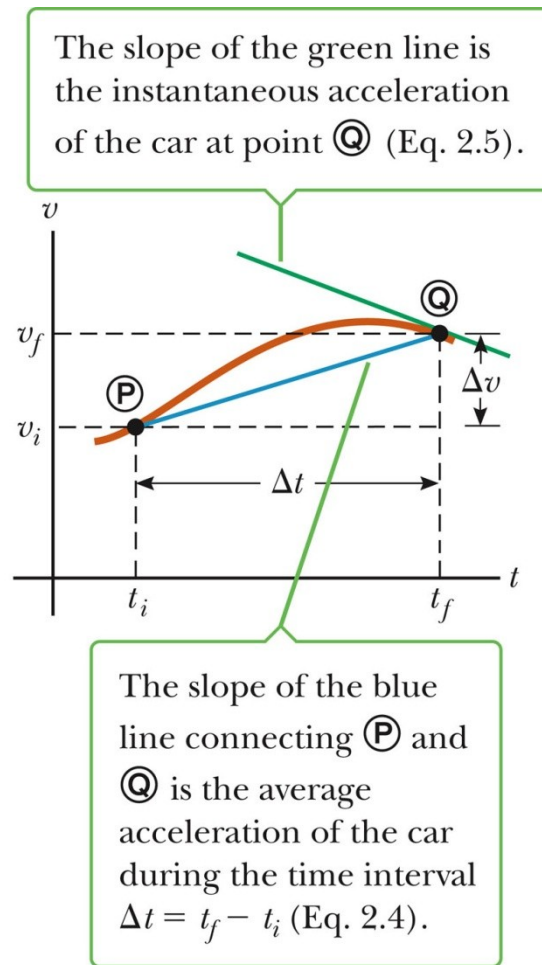
- **True or False?** An object with constant nonzero acceleration can never stop and remain at rest.

True, if acceleration remains constant, then even if the velocity reaches 0 it will continue into the opposite direction.

Graphical Interpretation of Acceleration

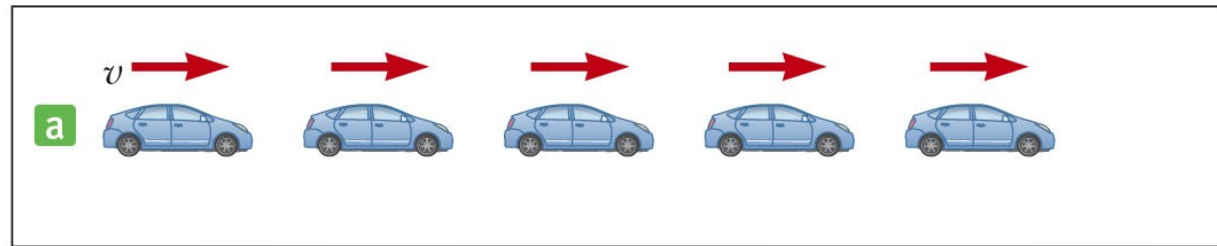
- Average acceleration is the slope of the line connecting the initial and final velocities on a velocity vs. time graph
- Instantaneous acceleration is the slope of the tangent to the curve of the velocity-time graph

Average Acceleration – Graphical Example



Relationship Between Acceleration and Velocity

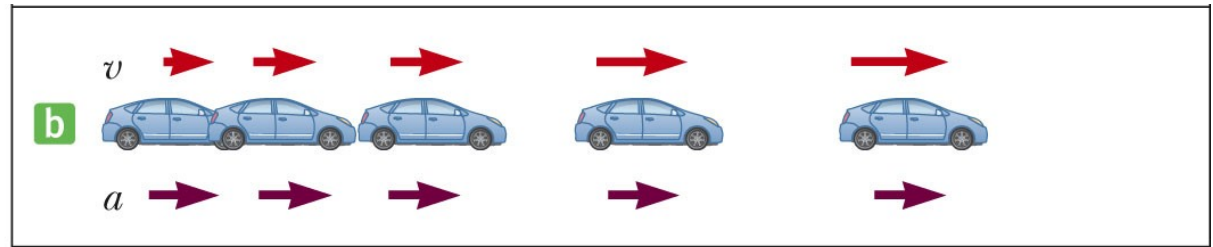
This car moves at constant velocity (zero acceleration).



- Uniform velocity (shown by red arrows maintaining the same size)
- Acceleration equals zero

Relationship Between Velocity and Acceleration

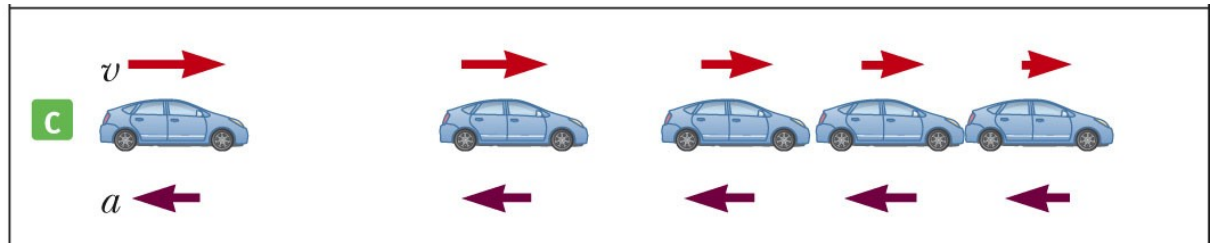
This car has a constant acceleration in the direction of its velocity.



- Velocity and acceleration are in the same direction
- Acceleration is uniform (violet arrows maintain the same length)
- Velocity is increasing (red arrows are getting longer)
- Positive velocity and positive acceleration

Relationship Between Velocity and Acceleration

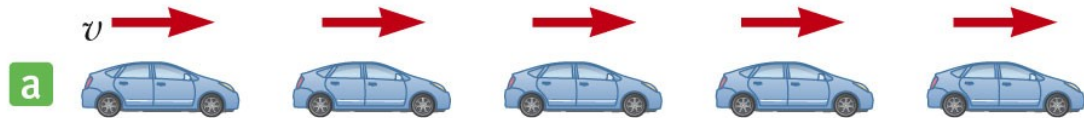
This car has a constant acceleration in the direction opposite its velocity.



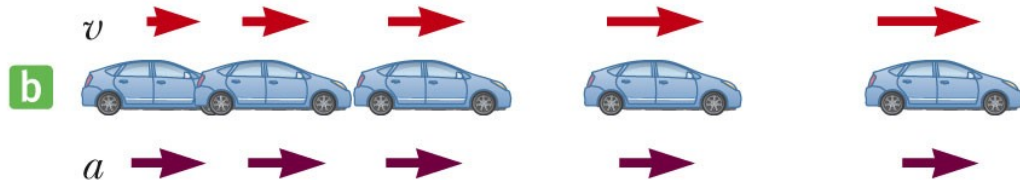
- Acceleration and velocity are in opposite directions
- Acceleration is uniform (violet arrows maintain the same length)
- Velocity is decreasing (red arrows are getting shorter)
- Velocity is positive and acceleration is negative

Motion Diagram Summary

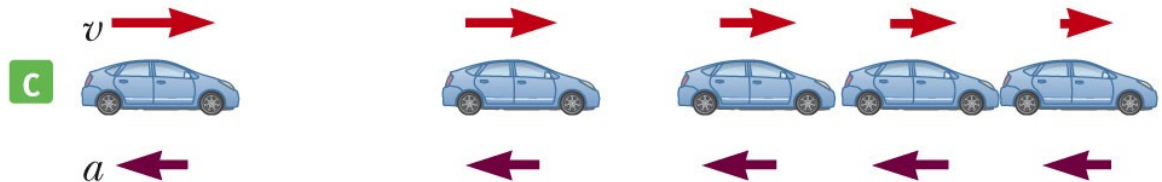
This car moves at constant velocity (zero acceleration).



This car has a constant acceleration in the direction of its velocity.



This car has a constant acceleration in the direction opposite its velocity.



Equations for Constant Acceleration

- These equations are used in situations with uniform acceleration

$$v = v_o + at$$

$$V_f = V_i + at$$

$$\Delta x = \bar{v}t = \frac{1}{2}(v_o + v)t$$

$$\Delta x = \frac{1}{2}(V_i + V_f)t$$

$$\Delta x = v_o t + \frac{1}{2}at^2$$

$$\Delta x = V_i t + \frac{1}{2}at^2$$

$$v^2 = v_o^2 + 2a\Delta x$$

$$V_f^2 = V_i^2 + 2a \Delta x$$

Notes on the equations

$$\Delta x = v_{average} t = \left(\frac{v_o + v}{2} \right) t$$

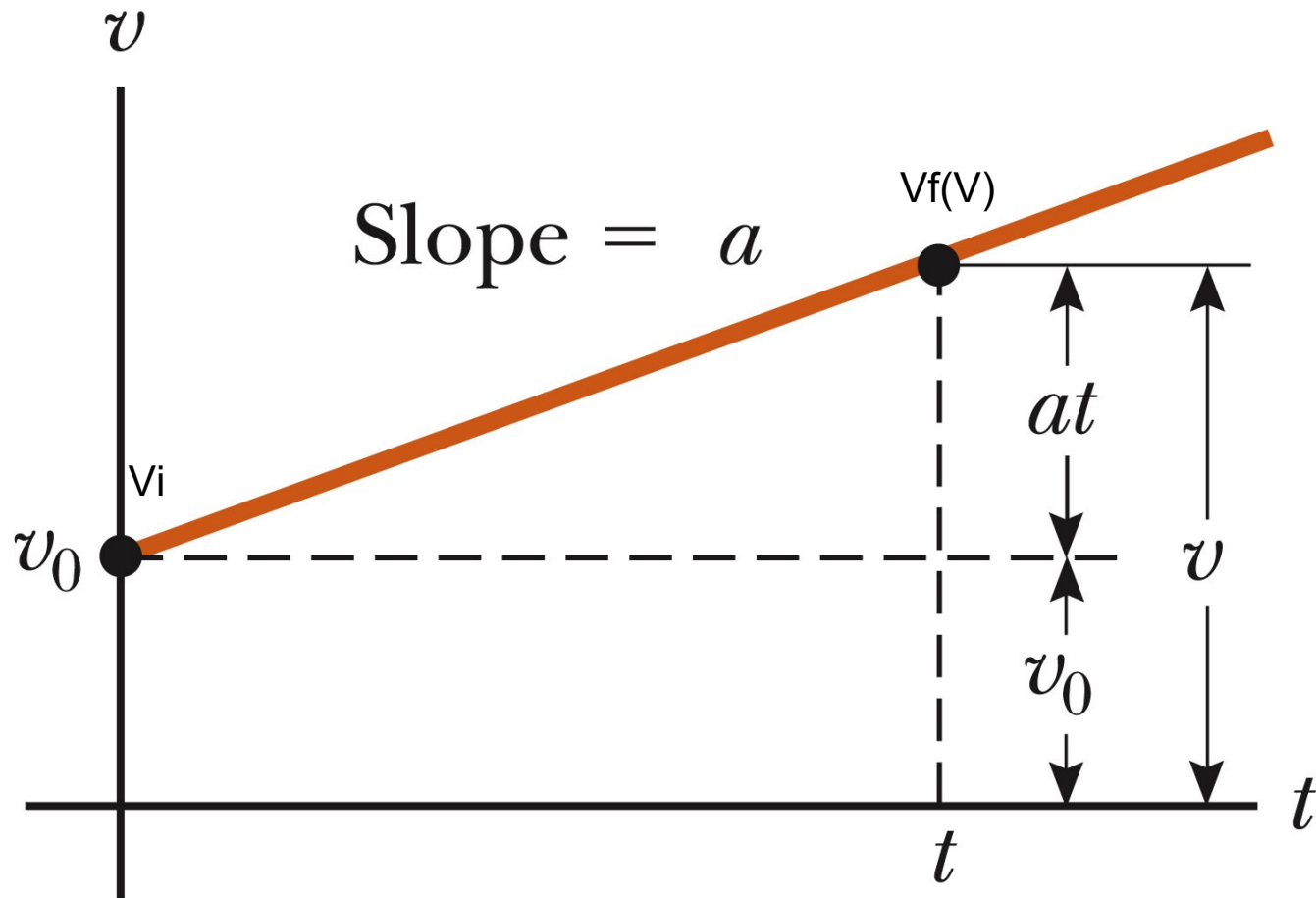
- Gives displacement as a function of velocity and time
- Use when you don't know and aren't asked for the acceleration

Notes on the equations

$$v = v_o + at$$

- Shows velocity as a function of acceleration and time
- Use when you don't know and aren't asked to find the displacement

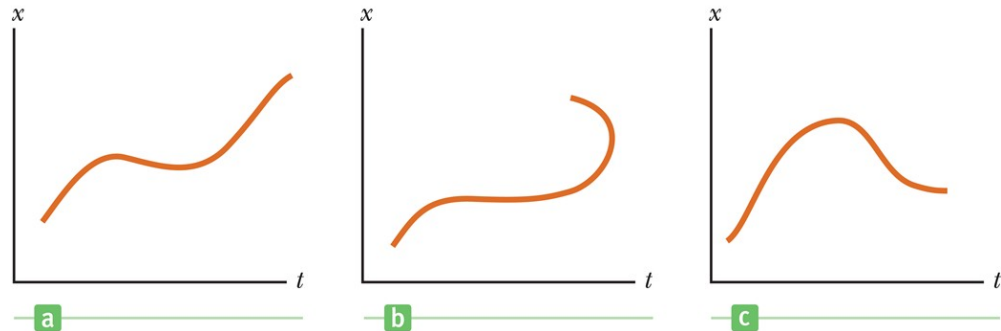
Graphical Interpretation of the Equation



Think – Pair – Share 9

- The three graphs in the figures below represent the position versus time for objects moving along the x-axis. Which, if any, of these graphs is **not** physically possible?
- Graph a is impossible.
 - Graph b is impossible.
 - Graph c is impossible.
 - All three are possible.

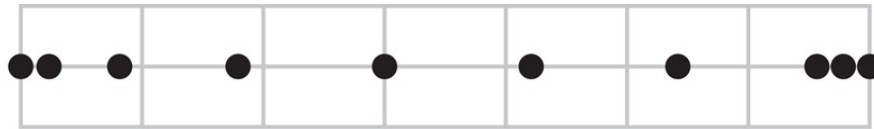
You can't go negative in time. Fails the vertical line test from Algebra.



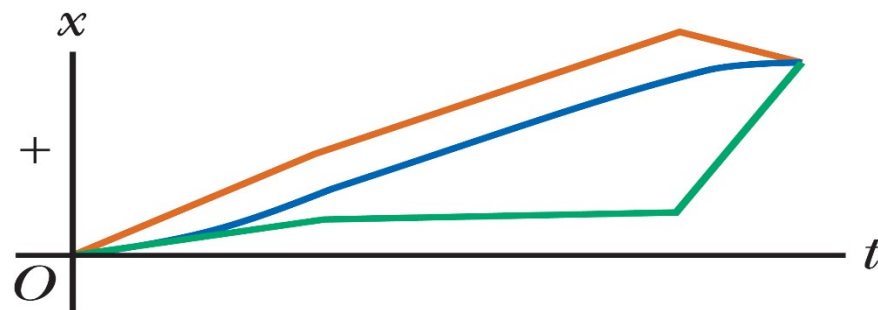
Think – Pair – Share 10

- This figure is a diagram of a multiframe image of an air puck moving to the right on a horizontal surface. The images sketched are separated by equal time intervals, and the first and last images show the puck at rest. In the graph, which color graph best shows the puck's position as a function of time?

1. red
2. green
3. blue



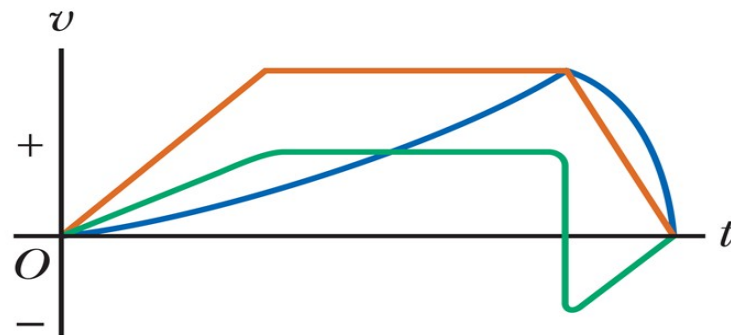
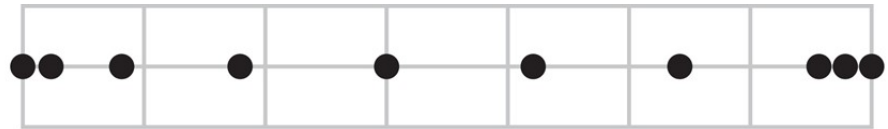
The steady rate between those 3-4 positions is a constant rate. Then the few close together are a rapid change, not necessarily decreasing.



Think – Pair – Share 11

- In the figure, which color graph best shows the puck's velocity as a function of time?

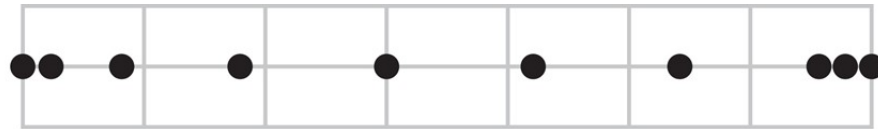
- red
- green
- blue



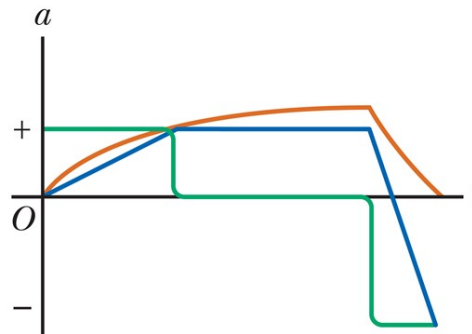
Think – Pair – Share 12

- In the figure, which color graph best shows the puck's acceleration as a function of time?

1. red
2. green
3. blue



This is with acceleration, so the short changes are decreases in velocity it results in a negative acceleration.

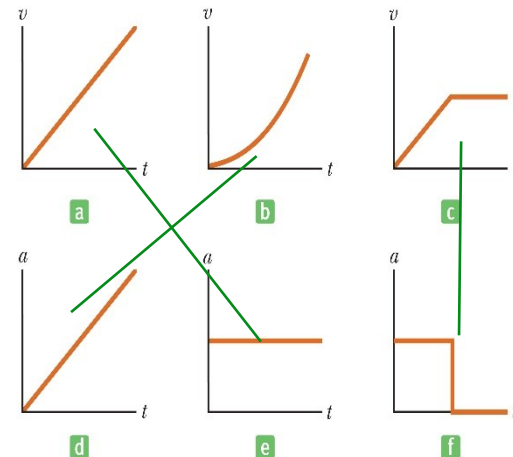


Think – Pair – Share 8

- Parts (a), (b), and (c) of the figure below represent three graphs of the velocities of different objects moving in straight-line paths as functions of time. The possible accelerations of each object as functions of time are shown in parts (d), (e), and (f). Match each velocity vs. time graph with the acceleration vs. time graph that best describes the motion.

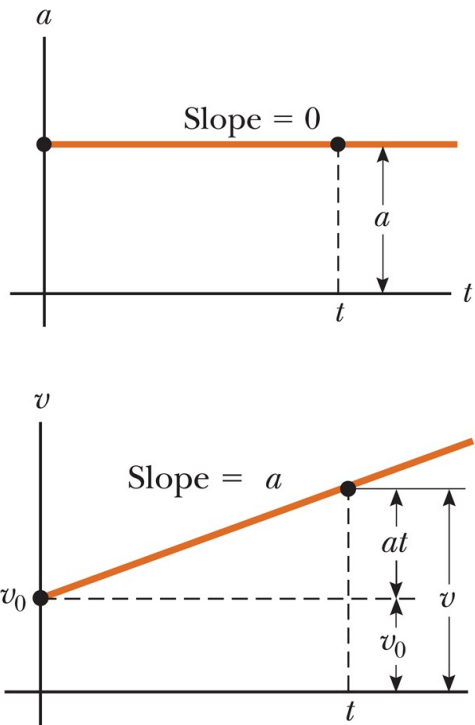
1. a and e, b and f, c and d
2. a and d, b and f, c and e
3. a and e, b and d, c and f

Constant velocity = Zero acceleration



Lines are showing the correlation

One-Dimensional Motion with



- For constant acceleration, the instantaneous acceleration at any point in a time interval is equal to the value of the average acceleration over the entire time interval.

$$a = \frac{V_f - V_i}{t_f - t_i} \rightarrow a = \frac{V - V_0}{t}$$

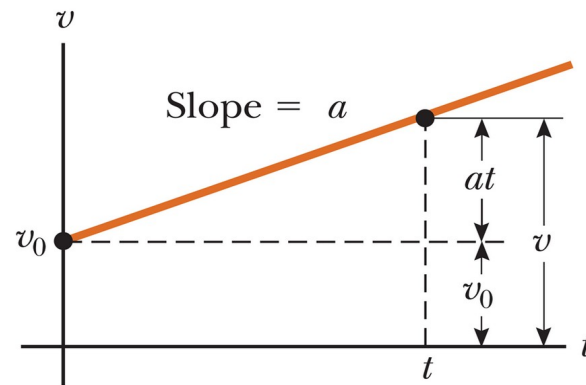
$$v = v_0 + at \quad (\text{for constant } a)$$

One-Dimensional Motion with

$$v_0 = +2.0 \text{ m/s}$$

$$a = +6.0 \text{ m/s}^2$$

$$t = 2.0 \text{ s}$$



$$v = v_0 + at = +2.0 \text{ m/s} + (6.0 \text{ m/s}^2)(2.0 \text{ s}) = +14 \text{ m/s}$$

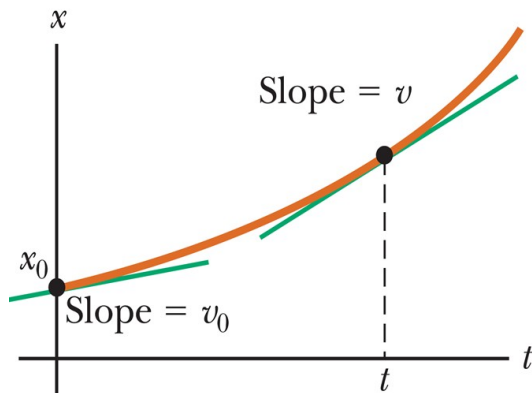
Notes on the equations

$$\Delta x = v_o t + \frac{1}{2} a t^2$$

- Gives displacement as a function of time, velocity and acceleration
- Use when you don't know and aren't asked to find the final velocity
- The area under the graph of v vs. t for any object is equal to the displacement of the object

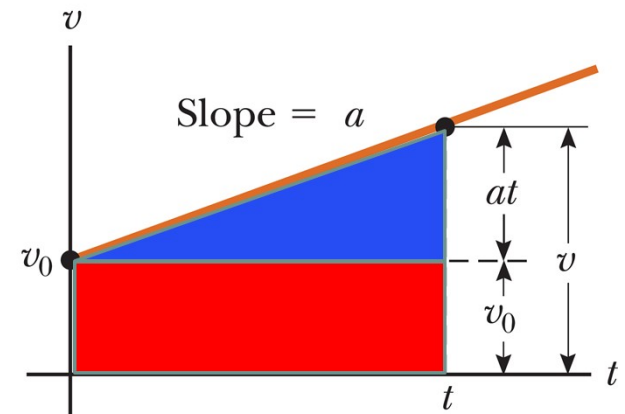
One-Dimensional Motion with

$$x = x_0 + v_0 t + \frac{1}{2} a t^2$$



$$\Delta x = v_0 t + \frac{1}{2} a t^2$$

$$v = v_0 + a t$$



- The area under the graph of v versus t for any object is equal to the displacement Δx of the object.

Notes on the equations

$$v^2 = v_o^2 + 2a\Delta x$$

- Gives velocity as a function of acceleration and displacement
- Use when you don't know and aren't asked for the time

Problem-Solving Hints

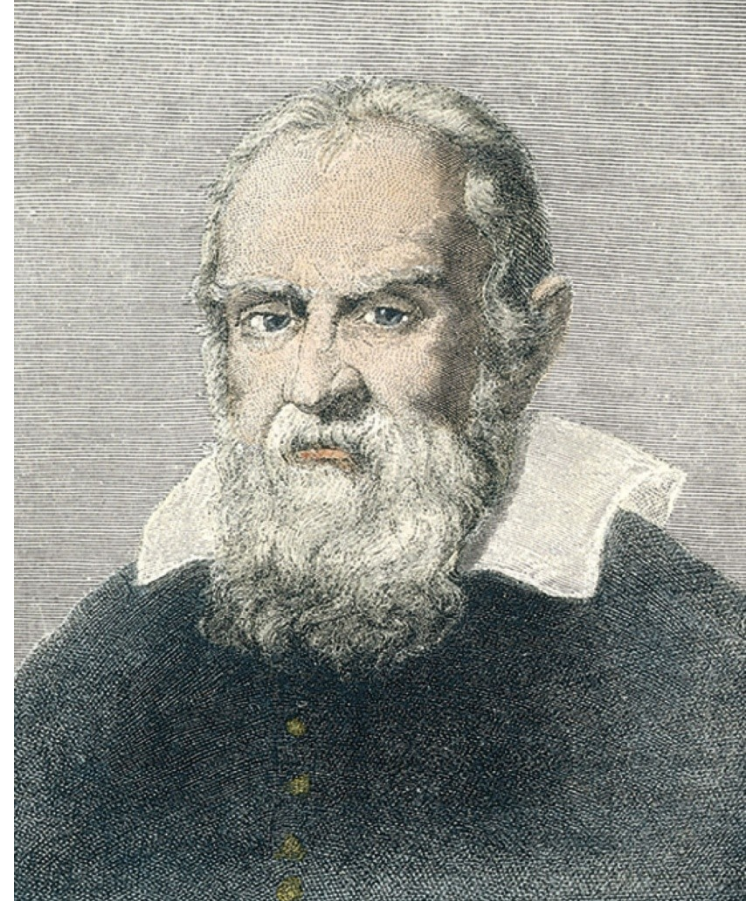
- Read the problem
- Draw a diagram
 - Choose a coordinate system
 - Label initial and final points
 - Indicate a positive direction for velocities and accelerations
- Label all quantities, be sure all the units are consistent
 - Convert if necessary
- Choose the appropriate kinematic equation

Problem-Solving Hints, cont

- Solve for the unknowns
 - You may have to solve two equations for two unknowns
- Check your results
 - Estimate and compare
 - Check units

Galileo Galilei

- 1564 - 1642
- Galileo formulated the laws that govern the motion of objects in free fall
- Also looked at:
 - Inclined planes
 - Relative motion
 - Thermometers
 - Pendulum



Free Fall

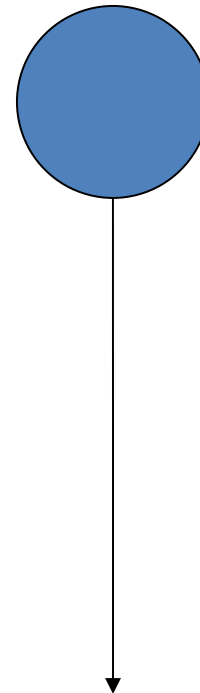
- A freely falling object is any object moving freely under the influence of gravity alone
 - Free fall does not depend on the object's original motion
- All objects falling near the earth's surface fall with a constant acceleration
- The acceleration is called the acceleration due to gravity, and indicated by g

Acceleration due to Gravity

- Symbolized by g
- $g = 9.80 \text{ m/s}^2$
 - When estimating, use $g \approx 10 \text{ m/s}^2$
- g is always directed downward
 - Toward the center of the earth
- Ignoring air resistance and assuming g doesn't vary with altitude over short vertical distances, free fall is constantly accelerated motion

Free Fall – an object dropped

- Initial velocity is zero
- Let up be positive
 - Conventional
- Use the kinematic equations
 - Generally use y instead of x since vertical
- Acceleration is $g = 9.80 \text{ m/s}^2$

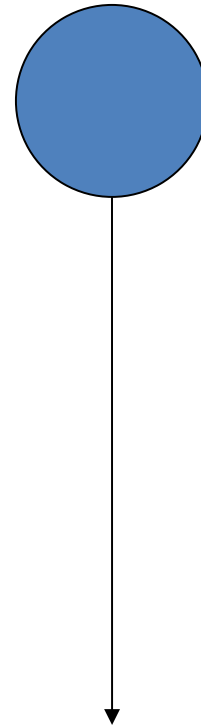


$$v_o = 0$$

$$a = -g$$

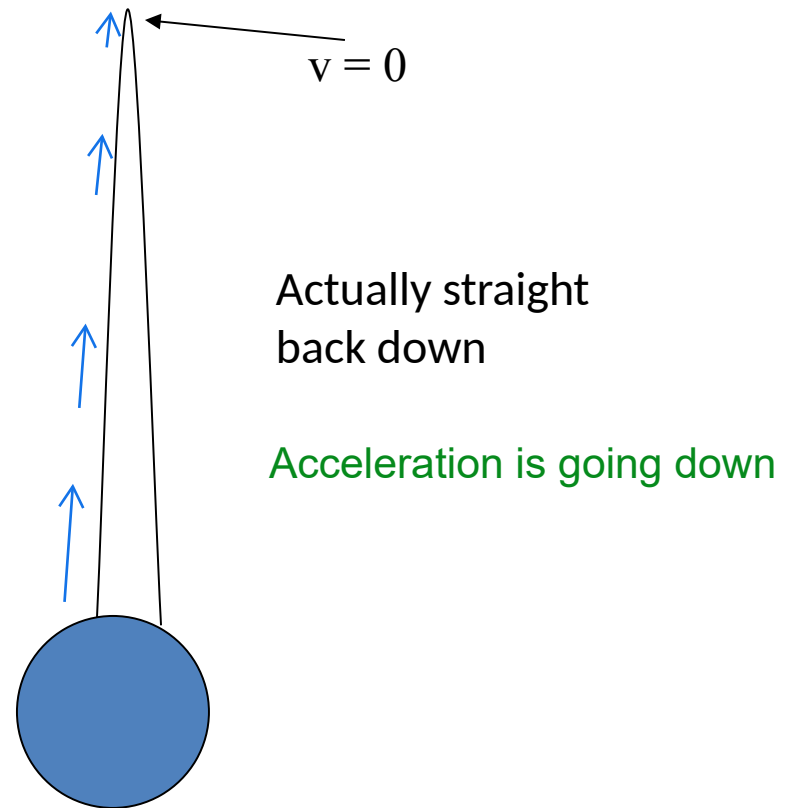
Free Fall – an object thrown downward

- $a = -g = -9.80 \text{ m/s}^2$
- Initial velocity $\neq 0$
 - With upward being positive, initial velocity will be negative



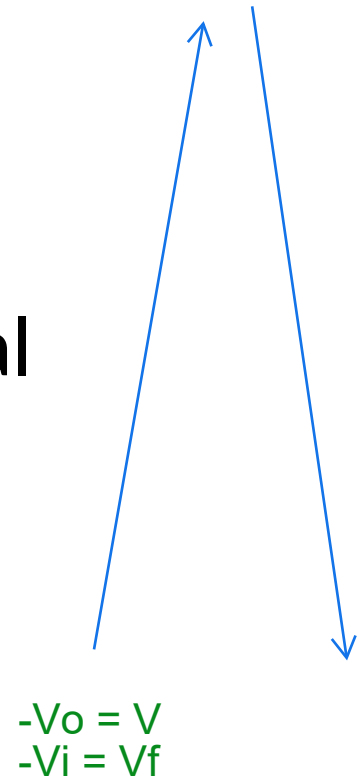
Free Fall – object thrown upward

- Initial velocity is upward, so positive
- The instantaneous velocity at the maximum height is zero
- $a = -g = -9.80 \text{ m/s}^2$ everywhere in the motion



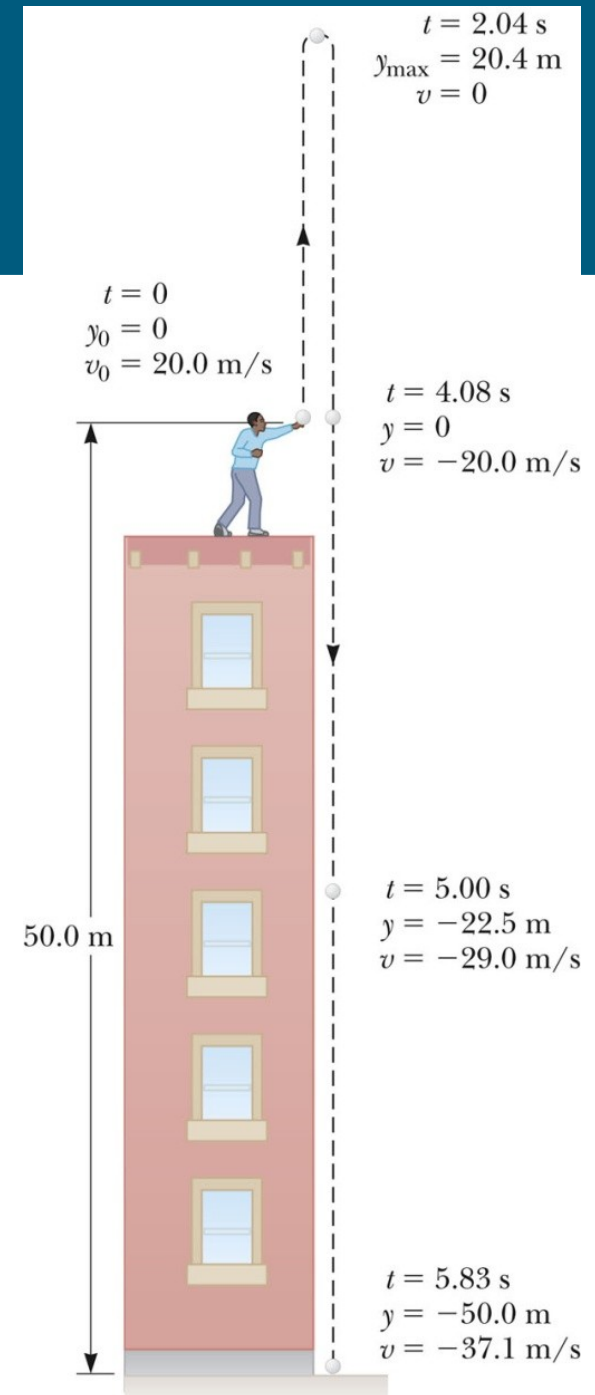
Thrown upward, cont.

- The motion may be symmetrical
 - Then $t_{\text{up}} = t_{\text{down}}$
 - Then $v = -v_o$
- The motion may not be symmetrical
 - Break the motion into various parts
 - Generally up and down



Non-symmetrical Free Fall Example

- Need to divide the motion into segments
- Possibilities include
 - Upward and downward portions
 - The symmetrical portion back to the release point and then the non-symmetrical portion



Combination Motions

