Sec.4.5

p.296-298: Antidifferentiation of a Composite Function

$\int f(u)du = F(u) + C, \quad where\ u = g(x) \to du = g'(x)dx;$ Examples 1 – 3

p. 301: General Power Rule: $\int u^n du = \frac{u^{n+1}}{n+1} + C, n \neq -1$; Example 7

p.305-306: Find the indefinite integral and check the result by differentiation. ✓

10. $\int (x^2-9)^3(2x)\,dx$ $\quad u = x^2-9 \to du = 2x\,dx$ ✓

$= \int u^3 du = \frac{u^4}{4} + C = \boxed{\frac{1}{4}(x^2-9)^4 + C}$

16. $\int x(5x^2+4)^3\,dx$ $\quad u = 5x^2+4 \to du = 10x\,dx$

$= \frac{1}{10}\int (5x^2+4)^3(10x\,dx) = \frac{1}{10}\int u^3 du = \frac{1}{10}\cdot\frac{u^4}{4} + C$

$= \boxed{\frac{1}{40}(5x^2+4)^4 + C}$

44. $\int x \sin x^2\,dx$ $\quad u = x^2 \to du = 2x\,dx$

$= \frac{1}{2}\int \sin x^2 (2x\,dx) = \frac{1}{2}\int \sin u\,du = \frac{1}{2}(-\cos u) + C$

$= \boxed{-\frac{1}{2}\cos x^2 + C}$

46. $\int \sqrt[3]{\tan x}\,\sec^2 x\,dx$ $\quad u = \tan x \to du = \sec^2 x\,dx$

$= \int \tan^{1/3} x (\sec^2 x\,dx) = \int u^{1/3} du = \frac{u^{4/3}}{\frac{4}{3}} + C = \boxed{\frac{3}{4}\tan^{4/3} x + C}$

p.299 - 300: Change of Variables for Indefinite Integrals – Guidelines ; Example 4

p.302 - 303: Change of Variables for Definite Integrals; Example 8

$\frac{1}{2\cdot 4}\cdot\frac{2}{5} = \frac{1}{10}$

$\frac{1}{2\cdot 4}\cdot\frac{2}{3} = \frac{1}{6}$

Example 5:

Find $\int x\sqrt{2x-1}\,dx.$

$u = 2x-1 \to du = 2dx \to \frac{du}{2} = dx$

$u+1 = 2x \to x = \frac{u+1}{2}$

$= \int \frac{u+1}{2}\cdot u^{1/2}\cdot\frac{du}{2} = \frac{1}{4}\int (u+1)u^{1/2}du = \frac{1}{4}\int (u^{3/2} + u^{1/2})du$

$= \frac{1}{4}\left[\frac{u^{5/2}}{5/2} + \frac{u^{3/2}}{3/2}\right] + C = \frac{1}{4}\left[\frac{2}{5}u^{5/2} + \frac{2}{3}u^{3/2}\right] + C$

$= \boxed{\frac{1}{10}(2x-1)^{5/2} + \frac{1}{6}(2x-1)^{3/2} + C}$

Example 9: Evaluate the definite integral.

$\int_1^5 \frac{x}{\sqrt{2x-1}}\,dx$

$u = 2x-1 \to du = 2dx \to dx = \frac{du}{2}$

$\to x = \frac{u+1}{2}$

$x = 1: \ u = 2(1)-1 = 1$

$x = 5: \ u = 2(5)-1 = 9$

$= \int_1^9 \frac{\frac{u+1}{2}}{u^{1/2}}\cdot\frac{du}{2} = \int_1^9 \frac{u+1}{2}\cdot u^{-1/2}\cdot\frac{du}{2}$

$$= \int_1^9 \frac{\frac{u+1}{2}}{u^{1/2}} \cdot \frac{du}{2} = \int_1^9 \frac{u+1}{2} \cdot u^{-1/2} \cdot \frac{du}{2}$$

$x=5: \; u = 2(5)-1 = 9$

$$= \frac{1}{4}\int_1^9 (u+1)u^{-1/2}\,du = \frac{1}{4}\int_1^9 (u^{1/2}+u^{-1/2})\,du$$

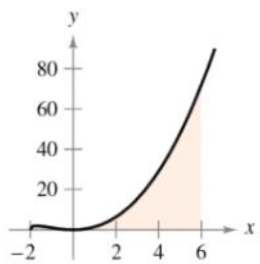
$$= \frac{1}{4}\Big[\underbrace{\tfrac{2}{3}u^{3/2}+2u^{1/2}}_{F(u)}\Big]_1^9 = \frac{1}{4}\left[\left(\tfrac{2}{3}\cdot 9^{3/2}+2\cdot 9^{1/2}\right)-\left(\tfrac{2}{3}\cdot 1^{3/2}+2\cdot 1^{1/2}\right)\right]$$

$$= \frac{1}{4}\left[\tfrac{2}{3}\cdot 27+2\cdot 3-\tfrac{2}{3}-2\right] = \frac{1}{4}\left[18+6-\tfrac{2}{3}-2\right] = \frac{1}{4}\left[\tfrac{64}{3}\right]$$

$$= \frac{16}{3}$$

p.306: Find the area of the region.

72. $\int_{-2}^{6} x^2\sqrt[3]{x+2}\,dx$



$$\int_{-2}^{6} x^2\sqrt[3]{x+2}\,dx$$

$u = x+2$

$du = dx \to x = u-2$

$x=-2: \; u = -2+2 = 0$

$x=6: \; u = 6+2 = 8$

$$= \int_0^8 (u-2)^2 \cdot u^{1/3}\,du$$

$$= \int_0^8 (u^2-4u+4)u^{1/3}\,du = \int_0^8 (u^{7/3}-4u^{4/3}+4u^{1/3})\,du$$

$$= \left[\tfrac{3}{10}u^{10/3}-4\cdot\tfrac{3}{7}u^{7/3}+4\cdot\tfrac{3}{4}u^{4/3}\right]_0^8$$

$$= \Big[\underbrace{\tfrac{3}{10}u^{10/3}-\tfrac{12}{7}u^{7/3}+3u^{4/3}}_{F(u)}\Big]_0^8 = \left[\tfrac{3}{10}\cdot 8^{10/3}-\tfrac{12}{7}\cdot 8^{7/3}+3\cdot 8^{4/3}\right]-[0]$$

$$= \frac{3}{10}\cdot 2^{10}-\frac{12}{7}\cdot 2^7+3\cdot 2^4 = \frac{4752}{35}$$

p.305: Find the general solution (G.S.) of the differential equation (DE).

32. $\frac{dy}{dx} = \frac{10x^2}{\sqrt{1+x^3}}$

$u = 1+x^3 \to du = 3x^2dx$

$$y = \int \frac{10x^2}{\sqrt{1+x^3}}\,dx = 10\int \frac{x^2}{\sqrt{1+x^3}}\,dx = \frac{10}{3}\int \frac{1}{(1+x^3)^{1/2}}(3x^2dx)$$

$$= \frac{10}{3}\int u^{-1/2}\,du = \frac{10}{3}\cdot 2u^{1/2}+C = \frac{20}{3}\sqrt{1+x^3}+C$$

$$\boxed{y = \frac{20}{3}\sqrt{1+x^3}+C}$$ G.S.

p.306: Find an equation for the function f that has the given derivative and whose graph passes through the given point.

50. $f'(x) = \sec^2 2x$ DE. $\left(\frac{\pi}{2}, 2\right)$ I.C

let $u = 2x$, *then* $du = 2dx$

$f(x) = \int sec^2 2x dx = \frac{1}{2}\int sec^2 2x(2dx) = \frac{1}{2}\int sec^2 u(du) = \frac{1}{2} tanu + C = \frac{1}{2} tan2x + C$

$\boxed{f(x) = \frac{1}{2} tan2x + C}$ (GS), IC: *point* $(\frac{\pi}{2}, 2)$

$\boxed{f(x) = \frac{1}{2}\tan 2x + C}$ (*GS*), *IC*: *point* $(\frac{\pi}{2}, 2)$

$\frac{1}{2}\tan 2\left(\frac{\pi}{2}\right) + C = 2 \to \frac{1}{2}\tan\pi + C = 2 \to C = 2 \to \boxed{f(x) = \frac{1}{2}\tan 2x + 2}$ (*PS*) $\cos x, \sec x$

0

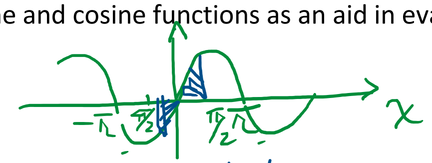
p.304: Integration of Even and Odd Functions: Theorem 4.16; Example 10

Even $f(-x) = f(x)$

Odd $f(-x) = -f(x)$

p.306:

80: Use the symmetry of the graphs of the sine and cosine functions as an aid in evaluating each definite integral. $\sin x, \csc x$, $\tan x$ $\cot x$



Odd

(a) $\int_{-\pi/4}^{\pi/4} \sin x\,dx = 0$

Even

(b) $\int_{-\pi/4}^{\pi/4} \cos x\,dx = 2\int_0^{\pi/4} \cos x\,dx = 2\left[\sin x\right]_0^{\pi/4} = 2\left[\sin\frac{\pi}{4} - \sin 0\right] = 2\left[\frac{\sqrt{2}}{2} - 0\right] = \sqrt{2}$

$\sin 2x = 2\sin x\cos x$

(d) $\int_{-\pi/2}^{\pi/2} \sin x\cos x\,dx = \int_{-\pi/2}^{\pi/2} \frac{1}{2}\sin 2x\,dx = \frac{1}{2}\int_{-\pi/2}^{\pi/2} \sin 2x\,dx \overset{\text{Odd}}{=} 0$