Sec.3.7

p.220: Guidelines for Solving Applied Min. and Max. Problems

p.224: In Exercises 5–10, find two positive numbers that satisfy the given requirements.

**6.** The product is 185 and the sum is a minimum.

S4 $x > 0$, $y > 0$

S1 Let $x = 1st\ number,\ \ y = 2nd\ number,\ \ S = sum$

S2 Primary equation: $S = x + y$

S3 Secondary equation: $xy = 185 \ \rightarrow y = \frac{185}{x}$

$S = x + \dfrac{185}{x} = x + 185x^{-1}$

S5 $S' = 1 + 185(-1)x^{-2} = 1 - 185x^{-2} = 1 - \dfrac{185}{x^2}$

Let $S' = 0$: $1 - \frac{185}{x^2} = 0 \rightarrow \frac{185}{x^2} = 1 \rightarrow x^2 = 185 \rightarrow x = +\sqrt{185}$ Critical #

$S'' = -185(-2)x^{-3} = \frac{370}{x^3}$  $S''(\sqrt{185}) = \dfrac{370}{(\sqrt{185})^3} > 0$ R.min

When $x = \sqrt{185}$: $S'' > 0$, concave upward ∪, $S\ has\ a\ R.min \rightarrow y = \frac{185}{\sqrt{185}} = \sqrt{185}$

c #

Two positive numbers are both $\sqrt{185}$.

**8.** The sum of the first number squared and the second number is 54 and the product is a maximum.

S4 $x > 0$, $y > 0$

S1 Let $x = 1st\ number,\ \ y = 2nd\ number,\ \ P = Product$

S2 Primary equation: $P = xy$

S3 Secondary equation: $x^2 + y = 54 \ \rightarrow y = 54 - x^2$

$P = x(54 - x^2) = 54x - x^3 \ \rightarrow \ P' = 54 - 3x^2$ S5

Let $P' = 0$: $54 - 3x^2 = 0 \rightarrow x^2 = 18 \rightarrow x = +\sqrt{18} = 3\sqrt{2}$ Critical #

$3x^2 = 54$  $9 \times 2$

R. max

$P'' = -6x$  $P''(3\sqrt{2}) = -6(3\sqrt{2}) < 0$

When $x = 3\sqrt{2}$: $P'' < 0$, concave downward ∩, $P\ has\ a\ R.max \rightarrow y = 54 - 18 = 36$

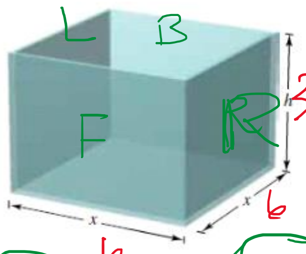
Two positive numbers are $3\sqrt{2}$ *and* 36.

p.219: Example 1

A manufacturer wants to design an open box having a square base and a surface area p.219: Example 1

A manufacturer wants to design an open box having a square base and a surface area of 108 square inches, as shown in Figure 3.53. What dimensions will produce a box with maximum volume? S2

$4xh = 108 - x^2$, $\frac{4xh}{4x} = \frac{108 - x^2}{4x}$




S3

Primary equation: $V = x^2 h$

Secondary equation: $x^2 + 4xh = 108 \rightarrow h = \frac{108-x^2}{4x}$ ✓

$$V = x^2\left(\frac{108 - x^2}{4x}\right) = \frac{108x - x^3}{4} = 27x - \frac{1}{4}x^3$$

$\frac{1}{4} \cdot 3x^2$

S1

S5

$V' = 27 - \frac{3}{4}x^2$ let $V' = 0$: $27 - \frac{3}{4}x^2 = 0 \rightarrow x^2 = 36 \rightarrow$

$9 \quad 27 = \frac{3}{4}x^2$

$x = {}^{+}6$ - Critical #

S4 $x > 0$

$h > 0$

$V'' = -\frac{3}{\cancel{4}_2}(\cancel{2}x) = -\frac{3}{2}x$, @ $x = 6$: $V'' = -\frac{3}{2}(6) < 0$

R.max

concave downward $\cap$, $V$ has a $R.\max \rightarrow h = \frac{108 - 6^2}{4(6)} = 3$

$\rightarrow$ $\boxed{Box: 6in \times 6in \times 3in}$

p.225:

**24. Maximum Area** A rectangle is bounded by the $x$- and $y$-axes and the graph of $y = (6 - x)/2$ (see figure). What length and width should the rectangle have so that its area is a maximum?

Let $P(x, y)$ be a point on the line ($Length = x$, $Width = y$)

$A = area$

S2

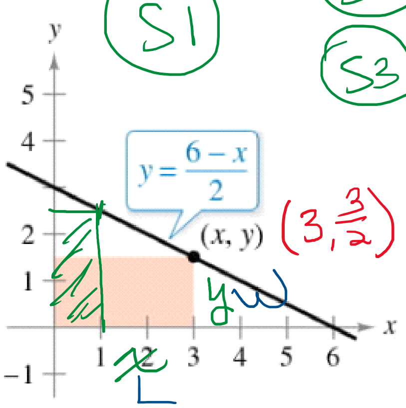
S1

S3

Primary equation: $A = xy$

Secondary equation: $y = \frac{6-x}{2}$

$\frac{1}{2} \cdot 2x$




$$A = x\left(\frac{6 - x}{2}\right) = \frac{6x - x^2}{2} = 3x - \frac{1}{2}x^2 \rightarrow A' = 3 - x$$

L

Let $A' = 0$: $3 - x = 0 \rightarrow x = 3$ - Critical #

R.max

$A'' = -1$, @ $x = 3$: $A'' < 0$, concave down, $A$ has a $R.max$

$\rightarrow y = \frac{6-3}{2} = \frac{3}{2}$ w

$\boxed{\text{The length and width are } 3 \text{ and } \frac{3}{2}.}$