Betingad sannolikhet repetition. S7 vecka 19

CONDITIONAL PROBABILITY WITH VENN DIAGRAMS

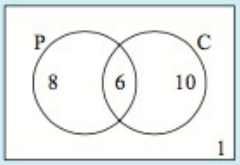
Suppose we have two events A and B, then

A | B is used to represent that 'A occurs knowing that B has occurred'.

Example 23

In a class of 25 students, 14 like Pizza and 16 like iced coffee. One student likes neither and 6 students like both. One student is randomly selected from the class. What is the probability that the student:

a likes Pizza b likes Pizza given that he/she likes iced coffee?



The Venn diagram of the situation is shown.

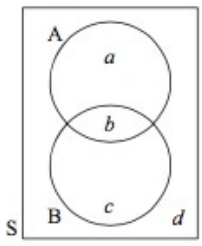
a $P(\text{Pizza}) = \frac{14}{25}$ {of the 25 students 14 like Pizza}

b $P(\text{Pizza/iced coffee}) = \frac{6}{16}$
{of the 16 who like iced coffee, 6 like Pizza}

If A and B are events then

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}.$$

Proof:



$$P(A \mid B) = \frac{b}{b+c} \quad \{\text{Venn diagram}\}$$

$$= \frac{b/(a+b+c+d)}{(b+c)/(a+b+c+d)}$$

$$= \frac{P(A \cap B)}{P(B)}$$

It follows that $P(A \cap B) = P(A \mid B)P(B)$ or $P(A \cap B) = P(B \mid A)P(A)$

I uppgift 1 b ii. Nedan så ska ni räkna ut sannolikheten för att någon studerar fysik givet att den personen studerar matematik. Då måste ni ju ta bort alla som inte studerar matematik.
Och räkna med de som är kvar.

Hinner ni klart snabbt. Översätt även till 2-way-tables.

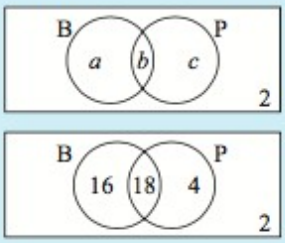
Example 24

In a class of 40, 34 like bananas, 22 like pineapples and 2 dislike both fruits. If a student is randomly selected, find the probability that the student:

a likes both fruits **b** likes at least one fruit

c likes bananas given that he/she likes pineapples

d dislikes pineapples given that he/she likes bananas.



B represents students who like bananas
P represents students who like pineapples

We are given that $a + b = 34$
$b + c = 22$
$a + b + c = 38$

$\therefore \quad c = 38 - 34$ and so $b = 18$
$= 4$ and $a = 16$

a P(likes both) $= \frac{18}{40} = \frac{9}{20}$

b P(likes at least one) $= \frac{38}{40} = \frac{19}{20}$

c $P(B \mid P) = \frac{18}{22} = \frac{9}{11}$

d $P(P' \mid B) = \frac{16}{34} = \frac{8}{17}$

EXERCISE 14K

1 In a group of 50 students, 40 study Mathematics, 32 study Physics and each student studies at least one of these subjects.

a From a Venn diagram find how many students study both subjects.

b If a student from this group is randomly selected, find the probability that he/she:

i studies Mathematics but not Physics

ii studies Physics given that he/she studies Mathematics.

2 In a class of 40 students, 23 have dark hair, 18 have brown eyes, and 26 have dark hair, brown eyes or both. A child is selected at random. Determine the probability that the child has:

a dark hair and brown eyes **b** neither dark hair nor brown eyes

c dark hair but not brown eyes **d** brown eyes given that the child has dark hair.