

Adder Tree / MAC

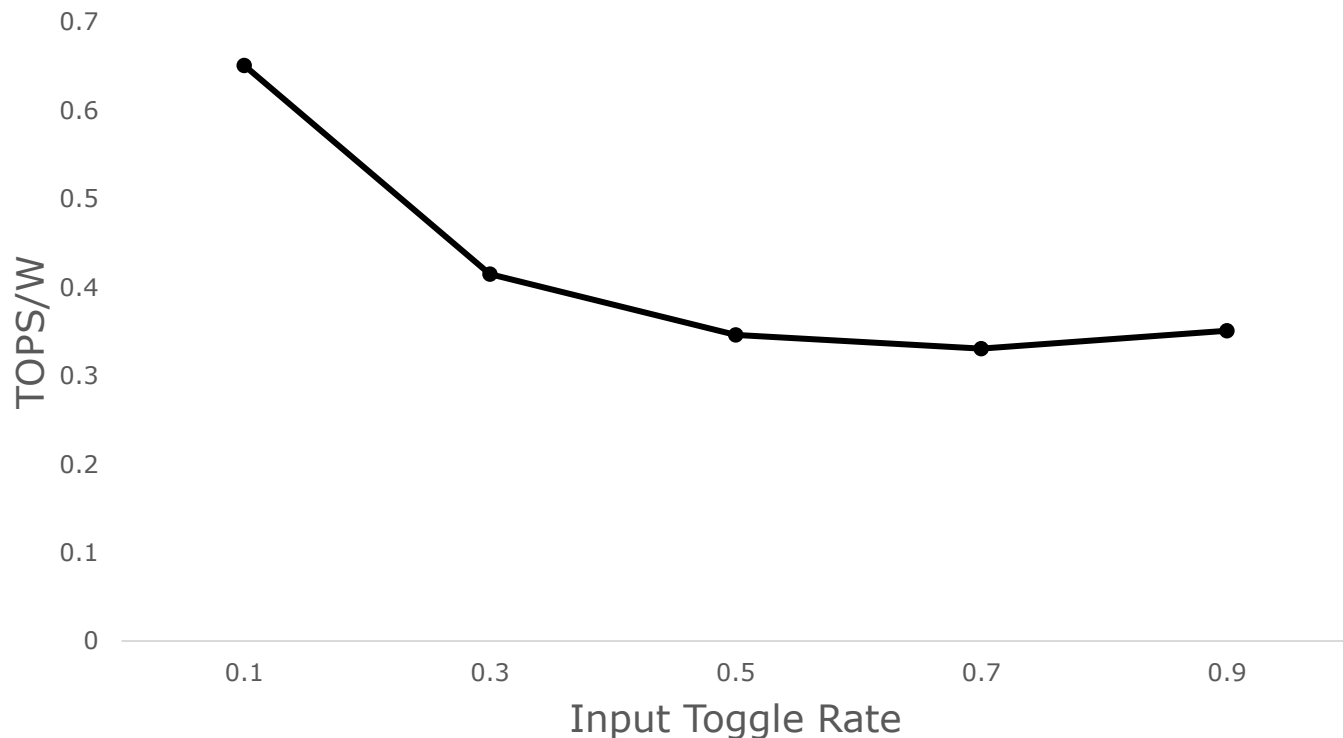


B11901027 王仁軒

Energy Efficiency

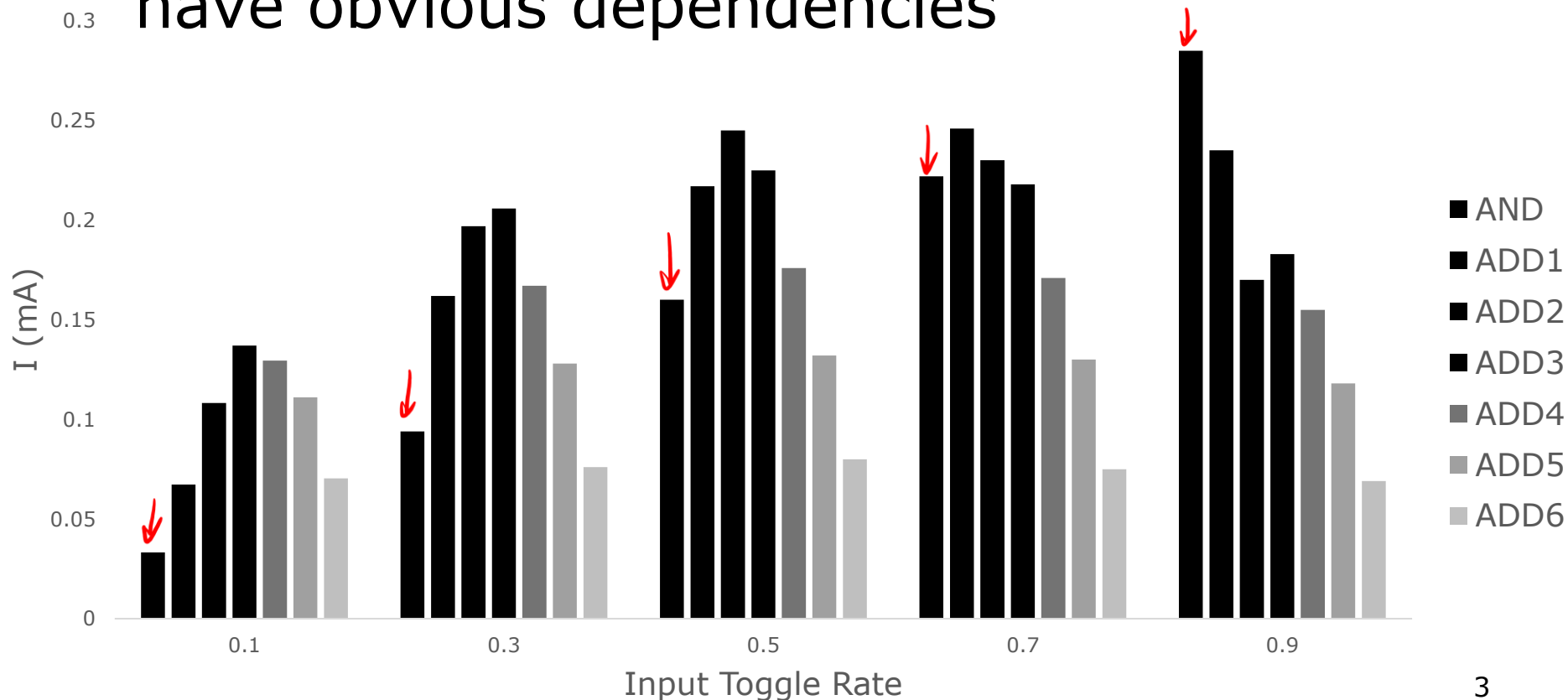
□ TOPS/W drops from 0.65 to 0.33 with the increase of input toggle rate

■ $T_{\text{cycle}} = 2.6\text{ns}$, 2.30 TOPS/mm²



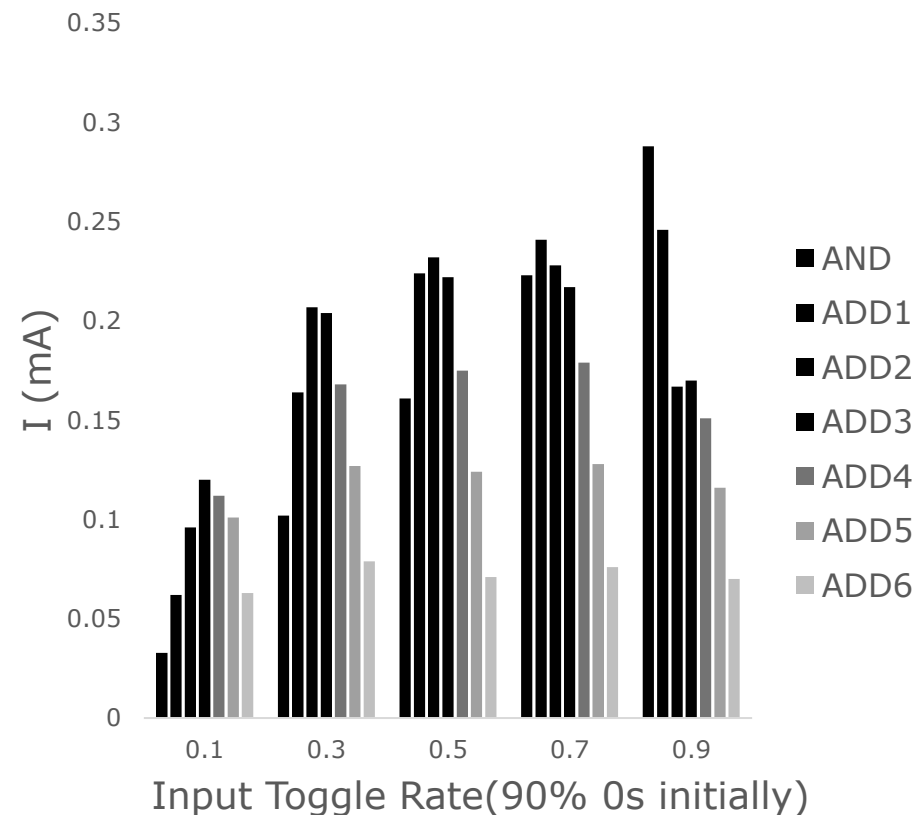
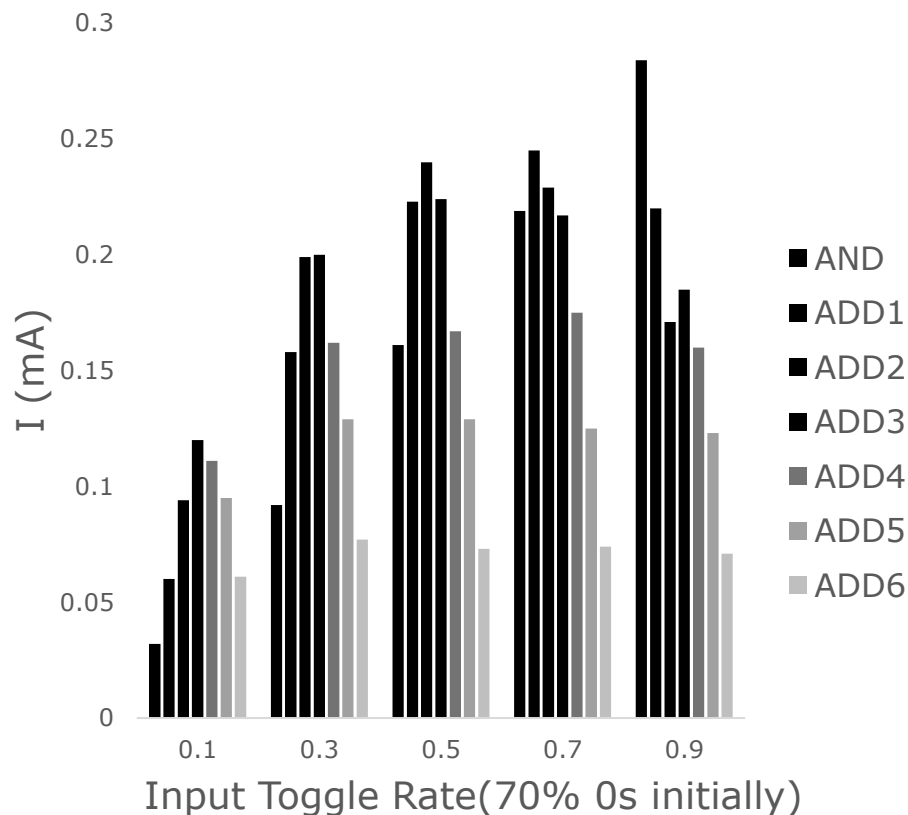
Power of All Adder Tree Stages

- Power of first few stages greatly depend on input toggle rate, while other stages do not have obvious dependencies



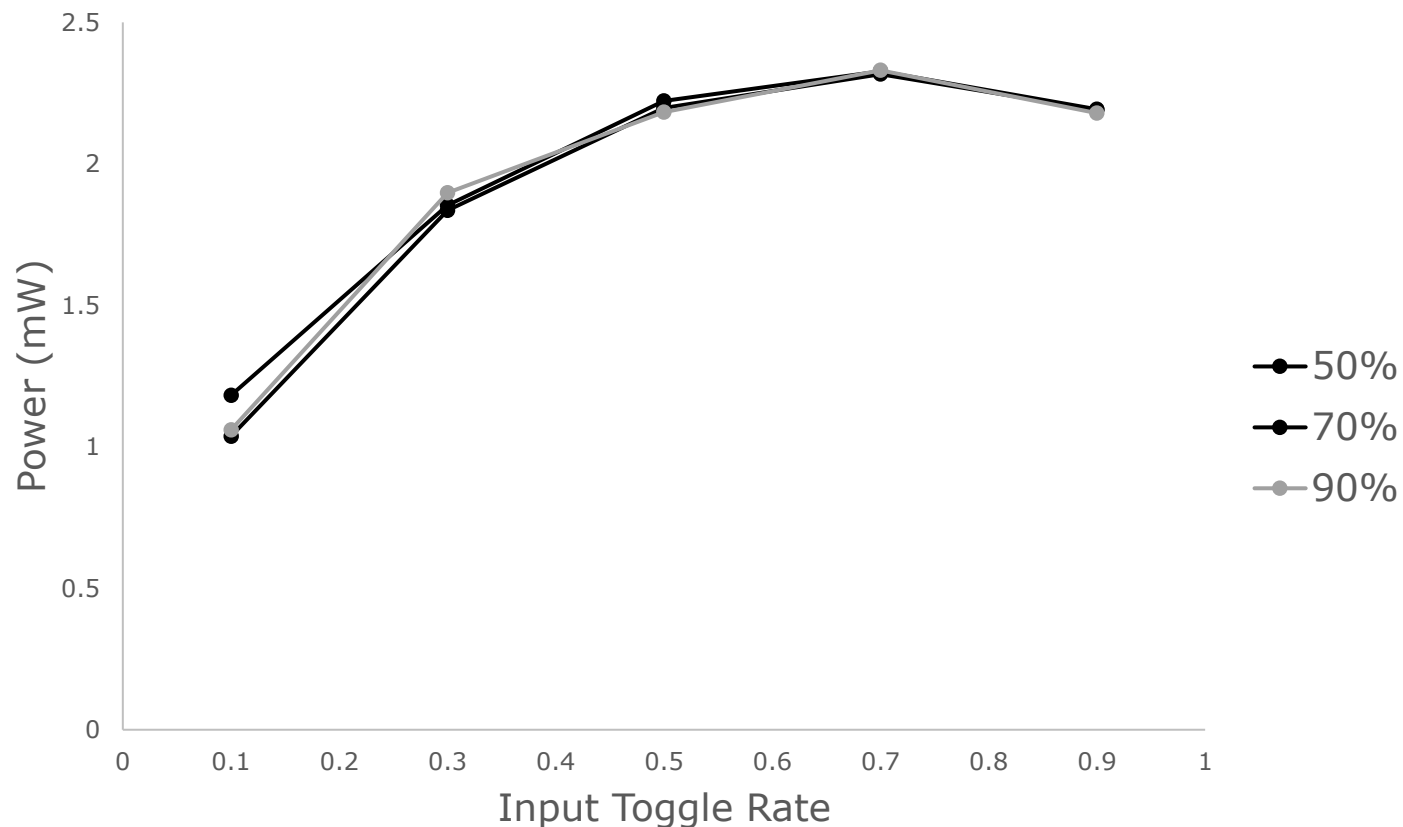
Input Initial condition

- The power distribution in all stages are same as 50% 0's input pattern's result



Input Initial condition

- The proportion of 0's in first input pattern does not affect the power of MAC



Timing diagram?

□ AND

□ VDD1

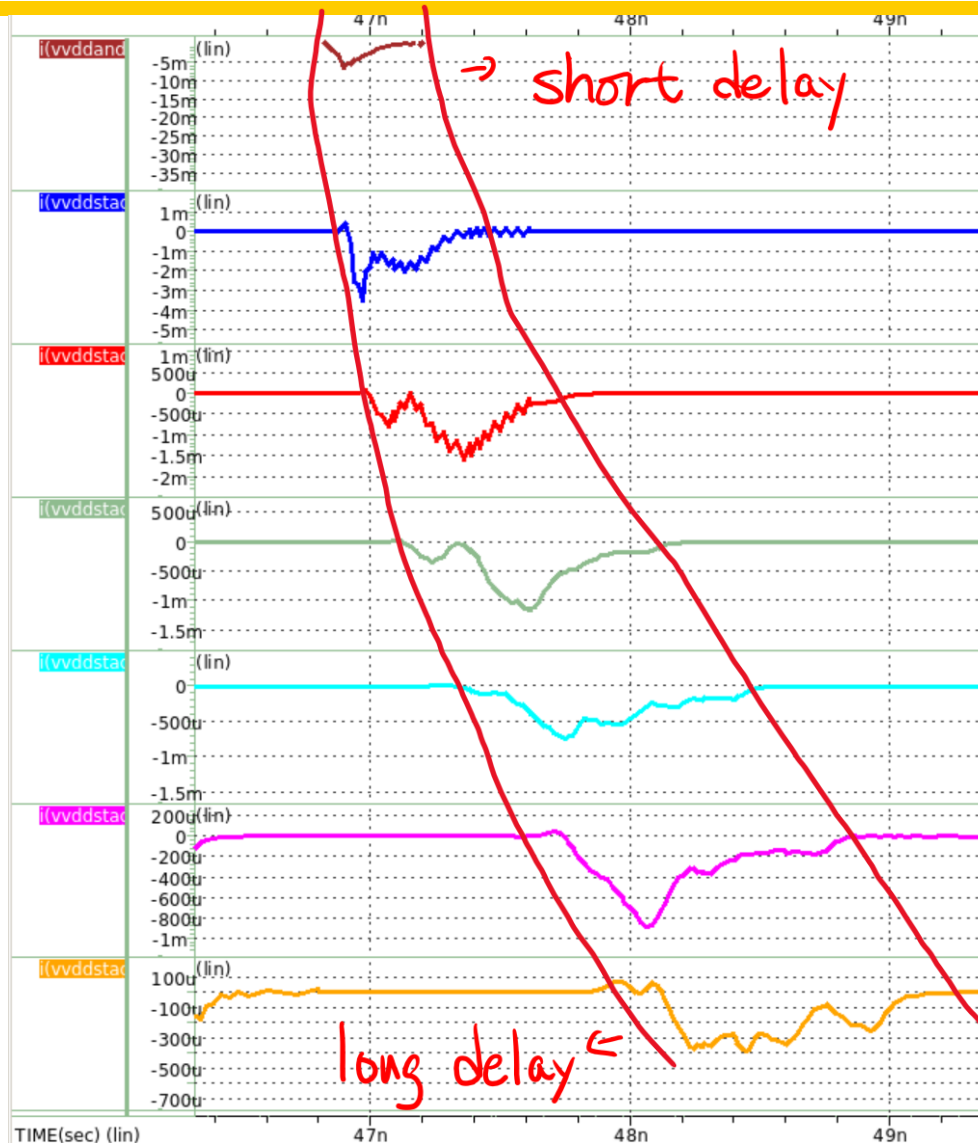
□ VDD2

□ VDD3

□ VDD4

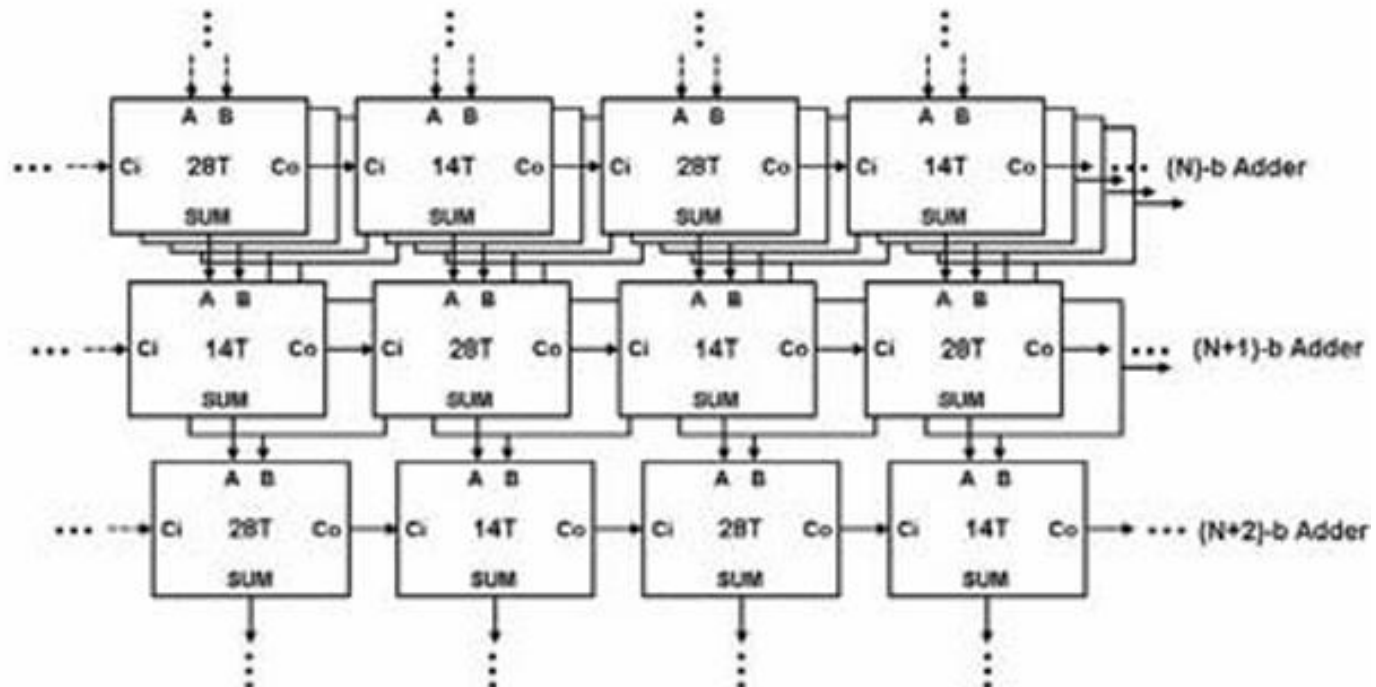
□ VDD5

□ VDD6



Interleaved FA

- ❑ Interleave 28T FAs along with 14T FAs
 - 30% smaller than the original design
 - But require longer cycle time (2.9ns)
 - Consumes 18% more power in average

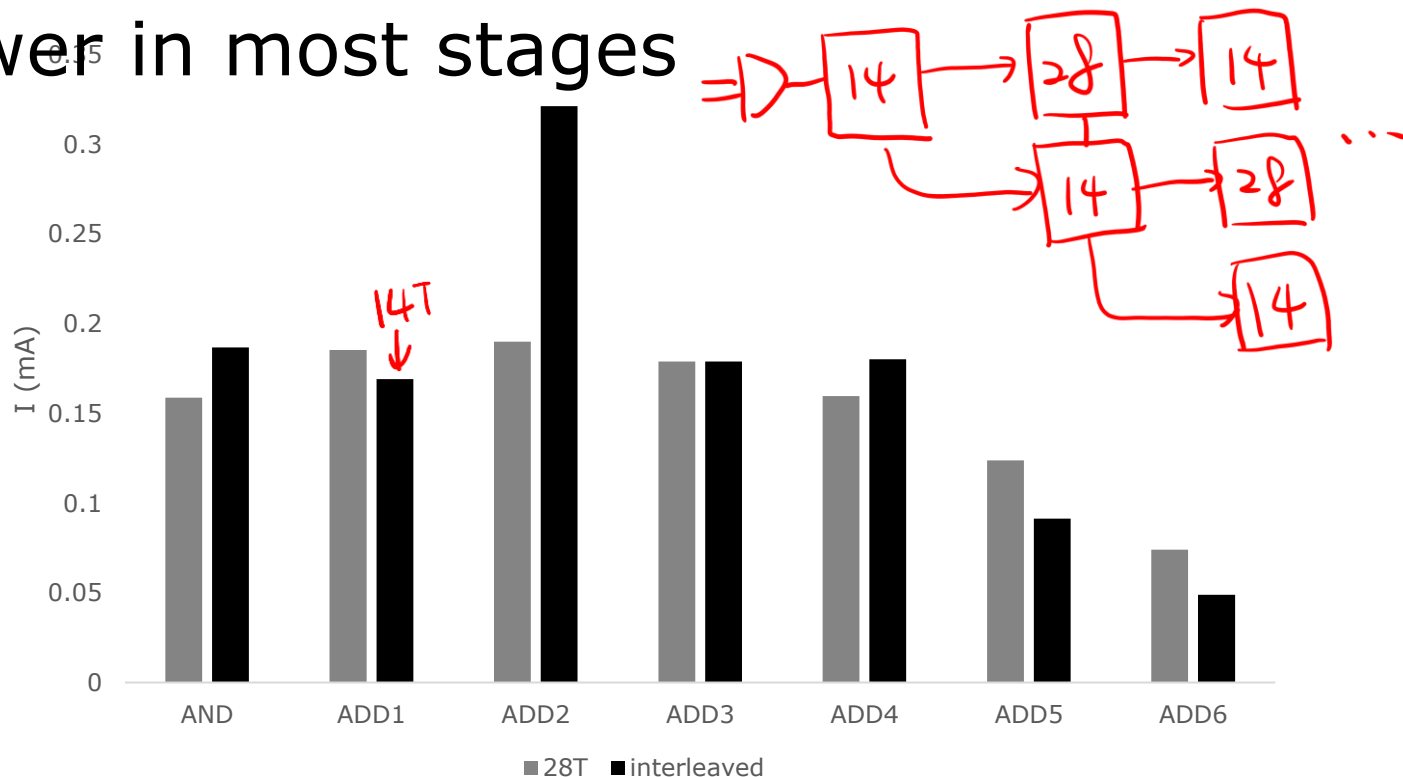


Interleaved FA

Energy Efficiency and Area Efficiency

0.26 TOPS/W, 2.97 TOPS/mm²

Interleaved FA structure consumes more power in most stages



Interleaved FA

$I_{peak} \downarrow$, but $I_{avg} \uparrow$

□ AND

□ VDD1

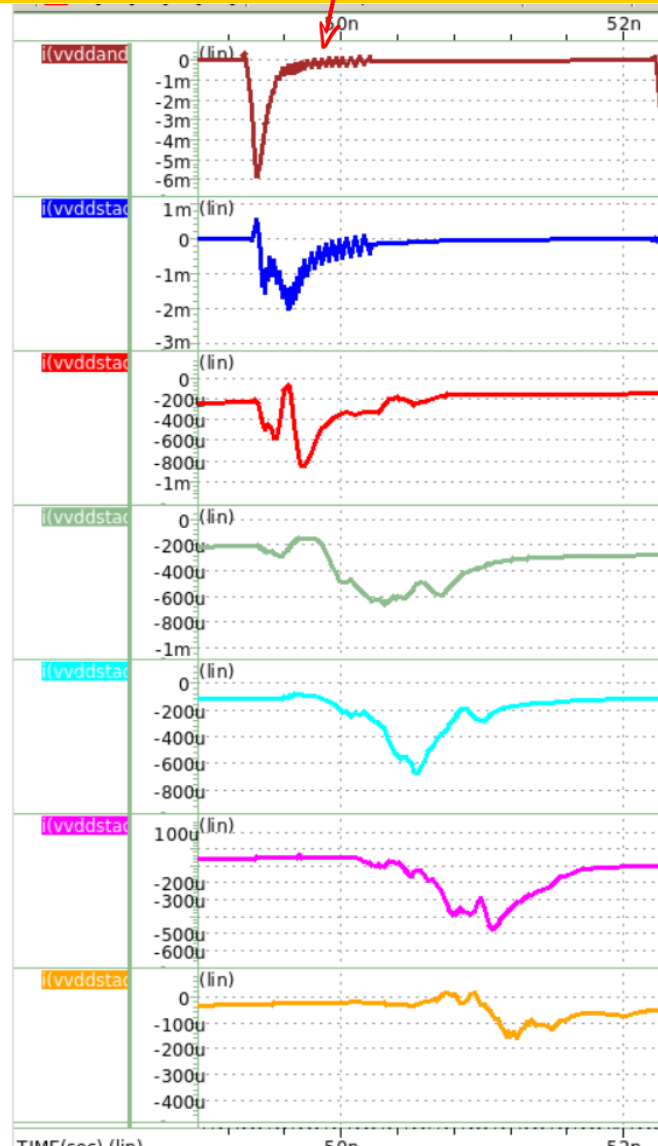
□ VDD2

□ VDD3

□ VDD4

□ VDD5

□ VDD6



6(7) mA

2(3.5) mA

0.8(1.6) mA

0.6(1.2) mA

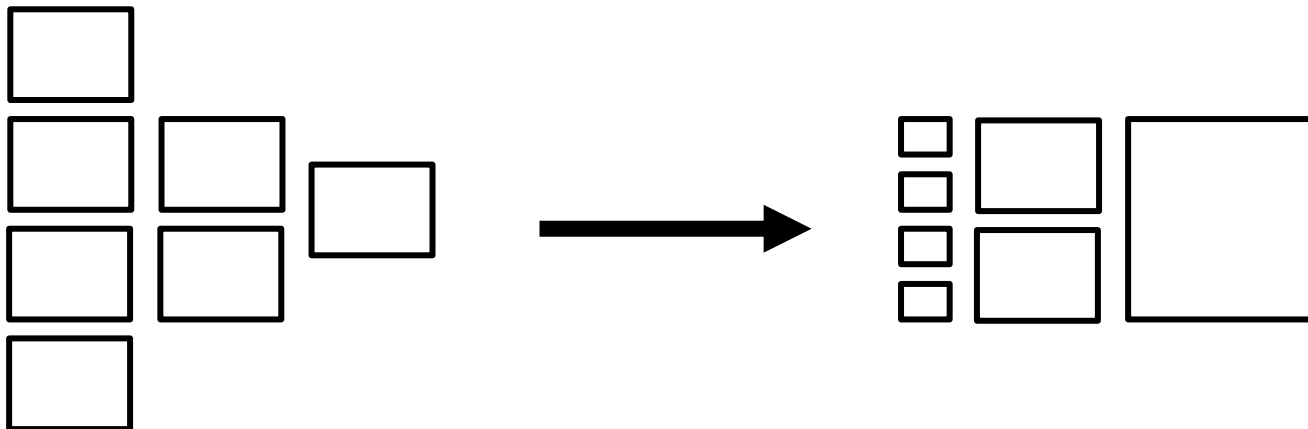
0.7(0.7) mA

0.5(0.9) mA

0.15(0.3) mA

FA Sizing

- High V_{th} for first few stages
 - But I don't know how to modify V_{th}
- Design first few stages smaller and last 2 stages larger
 - faster* *less power*
 - Alternative approach
 - May require more area



FA Sizing

- Applied on the interleaved design
 - Energy efficiency: 0.26 -> 0.29 TOPS/W
 - Area efficiency: 2.97 -> 2.81 TOPS/mm₂
- Slight improvement on energy efficiency
 - Area efficiency dropped, but not much

FA Sizing

AND

VDD1

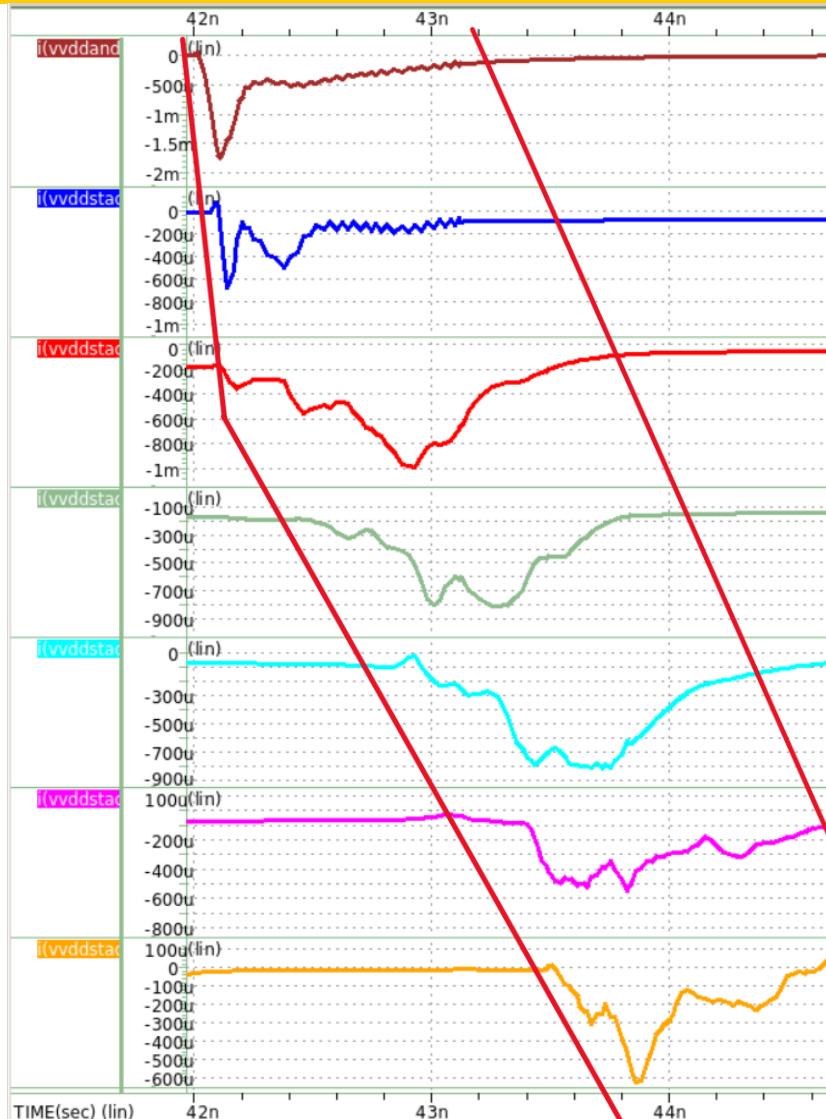
VDD2

VDD3

VDD4

VDD5

VDD6



1.8(6) mA

0.7(2) mA

1(0.8) mA

0.8(0.6) mA

0.8(0.7) mA

0.6(0.5) mA

0.6(0.15) mA

FA Sizing

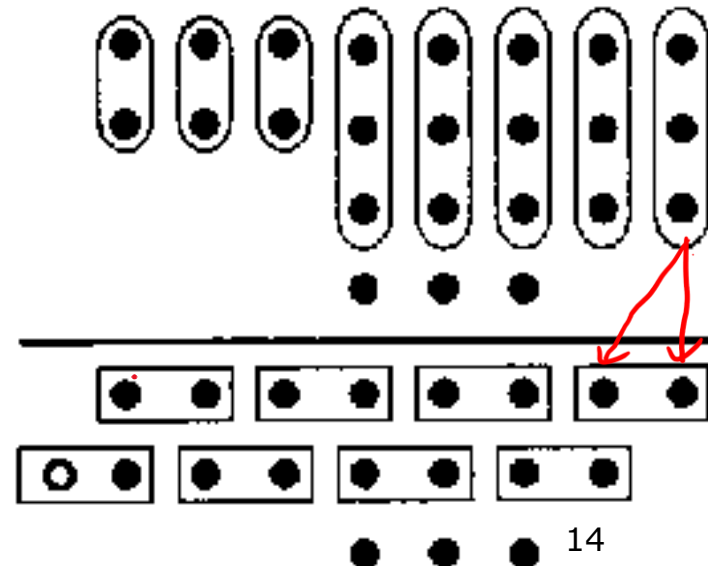
- Apply on the original design
- Energy efficiency
 - 0.33 -> 0.36 TOPS/W
- Area efficiency
 - 2.30 -> 1.27 TOPS/mm₂ *cannot further lower width*
 - Since minimum width is 250nm, increase L to lower drain current -> more area -> bad result

Adder Tree

- Calculate 64 1-bit input with groups of 2 input adders
 - FA has 3 degree of freedom at input but we only utilize 2 of them
 - We should use the FA fully
- Carry save adder design
 - Multiple input

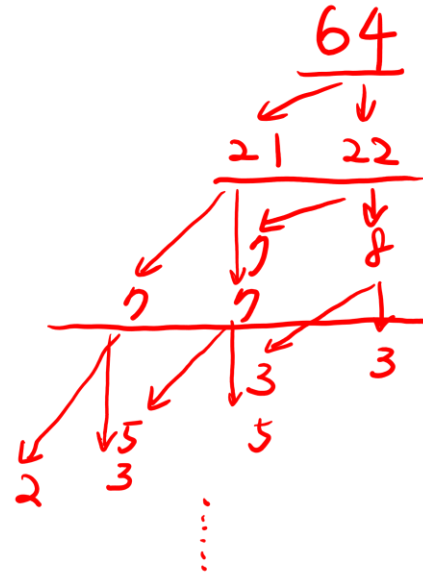
Inputs to the
second stage

Result of the
second stage

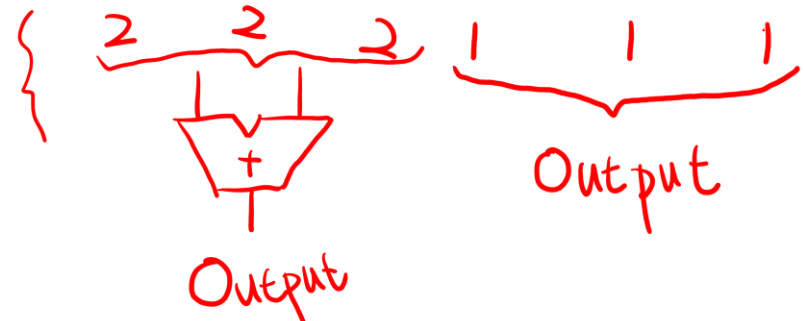


Adder Tree

Carry-save adder



Ripple-carry adder



Carry Save Adder Tree

- ❑ For regular 64b to 6b adder tree
 - Require 120 Full adders
 - Ripple carry adders propagate carries in every stage
 - Long critical path
- ❑ Carry save adder design
 - Only 67 full adders is used → 64 as CSA
 - Only need to propagate carry in the last stage
 - Relatively short critical path

CSA Tree Performance

□ 0.45 TOPS/W

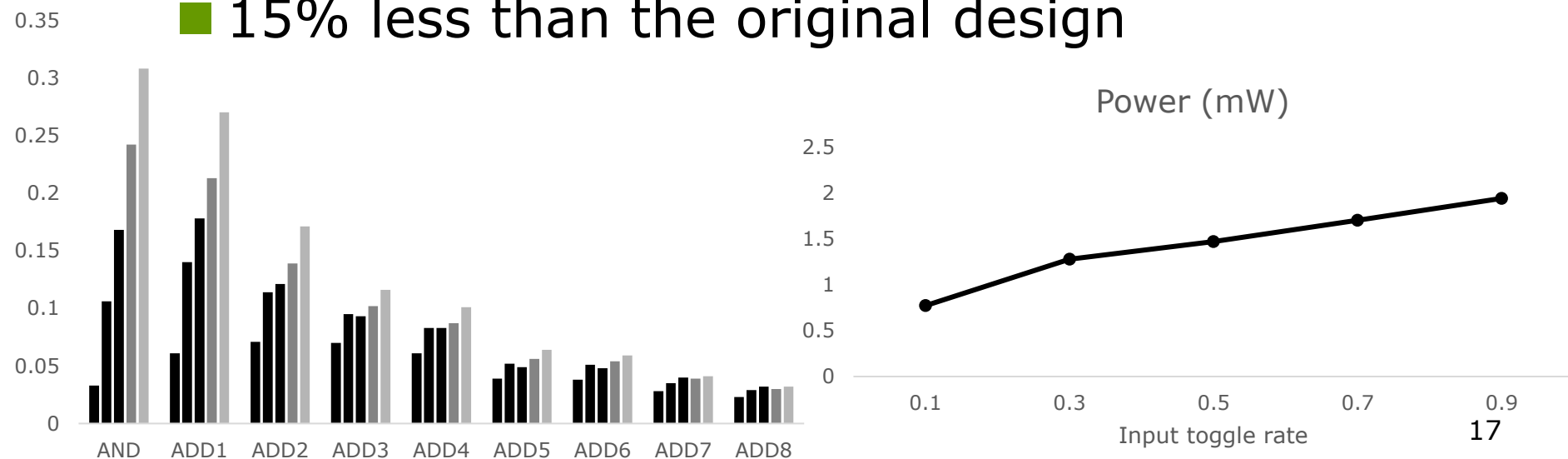
■ 36% improvement w.r.t. the original design

□ 4.33 TOPS/mm²

■ 88% improvement w.r.t. the original design

□ 2.3 ns cycle time

■ 15% less than the original design



$$32 \times 1.5 + 16 \times \frac{5}{4} \dots$$

$$S = A \oplus B \quad p$$

CSA Tree with interleaving $C = AB \quad 6 \quad 14$

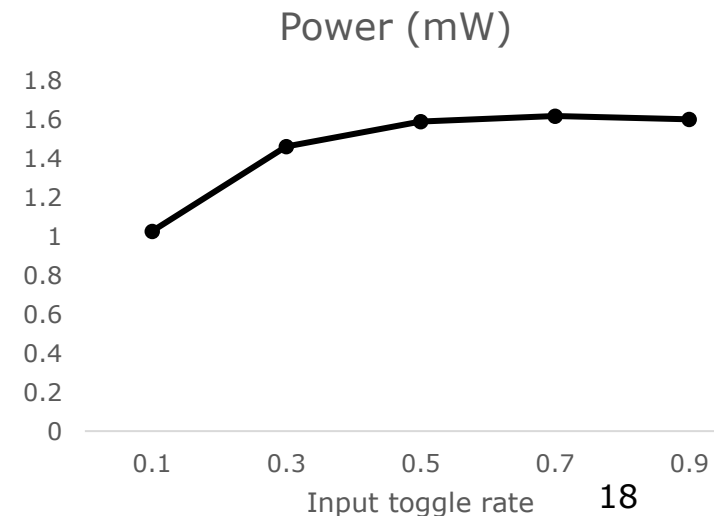
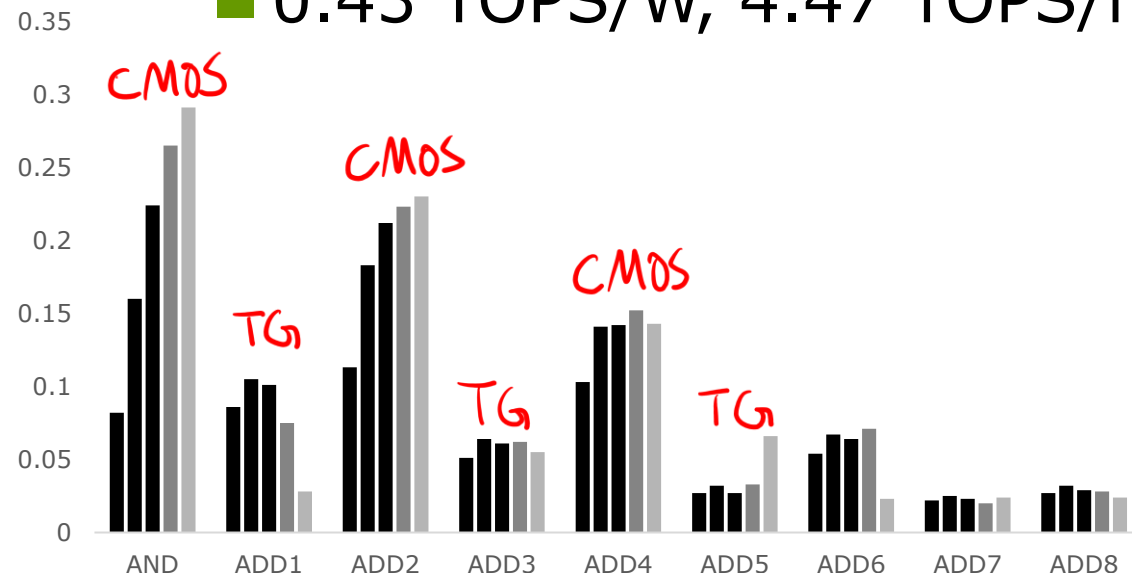
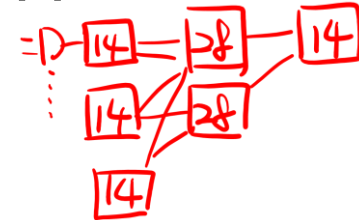
1st stage: 14T, 2nd stage: 28T

But ensure output are driven by 28T FA

37/67 FAs are replaced by 14T FA

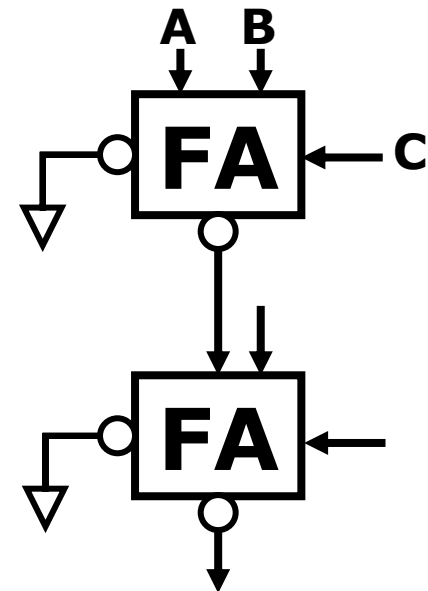
Performance

0.43 TOPS/W, 4.47 TOPS/mm²



CSA Tree with inverting FA

- More than better
- Use inverting full adders to remove the redundant inverters in FA's output stage
 - 28T -> 24T *but 14T remains 14 T*
 - May require extra inverters in some stages and at output
 - Driving capability of FA may drop due to no buffer at output
 - Saves some time consumed by INVs but smaller drain currents require more time to charge nodes



Performance of CSA Tree w/ invFA

- 0.55 TOPS/W, 5.14 TOPS/mm²
 - 22% improvement in energy efficiency
 - 15% improvement in area efficiency
 - Cycle time = 2.3 ns
- What about the design with interleaving
 - 0.51 TOPS/W, 5.35 TOPS/mm²
 - 19% improvement in energy efficiency
 - 19% improvement in area efficiency
 - Cycle time = 2.9ns

Summary of All Designs

Adder Type	Additional Feature	Energy Efficiency	Area Efficiency	Cycle Time
Ripple Carry	No	0.33	2.30	2.6
Ripple Carry	Interleaving	0.26	2.97	2.9
Ripple Carry	Sizing	0.36	1.27	2.7
Ripple Carry	Interleaving Sizing	0.29	2.81	3.0
Carry Save	No	0.45	4.33	2.3
Carry Save	Interleaving	0.43	4.47	2.9
Carry Save	Inverting	0.55 +66%	5.14 +123%	2.3
Carry Save	Interleaving Inverting	0.51	5.35 +133%	2.9

Summary

- ❑ Ripple carry adder tree
 - Longer delay, large area, and higher power
 - Easy to implement and debug
 - Compatible with input size scaling
- ❑ Carry save adder tree
 - Shorter delay, small area, and lower power
 - Hard to implement and debug
 - Cannot be made to be compatible with input size scaling easily (I don't know how to)
- ❑ Maybe even better? *↪ I tried but performance are poor*
 - Use abc+yosys to further optimize adder tree

Summary

- ❑ Standard (do nothing)
 - Old friends are always reliable
- ❑ Interleaving
 - Energy efficiency + Speed \leftrightarrow Area efficiency
- ❑ ~~V_{th} scaling~~ Transistor Sizing
 - As pathetic as my electric circuit design grade
- ❑ Inverting Full Adders
 - Increasing both energy and area efficiency
 - Very very good idea