Esercizi di Analisi Matematica

① $\frac{1}{x}+\frac{1}{x-1}+\frac{1}{x+2}<0 \Rightarrow$

CE
$\boxed{x\neq 0}$
$x-1\neq 0$ $\boxed{x\neq 1}$
$x+2\neq 0$ $\boxed{x\neq -2}$

$$\frac{(x-1)(x+2)+x(x+2)+x(x-1)}{x(x-1)(x+2)}<0$$

$$\frac{x^2+2x-x-2+x^2+2x+x^2-x}{x(x-1)(x+2)}<0$$

$$\frac{\overbrace{3x^2+2x-2}^{N}}{\underbrace{x(x-1)(x+2)}_{D}}<0$$

$N>0$
$D>0$

$N>0$ $\quad 3x^2+2x-2>0 \Rightarrow$

$$\underset{a}{3}x^2+\underset{b}{2}x\underset{c}{-2}=0$$

$$\boxed{x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}}$$
equazione 2° grado

$$x=\frac{-2\pm\sqrt{4+24}}{6}=\frac{-2\pm\sqrt{28}}{6}\Rightarrow$$

$$=\frac{-2\pm2\sqrt{7}}{6}=\frac{\cancel{2}^{1}(-1\pm\sqrt{7})}{\cancel{6}_{3}}=\frac{-1\pm\sqrt{7}}{3}$$

secondo la regola del DI | CE

In questo caso abbiamo

$\left[\frac{-1 \pm \sqrt{7}}{3}\right]$ sol di

$3x^2+2x-2>0$

DISCORDI INTERNI

CONCORDI ESTERNI

concordi esterni

$\frac{-1-\sqrt{7}}{3}$ $\frac{-1+\sqrt{7}}{3}$

$\left(-\infty; \frac{-1-\sqrt{7}}{3}\right) \cup \left(\frac{-1+\sqrt{7}}{3}; +\infty\right)$

$\boxed{D>0}$

$\overset{F_1}{x}\overset{F_2}{(x-1)}\overset{F_3}{(x+2)}>0$

F_1 $x>0$

F_2 $x-1>0 \Rightarrow x>1$

F_3 $x+2>0 \Rightarrow x>-2$

		-2		0		1	
F_1	−		−		+		+
F_2	−		−		−		+
F_3	−		+		+		+
	−		+		−		+

$(-2; 0) \cup (1; +\infty)$

D: $(-2; 0) \cup (1; +\infty)$

N: $\left(-\infty; \frac{-1-\sqrt{7}}{3}\right) \cup \left(\frac{-1+\sqrt{7}}{3}; +\infty\right)$

	-2	$\frac{-1-\sqrt{7}}{3}$	0	$\frac{-1+\sqrt{7}}{3}$	1	
N	+	+	−	−	+	+
D	−	+	+	−	−	+
	−	+	−	+	−	+

Soluzione

$(-\infty; -2) \cup \left(\frac{-1-\sqrt{7}}{3}; 0\right) \cup \left(\frac{-1+\sqrt{7}}{3}; 1\right)$

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1) $$\frac{x(x+1)^2}{x^2-16} \leq \frac{(x+1)^3}{x^2+2x-24}$$

⚠ DA RIFARE

CE $x^2 - 16 \neq 0 \Rightarrow x^2 \neq 16 \quad x \neq \pm 4$

$\boxed{x \neq 4 \quad x \neq -4}$

$$x^2 + 2x - 24 \neq 0$$

$$-24 \times 4 = 96$$

$$x = \frac{-2 \pm \sqrt{4+96}}{2} = \frac{-2 \pm 10}{2}$$

$+$: $\frac{-2+10}{2} = \frac{8}{2} = 4$

$-$: $\frac{-2-10}{2} = \frac{-12}{2} = -6$

$$\left\{ \boxed{\begin{array}{l} x \neq 4 \\ x \neq -6 \end{array}} \right.$$

$$x^2 + 2x - 24 = (x-4)(x+6)$$

$$\frac{x(x+1)^2}{x^2-16} \leq \frac{(x+1)^3}{x^2+2x-24} \Rightarrow \frac{x(x+1)^2}{(x-4)(x+4)} \leq \frac{(x+1)^3}{(x-4)(x+6)}$$

$$x^2 - 16 \rightarrow (x+4)(x-4)$$

$$\frac{x(x+6)(x+1)^2}{D\;(x+4)(x-4)(x+6)} \leq \frac{(x+4)(x+1)^3}{D\;(x+4)(x-4)(x+6)} \Rightarrow$$

$$\Rightarrow \frac{(x^2+6x)(x+1)^2}{D} \leq \frac{(x+4)(x^3+1+3x^2+x)}{D} \rightarrow$$

$$\Rightarrow \frac{(x^2+6x)(x^2+1+2x)}{D} \leq \frac{x^4+x+3x^3+x^2+4x^3+4+12x^2+4}{D} \Rightarrow$$

$$\Rightarrow \frac{x^4 + x + 2x^2 + 6x^3 + 6x + 12x^2}{D} \le \frac{x^4 + x + 7x^3 + 13x^2 + 8}{D} \Rightarrow$$

$$\Rightarrow \frac{x^4 + 7x + 14x^2 + 6x^3}{D} \le \frac{x^4 + x + 7x^3 + 13x^2 + 8}{D} \Rightarrow$$

$$\Rightarrow \frac{\cancel{x^4} + 7x + 14x^2 + 6x^3 \cancel{- x^4} - x - 7x^3 - 13x^2 - 8}{D} \le 0 \Rightarrow$$

$$\Rightarrow \frac{-x^3 + x^2 + 6x - 8}{D} \le 0 \Rightarrow \frac{-x^3 + x^2 + 6x - 8}{(x+4)(x-4)(x+6)} \le 0$$

N

D

$$\boxed{N > 0}$$

$$-x^3 + x^2 + 6x - 8 \ge 0 \Rightarrow x^3 - x^2 - 6x + 8 \le 0$$

$$x^3 - 2x^2 + x^2 - 2x - 4x + 8 = 0$$

$$x^2(x-2) + x(x-2) - 4(x-2) = 0$$

ricordiamoci qui la regola del DICE

DISCORDI INTERNI $\quad a \quad b$

CONCORDI ESTERNI $\quad a \quad b$

$$(x^2 + x - 4)(x - 2) = 0 \Rightarrow$$

$\downarrow F_1 \quad \downarrow F_2$
$= 0 \quad\quad = 0$

- $x^2 + x - 4 = 0$

$$x = \frac{-1 \pm \sqrt{1+16}}{2} = \boxed{-1 \pm \quad 7}$$

- $x - 2 = 0 \Rightarrow \boxed{x = 2}$

	$\frac{-1-\sqrt{17}}{2}$	$\frac{-1+\sqrt{17}}{2}$	2	
$-$	$+$	$-$	$-$	F_1
$+$	$+$	$+$	$-$	F_2
$-$	$+$	$-$	$+$	

$$\left(-\infty; \frac{-1-\sqrt{17}}{2}\right) \cup \left(\frac{-1+\sqrt{17}}{2}; 2\right)$$

$$\boxed{N \ge 0}$$

$\boxed{D>0}$

$\underset{F_1}{(x+4)}\underset{F_2}{(x-4)}\underset{F_3}{(x+6)} > 0$

$F_1 > 0 \quad x+4>0 \Rightarrow \underline{x>-4}$

$F_2 > 0 \quad x-4>0 \Rightarrow \underline{x>4}$

$F_3 > 0 \quad x+6>0 \Rightarrow \underline{x>-6}$

		-6		-4		4	
F_1	$-$		$-$		$+$		$+$
F_2	$-$		$-$		$-$		$+$
F_3	$-$		$+$		$+$		$+$
	$-$		$+$		$-$		$+$

$(-6;-4) \quad (4;+\infty)$

$\boxed{D>0}$

Unione delle soluzioni N e D

N $\left(-\infty; \frac{-1-\sqrt{17}}{2}\right) \cup \left(\frac{-1+\sqrt{17}}{2}; 2\right)$

D $(-6;-4) \cup (4;+\infty)$

		-6		-4		$\frac{-1-\sqrt{17}}{2}$		0		$\frac{-1+\sqrt{17}}{2}$		2		4	
N	$-$		$-$		$-$		$+$		$+$		$-$		$+$		$+$
D	$+$		$+$		$-$		$-$		$-$		$-$		$-$		
	$-$		$-$		$+$		$-$		$-$		$+$		$-$		$+$

$$(-\infty; -4) \cup \left(\frac{-1-\sqrt{17}}{2}; \frac{-1+\sqrt{17}}{2}\right) \cup (2; 4)$$