Disequazioni con moduli

① $|(x-4)(x-1)-x^2| < 1$

$|\cancel{x^2}-x-4x+4-\cancel{x^2}| < 1$

$|-5x+4| < 1$

a $\begin{cases} -5x+4 \geq 0 \\ -5x+4 < 1 \end{cases}$ $\cup$ b $\begin{cases} -5x+4 < 0 \\ 5x-4 < 1 \end{cases}$

Osservazione per alcune particolari disequazioni con MODULI

90% dei casi si risolve così: proseguiamo con 2 sistemi ma osservando notiamo che c'è un TRUCCO

Osser. termine di dx e' un num.

$[|a| < 1]$ → ho polinomio con x ma un numero

$-1 < a < 1$

$a = -100$ $a < 1$

$|-100| < 1$ no

$a > 1$ $a = 100$

$|100| < 1$

$100 < 1$ NO

$-1 < a < 1$

-1 0 1

$a = \frac{1}{4}$

$|a| < 1$

$\frac{1}{4} < 1$

$-\frac{1}{4} = a$

$|-\frac{1}{4}| < 1$

si $\frac{1}{4} < 1$

$|a| > 1$

$a < -1 \vee a > 1$

-1 1

si

$a = -3$

$|-3| > 1$

$3 > 1$

$a = 7$

$|7| > 1$ $7 > 1$

$|a| < -7$ modulo positivo ⚠

mai soddisfatto

$|a| > -7$ sempre $\forall a \in \mathbb{R}$

qualunque valore di a sarà sempre positivo → quindi > -7

$$|-5x+4| < 1$$

$$-1 < -5x+4 < 1$$

quando un modulo è minore di 1 → $|a| < 1$

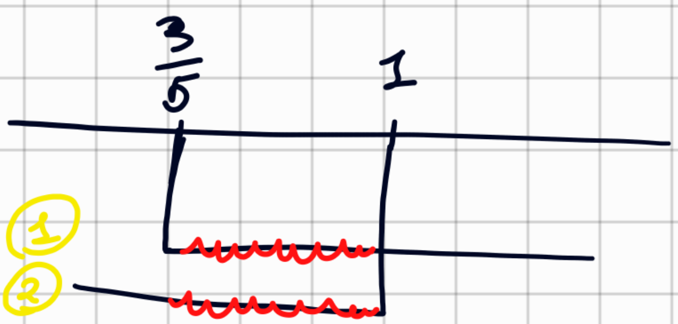
$$-1 < a < 1$$

$$\begin{cases} -5x+4 < 1 & (1) \\ -5x+4 > -1 & (2) \end{cases}$$

1 solo sistema

(1) $-5x+4 < 1 \Rightarrow -5x < -4+1 \Rightarrow -5x < -3 \Rightarrow \frac{\cancel{5}x}{\cancel{5}} > \frac{3}{5}$ $\quad \boxed{x > \frac{3}{5}}$

(2) $-5x+4 > -1 \Rightarrow -5x > -4-1 \Rightarrow -5x > -5 \Rightarrow \frac{\cancel{5}x}{\cancel{5}} < \frac{\cancel{5}}{\cancel{5}}\,1$ $\quad \boxed{x < 1}$



$$\boxed{\frac{3}{5} < x < 1}$$

Disequazioni con più moduli

① $|x-1| - \left|\frac{1}{2}x - 8\right| < x$

m_1 $\qquad$ m_2

studiamo i segni dei moduli (m_1, m_2)

- m_1 $\quad x-1 \geq 0 \quad \boxed{x \geq 1}$
- m_2 $\quad \frac{1}{2}x - 8 \geq 0 \quad \frac{1}{2}x \geq 8 \quad \times 2 \quad \boxed{x \geq 16}$

Studio del segno

	① $x \leq 1$	② $1 < x \leq 16$	③ $x > 16$
m_1	$-$	$+$	$+$
m_2	$-$	$-$	$+$

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① $\begin{cases} x \leq 1 \\ -x + 1 + \frac{1}{2}x - 8 < 1 \end{cases}$

qui i due moduli avevano argomenti negativi (-) e quindi posso togliere più i moduli e cambiare i segni agli argomenti

② $\begin{cases} 1 < x \leq 16 \\ x - 1 + \frac{1}{2}x - 8x < x \end{cases}$

qui il modulo 1 era positivo (quindi non cambia di segno)

m2 il modulo 2 era negativo e quindi si cambia di segno al suo argomento

③ $\begin{cases} x > 16 \\ x - 1 - \frac{1}{2}x + 8 < x \end{cases}$

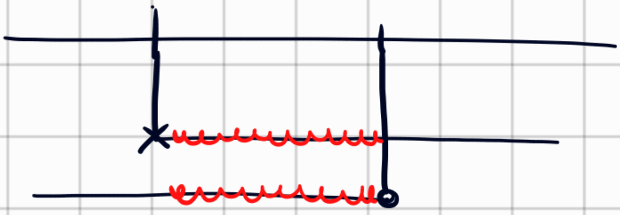
qui invece erano (m_1 e m_2) positivi entrambi e quindi non cambiano di segno

Risoluzione separata dei sistemi ① ② ③

① $\begin{cases} x \leq 1 \\ -x + 1 + \frac{1}{2}x - 8 < x \end{cases} \Rightarrow \begin{cases} x \leq 1 \\ -x + \frac{x}{2} - x < -1 + 8 \end{cases} \Rightarrow$

$\Rightarrow -\frac{3}{2}x < 7 \Rightarrow -3x < 14 \Rightarrow 3x > -14 \quad \frac{3x}{3} > -\frac{14}{3}$

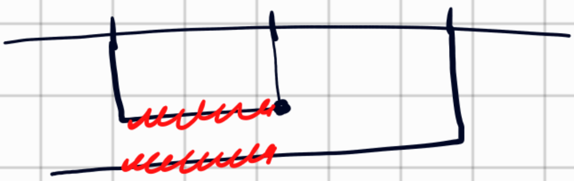
$\boxed{x > -\frac{14}{3}}$



① $\boxed{-\frac{14}{3} < x \leq 1}$

② $\begin{cases} 1 < x \leq 16 \\ \cancel{x} - 1 + \frac{x}{2} - 8 < \cancel{x} \end{cases} \Rightarrow \begin{cases} // \quad // \\ \frac{x}{2} < +1+8 \end{cases} \Rightarrow$

$\frac{x}{2} < 9 \Rightarrow \boxed{x < 18}$



② $\boxed{1 < x \leq 16}$

③ $\begin{cases} \boxed{x > 16} \\ \cancel{x} - 1 - \frac{1}{2}x + 8 < \cancel{x} \end{cases} \Rightarrow \begin{cases} // \quad // \\ -\frac{x}{2} < -8+1 \end{cases} \Rightarrow -\frac{x}{2} < -7$

$\frac{x}{2} > 7 \Rightarrow \boxed{x > 14}$

③ $\boxed{x > 16}$

① $\boxed{-\frac{14}{3} < x \le 1}$

② $\boxed{1 < x \le 16}$

③ $\boxed{x > 16}$

Unione delle soluzioni

$$\boxed{x > -\frac{14}{3}}$$

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① $\frac{|x| - x}{2x^2 - 1} \ge -2$

CE

• $2x^2 - 1 \neq 0$

$\frac{2x^2}{2} \neq \frac{1}{2} \Rightarrow x^2 \neq \frac{1}{2} \Rightarrow$

$\Rightarrow x \neq \frac{\sqrt{1}}{\sqrt{2}} = \frac{1}{\sqrt{2}}$

$\Rightarrow \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \boxed{\frac{\sqrt{2}}{2}}$

$\boxed{x \neq \pm\frac{\sqrt{2}}{2}}$

$$\Rightarrow \frac{|x| - x}{2x^2 - 1} + 2 \ge 0 \Rightarrow$$

$$\Rightarrow \frac{|x| - x + 2(2x^2 - 1)}{2x^2 - 1} \ge 0 \Rightarrow$$

$$\Rightarrow \frac{|x| - x + 4x^2 - 2 \quad (N)}{2x^2 - 1 \quad (D)} \ge 0$$

(N) > 0

$|x| - x + 4x^2 - 2 > 0$

$$\underbrace{\begin{cases} x \ge 0 \\ x - x + 4x^2 - 2 > 0 \end{cases}}_{①} \cup \underbrace{\begin{cases} x < 0 \\ -x - x + 4x^2 - 2 > 0 \end{cases}}_{②}$$

1

$$\begin{cases} x \geq 0 \\ \cancel{x} - \cancel{x} + 4x^2 - 2 > 0 \end{cases} \Rightarrow \begin{cases} x \geq 0 \\ 4x^2 - 2 > 0 \end{cases} \Rightarrow$$

equazione associata

$$\Rightarrow 4x^2 - 2 = 0$$

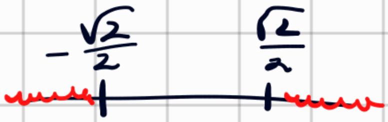
$$\Rightarrow \frac{\cancel{4}x^2}{\cancel{4}} = \frac{\cancel{2}^{1}}{\cancel{4}_{2}} \Rightarrow$$

$$\Rightarrow x^2 = \frac{1}{2} \Rightarrow x = \pm\sqrt{\frac{1}{2}}$$

$$\Rightarrow x = \pm\frac{1}{\sqrt{2}} \Rightarrow$$

$$\boxed{x = \pm\frac{\sqrt{2}}{2}}$$

concordi esterni



$$\begin{cases} x \geq 0 \\ x < -\frac{\sqrt{2}}{2} \vee x > \frac{\sqrt{2}}{2} \end{cases}$$

2

$$\begin{cases} x < 0 \\ -x - x + 4x^2 - 2 > 0 \end{cases} \Rightarrow \begin{cases} x < 0 \\ -2x + 4x^2 - 2 > 0 \end{cases}$$

$$\Downarrow$$

$$\begin{cases} x < 0 \\ 4x^2 - 2x - 2 > 0 \end{cases}$$

$$\frac{4x^2}{2} - \frac{2x}{2} - \frac{2}{2} = 0$$

$$2x^2 - x - 1 = 0$$

$$x = \frac{1 \pm \sqrt{1+8}}{4} =$$

$$= \frac{1 \pm 3}{4} = \begin{cases} \frac{4}{4} = 1 \\ \frac{-\cancel{2}^{1}}{\cancel{4}_{2}} = -\frac{1}{2} \end{cases}$$

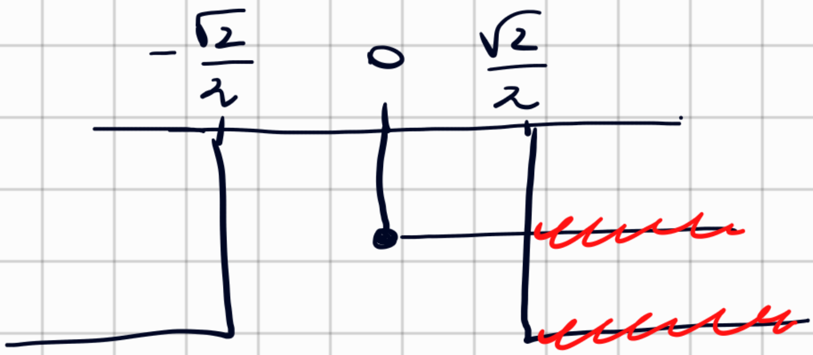
concordi esterni

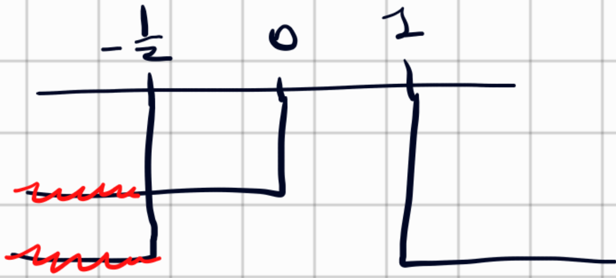
$$\begin{cases} x < 0 \\ x < -\frac{1}{2} \vee x > 1 \end{cases}$$

1

$$\begin{cases} x \geq 0 \\ x < -\frac{\sqrt{2}}{2} \vee x > \frac{\sqrt{2}}{2} \end{cases} \cup \begin{cases} x < 0 \\ x < -\frac{1}{2} \vee x > 1 \end{cases}$$

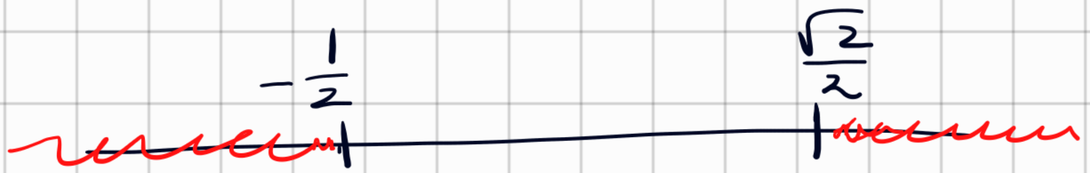
2





$$\boxed{x > \frac{\sqrt{2}}{2}}$$

$$\boxed{x < -\frac{1}{2}}$$



$N>0$ $\boxed{x < -\frac{1}{2} \vee x > \frac{\sqrt{2}}{2}}$

$\boxed{D>0}$ $2x^2-1>0 \Rightarrow 2x^2-1=0$

$\Rightarrow 2x^2=1 \Rightarrow x=\pm\frac{\sqrt{2}}{2}$

$\boxed{x < -\frac{\sqrt{2}}{2} \vee x > \frac{\sqrt{2}}{2}}$

concordi
esterni

		$-\frac{\sqrt{2}}{2}$		$-\frac{1}{2}$		$\frac{\sqrt{2}}{2}$	
N	+		+		−		+
D	+		−		−		+
	+		−		+		+

$$\left(-\infty; -\frac{\sqrt{2}}{2}\right) \cup \left(-\frac{1}{2}; \frac{\sqrt{2}}{2}\right) \cup \left(\frac{\sqrt{2}}{2}; +\infty\right)$$