

Esercizi d: Analisi Matematica

$$\textcircled{1} \quad \frac{1}{x} + \frac{1}{x-1} + \frac{1}{x+2} < 0 \Rightarrow$$

$$\frac{(x-1)(x+2) + x(x+2) + x(x-1)}{x(x-1)(x+2)} < 0$$

| CE

$$x \neq 0$$

$$x-1 \neq 0$$

$$x+2 \neq 0$$

$$x \neq 1$$

$$x \neq -2$$

$$\frac{x^2 + 2x - x - 2 + x^2 + 2x + x^2 - x}{x(x-1)(x+2)} < 0$$

$$\frac{N}{D} < 0 \quad N > 0 \quad D > 0$$

$$N > 0 \quad 3x^2 + 2x - 2 > 0 \Rightarrow$$

$$3x^2 + 2x - 2 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-2 \pm \sqrt{4 + 24}}{6} = \frac{-2 \pm \sqrt{28}}{6} \Rightarrow 2^{\text{grado}}$$

$$= \frac{-2 \pm 2\sqrt{7}}{6} = \frac{2(-1 \pm \sqrt{7})}{6} = \frac{-1 \pm \sqrt{7}}{3}$$

secondo la regola del

In questo caso abbiamo

$$\frac{-1 \pm \sqrt{7}}{3}$$

sol di

$$3x^2 + 2x - 2 > 0$$

concordi esterni

Di | CE

DISCORDI INTERNI

CONCORDI ESTERNI

$$\frac{-1 - \sqrt{7}}{3}$$

$$\frac{-1 + \sqrt{7}}{3}$$

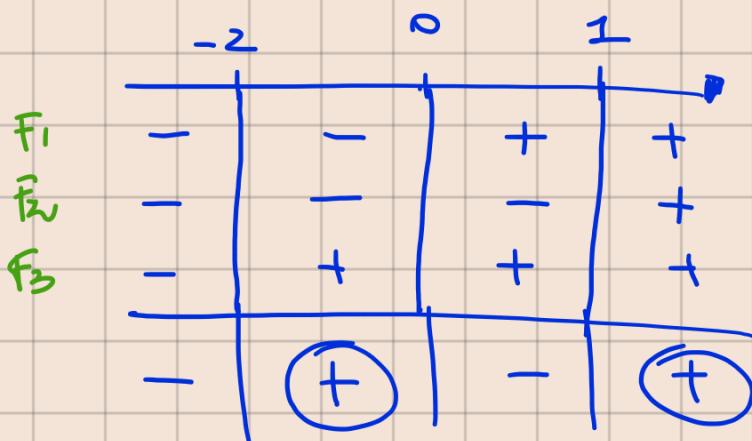
$$\bullet \left(-\infty; \frac{-1 - \sqrt{7}}{3} \right) \cup \left(\frac{-1 + \sqrt{7}}{3}; +\infty \right)$$

$$D > 0 \quad x(x-1)(x+2) > 0$$

$$F_1 \bullet x > 0$$

$$F_2 \bullet x - 1 > 0 \Rightarrow x > 1$$

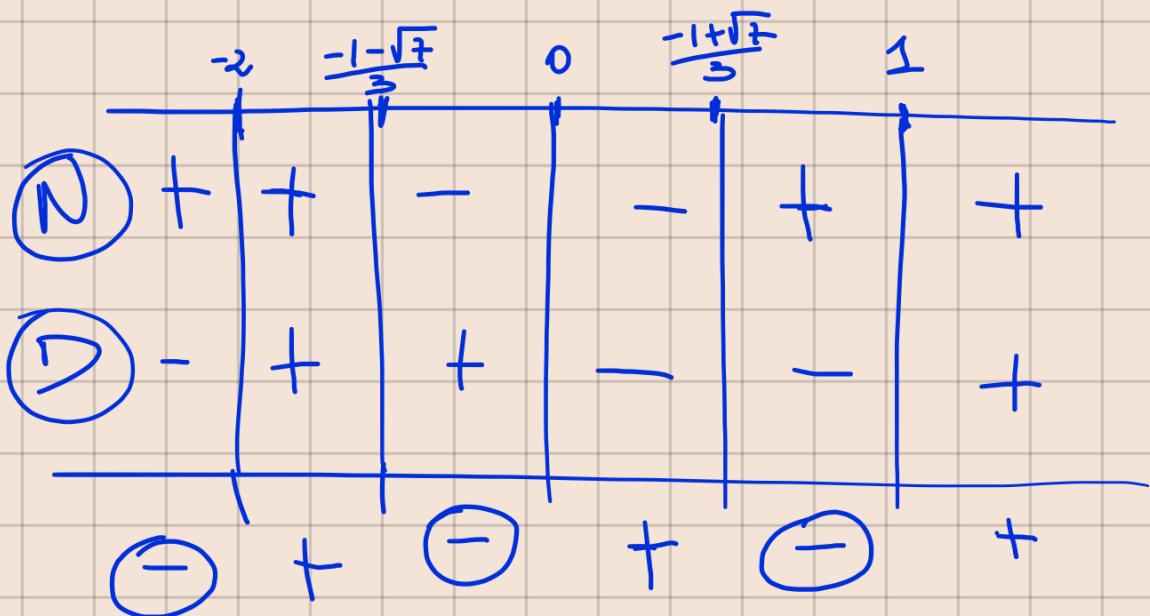
$$F_3 \bullet x + 2 > 0 \Rightarrow x > -2$$



$$\bullet (-2; 0) \cup (1; +\infty)$$

$$D \bullet (-\infty; 0) \cup (1; +\infty)$$

$$N \bullet \left(-\infty; \frac{-1-\sqrt{7}}{3}\right) \cup \left(\frac{-1+\sqrt{7}}{3}; +\infty\right)$$



Soluzione

$$(-\infty; -2) \cup \left(-\frac{1-\sqrt{7}}{3}; 0\right) \cup \left(\frac{-1+\sqrt{7}}{3}; 1\right)$$

$$\textcircled{1} \quad \frac{x(x+1)^2}{x^2-16} \leq \frac{(x+1)^3}{x^2+2x-24}$$

DA RIFARE

$$\textcircled{CE} \quad x^2-16 \neq 0 \Rightarrow x^2 \neq 16 \quad x \neq \pm 4$$

$$\boxed{\begin{array}{l} x \neq 4 \\ x \neq -4 \end{array}}$$

$$x^2 + 2x - 24 \neq 0$$

$$-24x$$

$$4 =$$

$$\frac{1}{96}$$

$$x = \frac{-2 \pm \sqrt{4 + 96}}{2} = \frac{-2 \pm 10}{2}$$

$$\begin{array}{l} + \\ \frac{-2 + 10}{2} = \frac{8}{2} = 4 \\ - \\ \frac{-2 - 10}{2} = \frac{-12}{2} = -6 \end{array}$$

$$\left. \begin{array}{l} x \neq 4 \\ x \neq -6 \end{array} \right\}$$

$$x^2 + 2x - 24 = (x-4)(x+6)$$

$$\frac{x(x+1)^2}{x^2-16} \leq \frac{(x+1)^3}{x^2+2x-24} \Rightarrow \frac{x(x+1)^2}{(x-4)(x+4)} \leq \frac{(x+1)^3}{(x-4)(x+6)}$$

$$\downarrow$$

$$(x+4)(x-4)$$

$$\frac{x(x+6)(x+1)^2}{(x+4)(x-4)(x+6)} \leq \frac{(x+4)(x+1)^3}{(x+4)(x-4)(x+6)} \Rightarrow$$

$$\frac{(x^2+6x)(x+1)^2}{(x+4)(x-4)(x+6)} \leq \frac{(x+4)(x^3+1+3x^2+x)}{(x+4)(x-4)(x+6)} \Rightarrow$$

$$\frac{(x^2+6x)(x^2+1+2x)}{(x+4)(x-4)(x+6)} \leq \frac{x^4+x^3+3x^3+x^2+x^3+4x^3+4+12x^2+4}{(x+4)(x-4)(x+6)} \Rightarrow$$

$$\Rightarrow \frac{x^4 + x^2 + 2x^2 + 6x^3 + 6x + 12x^2}{D} \leq \frac{x^4 + x^2 + 7x^3 + 13x^2 + 8}{D} \Rightarrow$$

$$\Rightarrow \frac{x^4 + 7x^2 + 14x^2 + 6x^3}{D} \leq \frac{x^4 + x^2 + 7x^3 + 13x^2 + 8}{D} \Rightarrow$$

$$\Rightarrow \frac{\cancel{x^4} + \cancel{7x^2} + \cancel{14x^2} + \cancel{6x^3} - \cancel{x^4} - \cancel{7x^3} - \cancel{13x^2} - 8}{D} \leq 0 \Rightarrow$$

$$\Rightarrow \frac{-x^3 + x^2 + 6x - 8}{D} \leq 0 \Rightarrow$$

N

$$\frac{-x^3 + x^2 + 6x - 8}{(x+4)(x-4)(x+6)} \leq 0$$

D

$N > 0$

$$-x^3 + x^2 + 6x - 8 \geq 0 \Rightarrow \frac{x^3 - x^2 - 6x + 8}{D} \leq 0$$

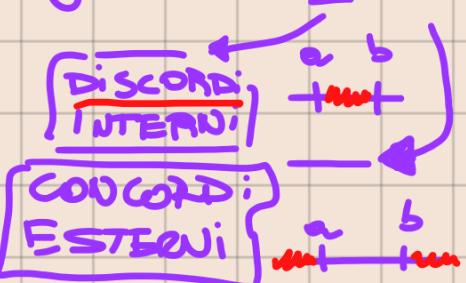
$$\cancel{x^3} - \cancel{2x^2} + \cancel{x^2} - 2x - 4x + 8 = 0$$

$$\cancel{x^2}(x-2) + x(x-2) - 4(x-2) = 0$$

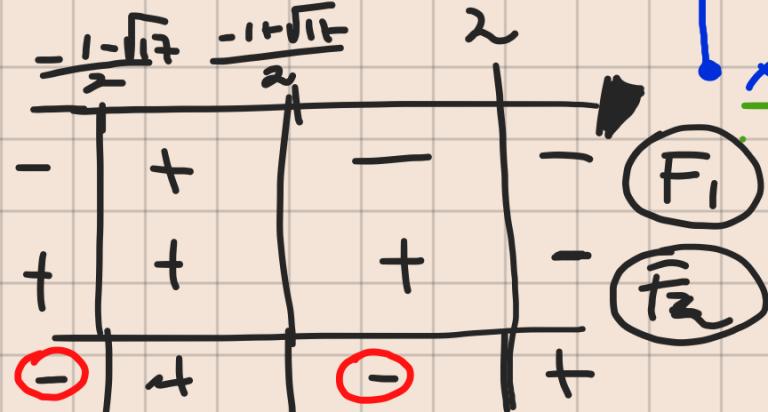
$$(x^2 + x - 4)(x - 2) = 0 \Rightarrow \frac{x^2 + x - 4}{D} = 0$$

$F_1 \quad F_2$

ricordiamoci qui la regola del DICE



$$x = \frac{-1 \pm \sqrt{1+16}}{2} = -1 \pm 3$$



$$(-\infty; \frac{-1-\sqrt{17}}{2}) \cup (\frac{-1+\sqrt{17}}{2}; 2)$$

$N > 0$

$D > 0$

$$(x+4)(x-4)(x+6) > 0$$

F_1 F_2 F_3

$$F_1 > 0$$

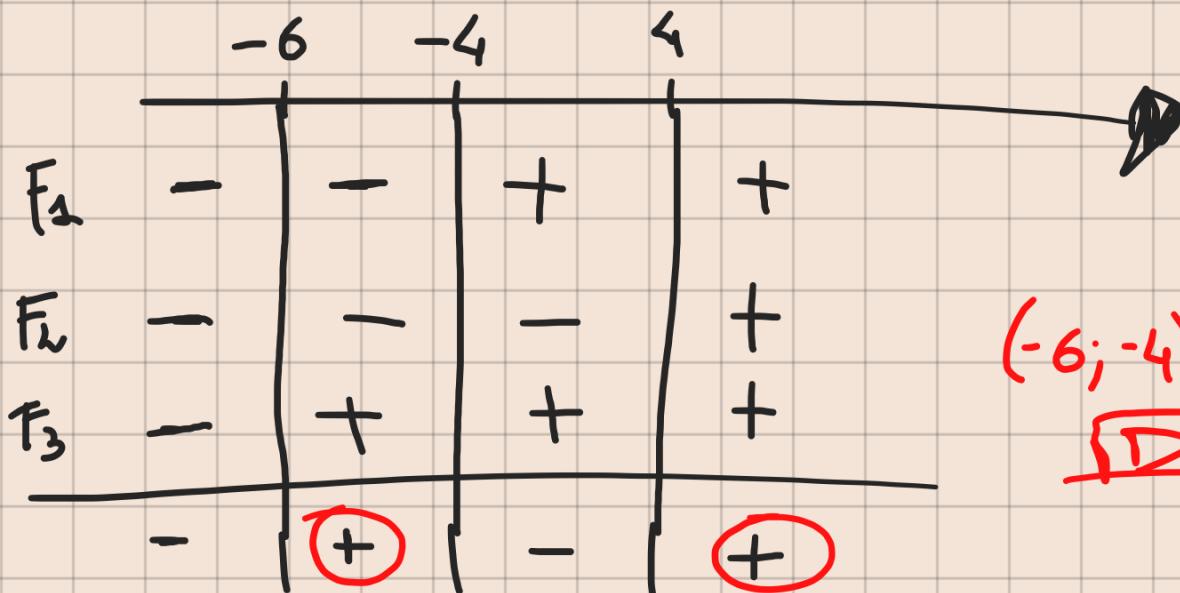
$$x+4 > 0 \Rightarrow x > -4$$

$$F_2 > 0$$

$$x-4 > 0 \Rightarrow x > 4$$

$$F_3 > 0$$

$$x+6 > 0 \Rightarrow x > -6$$



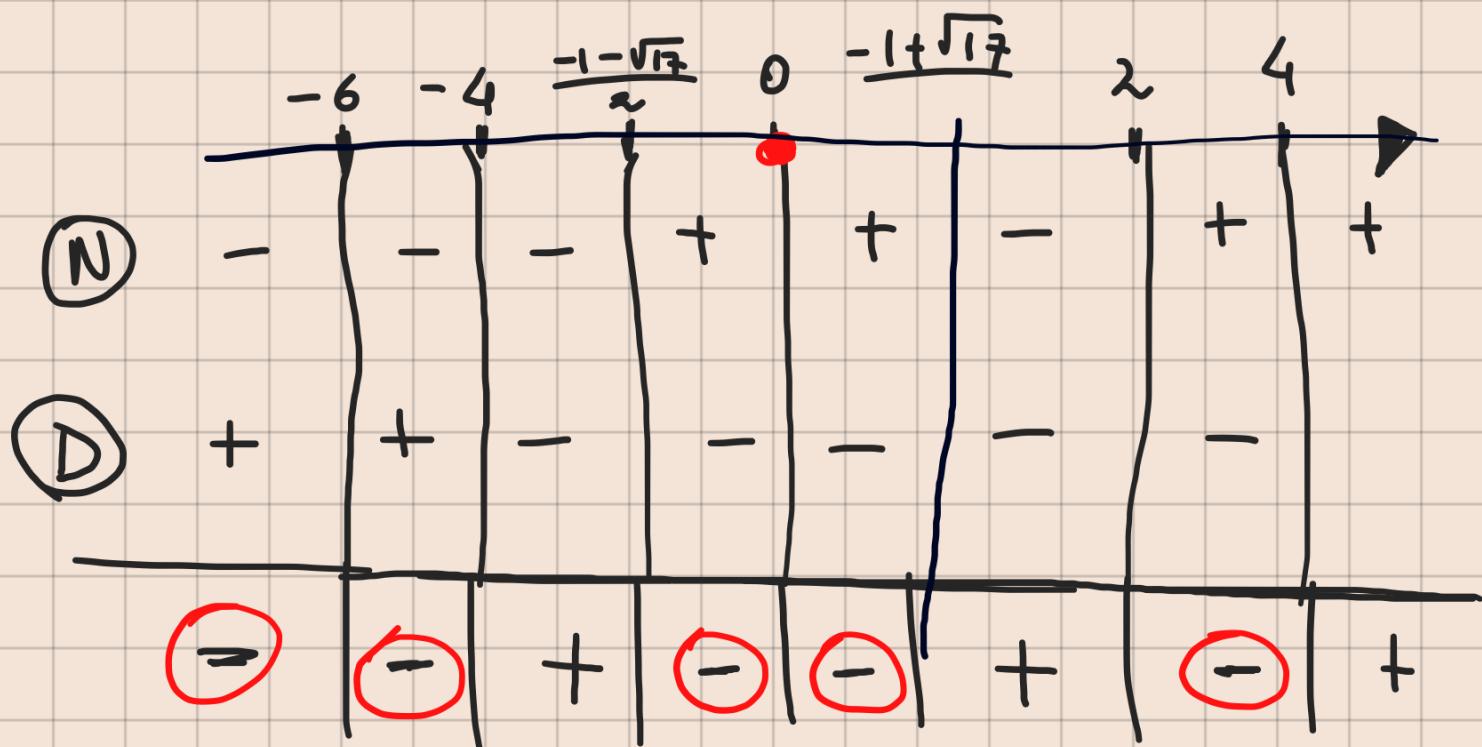
$$(-6; -4) \cup (4; +\infty)$$

$\boxed{D > 0}$

Unione delle soluzioni:
 $N \in D$

$$N \left(-\infty; \frac{-1-\sqrt{17}}{2} \right) \cup \left(\frac{-1+\sqrt{17}}{2}; +\infty \right)$$

$$D (-6; -4) \cup (4; +\infty)$$



$$(-\infty; -4) \cup \left(\frac{-1-\sqrt{17}}{2}; \frac{-1+\sqrt{17}}{2} \right) \cup (2; 4)$$



