

HW3: Linear Regression
CS383 - Machine Learning
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Part 1.1: Theory

$$H(a,b) = -\frac{a}{a+b} \log_2 \frac{a}{a+b} - \frac{b}{a+b} \log_2 \frac{b}{a+b}$$

A.

$$Y_{positive} = 3 + 4 + 4 + 1 = 12$$

$$Y_{negative} = 0 + 1 + 3 + 5 = 9$$

$$H(Y) = H(9,12) = 0.985$$

B.

Feature X1:

$$P_T = 7, N_T = 1$$

$$P_F = 5, N_F = 8$$

$$E(H(X1)) = \frac{8}{21}H(7,1) + \frac{13}{21}H(5,8) = 0.20707 + 0.59505 = 0.802$$

$$IG(X1) = H(Y) - E(H(X1)) = 0.985 - 0.802 = 0.183$$

Feature X2:

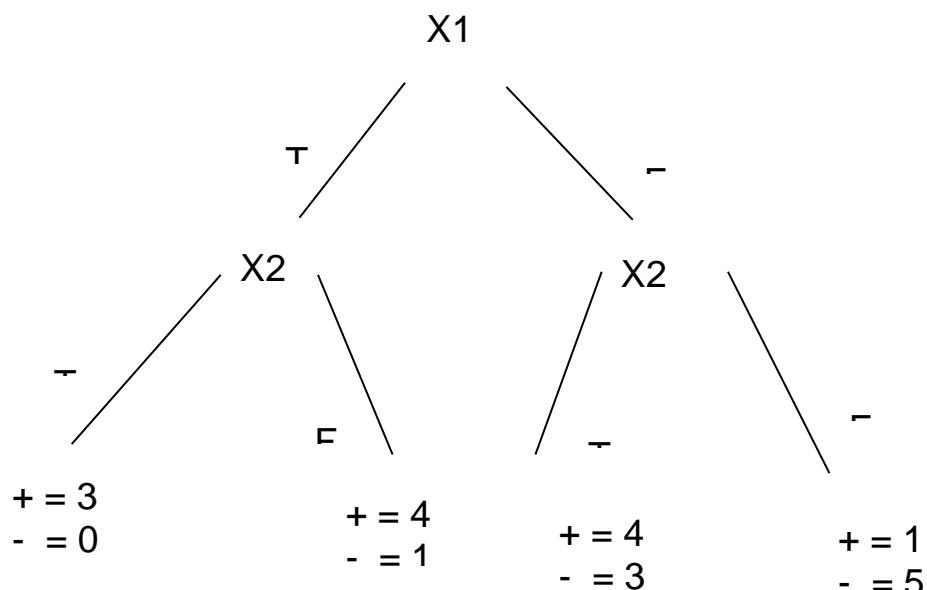
$$P_T = 7, N_T = 3$$

$$P_F = 5, N_F = 6$$

$$E(H(X1)) = \frac{10}{21}H(7,3) + \frac{11}{21}H(5,6) = 0.420 + 0.5207 = 0.940$$

$$IG(X1) = H(Y) - E(H(X1)) = 0.985 - 0.940 = 0.045$$

C.



Part 1.2: Theory

A.

$$P(A = Yes) = 3/5$$

$$P(A = No) = 2/5$$

B.

X =

[[216. 5.68]
[69. 4.78]
[302. 2.31] => Standardize =>
[60. 3.16]
[393. 4.2]]

X =

[[0.05509059 1.24771393]
[-0.95719904 0.56878857]
[0.64731446 -1.29448434]
[-1.01917595 -0.65327706]
[1.27396994 0.1312589]]

Class A:

[[0.05509059 1.24771393]
[-0.95719904 0.56878857]
[-1.01917595 -0.65327706]
mean_A = [-0.640 0.387]
Stdev_A = [0.603 0.963]

mean = [208 4.026]

Stdev = [145.2 1.326]

Class non A:

[0.64731446 -1.29448434]
[1.27396994 0.1312589]
mean_nonA = [0.961 -0.582]
Stdev_nonA = [0.443 1.008]

C.

$$Gaussian(x_k, \mu_i, \sigma_i) = \frac{1}{\sigma_i \sqrt{2\pi}} e^{-\frac{(x_k - \mu_i)^2}{2\sigma_i^2}}$$

$$F = [242 \ 4.56] \Rightarrow \text{standardize} \Rightarrow F = [0.234 \ 0.403]$$

$$P(Y = A) = P(A = Yes) Gaussian(0.234, -0.640, 0.603) Gaussian(0.403, 0.387, 0.963)$$

$$= 0.6 * 0.231 * 0.414 = 0.0574$$

$$P(Y \neq A) = P(A = No) Gaussian(0.234, 0.961, 0.443) Gaussian(0.403, -0.582, 1.008)$$

$$= 0.4 * 0.234 * 0.2455 = 0.0229$$

P(Y = A) > P(Y != A) => That student get an A

Part 1.3 Theory

A.

Since g(x) has to be in range [0,1] but tanh(x) is in range [-1,1] =>

$$g(x\theta) = \frac{\tanh(x\theta) + 1}{2}$$

B.

$$\tanh(z) = \frac{e^z - e^{-z}}{e^z + e^{-z}}$$

Proof: $\frac{\partial}{\partial \theta_j} (\tanh(x\theta)) = x_j (1 - \tanh(x\theta)^2).$

Solve:

$$\text{Note } \frac{d}{d\theta_j} e^{x\theta} = x_j e^{x\theta} \quad \text{and} \quad \frac{d}{d\theta_j} e^{-x\theta} = -x_j e^{-x\theta}$$

$$\begin{aligned} & \frac{d}{d\theta_j} \tanh(x\theta) \\ &= \frac{\frac{d}{d\theta_j} (e^{x\theta} - e^{-x\theta})(e^{x\theta} + e^{-x\theta}) - (e^{x\theta} - e^{-x\theta}) \frac{d}{d\theta_j} (e^{x\theta} + e^{-x\theta})}{(e^{x\theta} + e^{-x\theta})^2} \end{aligned}$$

$$\begin{aligned} & \frac{d}{d\theta_j} \tanh(x\theta) \\ &= \frac{(x_j e^{x\theta} + x_j e^{-x\theta})(e^{x\theta} + e^{-x\theta}) - (e^{x\theta} - e^{-x\theta})(x_j e^{x\theta} - x_j e^{-x\theta})}{(e^{x\theta} + e^{-x\theta})^2} \end{aligned}$$

$$\begin{aligned} & \frac{d}{d\theta_j} \tanh(x\theta) \\ &= x_j \frac{(e^{x\theta} + e^{-x\theta})(e^{x\theta} + e^{-x\theta}) - (e^{x\theta} - e^{-x\theta})(e^{x\theta} - e^{-x\theta})}{(e^{x\theta} + e^{-x\theta})^2} \end{aligned}$$

$$\frac{d}{d\theta_j} \tanh(x\theta) = x_j \left(1 - \frac{(e^{x\theta} - e^{-x\theta})^2}{(e^{x\theta} + e^{-x\theta})^2} \right) = x_j (1 - \tanh(x\theta)^2)$$

Part 2: Naive Bayes

Precision: 0.719660

Recall: 0.962662

F-Measure: 0.823611

Accuracy: 0.834312

Part 3: Recall and Precision:

Dataset:

<https://www.kaggle.com/yersever/500-person-gender-height-weight-bodymassindex/version/2?#500+Person+Gender+Height+Weight+Index.csv>

This dataset was modified to be similar to given format (no header and last column is label) in order to be used in our function

Precision vs Recall

