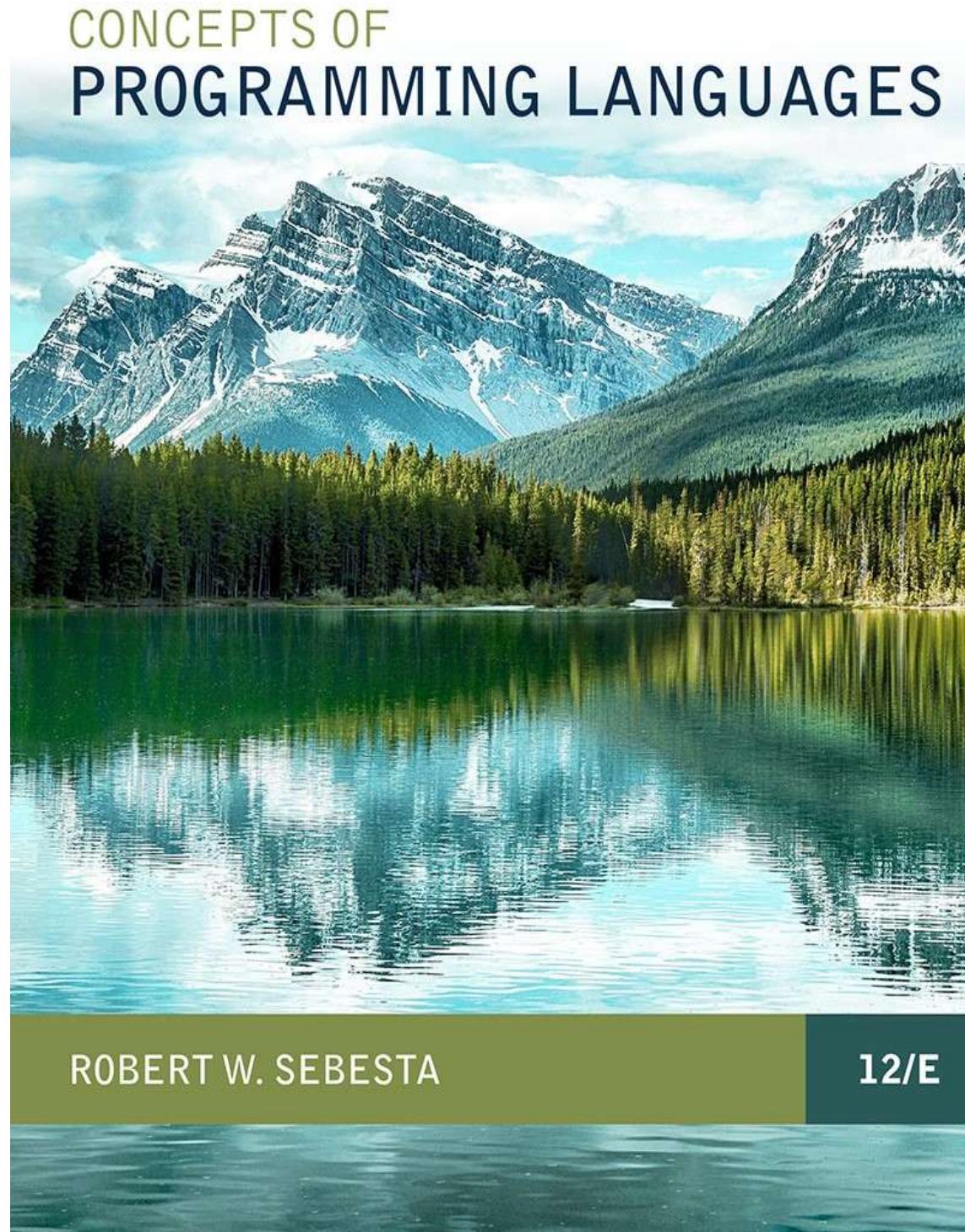


Lecture 2

Describing Syntax

Chapter 3 – Part 1

First Semester
1447 - 2025



Lecture 2 Topics:

- Introduction
- The General Problem of Describing **Syntax**:
 - Language **Recognizers**
 - Language **Generators**
- Formal Methods of Describing **Syntax**:
 - Backus–Naur Form (**BNF**):
 - Extended BNF (**EBNF**)
 - Context–Free **Grammars** (**CFG**):
 - Grammars
 - **Derivations**
 - Parse Trees
 - **Ambiguity**
- Attribute Grammars:
 - **Static Semantics**

Introduction

- The study of programming languages, like the study of natural languages, can be divided into:
 - Examinations of **syntax**.
 - Examinations of **semantics**.
- **Syntax**: the form or structure of the expressions, statements, and program units.
- **Semantics**: the meaning of the expressions, statements, and program units.
- **Syntax** and **semantics** provide a language's definition.

Introduction: Example

- Example:
 - The **syntax** of a Java “while” statement is:
 - while (boolean_expr) statement
 - The **semantics** of the same statement is:
 - When the current value of the Boolean expression is true, the embedded statement is executed.
 - Then control implicitly returns to the Boolean expression to repeat the process.
 - If the Boolean expression is false, control transfers to the statement following the while construct.

Introduction: Language Users

- **Users** of a **language definition**:
 - Other language designers (evaluators).
 - Implementers.
 - Programmers (the users of the language).

The General Problem of Describing Syntax: Terminology

- A *sentence* (statement) is a string of characters over some alphabet.
- A *language* is a set of sentences (statements).
- A *lexeme* is the lowest level syntactic unit of a language (e.g., *, sum, begin).
 - The language operators.
 - The language special words.
 - The language numerical literals.
 - Etc.
- A *token* is a category of lexemes (e.g., identifier).

Example

- Consider the following Java statement:

➤ `index = 2 * count + 17;`

Lexemes	Tokens
index	identifier
=	equal_sign
2	int_literal
*	mult_op
count	identifier
+	plus_op
17	int_literal
;	semicolon

Formal Definition of Languages

- **Recognizers:**

- A recognition device reads input strings over the alphabet of the language and decides whether the input strings belong to the language (accept or reject the given input strings).
- Example: syntax analysis (parsing) part of a compiler.
 - The syntax analyzer determines whether the given programs are syntactically correct.
 - Detailed discussion of syntax analysis appears in Chapter 4.

- **Generators:**

- A device that generates sentences of a language.
- One can determine if the syntax of a particular sentence is syntactically correct by comparing it to the structure of the generator.

BNF and Context-Free Grammars

- Context-Free Grammars:
 - Developed by Noam Chomsky in the mid-1950s.
 - Language **generators**, meant to describe the syntax of natural languages.
 - Define a class of languages called **context-free languages**.
- Backus–Naur Form (1959):
 - Invented by John Backus to describe the syntax of Algol 58.
 - BNF is equivalent to context-free grammars.

BNF Fundamentals

- BNF is a natural notation for describing syntax.
- In BNF, abstractions are used to represent classes of syntactic structures.
 - They act like syntactic variables (also called *nonterminal symbols*, or just *terminals*).
- Example:
 - A simple Java assignment statement might be represented by the abstraction `<assign>`
 - Pointed brackets are often used to delimit names of abstractions.
 - The actual definition of `<assign>` can be given by:
 - `<assign> → <var> = <expression>`

BNF Fundamentals (continued)

- **Terminals** are **lexemes** or **tokens**.
- A **rule** has a left-hand side (LHS), which is a **nonterminal**, and a right-hand side (RHS), which is a string of **terminals** and/or **nonterminals**.
- **Example:**
 - See the next slide!

<assign> → <var> = <expression>

- The **text** on the left side of the arrow, which is aptly called the **left-hand side (LHS)**, is the **abstraction** being defined.
- The **text** to the right of the arrow is the **definition** of the **LHS**.
- It is called the **right-hand side (RHS)** and consists of some mixture of tokens, lexemes, and references to **other abstractions**. (Actually, tokens are also **abstractions**.)
- Altogether, the definition is called a **rule**, or **production**.
- In the example **rule** just given, the **abstractions** **<var>** and **<expression>** obviously must be defined for the **<assign>** definition to be useful.

How a rule can be read?

<assign> → **<var>** = **<expression>**

- This particular **rule** specifies that the **abstraction** **<assign>** is defined as an instance of the **abstraction** **<var>**, followed by the lexeme **=**, followed by an instance of the **abstraction** **<expression>**.
- One example **sentence** whose syntactic structure is described by the **rule** above is:
 - total = subtotal1 + subtotal2

BNF Fundamentals (continued)

- Nonterminals are often enclosed in “*angle brackets*” (abstraction).
 - Examples of BNF rules:

`<ident_list> → identifier | identifier, <ident_list>`

`<if_stmt> → if <logic_expr> then <stmt>`

- Grammar: a finite non-empty set of rules.
- A *start symbol* is a special element of the nonterminals of a grammar.

BNF Rules

- An **abstraction** (or **nonterminal** symbol) can have more than one RHS:

`<stmt> → <single_stmt>`

`<stmt> → begin <stmt_list> end`

- These two **rules** can be written as:

`<stmt> → <single_stmt> | begin <stmt_list> end`

BNF Rules: More Examples

- a Java if statement can be described with the **rules**:

`<if_stmt> → if (<logic_expr>) <stmt>`

`<if_stmt> → if (<logic_expr>) <stmt> else <stmt>`

or with the rule

`<if_stmt> → if (<logic_expr>) <stmt>`

`| if (<logic_expr>) <stmt> else <stmt>`

Describing Lists

- Example of a **list**:
 - a list of identifiers appearing on a data declaration statement.
- **Syntactic lists** are described using **recursion**:

$$\begin{aligned} <\text{ident_list}> \rightarrow & \text{ ident} \\ | & \text{ ident, } <\text{ident_list}> \end{aligned}$$

- A **rule** is **recursive** if its LHS appears in its RHS.

Derivation

- A **grammar** is a generative device for defining languages.
- A **derivation** is a repeated application of **rules** (grammars), starting with the **start symbol** and ending with a **sentence** (all terminal symbols).
- The **start symbol** represents a complete program and is often named <program>.

An Example Grammar

<program> → <stmts>

<stmts> → <stmt> | <stmt> ; <stmts>

<stmt> → <var> = <expr>

<var> → **a** | **b** | **c** | **d**

<expr> → <term> + <term> | <term> - <term>

<term> → <var> | const

- The **language** described by the **grammar** of has only one statement form:
 - assignment.

An Example Derivation

- How can we **derive** the following **assignment** statement using the previous **grammars** (rules)?

- **a = b + const**

<program> \Rightarrow <stmts>

\Rightarrow <stmt>

\Rightarrow <var> = <expr>

\Rightarrow **a** = <expr>

\Rightarrow **a** = <term> + <term>

\Rightarrow **a** = <var> + <term>

\Rightarrow **a** = **b** + <term>

\Rightarrow **a** = **b** + **const**

The symbol \Rightarrow is
read “**derives**.”

Another Example Grammar

A Grammar for a Small Language

`<program> → begin <stmt_list> end`

`<stmt_list> → <stmt>`
 `| <stmt> ; <stmt_list>`

`<stmt> → <var> = <expression>`

`<var> → A | B | C`

`<expression> → <var> + <var>`
 `| <var> - <var>`
 `| <var>`

How can we **derive** the following small program?

begin

A = **B** + **C** ;

B = **C**

end

Another Example Derivation

```
<program> => begin <stmt_list> end  
          => begin <stmt> ; <stmt_list> end  
          => begin <var> = <expression> ; <stmt_list> end  
          => begin A = <expression> ; <stmt_list> end  
          => begin A = <var> + <var> ; <stmt_list> end  
          => begin A = B + <var> ; <stmt_list> end  
          => begin A = B + C ; <stmt_list> end  
          => begin A = B + ; <stmt> end  
          => begin A = B + C ; <var> = <expression> end  
          => begin A = B + C ; B = <expression> end  
          => begin A = B + C ; B = <var> end  
          => begin A = B + C ; B = C end
```

Derivations

- Every string of symbols in a derivation is a *sentential form*.
- A *sentence* (statement) is a sentential form that has only terminal symbols.
- A *leftmost derivation* is one in which the leftmost nonterminal in each sentential form is the one that is expanded.
- A derivation may be neither *leftmost* nor *rightmost*.

Yet Another Example Grammar

A Grammar for Simple Assignment Statements

$\langle \text{assign} \rangle \rightarrow \langle \text{id} \rangle = \langle \text{expr} \rangle$

$\langle \text{id} \rangle \rightarrow A \mid B \mid C$

$\langle \text{expr} \rangle \rightarrow \langle \text{id} \rangle + \langle \text{expr} \rangle$

$\mid \langle \text{id} \rangle * \langle \text{expr} \rangle$

$\mid (\langle \text{expr} \rangle)$

$\mid \langle \text{id} \rangle$

How can we **derive** the following **assignment** statement?

$A = B * (A + C)$

Yet Another Example Derivation

- The statement $A = B * (A + C)$ is generated by the following **leftmost derivation**:

```
<assign> => <id> = <expr>
            => A   = <expr>
            => A   = <id> * <expr>
            => A   = B   * <expr>
            => A   = B   * ( <expr> )
            => A   = B   * ( <id> + <expr> )
            => A   = B   * ( A   + <expr> )
            => A   = B   * ( A   + <id> )
            => A   = B   * ( A   + C   )
```

Parse Tree

- A **hierarchical** representation of a derivation.

A **parse tree** for the simple statement
a = b + const

<program>

|

<stmts>

|

<stmt>

| / \

<var> = <expr>

| / \ | \

a <term> + <term>

| |

<var> const

|

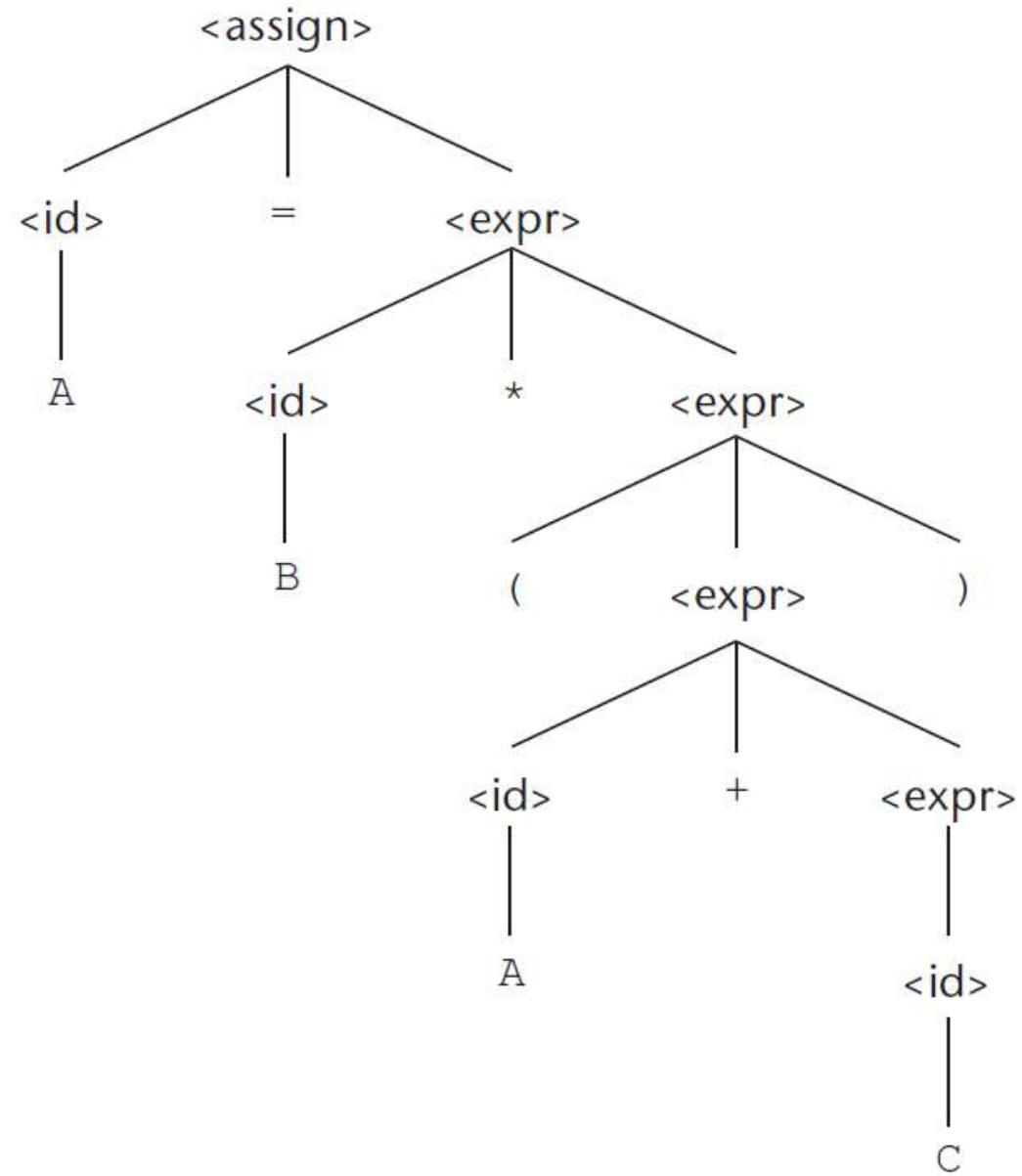
b

See slide 20

Parse Tree

A **parse tree** for the simple statement

$A = B * (A + C)$



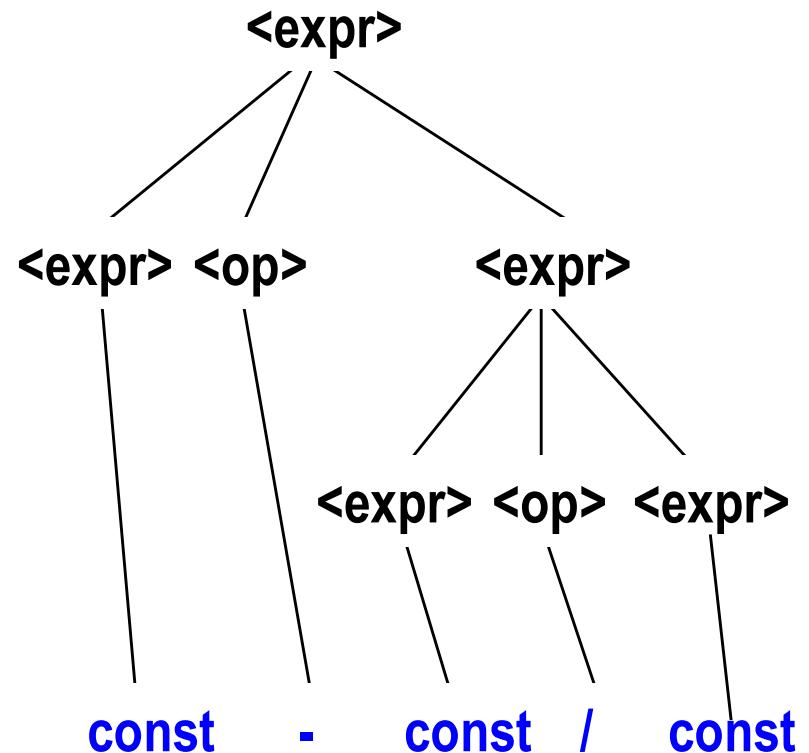
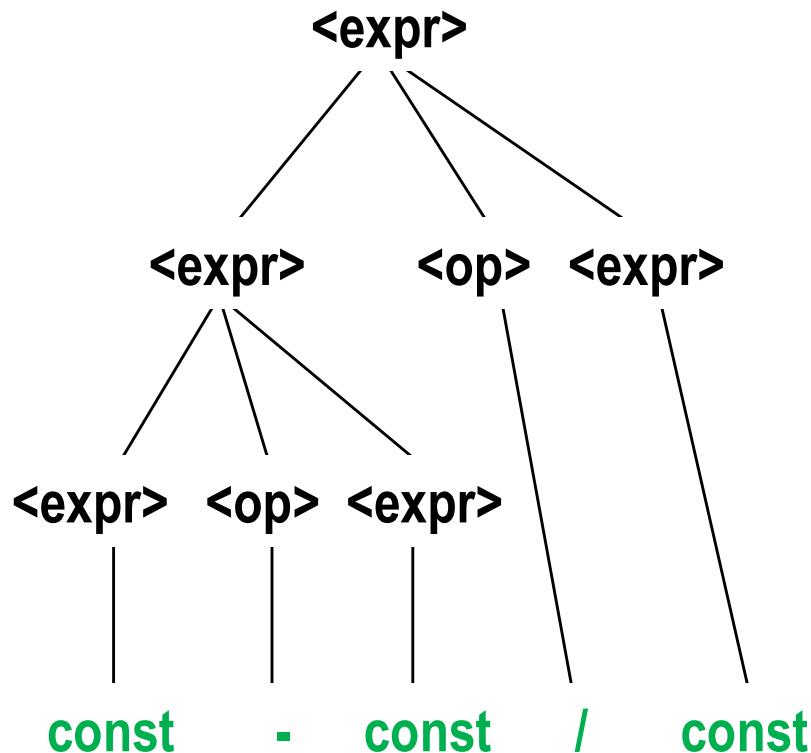
Ambiguity in Grammars

- A **grammar** is *ambiguous* if and only if it generates a **sentential form** that has two or more distinct parse trees.
- This type of **grammar** allows the **parse tree** of an expression to grow on both **left** and **right**.
 - It should allow the tree to grow on the right only in such cases.
- How can this be a problem?
 - It confuses compilers during syntax analysis as compilers use the **parse tree** to generate code.
 - So, the meaning of the structure cannot be determined uniquely.

An Ambiguous Expression Grammar

$\langle \text{expr} \rangle \rightarrow \langle \text{expr} \rangle \langle \text{op} \rangle \langle \text{expr} \rangle \quad | \quad \text{const}$

$\langle \text{op} \rangle \rightarrow / \quad | \quad -$



Another Ambiguous Grammar

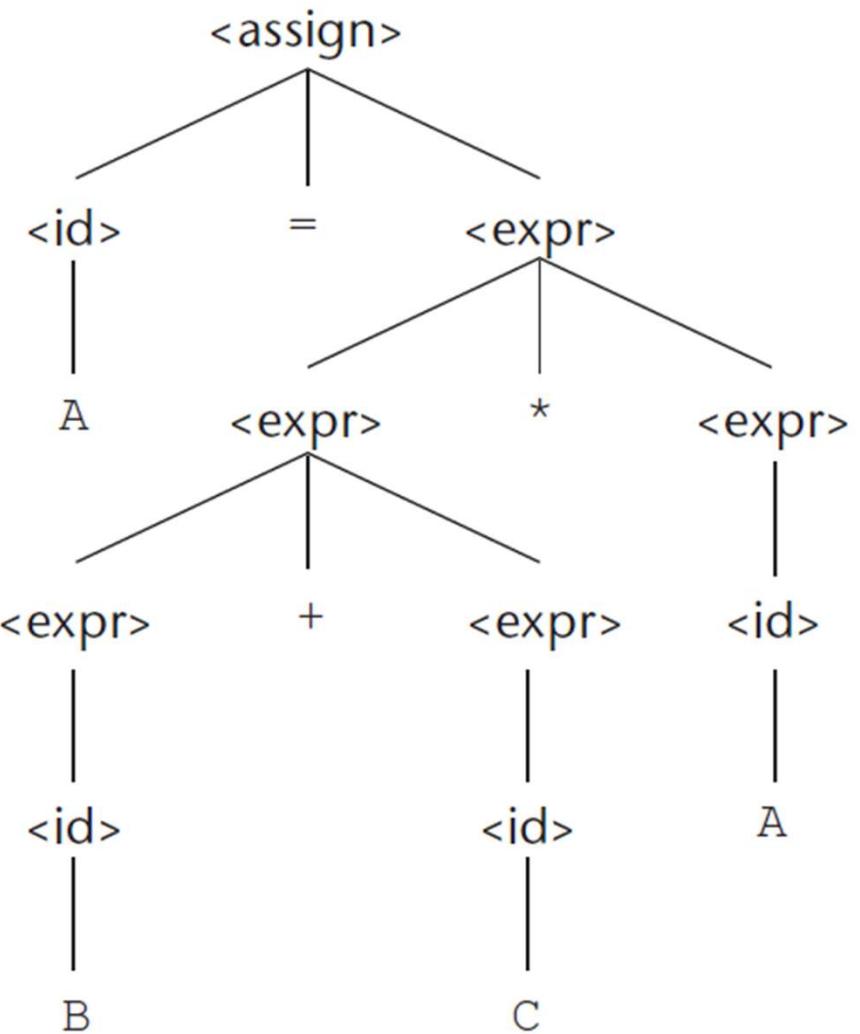
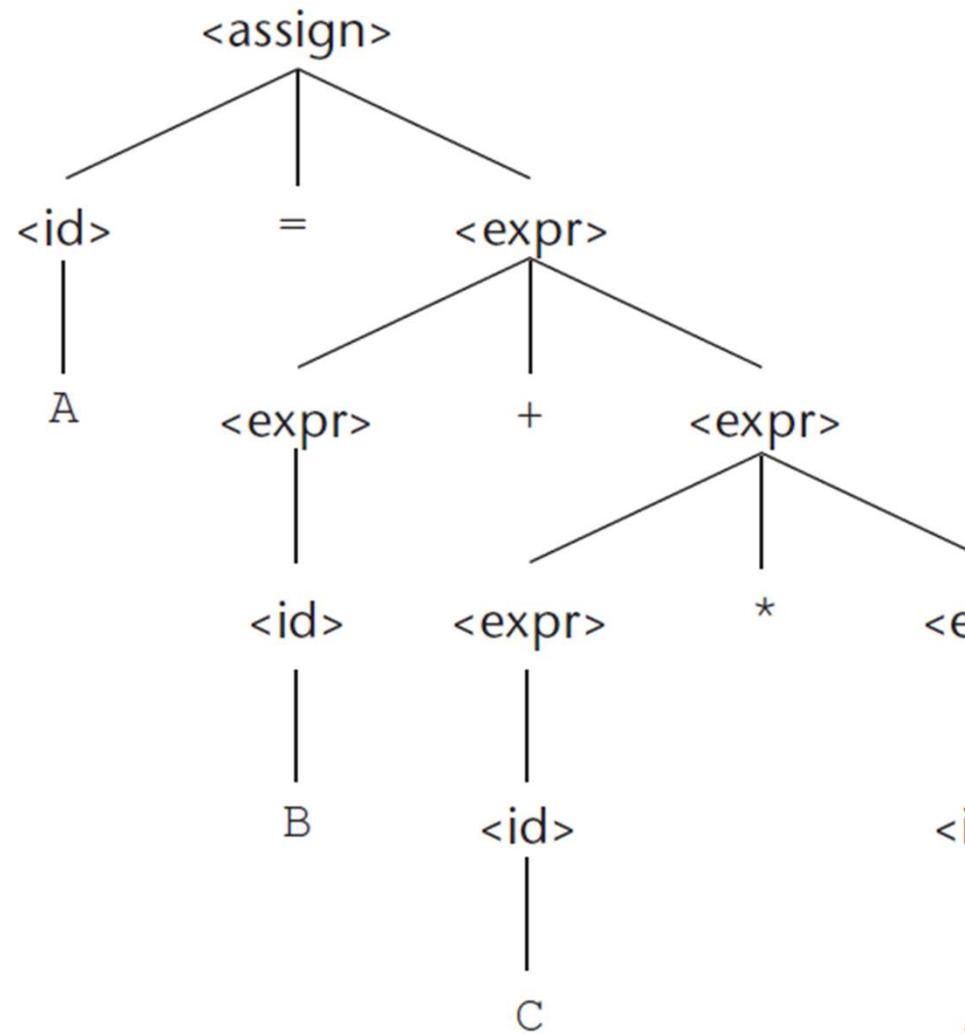
An Ambiguous Grammar for Simple Assignment Statements

$\langle \text{assign} \rangle \rightarrow \langle \text{id} \rangle = \langle \text{expr} \rangle$

$\langle \text{id} \rangle \rightarrow A \mid B \mid C$

$\langle \text{expr} \rangle \rightarrow \langle \text{expr} \rangle + \langle \text{expr} \rangle$
 | $\langle \text{expr} \rangle * \langle \text{expr} \rangle$
 | $(\langle \text{expr} \rangle)$
 | $\langle \text{id} \rangle$

- This grammar is ambiguous because the sentence $A = B + C * A$ has two *distinct parse trees*.
 - See the figure in the next slide.

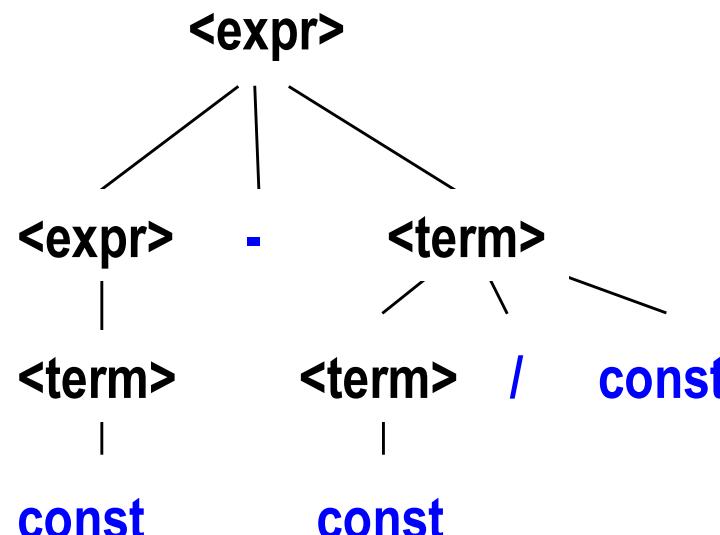


Two distinct parse trees for the same sentence, $A = B + C * A$

An Unambiguous Expression Grammar

- If we use the **parse tree** to indicate precedence levels of the operators, we cannot have **ambiguity**.

```
<expr> → <expr> - <term> | <term>  
<term> → <term> / const | const
```



Operators Precedence

- Given the expression $x + y * z$, one obvious **semantic_issue** is the order of evaluation of the two operators.
 - Is it add and then multiply, or vice versa?
- This **semantic** question can be answered by assigning different precedence levels to operators.
- As **grammar** can describe a certain syntactic structure so that part of the **meaning of the structure** can be determined from its parse tree.

Operators Precedence

- It is a fact that an **operator** in an arithmetic expression is generated **lower** in the **parse tree** must be evaluated first.
- This fact can be used to indicate that a lower operator in a parse tree has precedence over an operator produced higher up in the tree.
- **However, is this fact sufficient to solve the operator precedence problem?**
 - Not always! See the **unambiguous grammar** in slide 24.

Operators Precedence

- For example, using the **grammar** in slide 24, try to sketch the **parse trees** for these two expressions:
 - $A + B * C$
 - $A * B + C$
- **What have you noticed?**
- You will see that:
 - For $A + B * C$, the (*) operator is the lowest in the tree, which will lead to a **correct** evaluation.
 - **However**, for $A * B + C$ instead, the (+) operator is the lowest (indicating it is to be done first), which will lead to an **incorrect** evaluation.
- So, the **grammar** (slide 24) is **sensitive** to the **order** of the **operators** in the expressions.

Operators Precedence

- So, how this problem can be solved?
 - Simply, take the **order** into consideration when designing the **grammar** by:
 - Use separate nonterminal symbols to represent the operands of the operators that have different precedence.
 - This requires additional nonterminals and some new rules.
 - For example, to correct the **grammar** in slide 24, we could use three nonterminals to represent operands, which allows the grammar to force different operators to different levels in the parse tree.
 - But, **how?** See the next slide!

Operators Precedence

- If $\langle \text{expr} \rangle$ is the root symbol for expressions, + can be forced to the top of the parse tree by having $\langle \text{expr} \rangle$ directly generate only + operators, using the new nonterminal, $\langle \text{term} \rangle$, as the right operand of +.
- Next, we can define $\langle \text{term} \rangle$ to generate * operators, using $\langle \text{term} \rangle$ as the left operand and a new nonterminal, $\langle \text{factor} \rangle$, as its right operand.
- Now, * will always be lower in the parse tree, simply because it is farther from the start symbol than + in every derivation.

A Grammar for Simple Assignment Statements

$\langle \text{assign} \rangle \rightarrow \langle \text{id} \rangle = \langle \text{expr} \rangle$

$\langle \text{id} \rangle \rightarrow A \mid B \mid C$

$\langle \text{expr} \rangle \rightarrow \langle \text{id} \rangle + \langle \text{expr} \rangle$

$\mid \langle \text{id} \rangle * \langle \text{expr} \rangle$

$\mid (\langle \text{expr} \rangle)$

$\mid \langle \text{id} \rangle$

An Unambiguous Grammar for Expressions

$\langle \text{assign} \rangle \rightarrow \langle \text{id} \rangle = \langle \text{expr} \rangle$

$\langle \text{id} \rangle \rightarrow A \mid B \mid C$

$\langle \text{expr} \rangle \rightarrow \langle \text{expr} \rangle + \langle \text{term} \rangle$

$\mid \langle \text{term} \rangle$

$\langle \text{term} \rangle \rightarrow \langle \text{term} \rangle * \langle \text{factor} \rangle$

$\mid \langle \text{factor} \rangle$

$\langle \text{factor} \rangle \rightarrow (\langle \text{expr} \rangle)$

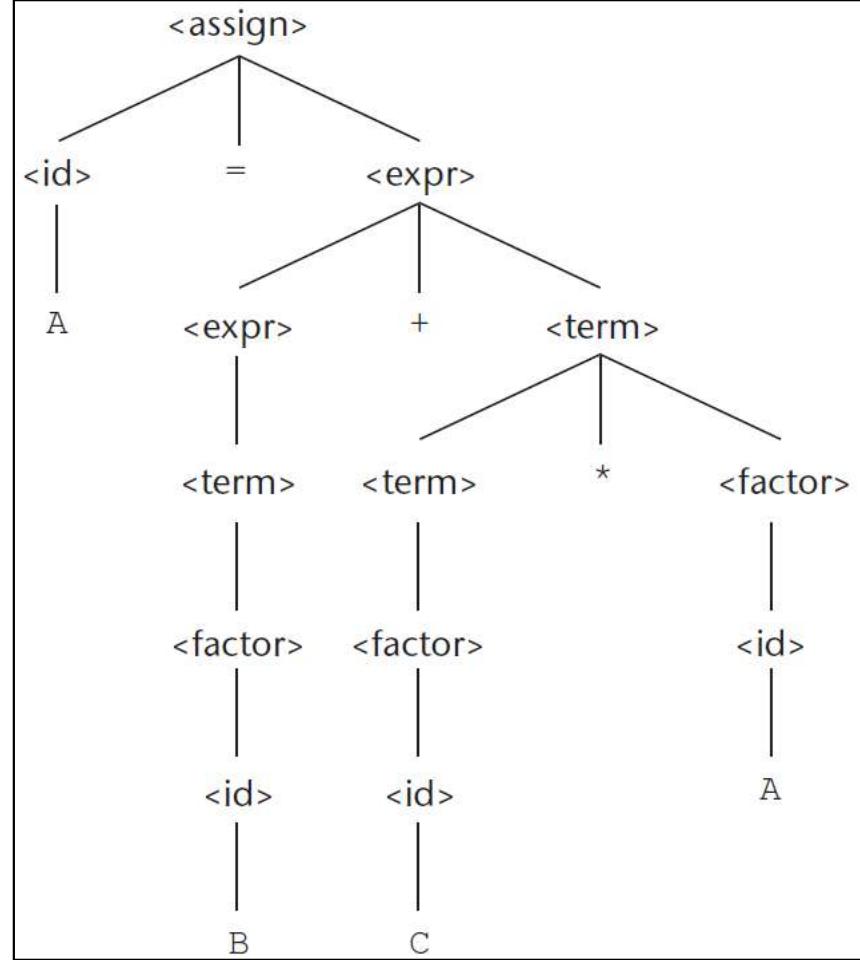
$\mid \langle \text{id} \rangle$

This grammar generates the same language as the above grammar. It is unambiguous and it specifies the usual precedence order of multiplication and addition operators.

Operators Precedence: Example (Leftmost Derivation)

A = B + C * A

<assign> => <id> = <expr>
=> A = <expr>
=> A = <expr> + <term>
=> A = <term> + <term>
=> A = <factor> + <term>
=> A = <id> + <term>
=> A = B + <term>
=> A = B + <term> * <factor>
=> A = B + <factor> * <factor>
=> A = B + <id> * <factor>
=> A = B + C * <factor>
=> A = B + C * <id>
=> A = B + C * A



The unique **parse tree** for A = B + C * A using an unambiguous grammar

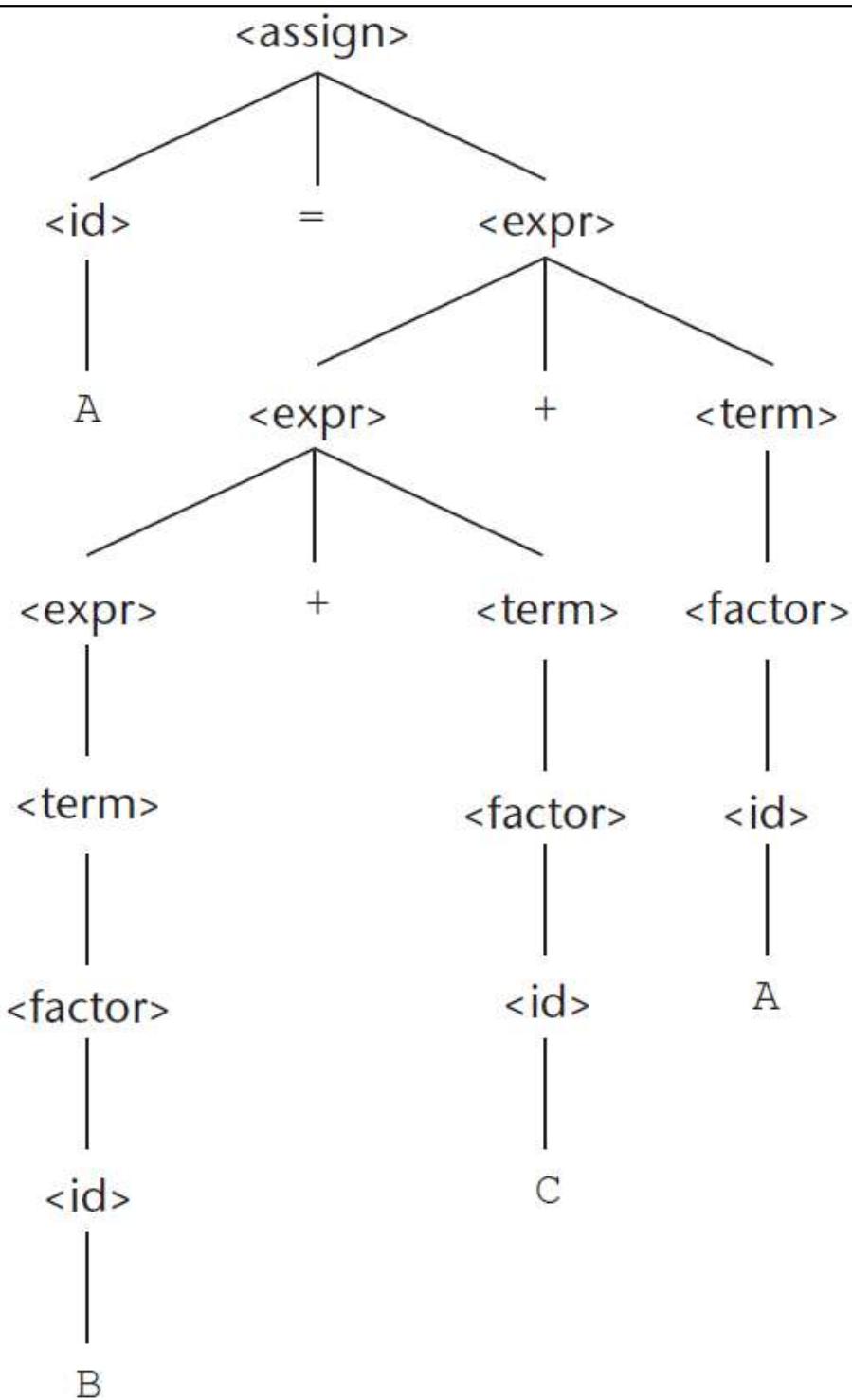
Operators Precedence: Example (Rightmost Derivation)

A = B + C * A

```
<assign> => <id> = <expr>
=> <id> = <expr> + <term>
=> <id> = <expr> + <term> * <factor>
=> <id> = <expr> + <term> * <id>
=> <id> = <expr> + <term> * A
=> <id> = <expr> + <factor> * A
=> <id> = <expr> + <id> * A
=> <id> = <expr> + C * A
=> <id> = <term> + C * A
=> <id> = <factor> + C * A
=> <id> = <id> + C * A
=> <id> = B + C * A
=> A = B + C * A
```

Associativity of Operators

- When an expression includes two **operators** that have the same precedence (as * and / usually have)—for example, “A / B * C”, then a semantic rule is required to specify which should have **precedence**.
- This rule is named **associativity**.
- A **grammar** for expressions may correctly imply **operator associativity**.
- Consider the following example of an assignment statement:
 - A = B + C + A
 - After using the **grammar** in the next slide for the **derivation** of this statement, then its **parse tree** will look like:



An Unambiguous Grammar for Expressions

$\text{<assign>} \rightarrow \text{<id>} = \text{<expr>}$
 $\text{<id>} \rightarrow \text{A} \mid \text{B} \mid \text{C}$
 $\text{<expr>} \rightarrow \text{<expr>} + \text{<term>}$
 $\quad \mid \text{<term>}$
 $\text{<term>} \rightarrow \text{<term>} * \text{<factor>}$
 $\quad \mid \text{<factor>}$
 $\text{<factor>} \rightarrow (\text{<expr>})$
 $\quad \mid \text{<id>}$

A parse tree for
 $\text{A} = \text{B} + \text{C} + \text{A}$
illustrating the
associativity of addition

Associativity of Operators

- The previous **parse tree** shows the **left** addition **operator** lower than the **right** addition **operator**.
 - This is the correct order if addition is meant to be left associative, which is typical.
 - In most cases, the **associativity** of addition in a computer is irrelevant.
- In mathematics, addition is associative, which means that **left** and **right** associative orders of evaluation mean the same thing.
 - That is, $(A + B) + C = A + (B + C)$
- **Subtraction** and **division** are not associative, whether in mathematics or in a computer.
 - Therefore, correct associativity may be essential for an expression that contains either of them.

Associativity of Operators

- When a **grammar rule** has its LHS also appearing at the beginning of its RHS, the rule is said to be **left recursive**.
 - This **left recursion** specifies left associativity.
- For example, the **left recursion** of the **rules** of the **grammar** below causes it to make both addition and multiplication left associative.

```
<assign> → <id> = <expr>
<id> → A | B | C
<expr> → <expr> + <term>
          | <term>
<term> → <term> * <factor>
          | <factor>
<factor> → ( <expr> )
           | <id>
```

Associativity of Operators

- The **exponentiation** operator is right associative in most languages that provide it.
- To indicate right associativity, right recursion can be used.
- A grammar rule is right recursive if the LHS appears at the right end of the RHS.
- Rules such as:

```
<factor> → <exp> ** <factor>
          | <exp>
<exp> → ( <expr> )
          | id
```

could be used to describe exponentiation as a right-associative operator.

Extended BNF

- **Optional** parts are placed in brackets []

$\langle \text{proc_call} \rangle \rightarrow \text{ident} \; [\langle \text{expr_list} \rangle]$

- **Alternative** parts of RHSs are placed inside parentheses and separated via vertical bars.

$\langle \text{term} \rangle \rightarrow \langle \text{term} \rangle \; (+ | -) \; \text{const}$

- **Repetitions** (zero or more) are placed inside braces { }

$\langle \text{ident} \rangle \rightarrow \text{letter} \; \{ \text{letter} | \text{digit} \}$

BNF and EBNF

- **BNF:**

$\langle \text{expr} \rangle \rightarrow \langle \text{expr} \rangle + \langle \text{term} \rangle$

| $\langle \text{expr} \rangle - \langle \text{term} \rangle$

| $\langle \text{term} \rangle$

$\langle \text{term} \rangle \rightarrow \langle \text{term} \rangle * \langle \text{factor} \rangle$

| $\langle \text{term} \rangle / \langle \text{factor} \rangle$

| $\langle \text{factor} \rangle$

- **EBNF:**

$\langle \text{expr} \rangle \rightarrow \langle \text{term} \rangle \{ (+ \mid -) \langle \text{term} \rangle \}$

$\langle \text{term} \rangle \rightarrow \langle \text{factor} \rangle \{ (* \mid /) \langle \text{factor} \rangle \}$

Recent Variations in EBNF

- Alternative RHSs are put on separate lines.
- Use of a **colon** instead of **=>**
- Use of **_{opt}** for **optional** parts.
- Use of **oneof** for **choices**.

Any Questions?

- Please, read chapter 3 (*first 2 sections*)
- I hope you were taking some notes!
- To test your understanding of this lecture, have a go with the “Review Questions” in page 156 of the textbook.
- We will do more exercises later on.
- Please, keep reviewing this lecture regularly.
- We may have a quiz (lecture 1) next week!
- Please, start doing your assignment.