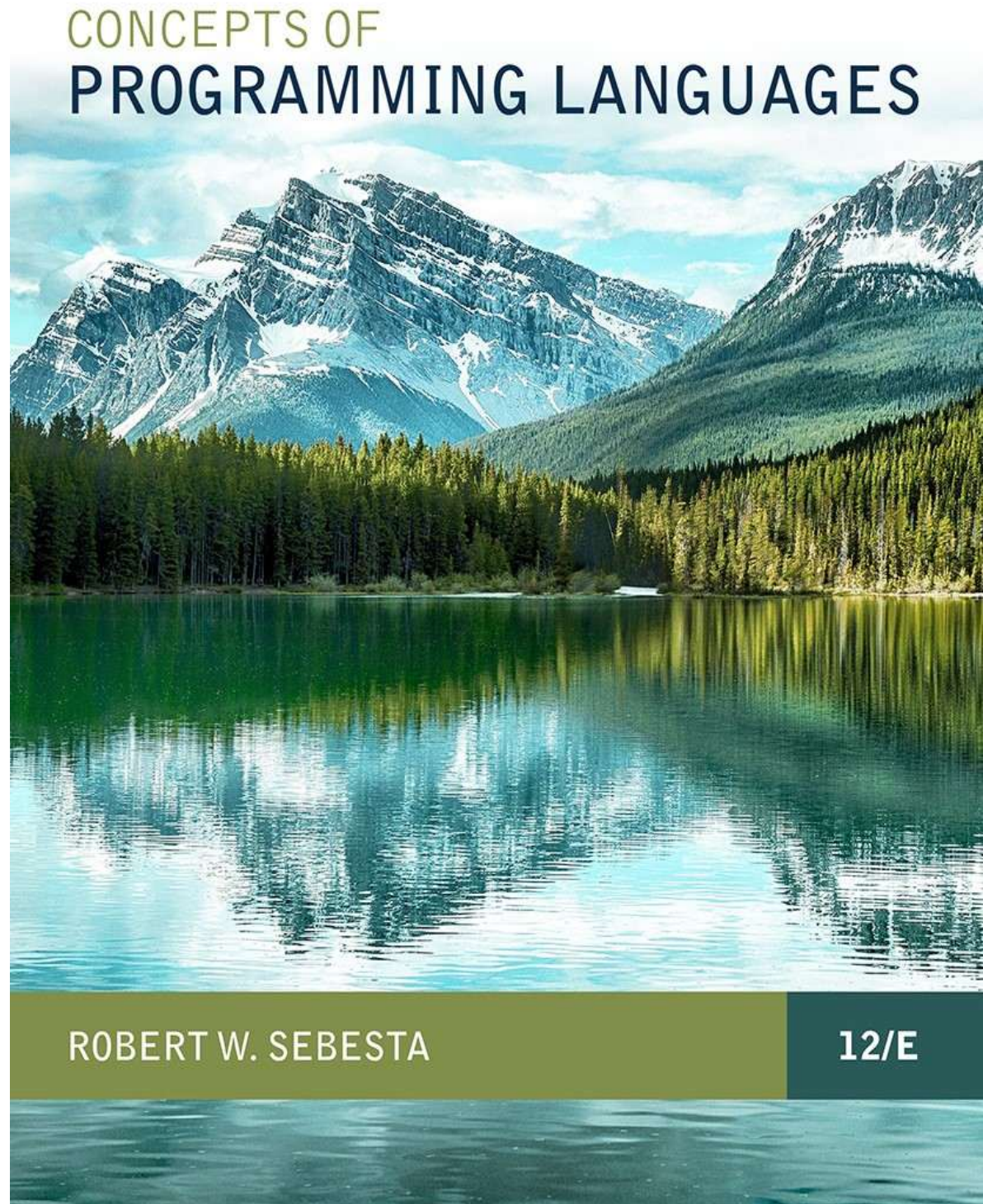


Lecture 2

Describing Syntax

Chapter 3 – Part 1

First Semester
1447 - 2025



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Lecture 2 Topics:

- Introduction
- The General Problem of Describing **Syntax**:
 - Language **Recognizers**
 - Language **Generators**
- Formal Methods of Describing **Syntax**:
 - Backus–Naur Form (**BNF**):
 - Extended BNF (**EBNF**)
 - Context–Free **Grammars** (**CFG**):
 - Grammars
 - **Derivations**
 - Parse Trees
 - **Ambiguity**
- **Attribute** Grammars:
 - **Static Semantics**

Introduction

- The study of programming languages, like the study of natural languages, can be divided into:
 - Examinations of **syntax**.
 - Examinations of **semantics**.
- **Syntax**: the **form** or **structure** of the expressions, statements, and program units.
- **Semantics**: the **meaning** of the expressions, statements, and program units.
- **Syntax** and **semantics** provide a language's definition.

Introduction: Example

- **Example:**
 - The **syntax** of a Java “while” statement is:
 - while (boolean_expr) statement
 - The **semantics** of the same statement is:
 - When the current value of the Boolean expression is true, the embedded statement is executed.
 - Then control implicitly returns to the Boolean expression to repeat the process.
 - If the Boolean expression is false, control transfers to the statement following the while construct.

Introduction: Language Users

- **Users** of a **language definition**:
 - Other language designers (evaluators).
 - Implementers.
 - Programmers (the users of the language).

The General Problem of Describing Syntax: Terminology

- A *sentence* (statement) is a string of characters over some alphabet.
- A *language* is a set of sentences (statements).
- A *lexeme* is the lowest level syntactic unit of a language (e.g., `*`, `sum`, `begin`).
 - The language operators.
 - The language special words.
 - The language numerical literals.
 - Etc.
- A *token* is a category of lexemes (e.g., identifier).

Example

- Consider the following Java statement:

➤ `index = 2 * count + 17;`

Lexemes	Tokens
<code>index</code>	identifier
<code>=</code>	equal_sign
<code>2</code>	int_literal
<code>*</code>	mult_op
<code>count</code>	identifier
<code>+</code>	plus_op
<code>17</code>	int_literal
<code>;</code>	semicolon

Formal Definition of Languages

■ Recognizers:

- A recognition device reads input strings over the alphabet of the language and decides whether the input strings belong to the language (accept or reject the given input strings).
- **Example:** syntax analysis (parsing) part of a compiler.
 - The syntax analyzer determines whether the given programs are syntactically correct.
- Detailed discussion of syntax analysis appears in Chapter 4.

■ Generators:

- A device that generates sentences of a language.
- One can determine if the syntax of a particular sentence is syntactically correct by comparing it to the structure of the generator.

BNF and Context-Free Grammars

- **Context-Free Grammars:**
 - Developed by Noam Chomsky in the mid-1950s.
 - Language **generators**, meant to describe the syntax of natural languages.
 - Define a class of languages called **context-free languages**.
- **Backus-Naur Form (1959):**
 - Invented by John Backus to describe the syntax of Algol 58.
 - **BNF** is equivalent to **context-free grammars**.

BNF Fundamentals

- BNF is a natural notation for describing **syntax**.
- In BNF, abstractions are used to represent classes of syntactic structures.
 - They act like syntactic variables (also called *nonterminal symbols*, or just *terminals*).
- **Example:**
 - A simple Java assignment statement might be represented by the **abstraction** `<assign>`
 - Pointed brackets are often used to delimit names of abstractions.
 - The actual definition of `<assign>` can be given by:
 - `<assign> → <var> = <expression>`

BNF Fundamentals (continued)

- **Terminals** are **lexemes** or **tokens**.
- A **rule** has a left-hand side (LHS), which is a **nonterminal**, and a right-hand side (RHS), which is a string of **terminals** and/or **nonterminals**.
- **Example:**
 - See the next slide!

<assign> → <var> = <expression>

- The **text** on the left side of the arrow, which is aptly called the **left-hand side (LHS)**, is the **abstraction** being defined.
- The **text** to the right of the arrow is the **definition** of the **LHS**.
- It is called the **right-hand side (RHS)** and consists of some mixture of tokens, lexemes, and references to **other abstractions**. (Actually, tokens are also **abstractions**.)
- Altogether, the definition is called a **rule**, or **production**.
- In the example **rule** just given, the **abstractions** **<var>** and **<expression>** obviously must be defined for the **<assign>** definition to be useful.

How a rule can be read?

<assign> → <var> = <expression>

- This particular **rule** specifies that the **abstraction** **<assign>** is defined as an instance of the **abstraction** **<var>**, followed by the lexeme **=**, followed by an instance of the **abstraction** **<expression>**.
- One example **sentence** whose syntactic structure is described by the **rule** above is:
 - `total = subtotal1 + subtotal2`

BNF Fundamentals (continued)

- **Nonterminals** are often enclosed in “*angle brackets*” (abstraction).

- Examples of **BNF rules**:

`<ident_list> → identifier | identifier, <ident_list>`

`<if_stmt> → if <logic_expr> then <stmt>`

- **Grammar**: a finite non-empty set of **rules**.
- A *start symbol* is a special element of the **nonterminals** of a grammar.

BNF Rules

- An **abstraction** (or **nonterminal** symbol) can have more than one RHS:

`<stmt> → <single_stmt>`

`<stmt> → begin <stmt_list> end`

- These two **rules** can be written as:

`<stmt> → <single_stmt> | begin <stmt_list> end`

BNF Rules: More Examples

- a **Java** if statement can be described with the **rules**:

`<if_stmt> → if (<logic_expr>) <stmt>`

`<if_stmt> → if (<logic_expr>) <stmt> else <stmt>`

or with the rule

`<if_stmt> → if (<logic_expr>) <stmt>`

`| if (<logic_expr>) <stmt> else <stmt>`

Describing Lists

- Example of a **list**:
 - a list of identifiers appearing on a data declaration statement.
- **Syntactic lists** are described using **recursion**:

`<ident_list> → ident`

`| ident, <ident_list>`

- A **rule** is **recursive** if its LHS appears in its RHS.

Derivation

- A **grammar** is a generative device for defining languages.
- A **derivation** is a repeated application of **rules** (grammars), starting with the **start symbol** and ending with a **sentence** (**all terminal symbols**).
- The **start symbol** represents a complete program and is often named `<program>`.

An Example Grammar

<program> \rightarrow <stmts>

<stmts> \rightarrow <stmt> | <stmt> ; <stmts>

<stmt> \rightarrow <var> = <expr>

<var> \rightarrow **a** | **b** | **c** | **d**

<expr> \rightarrow <term> + <term> | <term> - <term>

<term> \rightarrow <var> | const

- The **language** described by the **grammar** has only one statement form:
 - assignment.

An Example Derivation

- How can we **derive** the following **assignment** statement using the previous **grammars** (rules)?

- a = b + const**

The symbol \Rightarrow is read “**derives**.”

$\langle \text{program} \rangle \Rightarrow \langle \text{stmts} \rangle$
 $\Rightarrow \langle \text{stmt} \rangle$
 $\Rightarrow \langle \text{var} \rangle = \langle \text{expr} \rangle$
 $\Rightarrow \mathbf{a} = \langle \text{expr} \rangle$
 $\Rightarrow \mathbf{a} = \langle \text{term} \rangle + \langle \text{term} \rangle$
 $\Rightarrow \mathbf{a} = \langle \text{var} \rangle + \langle \text{term} \rangle$
 $\Rightarrow \mathbf{a} = \mathbf{b} + \langle \text{term} \rangle$
 $\Rightarrow \mathbf{a} = \mathbf{b} + \mathbf{const}$

Another Example Grammar

A Grammar for a Small Language

$\langle \text{program} \rangle \rightarrow \text{begin } \langle \text{stmt_list} \rangle \text{ end}$

$\langle \text{stmt_list} \rangle \rightarrow \langle \text{stmt} \rangle$
 $| \langle \text{stmt} \rangle ; \langle \text{stmt_list} \rangle$

$\langle \text{stmt} \rangle \rightarrow \langle \text{var} \rangle = \langle \text{expression} \rangle$

$\langle \text{var} \rangle \rightarrow A \mid B \mid C$

$\langle \text{expression} \rangle \rightarrow \langle \text{var} \rangle + \langle \text{var} \rangle$
 $| \langle \text{var} \rangle - \langle \text{var} \rangle$
 $| \langle \text{var} \rangle$

How can we **derive**
the following small
program?

begin

A = B + C ;

B = C

end

Another Example Derivation

```
<program> => begin <stmt_list> end
=> begin <stmt> ; <stmt_list> end
=> begin <var> = <expression> ; <stmt_list> end
=> begin A = <expression> ; <stmt_list> end
=> begin A = <var> + <var> ; <stmt_list> end
=> begin A = B + <var> ; <stmt_list> end
=> begin A = B + C ; <stmt_list> end
=> begin A = B + ; <stmt> end
=> begin A = B + C ; <var> = <expression> end
=> begin A = B + C ; B = <expression> end
=> begin A = B + C ; B = <var> end
=> begin A = B + C ; B = C end
```


Derivations

- Every string of symbols in a **derivation** is a *sentential form*.
- A *sentence* (statement) is a **sentential form** that has only **terminal** symbols.
- A *leftmost derivation* is one in which the **leftmost nonterminal** in each **sentential form** is the one that is expanded.
- A **derivation** may be neither **leftmost** nor **rightmost**.

Yet Another Example Grammar

A Grammar for Simple Assignment Statements

$\langle \text{assign} \rangle \rightarrow \langle \text{id} \rangle = \langle \text{expr} \rangle$

$\langle \text{id} \rangle \rightarrow A \mid B \mid C$

$\langle \text{expr} \rangle \rightarrow \langle \text{id} \rangle + \langle \text{expr} \rangle$

$\mid \langle \text{id} \rangle * \langle \text{expr} \rangle$

$\mid (\langle \text{expr} \rangle)$

$\mid \langle \text{id} \rangle$

How can we **derive** the following **assignment** statement?

$A = B * (A + C)$

Yet Another Example Derivation

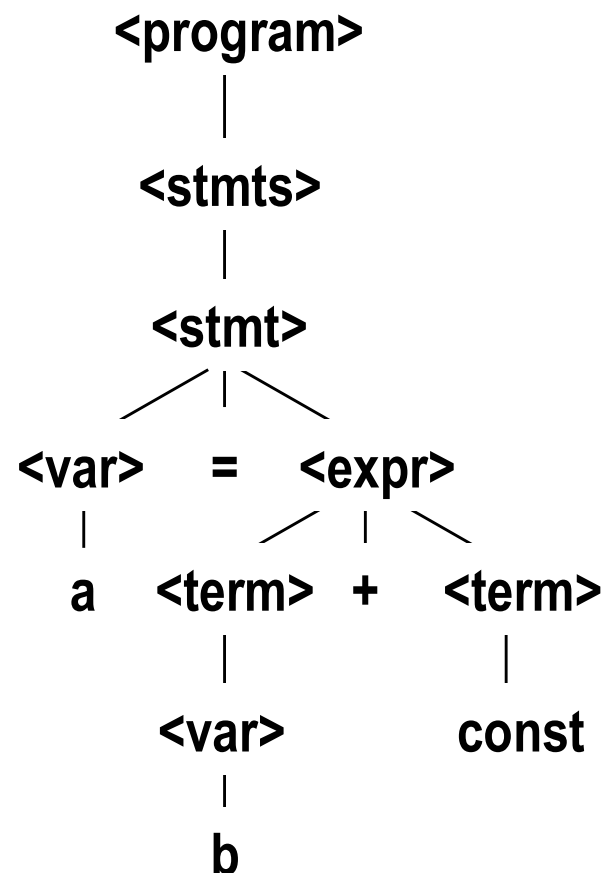
- The statement $A = B * (A + C)$ is generated by the following **leftmost derivation**:

$\langle \text{assign} \rangle \Rightarrow \langle \text{id} \rangle = \langle \text{expr} \rangle$
 $\Rightarrow A = \langle \text{expr} \rangle$
 $\Rightarrow A = \langle \text{id} \rangle * \langle \text{expr} \rangle$
 $\Rightarrow A = B * \langle \text{expr} \rangle$
 $\Rightarrow A = B * (\langle \text{expr} \rangle)$
 $\Rightarrow A = B * (\langle \text{id} \rangle + \langle \text{expr} \rangle)$
 $\Rightarrow A = B * (A + \langle \text{expr} \rangle)$
 $\Rightarrow A = B * (A + \langle \text{id} \rangle)$
 $\Rightarrow A = B * (A + C)$

Parse Tree

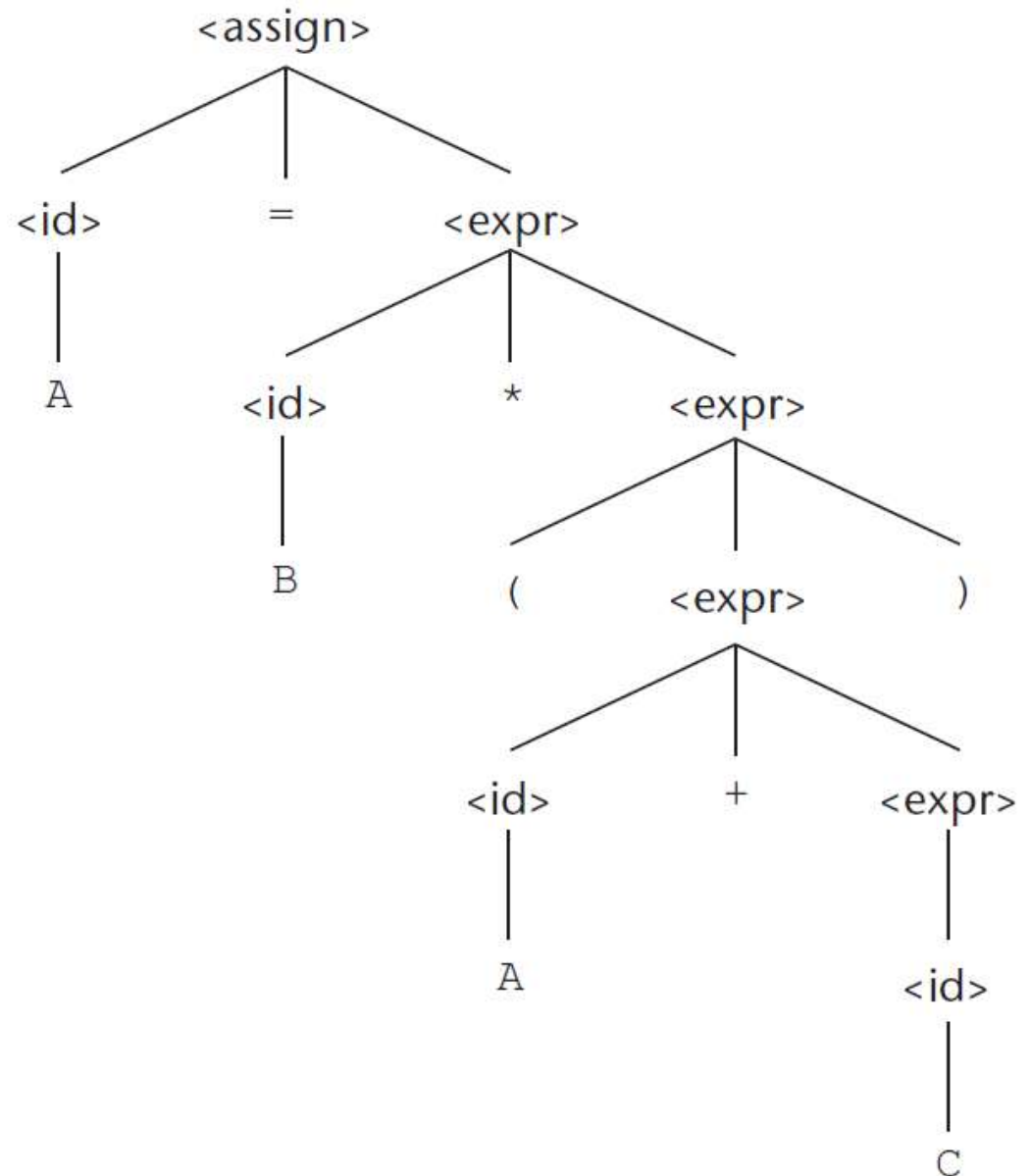
- A **hierarchical** representation of a derivation.

A **parse tree** for the
simple statement
a = b + const



See
slide 20

Parse Tree



A **parse tree** for the
simple statement

$A = B * (A + C)$

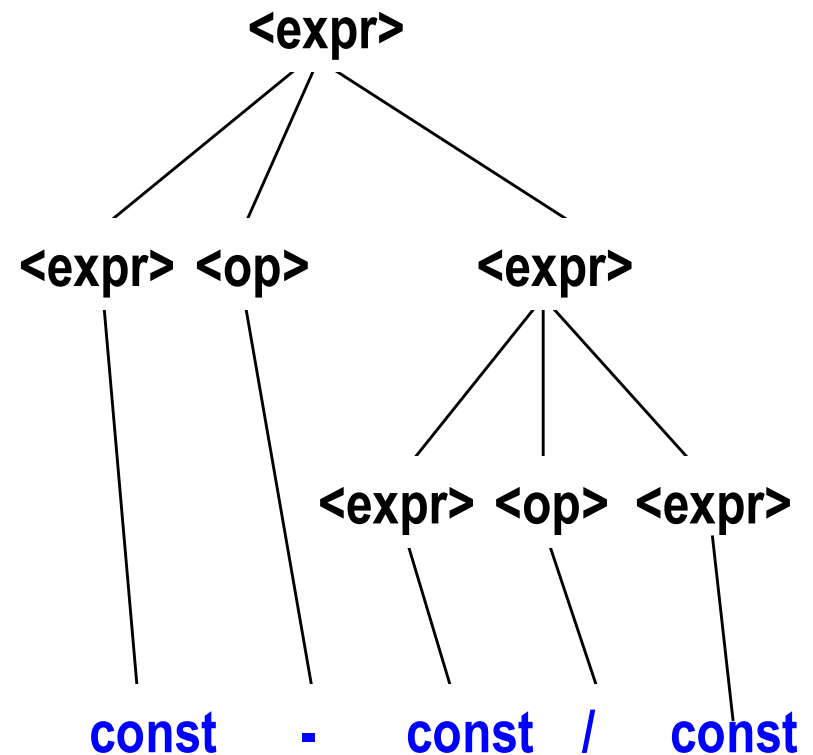
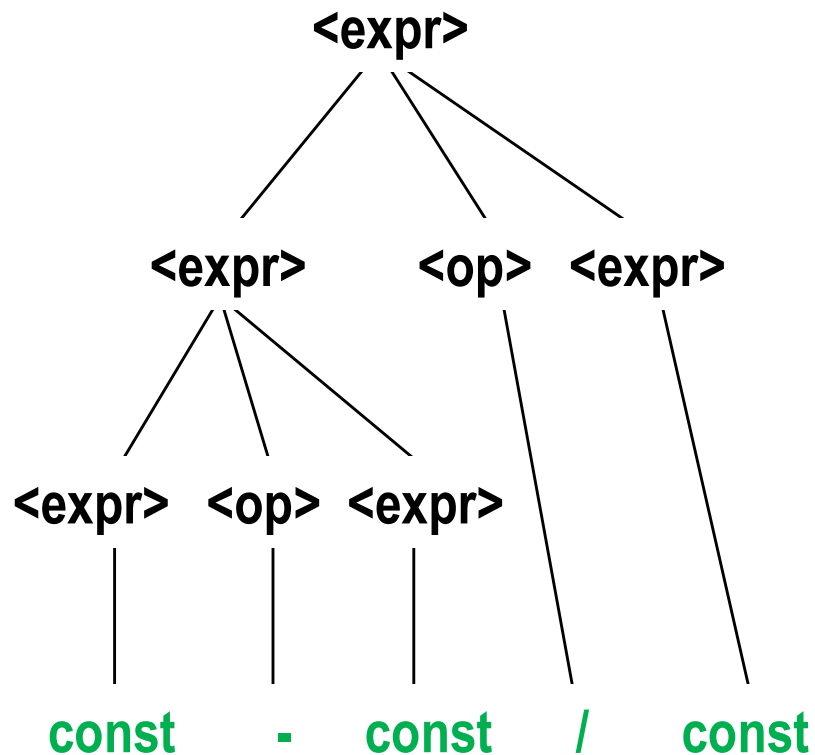
Ambiguity in Grammars

- A **grammar** is *ambiguous* if and only if it generates a **sentential form** that has two or more distinct parse trees.
- This type of **grammar** allows the **parse tree** of an expression to grow on both **left** and **right**.
 - It should allow the tree to grow on the right only in such cases.
- How can this be a problem?
 - It confuses compilers during syntax analysis as compilers use the **parse tree** to generate code.
 - So, the meaning of the structure cannot be determined uniquely.

An Ambiguous Expression Grammar

$\langle \text{expr} \rangle \rightarrow \langle \text{expr} \rangle \langle \text{op} \rangle \langle \text{expr} \rangle \mid \text{const}$

$\langle \text{op} \rangle \rightarrow / \mid -$

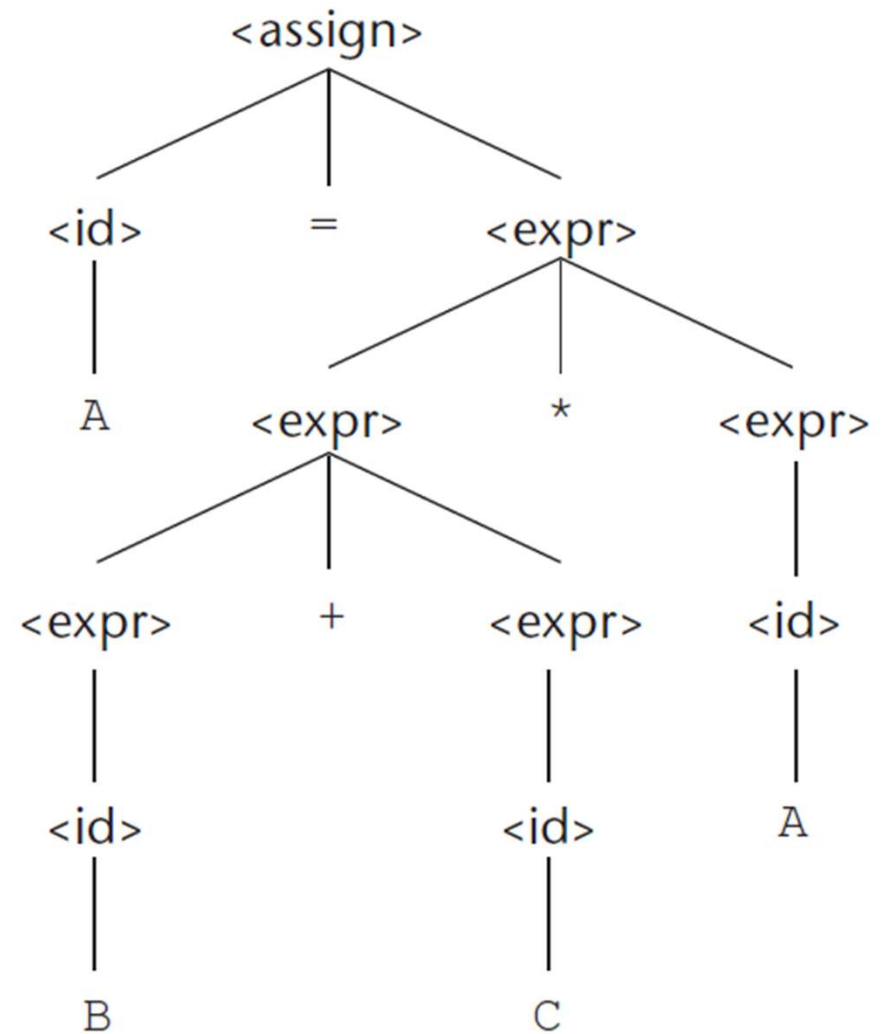
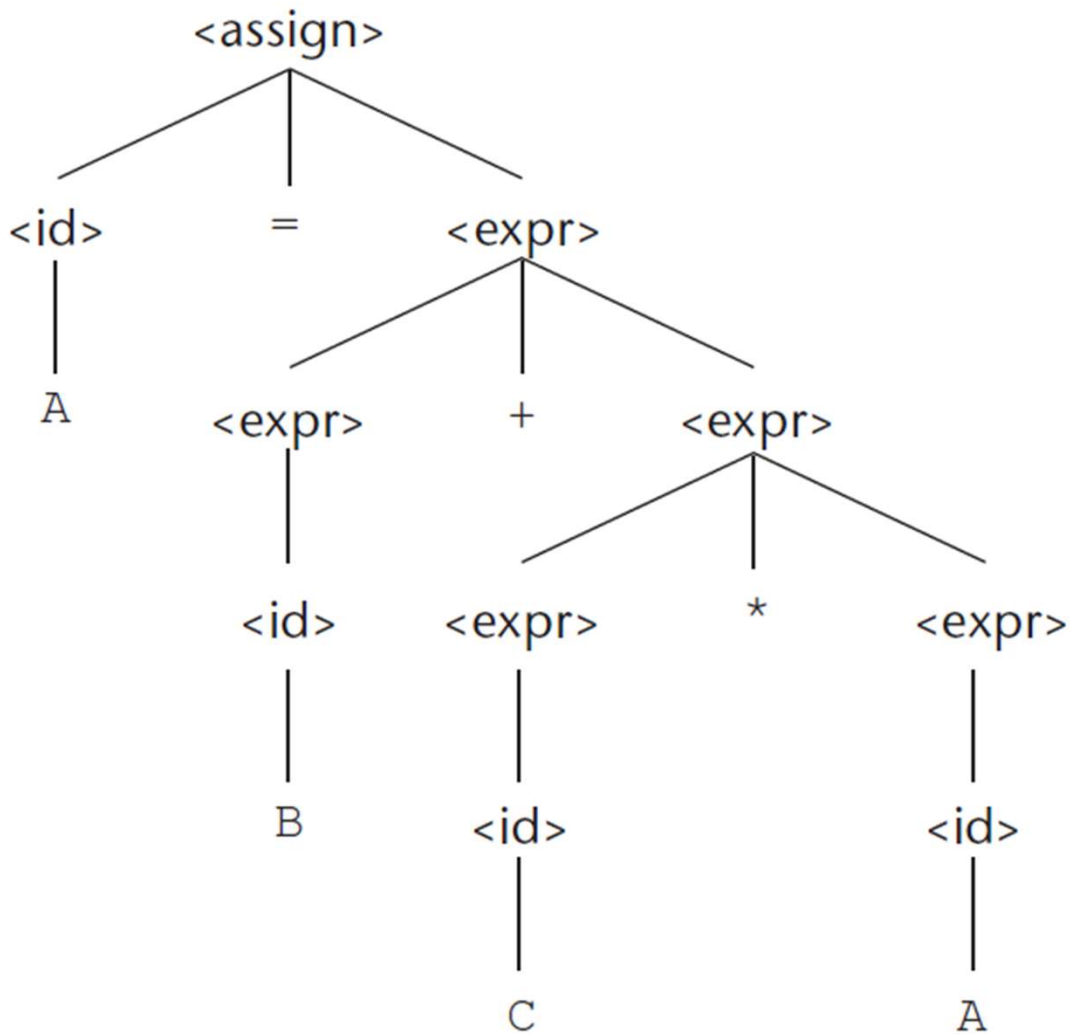


Another Ambiguous Grammar

An Ambiguous Grammar for Simple Assignment Statements

$\langle \text{assign} \rangle \rightarrow \langle \text{id} \rangle = \langle \text{expr} \rangle$
 $\langle \text{id} \rangle \rightarrow A \mid B \mid C$
 $\langle \text{expr} \rangle \rightarrow \langle \text{expr} \rangle + \langle \text{expr} \rangle$
 $\mid \langle \text{expr} \rangle * \langle \text{expr} \rangle$
 $\mid (\langle \text{expr} \rangle)$
 $\mid \langle \text{id} \rangle$

- This grammar is ambiguous because the sentence $A = B + C * A$ has two distinct parse trees.
 - See the figure in the next slide.

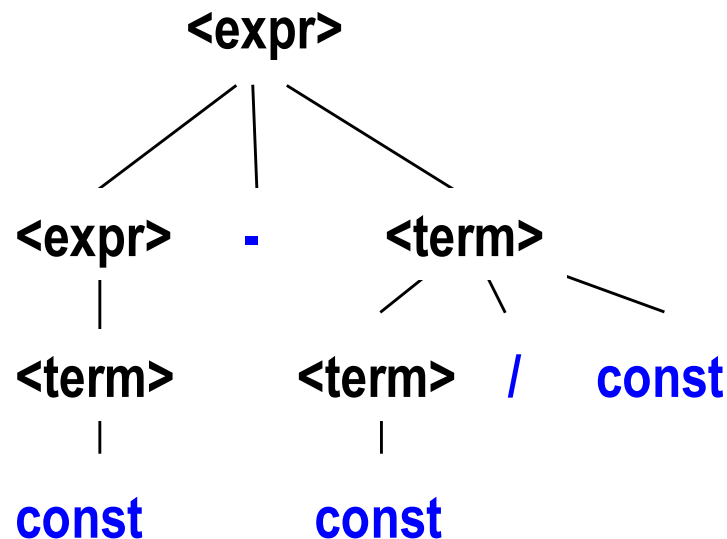


Two *distinct* **parse trees** for the same sentence, $A = B + C * A$

An Unambiguous Expression Grammar

- If we use the **parse tree** to indicate precedence levels of the operators, we cannot have **ambiguity**.

$\langle \text{expr} \rangle \rightarrow \langle \text{expr} \rangle - \langle \text{term} \rangle \mid \langle \text{term} \rangle$
 $\langle \text{term} \rangle \rightarrow \langle \text{term} \rangle / \text{const} \mid \text{const}$



Operators Precedence

- Given the expression $x + y * z$, one obvious **semantic_issue** is the order of evaluation of the two operators.
 - Is it add and then multiply, or vice versa?
- This **semantic** question can be answered by assigning different precedence levels to operators.
- As **grammar** can describe a certain syntactic structure so that part of the **meaning of the structure** can be determined from its parse tree.

Operators Precedence

- It is a fact that an **operator** in an arithmetic expression is generated **lower** in the **parse tree** must be evaluated first.
- This fact can be used to indicate that a lower operator in a parse tree has precedence over an operator produced higher up in the tree.
- **However**, *is this fact sufficient to solve the operator precedence problem?*
 - **Not always!** See the **unambiguous grammar** in slide 24.

Operators Precedence

- For example, using the **grammar** in slide 24, try to sketch the **parse trees** for these two expressions:
 - $A + B * C$
 - $A * B + C$
- **What have you noticed?**
- You will see that:
 - For $A + B * C$, the $(*)$ operator is the lowest in the tree, which will lead to a **correct** evaluation.
 - **However**, for $A * B + C$ instead, the $(+)$ operator is the lowest (indicating it is to be done first), which will lead to an **incorrect** evaluation.
- So, the **grammar** (slide 24) is **sensitive** to the **order** of the **operators** in the expressions.

Operators Precedence

- So, **how this problem can be solved?**
 - Simply, take the **order** into consideration when designing the **grammar** by:
 - Use separate nonterminal symbols to represent the operands of the operators that have different precedence.
 - This requires additional nonterminals and some new rules.
 - For example, to correct the **grammar** in slide 24, we could use three nonterminals to represent operands, which allows the grammar to force different operators to different levels in the **parse tree**.
 - But, **how?** See the next slide!

Operators Precedence

- If $\langle \text{expr} \rangle$ is the root symbol for expressions, $+$ can be forced to the top of the **parse tree** by having $\langle \text{expr} \rangle$ directly generate only $+$ operators, using the new nonterminal, $\langle \text{term} \rangle$, as the right operand of $+$.
- Next, we can define $\langle \text{term} \rangle$ to generate $*$ operators, using $\langle \text{term} \rangle$ as the left operand and a new nonterminal, $\langle \text{factor} \rangle$, as its right operand.
- Now, $*$ will always be lower in the **parse tree**, simply because it is farther from the start symbol than $+$ in every derivation.

A Grammar for Simple Assignment Statements

$\langle \text{assign} \rangle \rightarrow \langle \text{id} \rangle = \langle \text{expr} \rangle$

$\langle \text{id} \rangle \rightarrow A \mid B \mid C$

$\langle \text{expr} \rangle \rightarrow \langle \text{id} \rangle + \langle \text{expr} \rangle$

$\mid \langle \text{id} \rangle * \langle \text{expr} \rangle$

$\mid (\langle \text{expr} \rangle)$

$\mid \langle \text{id} \rangle$

An Unambiguous Grammar for Expressions

$\langle \text{assign} \rangle \rightarrow \langle \text{id} \rangle = \langle \text{expr} \rangle$

$\langle \text{id} \rangle \rightarrow A \mid B \mid C$

$\langle \text{expr} \rangle \rightarrow \langle \text{expr} \rangle + \langle \text{term} \rangle$

$\mid \langle \text{term} \rangle$

$\langle \text{term} \rangle \rightarrow \langle \text{term} \rangle * \langle \text{factor} \rangle$

$\mid \langle \text{factor} \rangle$

$\langle \text{factor} \rangle \rightarrow (\langle \text{expr} \rangle)$

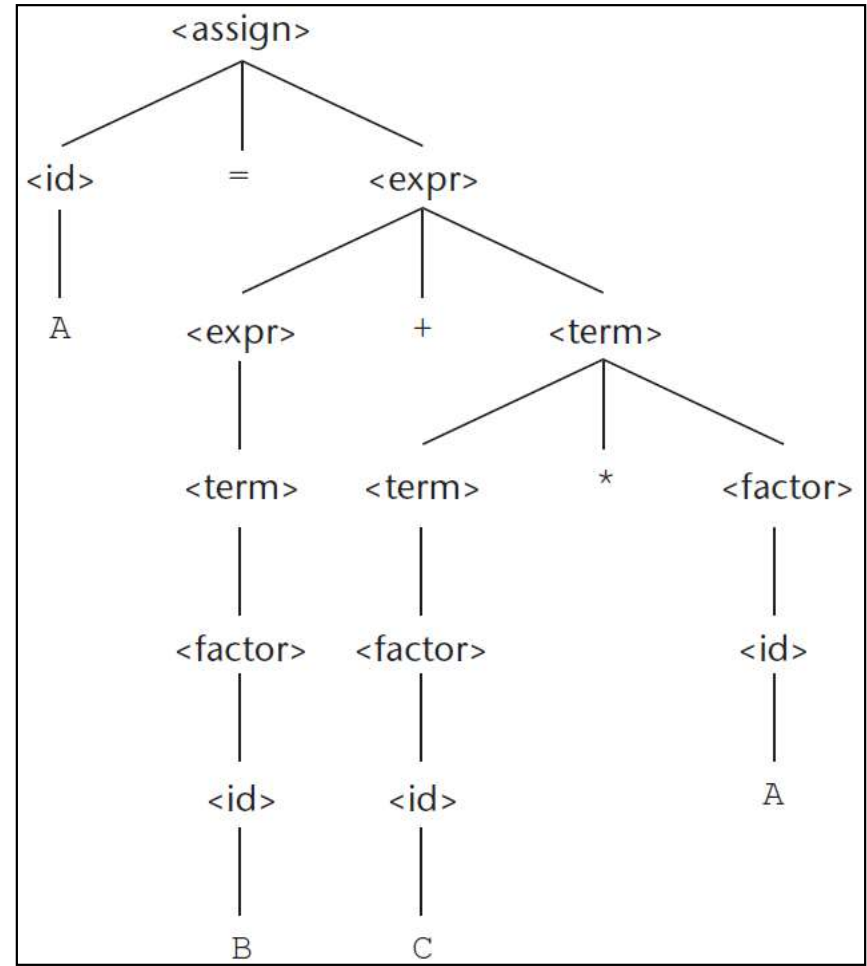
$\mid \langle \text{id} \rangle$

This grammar generates the same language as the above grammar. It is unambiguous and it specifies the usual precedence order of multiplication and addition operators.

Operators Precedence: Example (Leftmost Derivation)

$A = B + C * A$

$\langle \text{assign} \rangle \Rightarrow \langle \text{id} \rangle = \langle \text{expr} \rangle$
 $\Rightarrow A = \langle \text{expr} \rangle$
 $\Rightarrow A = \langle \text{expr} \rangle + \langle \text{term} \rangle$
 $\Rightarrow A = \langle \text{term} \rangle + \langle \text{term} \rangle$
 $\Rightarrow A = \langle \text{factor} \rangle + \langle \text{term} \rangle$
 $\Rightarrow A = \langle \text{id} \rangle + \langle \text{term} \rangle$
 $\Rightarrow A = B + \langle \text{term} \rangle$
 $\Rightarrow A = B + \langle \text{term} \rangle * \langle \text{factor} \rangle$
 $\Rightarrow A = B + \langle \text{factor} \rangle * \langle \text{factor} \rangle$
 $\Rightarrow A = B + \langle \text{id} \rangle * \langle \text{factor} \rangle$
 $\Rightarrow A = B + C * \langle \text{factor} \rangle$
 $\Rightarrow A = B + C * \langle \text{id} \rangle$
 $\Rightarrow A = B + C * A$



The unique **parse tree** for $A = B + C * A$ using an unambiguous grammar

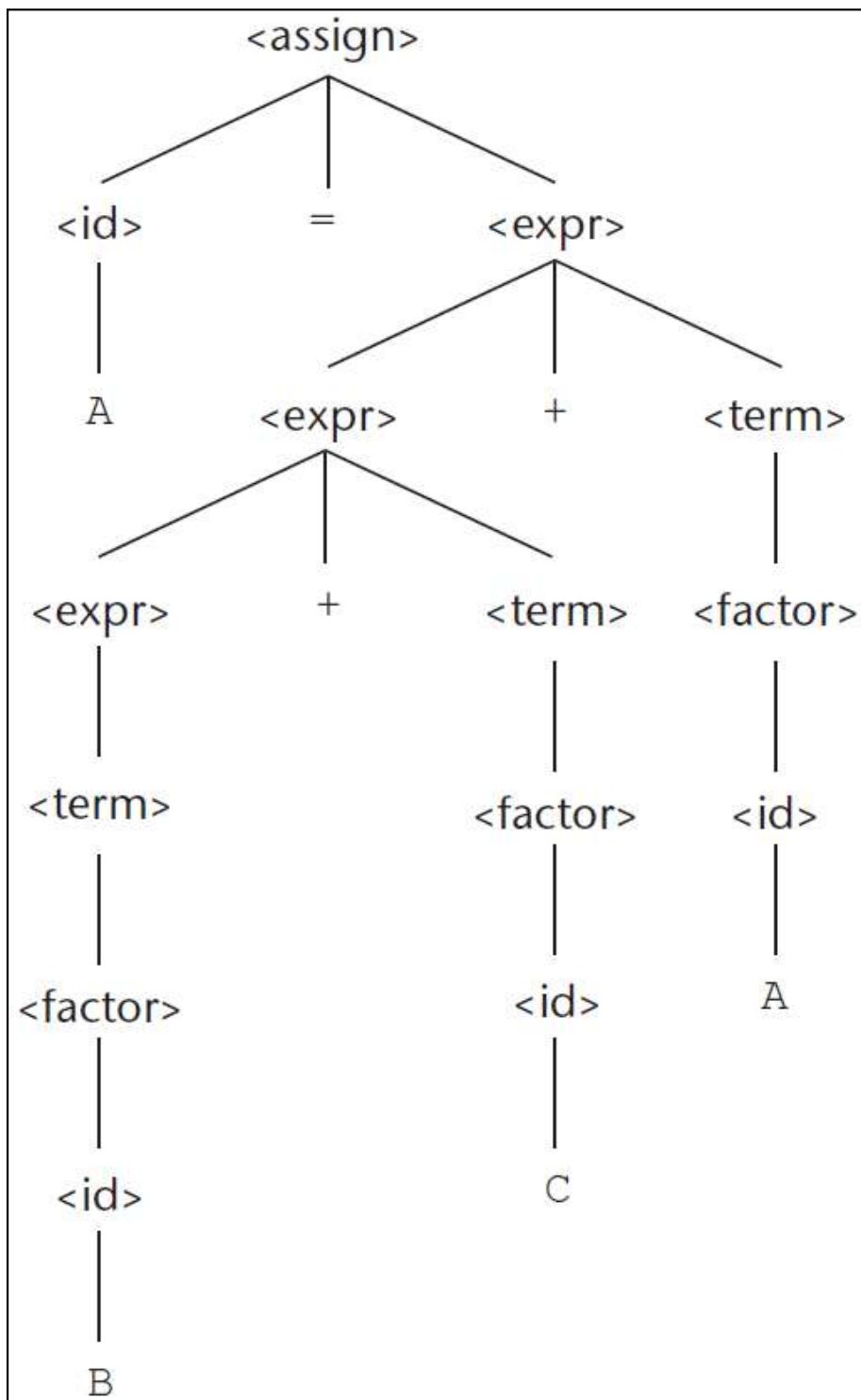
Operators Precedence: Example (Rightmost Derivation)

$$A = B + C * A$$

$\langle \text{assign} \rangle \Rightarrow \langle \text{id} \rangle = \langle \text{expr} \rangle$
 $\Rightarrow \langle \text{id} \rangle = \langle \text{expr} \rangle + \langle \text{term} \rangle$
 $\Rightarrow \langle \text{id} \rangle = \langle \text{expr} \rangle + \langle \text{term} \rangle * \langle \text{factor} \rangle$
 $\Rightarrow \langle \text{id} \rangle = \langle \text{expr} \rangle + \langle \text{term} \rangle * \langle \text{id} \rangle$
 $\Rightarrow \langle \text{id} \rangle = \langle \text{expr} \rangle + \langle \text{term} \rangle * A$
 $\Rightarrow \langle \text{id} \rangle = \langle \text{expr} \rangle + \langle \text{factor} \rangle * A$
 $\Rightarrow \langle \text{id} \rangle = \langle \text{expr} \rangle + \langle \text{id} \rangle * A$
 $\Rightarrow \langle \text{id} \rangle = \langle \text{expr} \rangle + C * A$
 $\Rightarrow \langle \text{id} \rangle = \langle \text{term} \rangle + C * A$
 $\Rightarrow \langle \text{id} \rangle = \langle \text{factor} \rangle + C * A$
 $\Rightarrow \langle \text{id} \rangle = \langle \text{id} \rangle + C * A$
 $\Rightarrow \langle \text{id} \rangle = B + C * A$
 $\Rightarrow A = B + C * A$

Associativity of Operators

- When an expression includes two **operators** that have the same precedence (as $*$ and $/$ usually have)—for example, “ $A / B * C$ ”, then a semantic rule is required to specify which should have **precedence**.
- This rule is named **associativity**.
- A **grammar** for expressions may correctly imply **operator associativity**.
- Consider the following example of an assignment statement:
 - $A = B + C + A$
 - After using the **grammar** in the next slide for the **derivation** of this statement, then its **parse tree** will look like:



An Unambiguous Grammar for Expressions

$\langle \text{assign} \rangle \rightarrow \langle \text{id} \rangle = \langle \text{expr} \rangle$

$\langle \text{id} \rangle \rightarrow A \mid B \mid C$

$\langle \text{expr} \rangle \rightarrow \langle \text{expr} \rangle + \langle \text{term} \rangle$

$\mid \langle \text{term} \rangle$

$\langle \text{term} \rangle \rightarrow \langle \text{term} \rangle * \langle \text{factor} \rangle$

$\mid \langle \text{factor} \rangle$

$\langle \text{factor} \rangle \rightarrow (\langle \text{expr} \rangle)$

$\mid \langle \text{id} \rangle$

A **parse tree** for
 $A = B + C + A$
 illustrating the
associativity of **addition**

Associativity of Operators

- The previous **parse tree** shows the **left** addition **operator** lower than the **right** addition **operator**.
 - This is the correct order if addition is meant to be left associative, which is typical.
 - In most cases, the **associativity** of addition in a computer is irrelevant.
- In mathematics, addition is associative, which means that **left** and **right** associative orders of evaluation mean the same thing.
 - That is, $(A + B) + C = A + (B + C)$
- **Subtraction** and **division** are not associative, whether in mathematics or in a computer.
 - Therefore, correct associativity may be essential for an expression that contains either of them.

Associativity of Operators

- When a **grammar rule** has its LHS also appearing at the beginning of its RHS, the rule is said to be **left recursive**.
 - This **left recursion** specifies left associativity.
- For example, the **left recursion** of the **rules** of the **grammar** below causes it to make both addition and multiplication left associative.

```
<assign> → <id> = <expr>
<id> → A | B | C
<expr> → <expr> + <term>
        | <term>
<term> → <term> * <factor>
        | <factor>
<factor> → ( <expr> )
          | <id>
```


Associativity of Operators

- The **exponentiation** operator is right associative in most languages that provide it.
- To indicate right associativity, right recursion can be used.
- A grammar rule is right recursive if the LHS appears at the right end of the RHS.
- Rules such as:

$$\begin{aligned} \langle \text{factor} \rangle &\rightarrow \langle \text{exp} \rangle ** \langle \text{factor} \rangle \\ &\quad | \langle \text{exp} \rangle \\ \langle \text{exp} \rangle &\rightarrow (\langle \text{expr} \rangle) \\ &\quad | \text{id} \end{aligned}$$

could be used to describe exponentiation as a right-associative operator.

Extended BNF

- **Optional** parts are placed in brackets []

`<proc_call> → ident [(<expr_list>)]`

- **Alternative** parts of **RHSs** are placed inside parentheses and separated via vertical bars.

`<term> → <term> (+|-) const`

- **Repetitions** (zero or more) are placed inside braces { }

`<ident> → letter {letter|digit}`

BNF and EBNF

- BNF:

$$\begin{aligned}\langle \text{expr} \rangle &\rightarrow \langle \text{expr} \rangle + \langle \text{term} \rangle \\ &| \langle \text{expr} \rangle - \langle \text{term} \rangle \\ &| \langle \text{term} \rangle\end{aligned}$$
$$\begin{aligned}\langle \text{term} \rangle &\rightarrow \langle \text{term} \rangle * \langle \text{factor} \rangle \\ &| \langle \text{term} \rangle / \langle \text{factor} \rangle \\ &| \langle \text{factor} \rangle\end{aligned}$$

- EBNF:

$$\begin{aligned}\langle \text{expr} \rangle &\rightarrow \langle \text{term} \rangle \{ (+ \mid -) \langle \text{term} \rangle \} \\ \langle \text{term} \rangle &\rightarrow \langle \text{factor} \rangle \{ (* \mid /) \langle \text{factor} \rangle \}\end{aligned}$$

Recent Variations in EBNF

- **Alternative** RHSs are put on separate lines.
- Use of a **colon** instead of **=>**
- Use of **opt** for **optional** parts.
- Use of **oneof** for **choices**.

Any Questions?

- Please, read chapter 3 (*first 2 sections*)
- I hope you were taking some notes!
- To test your understanding of this lecture, have a go with the “Review Questions” in page 156 of the textbook.
- We will do more exercises later on.
- Please, keep reviewing this lecture regularly.
- We may have a quiz (lecture 1) next week!
- Please, start doing your assignment.